Computer algebra independent integration tests

1-Algebraic-functions/1.2-Trinomial-products/1.2.1-Quadratic/1.2.1.5-a+b-x+c-x^2-^p-d+e-x+f-x^2-^q

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September 20, 2021 Compiled on September 20, 2021 at 12:18 Noon

Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test. Elementary Algebraic integrals version.

The download section below contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [118]. This is test number [23].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.3.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.45 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)
9. `IntegrateAlgebraic` under Mathematica 12.3.1 on windows 10. `https://github.com/stblake/algebraic_integration`. September 15, 2021 version.

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (118)	0.00 (0)
Mathematica	100.00 (118)	0.00 (0)
Maple	100.00 (118)	0.00 (0)
Fricas	94.07 (111)	5.93 (7)
Giac	76.27 (90)	23.73 (28)
IntegrateAlgebraic	61.02 (72)	38.98 (46)
Maxima	56.78 (67)	43.22 (51)
Mupad	44.92 (53)	55.08 (65)
Sympy	36.44 (43)	% 63.56 (75)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

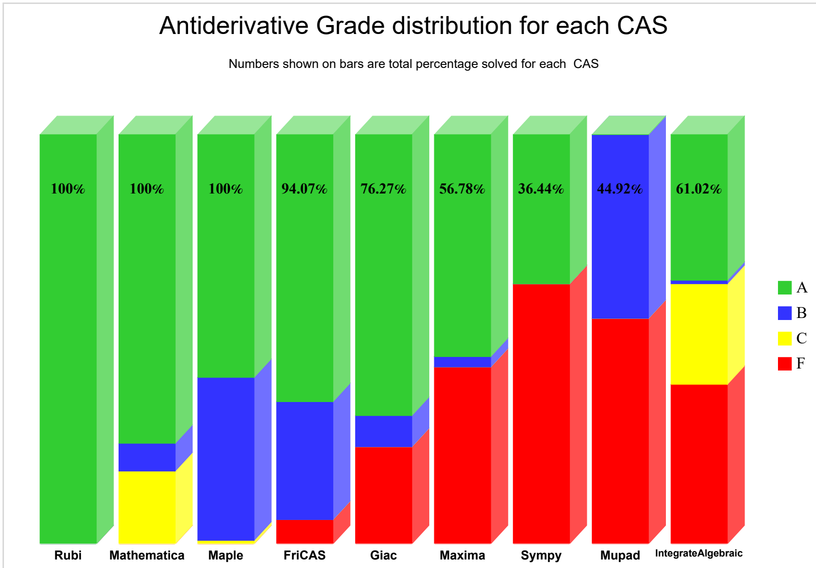
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

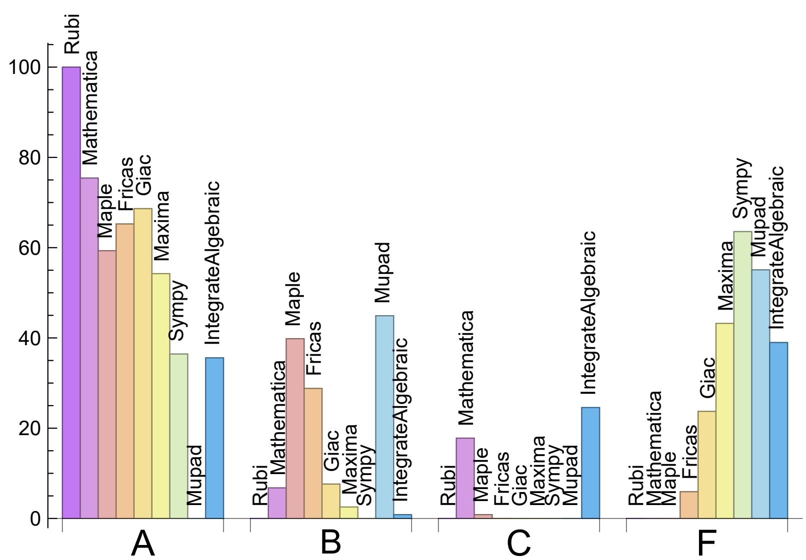
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	75.42	6.78	17.80	0.00
Giac	68.64	7.63	0.00	23.73
Fricas	65.25	28.81	0.00	5.93
Maple	59.32	39.83	0.85	0.00
Maxima	54.24	2.54	0.00	43.22
Sympy	36.44	0.00	0.00	63.56
IntegrateAlgebraic	35.59	0.85	24.58	38.98
Mupad	N/A	44.92	0.00	55.08

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F**(**-1**).

The third is due to an exception generated. Assigned **F**(**-2**). This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	0	0.00 %	0.00 %	0.00 %
Fricas	7	0.00 %	100.00 %	0.00 %
IntegrateAlgebraic	46	93.48 %	6.52 %	0.00 %
Giac	28	3.57 %	17.86 %	78.57 %
Maxima	51	60.78 %	0.00 %	39.22 %
Sympy	75	84.00 %	16.00 %	0.00 %
Mupad	65	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

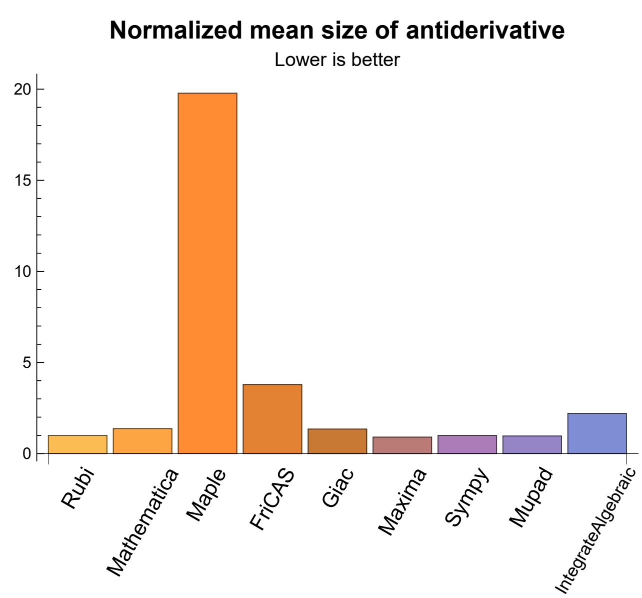
1.3 Performance

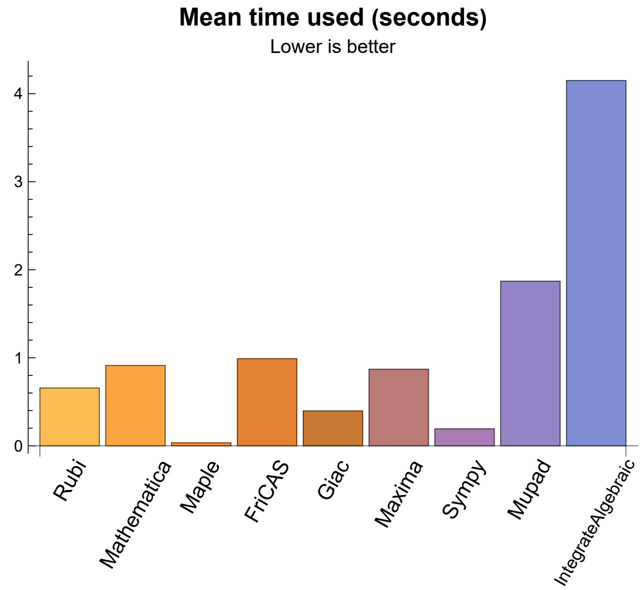
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.66	184.69	1.00	124.00	1.00
Mathematica	0.91	319.82	1.37	94.00	1.00
Maple	0.03	6393.43	19.78	134.00	0.83
Maxima	0.87	90.12	0.90	72.00	0.83
Fricas	0.99	827.68	3.79	112.00	1.24
Sympy	0.19	76.86	0.99	73.00	0.98
Giac	0.40	203.17	1.35	72.50	0.79
Mupad	1.87	105.40	0.96	64.00	0.83
IntegrateAlgebraic	4.15	1095.07	2.20	187.50	1.10

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.




Mean time used (seconds)
Lower is better
0
1
2
3
4
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad
IntegrateAlgebraic

1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

IntegrateAlgebraic {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {6, 103, 104, 105, 110}

IntegrateAlgebraic {}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
```

```
    # 1.7 is a fudge factor since it is low side from actual leaf count
    leafCount = round(1.7*count_ops(anti))

    except Exception as ee:
           leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

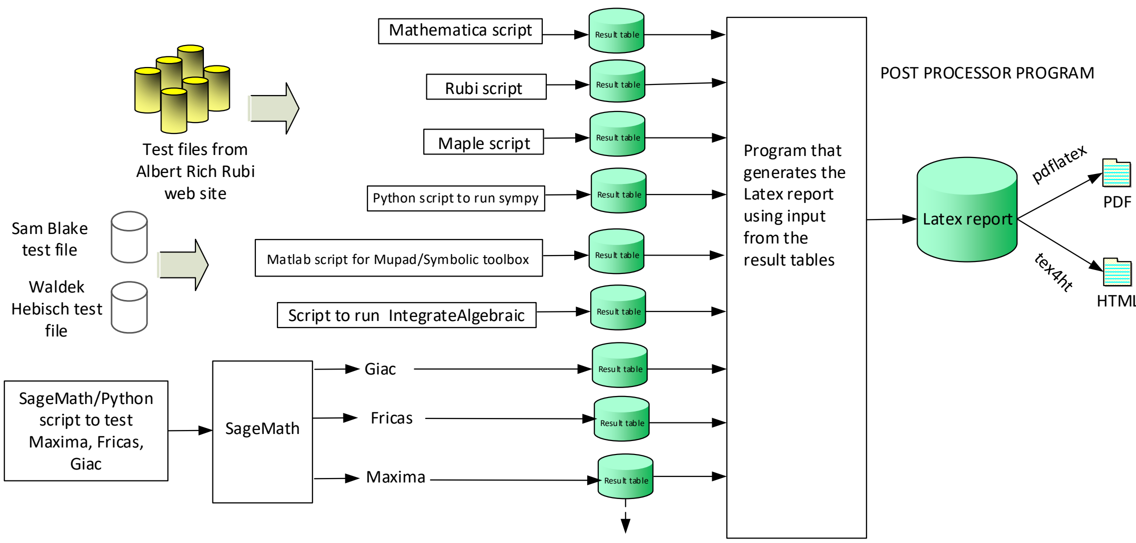
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"

    The following field present only in Rubi and Mathematica Tables

13. integer. 1 if result was verified or 0 if not verified.

    The following fields present only in Rubi Tables

14. integer. Number of rules used.
15. integer. Integrand leaf size.
16. real number. Ratio of field 14 over field 15
17. integer. 1 if result was verified or 0 if not verified.
18. String of form "{n,n,..}" which is list of the rules used by Rubi

High level overview of the CAS independent integration test build system

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May 11, 2021

Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 9, 10, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 62, 63, 64, 65, 69, 70, 71, 72, 76, 77, 78, 79, 83, 84, 85, 86, 90, 91, 92, 93, 97, 98, 99, 100, 101, 102, 103, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117 }

B grade: { 2, 3, 4, 5, 6, 7, 104, 105 }

C grade: { 8, 11, 59, 60, 61, 66, 67, 68, 73, 74, 75, 80, 81, 82, 87, 88, 89, 94, 95, 96, 118 }

F grade: { }

2.1.3 Maple

A grade: { 1, 8, 10, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 62, 63, 64, 65, 69, 70, 71, 72, 76, 77, 78, 79, 83, 84, 85, 86, 90, 93, 108, 117 }

B grade: { 2, 3, 4, 5, 6, 7, 11, 59, 60, 61, 66, 67, 68, 73, 74, 75, 80, 81, 82, 87, 88, 89, 91, 92, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 109, 110, 111, 112, 113, 114, 115, 116, 118 }

C grade: { 9 }

F grade: { }

2.1.4 Maxima

A grade: { 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 62, 63, 64, 65, 69, 70, 71, 72, 76, 77, 78, 79, 83, 84, 85, 86, 93 }

B grade: { 90, 91, 92 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 59, 60, 61, 66, 67, 68, 73, 74, 75, 80, 81, 82, 87, 88, 89, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118 }

2.1.5 FriCAS

A grade: { 1, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 62, 63, 64, 65, 69, 70, 71, 72, 76, 77, 78, 79, 83, 84, 85, 90, 91, 93, 97, 98, 102, 106, 107, 108, 116 }

B grade: { 2, 3, 4, 5, 6, 7, 59, 60, 61, 66, 67, 68, 73, 74, 75, 80, 81, 82, 86, 87, 88, 89, 92, 94, 95, 96, 101, 109, 111, 112, 113, 115, 117, 118 }

C grade: { }

F grade: { 99, 100, 103, 104, 105, 110, 114 }

2.1.6 Sympy

A grade: { 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54 }

B grade: { }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118 }

2.1.7 Giac

A grade: { 1, 9, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 62, 63, 64, 65, 69, 70, 71, 72, 76, 77, 78, 79, 83, 84, 85, 86, 90, 91, 92, 93, 97, 98, 102, 106, 107, 108, 111, 112, 113, 115, 116, 117 }

B grade: { 2, 3, 4, 5, 8, 10, 11, 101, 118 }

C grade: { }

F grade: { 6, 7, 59, 60, 61, 66, 67, 68, 73, 74, 75, 80, 81, 82, 87, 88, 89, 94, 95, 96, 99, 100, 103, 104, 105, 109, 110, 114 }

2.1.8 Mupad

A grade: { }

B grade: { 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 86, 93, 97, 98, 113, 117 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 87, 88, 89, 90, 91, 92, 94, 95, 96, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 114, 115, 116, 118 }

2.1.9 IntegrateAlgebraic

A grade: { 1, 8, 9, 10, 11, 55, 56, 57, 58, 62, 63, 64, 65, 69, 70, 71, 72, 76, 77, 78, 79, 83, 84, 85, 86, 90, 91, 92, 93, 97, 98, 102, 106, 107, 108, 111, 112, 113, 115, 116, 117, 118 }

B grade: { 101 }

C grade: { 2, 3, 4, 7, 59, 60, 61, 66, 67, 68, 73, 74, 75, 80, 81, 82, 87, 88, 89, 94, 95, 96, 99, 100, 103, 104, 109, 110, 114 }

F grade: { 5, 6, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 105 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, `Mathematica` was abbreviated to `MMA` and `IntegrateAlgebraic` to `I.A.`

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	102	102	87	136	0	205	0	84	-1	96
N.S.	1	1.00	0.85	1.33	0.00	2.01	0.00	0.82	-0.01	0.94
time (sec)	N/A	0.094	0.195	0.028	0.000	0.430	0.000	0.256	0.000	0.439

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	F(-2)	B	F	B	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	82	82	178	491	0	1079	0	847	-1	162
N.S.	1	1.00	2.17	5.99	0.00	13.16	0.00	10.33	-0.01	1.98
time (sec)	N/A	0.111	0.394	0.060	0.000	0.662	0.000	3.037	0.000	0.505

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	F(-2)	B	F	B	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	66	66	161	307	0	813	0	703	-1	146
N.S.	1	1.00	2.44	4.65	0.00	12.32	0.00	10.65	-0.02	2.21
time (sec)	N/A	0.079	0.228	0.029	0.000	0.516	0.000	1.898	0.000	0.444

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	F	B	F	B	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	129	129	296	829	0	1544	0	1166	-1	239
N.S.	1	1.00	2.29	6.43	0.00	11.97	0.00	9.04	-0.01	1.85
time (sec)	N/A	0.169	0.923	0.033	0.000	0.843	0.000	2.265	0.000	0.863

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	F	B	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	224	224	1746	1884	0	3818	0	2986	-1	0
N.S.	1	1.00	7.79	8.41	0.00	17.04	0.00	13.33	-0.00	0.00
time (sec)	N/A	0.427	6.382	0.032	0.000	4.820	0.000	4.870	0.000	180.014

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	F	B	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	328	328	3382	3695	0	8134	0	0	-1	0
N.S.	1	1.00	10.31	11.27	0.00	24.80	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.970	6.607	0.038	0.000	20.449	0.000	0.000	0.000	180.018

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	F	B	F(-1)	F(-2)	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	162	162	490	1377	0	2005	0	0	-1	266
N.S.	1	1.00	3.02	8.50	0.00	12.38	0.00	0.00	-0.01	1.64
time (sec)	N/A	0.306	1.965	0.033	0.000	1.573	0.000	0.000	0.000	1.337

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	28	84	27	0	38	0	52	-1	55
N.S.	1	1.00	3.00	0.96	0.00	1.36	0.00	1.86	-0.04	1.96
time (sec)	N/A	0.017	0.074	0.022	0.000	0.398	0.000	0.219	0.000	0.276

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	48	48	27	16	0	22	0	49	-1	41
N.S.	1	1.00	0.56	0.33	0.00	0.46	0.00	1.02	-0.02	0.85
time (sec)	N/A	0.015	0.020	0.046	0.000	0.404	0.000	0.229	0.000	0.141

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	70	70	66	84	0	82	0	143	-1	62
N.S.	1	1.00	0.94	1.20	0.00	1.17	0.00	2.04	-0.01	0.89
time (sec)	N/A	0.051	0.088	0.021	0.000	0.413	0.000	0.260	0.000	0.276

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	98	98	159	341	0	161	0	171	-1	80
N.S.	1	1.00	1.62	3.48	0.00	1.64	0.00	1.74	-0.01	0.82
time (sec)	N/A	0.210	0.395	0.030	0.000	0.424	0.000	0.215	0.000	0.288

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	68	68	68	55	54	54	65	54	54	0
N.S.	1	1.00	1.00	0.81	0.79	0.79	0.96	0.79	0.79	0.00
time (sec)	N/A	0.049	0.003	0.001	0.460	0.337	0.086	0.206	0.062	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	56	56	56	45	44	44	53	44	44	0
N.S.	1	1.00	1.00	0.80	0.79	0.79	0.95	0.79	0.79	0.00
time (sec)	N/A	0.038	0.002	0.001	0.447	0.342	0.081	0.181	0.034	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	44	44	44	35	34	34	41	34	34	0
N.S.	1	1.00	1.00	0.80	0.77	0.77	0.93	0.77	0.77	0.00
time (sec)	N/A	0.029	0.001	0.001	0.434	0.341	0.075	0.209	0.025	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	30	30	25	24	24	26	24	24	0
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.87	0.80	0.80	0.00
time (sec)	N/A	0.016	0.001	0.001	0.432	0.341	0.062	0.182	0.023	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	42	42	42	34	33	33	49	33	35	0
N.S.	1	1.00	1.00	0.81	0.79	0.79	1.17	0.79	0.83	0.00
time (sec)	N/A	0.040	0.016	0.005	0.975	0.393	0.141	0.176	3.393	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	43	43	43	34	36	45	42	36	35	0
N.S.	1	1.00	1.00	0.79	0.84	1.05	0.98	0.84	0.81	0.00
time (sec)	N/A	0.027	0.015	0.005	0.969	0.388	0.157	0.209	0.038	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	64	64	53	47	56	75	63	46	55	0
N.S.	1	1.00	0.83	0.73	0.88	1.17	0.98	0.72	0.86	0.00
time (sec)	N/A	0.037	0.026	0.005	0.958	0.390	0.182	0.175	0.049	0.001

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	80	80	80	65	64	64	76	64	64	0
N.S.	1	1.00	1.00	0.81	0.80	0.80	0.95	0.80	0.80	0.00
time (sec)	N/A	0.060	0.003	0.002	0.452	0.345	0.102	0.201	0.085	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	66	66	66	55	54	54	63	54	54	0
N.S.	1	1.00	1.00	0.83	0.82	0.82	0.95	0.82	0.82	0.00
time (sec)	N/A	0.048	0.002	0.001	0.438	0.338	0.086	0.215	0.054	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	54	54	54	45	44	44	51	44	44	0
N.S.	1	1.00	1.00	0.83	0.81	0.81	0.94	0.81	0.81	0.00
time (sec)	N/A	0.042	0.002	0.001	0.437	0.332	0.082	0.202	0.034	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	46	46	46	35	34	34	41	34	34	0
N.S.	1	1.00	1.00	0.76	0.74	0.74	0.89	0.74	0.74	0.00
time (sec)	N/A	0.030	0.001	0.001	0.435	0.337	0.072	0.192	0.025	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	56	56	53	44	43	43	63	43	45	0
N.S.	1	1.00	0.95	0.79	0.77	0.77	1.12	0.77	0.80	0.00
time (sec)	N/A	0.051	0.020	0.004	0.968	0.392	0.150	0.210	3.449	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	63	59	51	52	78	65	52	51	0
N.S.	1	1.00	0.94	0.81	0.83	1.24	1.03	0.83	0.81	0.00
time (sec)	N/A	0.061	0.032	0.006	0.961	0.396	0.198	0.172	0.050	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	64	64	53	47	56	75	63	46	55	0
N.S.	1	1.00	0.83	0.73	0.88	1.17	0.98	0.72	0.86	0.00
time (sec)	N/A	0.052	0.024	0.006	0.968	0.383	0.199	0.185	3.442	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	85	85	63	57	76	105	83	56	75	0
N.S.	1	1.00	0.74	0.67	0.89	1.24	0.98	0.66	0.88	0.00
time (sec)	N/A	0.063	0.044	0.007	0.962	0.377	0.229	0.183	3.468	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	96	96	96	75	74	74	92	74	74	0
N.S.	1	1.00	1.00	0.78	0.77	0.77	0.96	0.77	0.77	0.00
time (sec)	N/A	0.071	0.004	0.000	0.432	0.339	0.097	0.176	0.121	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	82	82	82	65	64	64	78	64	64	0
N.S.	1	1.00	1.00	0.79	0.78	0.78	0.95	0.78	0.78	0.00
time (sec)	N/A	0.056	0.002	0.001	0.426	0.329	0.090	0.205	0.081	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	68	68	68	55	54	54	65	54	54	0
N.S.	1	1.00	1.00	0.81	0.79	0.79	0.96	0.79	0.79	0.00
time (sec)	N/A	0.051	0.002	0.002	0.429	0.337	0.084	0.202	0.054	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	56	56	56	45	44	44	53	44	44	0
N.S.	1	1.00	1.00	0.80	0.79	0.79	0.95	0.79	0.79	0.00
time (sec)	N/A	0.032	0.001	0.001	0.434	0.332	0.077	0.201	0.033	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	70	70	63	54	53	53	76	53	55	0
N.S.	1	1.00	0.90	0.77	0.76	0.76	1.09	0.76	0.79	0.00
time (sec)	N/A	0.053	0.023	0.004	0.963	0.408	0.159	0.211	0.043	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	77	77	77	61	62	88	78	62	61	0
N.S.	1	1.00	1.00	0.79	0.81	1.14	1.01	0.81	0.79	0.00
time (sec)	N/A	0.072	0.027	0.008	0.961	0.408	0.188	0.206	3.431	0.001

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	84	84	78	63	72	118	85	62	71	0
N.S.	1	1.00	0.93	0.75	0.86	1.40	1.01	0.74	0.85	0.00
time (sec)	N/A	0.087	0.037	0.009	0.970	0.393	0.235	0.189	3.426	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	84	84	72	64	63	63	87	63	65	0
N.S.	1	1.00	0.86	0.76	0.75	0.75	1.04	0.75	0.77	0.00
time (sec)	N/A	0.057	0.028	0.006	0.976	0.397	0.167	0.211	3.441	0.001

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	70	70	63	54	53	53	73	53	55	0
N.S.	1	1.00	0.90	0.77	0.76	0.76	1.04	0.76	0.79	0.00
time (sec)	N/A	0.056	0.021	0.004	0.968	0.414	0.152	0.213	0.042	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	56	56	52	44	43	43	60	43	45	0
N.S.	1	1.00	0.93	0.79	0.77	0.77	1.07	0.77	0.80	0.00
time (sec)	N/A	0.050	0.017	0.004	0.958	0.434	0.155	0.182	3.445	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	42	42	42	34	33	33	46	33	35	0
N.S.	1	1.00	1.00	0.81	0.79	0.79	1.10	0.79	0.83	0.00
time (sec)	N/A	0.035	0.010	0.004	0.950	0.395	0.141	0.207	0.039	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	73	73	73	60	59	59	83	59	79	0
N.S.	1	1.00	1.00	0.82	0.81	0.81	1.14	0.81	1.08	0.00
time (sec)	N/A	0.053	0.032	0.006	0.963	0.407	0.244	0.191	0.187	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	94	94	94	77	78	117	102	78	95	0
N.S.	1	1.00	1.00	0.82	0.83	1.24	1.09	0.83	1.01	0.00
time (sec)	N/A	0.089	0.083	0.007	0.960	0.412	0.322	0.198	3.567	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	115	115	104	89	98	177	119	88	115	0
N.S.	1	1.00	0.90	0.77	0.85	1.54	1.03	0.77	1.00	0.00
time (sec)	N/A	0.124	0.155	0.008	0.959	0.402	0.363	0.180	0.179	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	91	91	91	71	72	98	90	72	72	0
N.S.	1	1.00	1.00	0.78	0.79	1.08	0.99	0.79	0.79	0.00
time (sec)	N/A	0.086	0.052	0.010	0.964	0.393	0.198	0.183	3.462	0.001

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	77	77	77	61	62	88	75	62	61	0
N.S.	1	1.00	1.00	0.79	0.81	1.14	0.97	0.81	0.79	0.00
time (sec)	N/A	0.073	0.028	0.008	0.960	0.432	0.191	0.195	3.424	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	63	63	51	52	78	61	52	52	0
N.S.	1	1.00	1.00	0.81	0.83	1.24	0.97	0.83	0.83	0.00
time (sec)	N/A	0.063	0.031	0.007	0.966	0.407	0.189	0.173	3.404	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	43	43	43	34	36	45	42	36	36	0
N.S.	1	1.00	1.00	0.79	0.84	1.05	0.98	0.84	0.84	0.00
time (sec)	N/A	0.026	0.015	0.004	0.964	0.401	0.154	0.189	0.040	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	94	94	94	77	78	117	102	78	96	0
N.S.	1	1.00	1.00	0.82	0.83	1.24	1.09	0.83	1.02	0.00
time (sec)	N/A	0.088	0.061	0.007	0.963	0.414	0.319	0.192	3.578	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	127	127	106	94	96	167	122	96	115	0
N.S.	1	1.00	0.83	0.74	0.76	1.31	0.96	0.76	0.91	0.00
time (sec)	N/A	0.123	0.057	0.010	0.961	0.434	0.359	0.193	0.177	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	148	148	136	106	118	237	143	110	135	0
N.S.	1	1.00	0.92	0.72	0.80	1.60	0.97	0.74	0.91	0.00
time (sec)	N/A	0.161	0.076	0.011	0.968	0.409	0.400	0.202	3.586	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	98	98	98	73	82	128	95	72	81	0
N.S.	1	1.00	1.00	0.74	0.84	1.31	0.97	0.73	0.83	0.00
time (sec)	N/A	0.113	0.038	0.008	0.951	0.411	0.241	0.207	0.045	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	84	84	84	63	72	118	82	62	72	0
N.S.	1	1.00	1.00	0.75	0.86	1.40	0.98	0.74	0.86	0.00
time (sec)	N/A	0.087	0.037	0.007	0.958	0.394	0.229	0.183	0.049	0.001

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	64	64	51	47	56	75	63	46	56	0
N.S.	1	1.00	0.80	0.73	0.88	1.17	0.98	0.72	0.88	0.00
time (sec)	N/A	0.053	0.029	0.006	0.961	0.387	0.201	0.186	3.474	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	64	64	51	47	56	75	61	46	55	0
N.S.	1	1.00	0.80	0.73	0.88	1.17	0.95	0.72	0.86	0.00
time (sec)	N/A	0.033	0.028	0.007	0.960	0.418	0.183	0.205	0.044	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	115	115	99	89	98	177	122	88	116	0
N.S.	1	1.00	0.86	0.77	0.85	1.54	1.06	0.77	1.01	0.00
time (sec)	N/A	0.124	0.161	0.008	0.973	0.431	0.357	0.211	3.580	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	160	160	136	106	116	227	143	110	136	0
N.S.	1	1.00	0.85	0.66	0.72	1.42	0.89	0.69	0.85	0.00
time (sec)	N/A	0.160	0.124	0.011	0.974	0.426	0.427	0.217	3.597	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	181	181	151	118	138	297	163	116	155	0
N.S.	1	1.00	0.83	0.65	0.76	1.64	0.90	0.64	0.86	0.00
time (sec)	N/A	0.204	0.095	0.013	0.972	0.428	0.470	0.219	3.588	0.001

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	208	208	85	166	177	98	0	93	221	100
N.S.	1	1.00	0.41	0.80	0.85	0.47	0.00	0.45	1.06	0.48
time (sec)	N/A	0.309	0.296	0.031	1.006	0.437	0.000	0.249	5.028	1.083

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	166	166	75	132	143	88	0	83	187	90
N.S.	1	1.00	0.45	0.80	0.86	0.53	0.00	0.50	1.13	0.54
time (sec)	N/A	0.181	0.171	0.008	0.982	0.434	0.000	0.254	4.692	0.869

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	124	124	65	98	109	78	0	73	153	80
N.S.	1	1.00	0.52	0.79	0.88	0.63	0.00	0.59	1.23	0.65
time (sec)	N/A	0.099	0.105	0.007	0.976	0.419	0.000	0.217	4.185	0.562

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	82	82	55	64	75	68	0	63	119	70
N.S.	1	1.00	0.67	0.78	0.91	0.83	0.00	0.77	1.45	0.85
time (sec)	N/A	0.040	0.052	0.006	0.961	0.411	0.000	0.238	3.835	0.366

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	F	F(-2)	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	174	174	185	2065	0	2016	0	0	-1	211
N.S.	1	1.00	1.06	11.87	0.00	11.59	0.00	0.00	-0.01	1.21
time (sec)	N/A	0.441	0.457	0.149	0.000	0.994	0.000	0.000	0.000	0.346

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	F	F(-2)	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	188	188	214	16357	0	2102	0	0	-1	423
N.S.	1	1.00	1.14	87.01	0.00	11.18	0.00	0.00	-0.01	2.25
time (sec)	N/A	0.393	1.036	0.232	0.000	1.194	0.000	0.000	0.000	0.705

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	F	F(-2)	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	223	223	299	44343	0	2182	0	0	-1	396
N.S.	1	1.00	1.34	198.85	0.00	9.78	0.00	0.00	-0.00	1.78
time (sec)	N/A	0.459	2.075	0.372	0.000	1.437	0.000	0.000	0.000	0.824

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	231	231	95	185	206	108	0	103	-1	110
N.S.	1	1.00	0.41	0.80	0.89	0.47	0.00	0.45	-0.00	0.48
time (sec)	N/A	0.342	0.367	0.035	1.022	0.479	0.000	0.255	0.000	1.372

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	189	189	85	151	172	98	0	93	-1	100
N.S.	1	1.00	0.45	0.80	0.91	0.52	0.00	0.49	-0.01	0.53
time (sec)	N/A	0.190	0.231	0.007	0.991	0.460	0.000	0.256	0.000	1.056

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	147	147	75	117	138	88	0	83	-1	90
N.S.	1	1.00	0.51	0.80	0.94	0.60	0.00	0.56	-0.01	0.61
time (sec)	N/A	0.122	0.141	0.009	0.983	0.455	0.000	0.257	0.000	0.824

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	105	105	65	83	104	78	0	73	-1	80
N.S.	1	1.00	0.62	0.79	0.99	0.74	0.00	0.70	-0.01	0.76
time (sec)	N/A	0.050	0.080	0.006	0.963	0.410	0.000	0.230	0.000	0.581

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	F	F(-2)	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	197	197	310	3460	0	2027	0	0	-1	235
N.S.	1	1.00	1.57	17.56	0.00	10.29	0.00	0.00	-0.01	1.19
time (sec)	N/A	0.488	0.693	0.053	0.000	0.994	0.000	0.000	0.000	0.530

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	F	F(-2)	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	232	232	530	28185	0	2150	0	0	-1	422
N.S.	1	1.00	2.28	121.49	0.00	9.27	0.00	0.00	-0.00	1.82
time (sec)	N/A	0.575	2.562	0.158	0.000	1.291	0.000	0.000	0.000	0.680

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	F	F(-2)	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	223	223	1262	81552	0	2183	0	0	-1	578
N.S.	1	1.00	5.66	365.70	0.00	9.79	0.00	0.00	-0.00	2.59
time (sec)	N/A	0.433	5.342	0.350	0.000	1.270	0.000	0.000	0.000	1.059

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	254	254	105	204	235	118	0	113	-1	120
N.S.	1	1.00	0.41	0.80	0.93	0.46	0.00	0.44	-0.00	0.47
time (sec)	N/A	0.373	0.452	0.040	1.021	0.419	0.000	0.536	0.000	1.834

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	212	212	95	170	201	108	0	103	-1	110
N.S.	1	1.00	0.45	0.80	0.95	0.51	0.00	0.49	-0.00	0.52
time (sec)	N/A	0.220	0.296	0.009	0.996	0.417	0.000	0.549	0.000	1.336

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	170	170	85	136	167	98	0	93	-1	100
N.S.	1	1.00	0.50	0.80	0.98	0.58	0.00	0.55	-0.01	0.59
time (sec)	N/A	0.130	0.183	0.007	1.028	0.415	0.000	0.516	0.000	1.048

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	128	128	75	102	133	88	0	83	-1	90
N.S.	1	1.00	0.59	0.80	1.04	0.69	0.00	0.65	-0.01	0.70
time (sec)	N/A	0.062	0.112	0.006	0.974	0.411	0.000	0.391	0.000	0.843

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	F	F(-2)	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	222	222	229	4860	0	2010	0	0	-1	245
N.S.	1	1.00	1.03	21.89	0.00	9.05	0.00	0.00	-0.00	1.10
time (sec)	N/A	0.539	1.047	0.053	0.000	1.040	0.000	0.000	0.000	0.762

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	F	F(-2)	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	255	255	685	40028	0	2161	0	0	-1	442
N.S.	1	1.00	2.69	156.97	0.00	8.47	0.00	0.00	-0.00	1.73
time (sec)	N/A	0.660	1.655	0.157	0.000	1.317	0.000	0.000	0.000	0.970

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	F	F(-2)	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	281	281	1009	119458	0	2240	0	0	-1	627
N.S.	1	1.00	3.59	425.12	0.00	7.97	0.00	0.00	-0.00	2.23
time (sec)	N/A	0.655	1.996	0.384	0.000	1.479	0.000	0.000	0.000	1.085

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	185	185	75	147	148	88	0	83	-1	90
N.S.	1	1.00	0.41	0.79	0.80	0.48	0.00	0.45	-0.01	0.49
time (sec)	N/A	0.312	0.241	0.019	0.999	0.433	0.000	0.503	0.000	0.802

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	143	143	65	113	114	78	0	73	-1	80
N.S.	1	1.00	0.45	0.79	0.80	0.55	0.00	0.51	-0.01	0.56
time (sec)	N/A	0.168	0.134	0.008	0.977	0.407	0.000	0.524	0.000	0.598

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	101	101	55	79	80	68	0	63	-1	70
N.S.	1	1.00	0.54	0.78	0.79	0.67	0.00	0.62	-0.01	0.69
time (sec)	N/A	0.088	0.075	0.008	0.965	0.408	0.000	0.487	0.000	0.490

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	59	59	45	45	46	58	0	53	-1	60
N.S.	1	1.00	0.76	0.76	0.78	0.98	0.00	0.90	-0.02	1.02
time (sec)	N/A	0.033	0.039	0.006	0.956	0.415	0.000	0.530	0.000	0.276

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	F	F(-2)	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	148	148	176	684	0	2002	0	0	-1	135
N.S.	1	1.00	1.19	4.62	0.00	13.53	0.00	0.00	-0.01	0.91
time (sec)	N/A	0.314	0.300	0.004	0.000	1.132	0.000	0.000	0.000	0.359

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	F	F(-2)	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	188	188	287	5225	0	2102	0	0	-1	230
N.S.	1	1.00	1.53	27.79	0.00	11.18	0.00	0.00	-0.01	1.22
time (sec)	N/A	0.429	0.996	0.006	0.000	1.250	0.000	0.000	0.000	0.543

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	F	F(-2)	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	223	223	1277	13040	0	2183	0	0	-1	396
N.S.	1	1.00	5.73	58.48	0.00	9.79	0.00	0.00	-0.00	1.78
time (sec)	N/A	0.469	6.230	0.007	0.000	1.284	0.000	0.000	0.000	0.843

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	166	166	95	166	148	112	0	82	-1	90
N.S.	1	1.00	0.57	1.00	0.89	0.67	0.00	0.49	-0.01	0.54
time (sec)	N/A	0.204	0.373	0.029	0.985	0.411	0.000	0.251	0.000	1.321

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	124	124	65	132	114	102	0	72	-1	80
N.S.	1	1.00	0.52	1.06	0.92	0.82	0.00	0.58	-0.01	0.65
time (sec)	N/A	0.127	0.232	0.008	0.973	0.419	0.000	0.281	0.000	0.844

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	82	82	55	98	80	92	0	62	-1	70
N.S.	1	1.00	0.67	1.20	0.98	1.12	0.00	0.76	-0.01	0.85
time (sec)	N/A	0.071	0.143	0.008	0.959	0.415	0.000	0.237	0.000	0.728

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	45	45	45	64	46	82	0	53	87	60
N.S.	1	1.00	1.00	1.42	1.02	1.82	0.00	1.18	1.93	1.33
time (sec)	N/A	0.029	0.076	0.006	0.955	0.415	0.000	0.234	0.227	0.424

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	F	F(-2)	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	176	176	202	718	0	2083	0	0	-1	199
N.S.	1	1.00	1.15	4.08	0.00	11.84	0.00	0.00	-0.01	1.13
time (sec)	N/A	0.408	1.287	0.034	0.000	1.168	0.000	0.000	0.000	0.493

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	F	F(-2)	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	211	211	740	5942	0	2173	0	0	-1	406
N.S.	1	1.00	3.51	28.16	0.00	10.30	0.00	0.00	-0.00	1.92
time (sec)	N/A	0.473	1.525	0.099	0.000	1.259	0.000	0.000	0.000	0.945

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	F	F(-2)	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	246	246	231	18981	0	2263	0	0	-1	611
N.S.	1	1.00	0.94	77.16	0.00	9.20	0.00	0.00	-0.00	2.48
time (sec)	N/A	0.525	2.206	0.208	0.000	1.386	0.000	0.000	0.000	1.513

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	147	147	75	214	253	132	0	81	-1	90
N.S.	1	1.00	0.51	1.46	1.72	0.90	0.00	0.55	-0.01	0.61
time (sec)	N/A	0.167	0.513	0.031	1.002	0.423	0.000	0.252	0.000	1.449

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	105	105	65	180	219	122	0	72	-1	80
N.S.	1	1.00	0.62	1.71	2.09	1.16	0.00	0.69	-0.01	0.76
time (sec)	N/A	0.105	0.338	0.008	1.020	0.426	0.000	0.268	0.000	1.028

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	68	68	55	146	185	112	0	61	-1	70
N.S.	1	1.00	0.81	2.15	2.72	1.65	0.00	0.90	-0.01	1.03
time (sec)	N/A	0.061	0.236	0.007	0.978	0.408	0.000	0.267	0.000	0.737

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	47	47	33	30	59	51	0	29	29	33
N.S.	1	1.00	0.70	0.64	1.26	1.09	0.00	0.62	0.62	0.70
time (sec)	N/A	0.022	0.103	0.005	0.432	0.404	0.000	0.232	0.090	0.515

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	F	F(-2)	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	199	199	218	751	0	2133	0	0	-1	209
N.S.	1	1.00	1.10	3.77	0.00	10.72	0.00	0.00	-0.01	1.05
time (sec)	N/A	0.456	0.868	0.039	0.000	1.239	0.000	0.000	0.000	0.826

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	F	F(-2)	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	234	234	296	5975	0	2253	0	0	-1	416
N.S.	1	1.00	1.26	25.53	0.00	9.63	0.00	0.00	-0.00	1.78
time (sec)	N/A	0.543	1.153	0.104	0.000	1.194	0.000	0.000	0.000	1.418

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	F	F(-2)	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	269	269	242	19014	0	2343	0	0	-1	601
N.S.	1	1.00	0.90	70.68	0.00	8.71	0.00	0.00	-0.00	2.23
time (sec)	N/A	0.589	1.958	0.209	0.000	1.327	0.000	0.000	0.000	1.611

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	436	436	657	1429	0	1269	0	638	1299	655
N.S.	1	1.00	1.51	3.28	0.00	2.91	0.00	1.46	2.98	1.50
time (sec)	N/A	0.789	0.935	0.023	0.000	0.739	0.000	0.365	5.312	2.823

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	175	175	173	453	0	465	0	212	320	207
N.S.	1	1.00	0.99	2.59	0.00	2.66	0.00	1.21	1.83	1.18
time (sec)	N/A	0.164	0.273	0.007	0.000	0.472	0.000	0.299	3.906	0.770

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	431	431	417	6019	0	0	0	0	-1	402
N.S.	1	1.00	0.97	13.97	0.00	0.00	0.00	0.00	-0.00	0.93
time (sec)	N/A	1.051	1.219	0.052	0.000	0.000	0.000	0.000	0.000	0.601

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	F(-1)	F(-1)	F(-1)	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	488	488	555	22287	0	0	0	0	-1	946
N.S.	1	1.00	1.14	45.67	0.00	0.00	0.00	0.00	-0.00	1.94
time (sec)	N/A	2.929	5.093	0.037	0.000	0.000	0.000	0.000	0.000	2.554

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	564	564	829	2458	0	2179	0	1150	-1	1210
N.S.	1	1.00	1.47	4.36	0.00	3.86	0.00	2.04	-0.00	2.15
time (sec)	N/A	0.935	1.762	0.025	0.000	1.939	0.000	0.598	0.000	7.792

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	236	236	392	862	0	839	0	417	-1	408
N.S.	1	1.00	1.66	3.65	0.00	3.56	0.00	1.77	-0.00	1.73
time (sec)	N/A	0.230	0.606	0.009	0.000	0.842	0.000	0.612	0.000	1.701

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F(-2)	F	C
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	679	678	1232	22523	0	0	0	0	-1	1160
N.S.	1	1.00	1.81	33.17	0.00	0.00	0.00	0.00	-0.00	1.71
time (sec)	N/A	11.033	4.576	0.029	0.000	0.000	0.000	0.000	0.000	1.670

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	F	F(-1)	F(-1)	F(-2)	F	C
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	704	704	2843	72576	0	0	0	0	-1	2427
N.S.	1	1.00	4.04	103.09	0.00	0.00	0.00	0.00	-0.00	3.45
time (sec)	N/A	11.950	6.831	0.039	0.000	0.000	0.000	0.000	0.000	5.181

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	F	F(-1)	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	671	669	4727	178044	0	0	0	0	-1	0
N.S.	1	1.00	7.04	265.34	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	11.597	7.229	0.066	0.000	0.000	0.000	0.000	0.000	180.012

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	717	717	615	1930	0	1583	0	824	-1	847
N.S.	1	1.00	0.86	2.69	0.00	2.21	0.00	1.15	-0.00	1.18
time (sec)	N/A	2.710	1.346	0.028	0.000	1.407	0.000	0.528	0.000	4.221

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	316	316	251	706	0	637	0	304	-1	296
N.S.	1	1.00	0.79	2.23	0.00	2.02	0.00	0.96	-0.00	0.94
time (sec)	N/A	0.626	0.503	0.014	0.000	0.800	0.000	0.632	0.000	1.089

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	116	116	96	185	0	227	0	98	-1	102
N.S.	1	1.00	0.83	1.59	0.00	1.96	0.00	0.84	-0.01	0.88
time (sec)	N/A	0.111	0.146	0.008	0.000	0.634	0.000	0.499	0.000	0.480

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F	F(-1)	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	374	374	376	761	0	11287	0	0	-1	211
N.S.	1	1.00	1.01	2.03	0.00	30.18	0.00	0.00	-0.00	0.56
time (sec)	N/A	0.579	1.454	0.025	0.000	7.139	0.000	0.000	0.000	0.442

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	F(-1)	F(-1)	F(-1)	F	C
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	789	787	1377	3858	0	0	0	0	-1	55521
N.S.	1	1.00	1.75	4.89	0.00	0.00	0.00	0.00	-0.00	70.37
time (sec)	N/A	8.210	6.689	0.031	0.000	0.000	0.000	0.000	0.000	175.847

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	649	649	745	2827	0	3143	0	1099	-1	1100
N.S.	1	1.00	1.15	4.36	0.00	4.84	0.00	1.69	-0.00	1.69
time (sec)	N/A	2.106	1.664	0.030	0.000	1.952	0.000	0.423	0.000	5.978

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	309	309	288	1011	0	1305	0	407	-1	375
N.S.	1	1.00	0.93	3.27	0.00	4.22	0.00	1.32	-0.00	1.21
time (sec)	N/A	0.447	0.733	0.015	0.000	1.238	0.000	0.352	0.000	2.163

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	111	111	113	249	0	429	0	122	143	111
N.S.	1	1.00	1.02	2.24	0.00	3.86	0.00	1.10	1.29	1.00
time (sec)	N/A	0.080	0.311	0.006	0.000	0.784	0.000	0.312	3.734	0.551

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	666	666	700	4099	0	0	0	0	-1	730
N.S.	1	1.00	1.05	6.15	0.00	0.00	0.00	0.00	-0.00	1.10
time (sec)	N/A	1.829	5.343	0.028	0.000	0.000	0.000	0.000	0.000	1.609

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	891	891	872	4635	0	3995	0	1401	-1	1411
N.S.	1	1.00	0.98	5.20	0.00	4.48	0.00	1.57	-0.00	1.58
time (sec)	N/A	1.768	2.191	0.031	0.000	5.385	0.000	0.455	0.000	37.604

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	444	444	387	1786	0	1581	0	587	-1	555
N.S.	1	1.00	0.87	4.02	0.00	3.56	0.00	1.32	-0.00	1.25
time (sec)	N/A	0.451	1.257	0.014	0.000	3.503	0.000	0.338	0.000	3.156

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	131	131	147	185	0	286	0	240	175	176
N.S.	1	1.00	1.12	1.41	0.00	2.18	0.00	1.83	1.34	1.34
time (sec)	N/A	0.085	0.378	0.007	0.000	2.597	0.000	0.322	3.708	1.165

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	51	51	81	144	0	154	0	205	-1	53
N.S.	1	1.00	1.59	2.82	0.00	3.02	0.00	4.02	-0.02	1.04
time (sec)	N/A	0.066	0.041	0.024	0.000	0.445	0.000	0.276	0.000	0.336

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [11] had the largest ratio of [.4444]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	4	4	1.00	29	0.138
2	A	2	2	1.00	31	0.065
3	A	2	2	1.00	27	0.074
4	A	4	4	1.00	27	0.148
5	A	5	5	1.00	27	0.185
6	A	6	5	1.00	27	0.185
7	A	4	4	1.00	31	0.129
8	A	2	2	1.00	23	0.087
9	A	3	3	1.00	22	0.136
10	A	6	5	1.00	18	0.278
11	A	16	12	1.00	27	0.444
12	A	2	1	1.00	23	0.043
13	A	2	1	1.00	23	0.043
14	A	2	1	1.00	23	0.043
15	A	2	1	1.00	21	0.048
16	A	6	5	1.00	23	0.217
17	A	4	4	1.00	23	0.174
18	A	5	5	1.00	23	0.217
19	A	2	1	1.00	25	0.040
20	A	2	1	1.00	25	0.040
21	A	2	1	1.00	25	0.040
22	A	2	1	1.00	23	0.043
23	A	6	5	1.00	25	0.200
24	A	7	6	1.00	25	0.240
25	A	5	4	1.00	25	0.160
26	A	6	5	1.00	25	0.200

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
27	A	2	1	1.00	25	0.040
28	A	2	1	1.00	25	0.040
29	A	2	1	1.00	25	0.040
30	A	2	1	1.00	23	0.043
31	A	6	5	1.00	25	0.200
32	A	7	6	1.00	25	0.240
33	A	8	6	1.00	25	0.240
34	A	6	5	1.00	25	0.200
35	A	6	5	1.00	25	0.200
36	A	6	5	1.00	25	0.200
37	A	6	5	1.00	23	0.217
38	A	9	5	1.00	25	0.200
39	A	10	6	1.00	25	0.240
40	A	11	7	1.00	25	0.280
41	A	7	6	1.00	25	0.240
42	A	7	6	1.00	25	0.240
43	A	7	6	1.00	25	0.240
44	A	4	4	1.00	23	0.174
45	A	10	6	1.00	25	0.240
46	A	11	7	1.00	25	0.280
47	A	12	7	1.00	25	0.280
48	A	8	6	1.00	25	0.240
49	A	8	6	1.00	25	0.240
50	A	5	4	1.00	25	0.160
51	A	5	5	1.00	23	0.217
52	A	11	7	1.00	25	0.280
53	A	12	7	1.00	25	0.280
54	A	13	7	1.00	25	0.280
55	A	11	5	1.00	27	0.185
56	A	9	5	1.00	27	0.185
57	A	7	5	1.00	27	0.185
58	A	5	5	1.00	25	0.200
59	A	8	7	1.00	27	0.259
60	A	6	5	1.00	27	0.185

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	7	6	1.00	27	0.222
62	A	12	5	1.00	27	0.185
63	A	10	5	1.00	27	0.185
64	A	8	5	1.00	27	0.185
65	A	6	5	1.00	25	0.200
66	A	9	8	1.00	27	0.296
67	A	10	9	1.00	27	0.333
68	A	7	6	1.00	27	0.222
69	A	13	5	1.00	27	0.185
70	A	11	5	1.00	27	0.185
71	A	9	5	1.00	27	0.185
72	A	7	5	1.00	25	0.200
73	A	10	9	1.00	27	0.333
74	A	11	9	1.00	27	0.333
75	A	11	10	1.00	27	0.370
76	A	10	4	1.00	27	0.148
77	A	8	4	1.00	27	0.148
78	A	6	4	1.00	27	0.148
79	A	4	4	1.00	25	0.160
80	A	5	4	1.00	27	0.148
81	A	6	5	1.00	27	0.185
82	A	7	6	1.00	27	0.222
83	A	9	5	1.00	27	0.185
84	A	7	5	1.00	27	0.185
85	A	5	5	1.00	27	0.185
86	A	4	4	1.00	25	0.160
87	A	6	5	1.00	27	0.185
88	A	7	6	1.00	27	0.222
89	A	8	6	1.00	27	0.222
90	A	8	5	1.00	27	0.185
91	A	6	5	1.00	27	0.185
92	A	5	4	1.00	27	0.148
93	A	3	3	1.00	25	0.120
94	A	7	6	1.00	27	0.222

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
95	A	8	6	1.00	27	0.222
96	A	9	6	1.00	27	0.222
97	A	7	5	1.00	27	0.185
98	A	5	5	1.00	25	0.200
99	A	8	5	1.00	27	0.185
100	A	6	4	1.00	27	0.148
101	A	8	5	1.00	27	0.185
102	A	6	5	1.00	25	0.200
103	A	9	6	1.00	27	0.222
104	A	10	7	1.00	27	0.259
105	A	7	5	1.00	27	0.185
106	A	8	4	1.00	27	0.148
107	A	6	4	1.00	27	0.148
108	A	4	4	1.00	25	0.160
109	A	5	3	1.00	27	0.111
110	A	6	4	1.00	27	0.148
111	A	7	5	1.00	27	0.185
112	A	5	5	1.00	27	0.185
113	A	4	4	1.00	25	0.160
114	A	6	4	1.00	27	0.148
115	A	6	5	1.00	27	0.185
116	A	5	4	1.00	27	0.148
117	A	3	3	1.00	25	0.120
118	A	5	4	1.00	27	0.148

Chapter 3

Listing of integrals

Local contents

3.1 $\int \frac{a+bx+\frac{bfx^2}{e}}{\sqrt{d+ex+fx^2}} \, dx$

Optimal. Leaf size=102

$$\frac{\left(8af-b\left(\frac{4df}{e}+e\right)\right)\tanh^{-1}\left(\frac{e+2fx}{2\sqrt{f}\sqrt{d+ex+fx^2}}\right)}{8f^{3/2}}+\frac{bx\sqrt{d+ex+fx^2}}{2e}+\frac{b\sqrt{d+ex+fx^2}}{4f}$$

Rubi [A] time = 0.09, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1661, 640, 621, 206}

$$\frac{\left(8af-b\left(\frac{4df}{e}+e\right)\right)\tanh^{-1}\left(\frac{e+2fx}{2\sqrt{f}\sqrt{d+ex+fx^2}}\right)}{8f^{3/2}}+\frac{bx\sqrt{d+ex+fx^2}}{2e}+\frac{b\sqrt{d+ex+fx^2}}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + (b*f*x^2)/e)/Sqrt[d + e*x + f*x^2],x]
```

```
[Out] (b*Sqrt[d + e*x + f*x^2])/(4*f) + (b*x*Sqrt[d + e*x + f*x^2])/(2*e) + ((8*a
*f - b*(e + (4*d*f)/e))*ArcTanh[(e + 2*f*x)/(2*Sqrt[f]*Sqrt[d + e*x + f*x^2
])])/(8*f^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+bx+\frac{bfx^2}{e}}{\sqrt{d+ex+fx^2}}\,dx &= \frac{bx\sqrt{d+ex+fx^2}}{2e}+\frac{\int \frac{\left(2a-\frac{bd}{e}\right)f+\frac{bfx}{2}}{\sqrt{d+ex+fx^2}}\,dx}{2f}\\
&= \frac{b\sqrt{d+ex+fx^2}}{4f}+\frac{bx\sqrt{d+ex+fx^2}}{2e}+\frac{\left(-be+8af-\frac{4bdf}{e}\right)\int \frac{1}{\sqrt{d+ex+fx^2}}\,dx}{8f}\\
&= \frac{b\sqrt{d+ex+fx^2}}{4f}+\frac{bx\sqrt{d+ex+fx^2}}{2e}+\frac{\left(-be+8af-\frac{4bdf}{e}\right)\text{Subst}\left(\int \frac{1}{4f-x^2}\,dx,x,\frac{e+2fx}{\sqrt{d+ex+}}\right.}{4f}\\
&= \frac{b\sqrt{d+ex+fx^2}}{4f}+\frac{bx\sqrt{d+ex+fx^2}}{2e}-\frac{\left(be-8af+\frac{4bdf}{e}\right)\tanh^{-1}\left(\frac{e+2fx}{2\sqrt{f}\sqrt{d+ex+fx^2}}\right)}{8f^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.19, size = 87, normalized size = 0.85

$$
\frac{2b\sqrt{f}\,(e+2fx)\sqrt{d+x(e+fx)}-\left(b\left(4df+e^2\right)-8aef\right)\tanh^{-1}\left(\frac{e+2fx}{2\sqrt{f}\sqrt{d+x(e+fx)}}\right)}{8ef^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + (b*f*x^2)/e)/Sqrt[d + e*x + f*x^2],x]
```

```
[Out] (2*b*Sqrt[f]*(e + 2*f*x)*Sqrt[d + x*(e + f*x)] - (-8*a*e*f + b*(e^2 + 4*d*f
))*ArcTanh[(e + 2*f*x)/(2*Sqrt[f]*Sqrt[d + x*(e + f*x)])])/(8*e*f^(3/2))
```

IntegrateAlgebraic [A] time = 0.44, size = 96, normalized size = 0.94

$$
\frac{\left(-8aef+4bdf+be^2\right)\log\left(-2ef^{3/2}\sqrt{d+ex+fx^2}+e^2f+2ef^2x\right)}{8ef^{3/2}}+\frac{b(e+2fx)\sqrt{d+ex+fx^2}}{4ef}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x + (b*f*x^2)/e)/Sqrt[d + e*x + f*x^2],x]
```

```
[Out] (b*(e + 2*f*x)*Sqrt[d + e*x + f*x^2])/(4*e*f) + ((b*e^2 + 4*b*d*f - 8*a*e*f
)*Log[e^2*f + 2*e*f^2*x - 2*e*f^(3/2)*Sqrt[d + e*x + f*x^2]])/(8*e*f^(3/2))
```

fricas [A] time = 0.43, size = 205, normalized size = 2.01

$$\left[-\frac{\left(be^{2}+4\,(bd-2\,ae)f\right)\sqrt{f}\,\log\left(-8\,f^{2}x^{2}-8\,efx-e^{2}-4\,\sqrt{fx^{2}+ex+d}\left(2\,fx+e\right)\sqrt{f}-4\,df\right)-4\left(2\,bf^{2}x+bef\right)\sqrt{fx^{2}+ex+d}}{16\,ef^{2}},\ \frac{\left(be^{2}+4\,(bd-2\,ae)f\right)\sqrt{-f}\,\arctan\left(\frac{\sqrt{fx^{2}+ex+d}\left(2\,fx+e\right)\sqrt{-f}}{2\left(f^{2}x^{2}+efx+df\right)}\right)+2\left(2\,bf^{2}x+bef\right)\sqrt{fx^{2}+ex+d}}{8\,ef^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x+b*f*x^2/e)/(f*x^2+e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*((b*e^2 + 4*(b*d - 2*a*e)*f)*sqrt(f)*log(-8*f^2*x^2 - 8*e*f*x - e^2
- 4*sqrt(f*x^2 + e*x + d)*(2*f*x + e)*sqrt(f) - 4*d*f) - 4*(2*b*f^2*x + b*e
*f)*sqrt(f*x^2 + e*x + d))/(e*f^2), 1/8*((b*e^2 + 4*(b*d - 2*a*e)*f)*sqrt(-
f)*arctan(1/2*sqrt(f*x^2 + e*x + d)*(2*f*x + e)*sqrt(-f)/(f^2*x^2 + e*f*x +
 d*f)) + 2*(2*b*f^2*x + b*e*f)*sqrt(f*x^2 + e*x + d))/(e*f^2)]
```

giac [A] time = 0.26, size = 84, normalized size = 0.82

$$\frac{1}{4}\,\sqrt{fx^{2}+xe+d}\left(2\,bxe^{(-1)}+\frac{b}{f}\right)+\frac{\left(4\,bdf-8\,afe+be^{2}\right)e^{(-1)}\log\left(\left|-2\left(\sqrt{f}\,x-\sqrt{fx^{2}+xe+d}\right)\sqrt{f}-e\right|\right)}{8\,f^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x+b*f*x^2/e)/(f*x^2+e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(f*x^2 + x*e + d)*(2*b*x*e^(-1) + b/f) + 1/8*(4*b*d*f - 8*a*f*e + b
*e^2)*e^(-1)*log(abs(-2*(sqrt(f)*x - sqrt(f*x^2 + x*e + d))*sqrt(f) - e))/f
^(3/2)
```

maple [A] time = 0.03, size = 136, normalized size = 1.33

$$\frac{a\ln\left(\frac{fx+\frac{e}{2}}{\sqrt{f}}+\sqrt{f\,x^{2}+ex+d}\right)}{\sqrt{f}}-\frac{bd\ln\left(\frac{fx+\frac{e}{2}}{\sqrt{f}}+\sqrt{f\,x^{2}+ex+d}\right)}{2e\sqrt{f}}-\frac{be\ln\left(\frac{fx+\frac{e}{2}}{\sqrt{f}}+\sqrt{f\,x^{2}+ex+d}\right)}{8f^{\frac{3}{2}}}+\frac{\sqrt{f\,x^{2}+ex+d}\;bx}{2e}+\frac{\sqrt{f\,x^{2}+ex+d}\;b}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x+b*f*x^2/e)/(f*x^2+e*x+d)^(1/2),x)
```

```
[Out] 1/2*b*x*(f*x^2+e*x+d)^(1/2)/e+1/4*b*(f*x^2+e*x+d)^(1/2)/f-1/8*e*b/f^(3/2)*l
n((1/2*e+f*x)/f^(1/2)+(f*x^2+e*x+d)^(1/2))-1/2/e*b/f^(1/2)*d*ln((1/2*e+f*x)
```

```
/f^(1/2)+(f*x^2+e*x+d)^(1/2))+a*ln((1/2*e+f*x)/f^(1/2)+(f*x^2+e*x+d)^(1/2))
/f^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x+b*f*x^2/e)/(f*x^2+e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re details)Is 4*d*f-e^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,x+\frac{b\,f\,x^2}{e}}{\sqrt{f\,x^2+e\,x+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + (b*f*x^2)/e)/(d + e*x + f*x^2)^(1/2),x)
```

```
[Out] int((a + b*x + (b*f*x^2)/e)/(d + e*x + f*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{ae}{\sqrt{d+ex+fx^2}}\,dx + \int \frac{bex}{\sqrt{d+ex+fx^2}}\,dx + \int \frac{bfx^2}{\sqrt{d+ex+fx^2}}\,dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x+b*f*x**2/e)/(f*x**2+e*x+d)**(1/2),x)
```

```
[Out] (Integral(a*e/sqrt(d + e*x + f*x**2), x) + Integral(b*e*x/sqrt(d + e*x + f*
x**2), x) + Integral(b*f*x**2/sqrt(d + e*x + f*x**2), x))/e
```

3.2 $\int \frac{1}{\sqrt{d+ex+fx^2}\left(a+bx+\frac{bfx^2}{e}\right)}\,dx$

Optimal. Leaf size=82

$$-\frac{2\sqrt{e}\tanh^{-1}\left(\frac{(e+2fx)\sqrt{bd-ae}}{\sqrt{e}\sqrt{be-4af}\sqrt{d+ex+fx^2}}\right)}{\sqrt{bd-ae}\sqrt{be-4af}}$$

Rubi [A] time = 0.11, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {982, 208}

$$-\frac{2\sqrt{e}\tanh^{-1}\left(\frac{(e+2fx)\sqrt{bd-ae}}{\sqrt{e}\sqrt{be-4af}\sqrt{d+ex+fx^2}}\right)}{\sqrt{bd-ae}\sqrt{be-4af}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d + e*x + f*x^2]*(a + b*x + (b*f*x^2)/e)),x]
```

```
[Out] (-2*Sqrt[e]*ArcTanh[(Sqrt[b*d - a*e]*(e + 2*f*x))/(Sqrt[e]*Sqrt[b*e - 4*a*f
]*Sqrt[d + e*x + f*x^2])])/(Sqrt[b*d - a*e]*Sqrt[b*e - 4*a*f])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 982

```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(
x_)^2]), x_Symbol] :> Dist[-2*e, Subst[Int[1/(e*(b*e - 4*a*f) - (b*d - a*e)
*x^2), x], x, (e + 2*f*x)/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0
]
```

Rubi steps

$$\int \frac{1}{\sqrt{d+ex+fx^2}\left(a+bx+\frac{bfx^2}{e}\right)}\,dx = -\left((2e)\,\text{Subst}\left(\int \frac{1}{e(be-4af)-(bd-ae)x^2}\,dx, x, \frac{e+2fx}{\sqrt{d+ex+fx^2}}\right)\right)$$

$$= -\frac{2\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{bd-ae}\,(e+2fx)}{\sqrt{e}\sqrt{be-4af}\sqrt{d+ex+fx^2}}\right)}{\sqrt{bd-ae}\sqrt{be-4af}}$$

Mathematica [B] time = 0.39, size = 178, normalized size = 2.17

$$\frac{\sqrt{e}\left(\tanh^{-1}\left(\frac{-\sqrt{e}(e+2fx)\sqrt{be-4af}-\sqrt{b}\left(e^2-4df\right)}{4f\sqrt{bd-ae}\sqrt{d+x(e+fx)}}\right)+\tanh^{-1}\left(\frac{\sqrt{b}\left(e^2-4df\right)-\sqrt{e}(e+2fx)\sqrt{be-4af}}{4f\sqrt{bd-ae}\sqrt{d+x(e+fx)}}\right)\right)}{\sqrt{bd-ae}\sqrt{be-4af}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d + e*x + f*x^2]*(a + b*x + (b*f*x^2)/e)),x]
```

```
[Out] (Sqrt[e]*(ArcTanh[(-(Sqrt[b]*(e^2 - 4*d*f)) - Sqrt[e]*Sqrt[b*e - 4*a*f]*(e
+ 2*f*x))/(4*Sqrt[b*d - a*e]*f*Sqrt[d + x*(e + f*x)])] + ArcTanh[(Sqrt[b]*(
e^2 - 4*d*f) - Sqrt[e]*Sqrt[b*e - 4*a*f]*(e + 2*f*x))/(4*Sqrt[b*d - a*e]*f*
Sqrt[d + x*(e + f*x)])]))/(Sqrt[b*d - a*e]*Sqrt[b*e - 4*a*f])
```

IntegrateAlgebraic [C] time = 0.50, size = 162, normalized size = 1.98

$$e\text{RootSum}\left[\#1^4bf-2\#1^3be\sqrt{f}+4\#1^2aef-2\#1^2bdf+\#1^2be^2-4\#1ae^2\sqrt{f}+2\#1bde\sqrt{f}+ae^3+bd^2f-bde^2\&,\frac{\log\left(-\#1+\sqrt{d+ex+fx^2}-\sqrt{f}x\right)}{\#1^2(-b)\sqrt{f}+\#1be-2ae\sqrt{f}+bd\sqrt{f}}\&\right]$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[d + e*x + f*x^2]*(a + b*x + (b*f*x^2)/e)),x]
```

```
[Out] e*RootSum[-(b*d*e^2) + a*e^3 + b*d^2*f + 2*b*d*e*Sqrt[f]*#1 - 4*a*e^2*Sqrt[
f]*#1 + b*e^2*#1^2 - 2*b*d*f*#1^2 + 4*a*e*f*#1^2 - 2*b*e*Sqrt[f]*#1^3 + b*f
*#1^4 & , Log[-(Sqrt[f]*x) + Sqrt[d + e*x + f*x^2] - #1]/(b*d*Sqrt[f] - 2*a
*e*Sqrt[f] + b*e*#1 - b*Sqrt[f]*#1^2) & ]
```

fricas [B] time = 0.66, size = 1079, normalized size = 13.16

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x+b*f*x^2/e)/(f*x^2+e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(e/(b^2*d*e - a*b*e^2 - 4*(a*b*d - a^2*e)*f))*log((8*b^2*d^2*e^4 -
 8*a*b*d*e^5 + a^2*e^6 + 16*a^2*d^2*e^2*f^2 + (b^2*e^4*f^2 + 16*(b^2*d^2 -
8*a*b*d*e + 8*a^2*e^2)*f^4 + 8*(3*b^2*d*e^2 - 4*a*b*e^3)*f^3)*x^4 + 2*(b^2*
e^5*f + 16*(b^2*d^2*e - 8*a*b*d*e^2 + 8*a^2*e^3)*f^3 + 8*(3*b^2*d*e^3 - 4*a
*b*e^4)*f^2)*x^3 + (b^2*e^6 - 32*(3*a*b*d^2*e - 4*a^2*d*e^2)*f^3 + 16*(3*b^
2*d^2*e^2 - 13*a*b*d*e^3 + 10*a^2*e^4)*f^2 + 2*(16*b^2*d*e^4 - 19*a*b*e^5)*
f)*x^2 - 8*(4*a*b*d^2*e^3 - 3*a^2*d*e^4)*f + 2*(4*b^2*d*e^5 - 3*a*b*e^6 - 1
6*(3*a*b*d^2*e^2 - 4*a^2*d*e^3)*f^2 + 8*(2*b^2*d^2*e^3 - 5*a*b*d*e^4 + 2*a^
2*e^5)*f)*x - 4*(2*b^3*d^2*e^4 - 3*a*b^2*d*e^5 + a^2*b*e^6 - 2*(16*(a*b^2*d
^2 - 3*a^2*b*d*e + 2*a^3*e^2)*f^4 - 4*(b^3*d^2*e - 4*a*b^2*d*e^2 + 3*a^2*b*
e^3)*f^3 - (b^3*d*e^3 - a*b^2*e^4)*f^2)*x^3 + 16*(a^2*b*d^2*e^2 - a^3*d*e^3
)*f^2 - 3*(16*(a*b^2*d^2*e - 3*a^2*b*d*e^2 + 2*a^3*e^3)*f^3 - 4*(b^3*d^2*e^
2 - 4*a*b^2*d*e^3 + 3*a^2*b*e^4)*f^2 - (b^3*d*e^4 - a*b^2*e^5)*f)*x^2 - 4*(
3*a*b^2*d^2*e^3 - 4*a^2*b*d*e^4 + a^3*e^5)*f + (b^3*d*e^5 - a*b^2*e^6 + 32*
(a^2*b*d^2*e - a^3*d*e^2)*f^3 - 40*(a*b^2*d^2*e^2 - 2*a^2*b*d*e^3 + a^3*e^4
)*f^2 + 2*(4*b^3*d^2*e^3 - 11*a*b^2*d*e^4 + 7*a^2*b*e^5)*f)*x)*sqrt(f*x^2 +
 e*x + d)*sqrt(e/(b^2*d*e - a*b*e^2 - 4*(a*b*d - a^2*e)*f)))/(b^2*f^2*x^4 +
 2*b^2*e*f*x^3 + 2*a*b*e^2*x + a^2*e^2 + (b^2*e^2 + 2*a*b*e*f)*x^2)), -sqrt
(-e/(b^2*d*e - a*b*e^2 - 4*(a*b*d - a^2*e)*f))*arctan(-1/2*(2*b*d*e^2 - a*e
^3 - 4*a*d*e*f + (b*e^2*f + 4*(b*d - 2*a*e)*f^2)*x^2 + (b*e^3 + 4*(b*d*e -
2*a*e^2)*f)*x)*sqrt(f*x^2 + e*x + d)*sqrt(-e/(b^2*d*e - a*b*e^2 - 4*(a*b*d
- a^2*e)*f))/(2*e*f^2*x^3 + 3*e^2*f*x^2 + d*e^2 + (e^3 + 2*d*e*f)*x))]
```

giac [B] time = 3.04, size = 847, normalized size = 10.33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x+b*f*x^2/e)/(f*x^2+e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(-4*a*b*d*f*e + b^2*d*e^2 + 4*a^2*f*e^2 - a*b*e^3)*log(abs(-4*(sqrt(f)*
x - sqrt(f*x^2 + x*e + d))^2*b*d*f^2 + 8*(sqrt(f)*x - sqrt(f*x^2 + x*e + d)
)^2*a*f^2*e - 4*(sqrt(f)*x - sqrt(f*x^2 + x*e + d))*b*d*f^(3/2)*e + 4*b*d^2
*f^2 - (sqrt(f)*x - sqrt(f*x^2 + x*e + d))^2*b*f*e^2 + 8*(sqrt(f)*x - sqrt(
f*x^2 + x*e + d))*a*f^(3/2)*e^2 + 4*sqrt(-4*a*b*d*f*e + b^2*d*e^2 + 4*a^2*f
*e^2 - a*b*e^3)*(sqrt(f)*x - sqrt(f*x^2 + x*e + d))^2*f^(3/2) - 3*b*d*f*e^2
 - (sqrt(f)*x - sqrt(f*x^2 + x*e + d))*b*sqrt(f)*e^3 + 4*sqrt(-4*a*b*d*f*e
+ b^2*d*e^2 + 4*a^2*f*e^2 - a*b*e^3)*(sqrt(f)*x - sqrt(f*x^2 + x*e + d))*f*
e + 2*a*f*e^3 + sqrt(-4*a*b*d*f*e + b^2*d*e^2 + 4*a^2*f*e^2 - a*b*e^3)*sqrt
(f)*e^2))/(4*a*b*d*f - b^2*d*e - 4*a^2*f*e + a*b*e^2) - sqrt(-4*a*b*d*f*e +
 b^2*d*e^2 + 4*a^2*f*e^2 - a*b*e^3)*log(abs(-4*(sqrt(f)*x - sqrt(f*x^2 + x*
e + d))^2*b*d*f^2 + 8*(sqrt(f)*x - sqrt(f*x^2 + x*e + d))^2*a*f^2*e - 4*(sq
rt(f)*x - sqrt(f*x^2 + x*e + d))*b*d*f^(3/2)*e + 4*b*d^2*f^2 - (sqrt(f)*x -
 sqrt(f*x^2 + x*e + d))^2*b*f*e^2 + 8*(sqrt(f)*x - sqrt(f*x^2 + x*e + d))*a
*f^(3/2)*e^2 - 4*sqrt(-4*a*b*d*f*e + b^2*d*e^2 + 4*a^2*f*e^2 - a*b*e^3)*(sq
```

```
rt(f)*x - sqrt(f*x^2 + x*e + d))^2*f^(3/2) - 3*b*d*f*e^2 - (sqrt(f)*x - sqr
t(f*x^2 + x*e + d))*b*sqrt(f)*e^3 - 4*sqrt(-4*a*b*d*f*e + b^2*d*e^2 + 4*a^2
*f*e^2 - a*b*e^3)*(sqrt(f)*x - sqrt(f*x^2 + x*e + d))*f*e + 2*a*f*e^3 - sqr
t(-4*a*b*d*f*e + b^2*d*e^2 + 4*a^2*f*e^2 - a*b*e^3)*sqrt(f)*e^2))/(4*a*b*d*
f - b^2*d*e - 4*a^2*f*e + a*b*e^2)
```

maple [B] time = 0.06, size = 491, normalized size = 5.99

$$-\frac{e\ln\left(\frac{-\frac{2(ae-bd)}{b}+\frac{\sqrt{-(4af-be)be}\left(x-\frac{-be+\sqrt{-(4af-be)be}}{2bf}\right)}{b}+2\sqrt{-\frac{ae-bd}{b}}\sqrt{\left(x-\frac{-be+\sqrt{-(4af-be)be}}{2bf}\right)^2 f+\frac{\sqrt{-(4af-be)be}\left(x-\frac{-be+\sqrt{-(4af-be)be}}{2bf}\right)}{b}-\frac{ae-bd}{b}}}{x-\frac{-be+\sqrt{-(4af-be)be}}{2bf}}\right)}{\sqrt{-(4af-be)be}\sqrt{-\frac{ae-bd}{b}}}+\frac{e\ln\left(\frac{-\frac{2(ae-bd)}{b}-\frac{\sqrt{-(4af-be)be}\left(x+\frac{be+\sqrt{-(4af-be)be}}{2bf}\right)}{b}+2\sqrt{-\frac{ae-bd}{b}}\sqrt{\left(x+\frac{be+\sqrt{-(4af-be)be}}{2bf}\right)^2 f-\frac{\sqrt{-(4af-be)be}\left(x+\frac{be+\sqrt{-(4af-be)be}}{2bf}\right)}{b}-\frac{ae-bd}{b}}}{x+\frac{be+\sqrt{-(4af-be)be}}{2bf}}\right)}{\sqrt{-(4af-be)be}\sqrt{-\frac{ae-bd}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*x+b*f*x^2/e)/(f*x^2+e*x+d)^(1/2),x)
```

```
[Out] -e/(-b*e*(4*a*f-b*e))^(1/2)/(-1/b*(a*e-b*d))^(1/2)*ln((-2/b*(a*e-b*d)+(-b*e
*(4*a*f-b*e))^(1/2)/b*(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)+2*(-1/b*(
a*e-b*d))^(1/2)*((x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)^2*f+(-b*e*(4*a
*f-b*e))^(1/2)/b*(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)-1/b*(a*e-b*d))
^(1/2))/(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f))+e/(-b*e*(4*a*f-b*e))^(
1/2)/(-1/b*(a*e-b*d))^(1/2)*ln((-2/b*(a*e-b*d)-(-b*e*(4*a*f-b*e))^(1/2)/b*(
x+1/2*(b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)+2*(-1/b*(a*e-b*d))^(1/2)*((x+1/2*
(b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)^2*f-(-b*e*(4*a*f-b*e))^(1/2)/b*(x+1/2*(
b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)-1/b*(a*e-b*d))^(1/2))/(x+1/2*(b*e+(-b*e*
(4*a*f-b*e))^(1/2))/b/f))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x+b*f*x^2/e)/(f*x^2+e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*f-b*e>0)', see `assume?` for mo
re details)Is 4*a*f-b*e    positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{f\,x^2+e\,x+d}\,\left(a+b\,x+\frac{b\,f\,x^2}{e}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d + e*x + f*x^2)^(1/2)*(a + b*x + (b*f*x^2)/e)),x)
```

```
[Out] int(1/((d + e*x + f*x^2)^(1/2)*(a + b*x + (b*f*x^2)/e)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e\int \frac{1}{ae\sqrt{d+ex+fx^2}+bex\sqrt{d+ex+fx^2}+bfx^2\sqrt{d+ex+fx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x+b*f*x**2/e)/(f*x**2+e*x+d)**(1/2),x)
```

```
[Out] e*Integral(1/(a*e*sqrt(d + e*x + f*x**2) + b*e*x*sqrt(d + e*x + f*x**2) + b
*f*x**2*sqrt(d + e*x + f*x**2)), x)
```

3.3 $\int \frac{1}{\sqrt{a+bx+cx^2}\left(d+bx+cx^2\right)} dx$

Optimal. Leaf size=66

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a-d}\,(b+2cx)}{\sqrt{b^2-4cd}\,\sqrt{a+bx+cx^2}}\right)}{\sqrt{a-d}\,\sqrt{b^2-4cd}}$$

Rubi [A] time = 0.08, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} =$ 0.074, Rules used = {982, 208}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a-d}\,(b+2cx)}{\sqrt{b^2-4cd}\,\sqrt{a+bx+cx^2}}\right)}{\sqrt{a-d}\,\sqrt{b^2-4cd}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x + c*x^2]*(d + b*x + c*x^2)),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[a - d]*(b + 2*c*x))/(Sqrt[b^2 - 4*c*d]*Sqrt[a + b*x + c*x
^2])])/(Sqrt[a - d]*Sqrt[b^2 - 4*c*d])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 982

```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(
x_)^2]), x_Symbol] :> Dist[-2*e, Subst[Int[1/(e*(b*e - 4*a*f) - (b*d - a*e)
*x^2), x], x, (e + 2*f*x)/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0
]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx+cx^2}\left(d+bx+cx^2\right)} dx = -\left((2b)\,\text{Subst}\left(\int \frac{1}{b\left(b^2-4cd\right)-(ab-bd)x^2} dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)\right)$$
$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a-d}\,(b+2cx)}{\sqrt{b^2-4cd}\,\sqrt{a+bx+cx^2}}\right)}{\sqrt{a-d}\,\sqrt{b^2-4cd}}$$

Mathematica [B] time = 0.23, size = 161, normalized size = 2.44

$$\frac{\tanh^{-1}\left(\frac{4ac-2cx\sqrt{b^2-4cd}-b\left(\sqrt{b^2-4cd}+b\right)}{4c\sqrt{a-d}\sqrt{a+x(b+cx)}}\right)+\tanh^{-1}\left(\frac{-2c\left(2a+x\sqrt{b^2-4cd}\right)-b\sqrt{b^2-4cd}+b^2}{4c\sqrt{a-d}\sqrt{a+x(b+cx)}}\right)}{\sqrt{a-d}\sqrt{b^2-4cd}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x + c*x^2]*(d + b*x + c*x^2)),x]
```

```
[Out] (ArcTanh[(4*a*c - b*(b + Sqrt[b^2 - 4*c*d]) - 2*c*Sqrt[b^2 - 4*c*d]*x)/(4*c
*Sqrt[a - d]*Sqrt[a + x*(b + c*x)])] + ArcTanh[(b^2 - b*Sqrt[b^2 - 4*c*d] -
 2*c*(2*a + Sqrt[b^2 - 4*c*d]*x))/(4*c*Sqrt[a - d]*Sqrt[a + x*(b + c*x)])])
/(Sqrt[a - d]*Sqrt[b^2 - 4*c*d])
```

IntegrateAlgebraic [C] time = 0.44, size = 146, normalized size = 2.21

$$\text{RootSum}\left[\#1^4c-2\#1^3b\sqrt{c}-2\#1^2ac+\#1^2b^2+4\#1^2cd+2\#1ab\sqrt{c}-4\#1b\sqrt{c}\,d+a^2c-ab^2+b^2d\&,\frac{\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}\,x\right)}{\#1^2\left(-\sqrt{c}\right)+\#1b+a\sqrt{c}-2\sqrt{c}\,d}\&\right]$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[a + b*x + c*x^2]*(d + b*x + c*x^2)),x]
```

```
[Out] RootSum[-(a*b^2) + a^2*c + b^2*d + 2*a*b*Sqrt[c]*#1 - 4*b*Sqrt[c]*d*#1 + b^
2*#1^2 - 2*a*c*#1^2 + 4*c*d*#1^2 - 2*b*Sqrt[c]*#1^3 + c*#1^4 & , Log[-(Sqrt
[c]*x) + Sqrt[a + b*x + c*x^2] - #1]/(a*Sqrt[c] - 2*Sqrt[c]*d + b*#1 - Sqrt
[c]*#1^2) & ]
```

fricas [B] time = 0.52, size = 813, normalized size = 12.32

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log((8*a^2*b^4 + (b^4*c^2 + 24*a*b^2*c^3 + 16*a^2*c^4 + 128*c^4*d^2 -
32*(b^2*c^3 + 4*a*c^4)*d)*x^4 + 2*(b^5*c + 24*a*b^3*c^2 + 16*a^2*b*c^3 + 12
8*b*c^3*d^2 - 32*(b^3*c^2 + 4*a*b*c^3)*d)*x^3 + (b^4 + 24*a*b^2*c + 16*a^2*
c^2)*d^2 + (b^6 + 32*a*b^4*c + 48*a^2*b^2*c^2 + 32*(5*b^2*c^2 + 4*a*c^3)*d^
2 - 2*(19*b^4*c + 104*a*b^2*c^2 + 48*a^2*c^3)*d)*x^2 - 4*(2*a*b^3 + 2*(b^2*
c^2 + 4*a*c^3 - 8*c^3*d)*x^3 + 3*(b^3*c + 4*a*b*c^2 - 8*b*c^2*d)*x^2 - (b^3
 + 4*a*b*c)*d + (b^4 + 8*a*b^2*c - 2*(5*b^2*c + 4*a*c^2)*d)*x)*sqrt(a*b^2 +
 4*c*d^2 - (b^2 + 4*a*c)*d)*sqrt(c*x^2 + b*x + a) - 8*(a*b^4 + 4*a^2*b^2*c)
```

```
*d + 2*(4*a*b^5 + 16*a^2*b^3*c + 16*(b^3*c + 4*a*b*c^2)*d^2 - (3*b^5 + 40*a
*b^3*c + 48*a^2*b*c^2)*d)*x)/(c^2*x^4 + 2*b*c*x^3 + 2*b*d*x + (b^2 + 2*c*d)
*x^2 + d^2))/sqrt(a*b^2 + 4*c*d^2 - (b^2 + 4*a*c)*d), -sqrt(-a*b^2 - 4*c*d^
2 + (b^2 + 4*a*c)*d)*arctan(-1/2*(2*a*b^2 + (b^2*c + 4*a*c^2 - 8*c^2*d)*x^2
 - (b^2 + 4*a*c)*d + (b^3 + 4*a*b*c - 8*b*c*d)*x)*sqrt(-a*b^2 - 4*c*d^2 + (
b^2 + 4*a*c)*d)*sqrt(c*x^2 + b*x + a)/(a^2*b^3 + 4*a*b*c*d^2 + 2*(a*b^2*c^2
 + 4*c^3*d^2 - (b^2*c^2 + 4*a*c^3)*d)*x^3 + 3*(a*b^3*c + 4*b*c^2*d^2 - (b^3
*c + 4*a*b*c^2)*d)*x^2 - (a*b^3 + 4*a^2*b*c)*d + (a*b^4 + 2*a^2*b^2*c + 4*(
b^2*c + 2*a*c^2)*d^2 - (b^4 + 6*a*b^2*c + 8*a^2*c^2)*d)*x))/(a*b^2 + 4*c*d^
2 - (b^2 + 4*a*c)*d)]
```

giac [B] time = 1.90, size = 703, normalized size = 10.65

$$-\frac{\log\left(\left|-\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)^2 b^2 c-4\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)^2 ac^2+8\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)^2 c^2 d-\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)b^3\sqrt{c}-4\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)abc^{\frac{3}{2}}+8\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)bc^{\frac{3}{2}}d-3ab^2c+4\sqrt{ab^2-b^2d-4acd+4cd^2}\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)^2 c^{\frac{3}{2}}+4a^2c^2+2b^2cd+4\sqrt{ab^2-b^2d-4acd+4cd^2}\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)bc+\sqrt{ab^2-b^2d-4acd+4cd^2}\,b^2\sqrt{c}\right|\right)}{\sqrt{ab^2-b^2d-4acd+4cd^2}}+\frac{\log\left(\left|-\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)^2 b^2 c-4\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)^2 ac^2+8\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)^2 c^2 d-\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)b^3\sqrt{c}-4\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)abc^{\frac{3}{2}}+8\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)bc^{\frac{3}{2}}d-3ab^2c-4\sqrt{ab^2-b^2d-4acd+4cd^2}\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)^2 c^{\frac{3}{2}}+4a^2c^2+2b^2cd-4\sqrt{ab^2-b^2d-4acd+4cd^2}\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)bc-\sqrt{ab^2-b^2d-4acd+4cd^2}\,b^2\sqrt{c}\right|\right)}{\sqrt{ab^2-b^2d-4acd+4cd^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] -log(abs(-(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^2*c - 4*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^2*a*c^2 + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*c^2*d
- (sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^3*sqrt(c) - 4*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))*a*b*c^(3/2) + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b*c^(3/2
)*d - 3*a*b^2*c + 4*sqrt(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2)*(sqrt(c)*x - sq
rt(c*x^2 + b*x + a))^2*c^(3/2) + 4*a^2*c^2 + 2*b^2*c*d + 4*sqrt(a*b^2 - b^2
*d - 4*a*c*d + 4*c*d^2)*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b*c + sqrt(a*b^
2 - b^2*d - 4*a*c*d + 4*c*d^2)*b^2*sqrt(c)))/sqrt(a*b^2 - b^2*d - 4*a*c*d +
 4*c*d^2) + log(abs(-(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^2*c - 4*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))^2*a*c^2 + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))^2*c^2*d - (sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^3*sqrt(c) - 4*(sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))*a*b*c^(3/2) + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))*b*c^(3/2)*d - 3*a*b^2*c - 4*sqrt(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2)*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))^2*c^(3/2) + 4*a^2*c^2 + 2*b^2*c*d - 4*sqrt(
a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2)*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b*c
- sqrt(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2)*b^2*sqrt(c)))/sqrt(a*b^2 - b^2*d
- 4*a*c*d + 4*c*d^2)
```

maple [B] time = 0.03, size = 307, normalized size = 4.65

$$-\frac{\ln\left(\frac{2a-2d+\sqrt{b^2-4cd}\left(x-\frac{-b+\sqrt{b^2-4cd}}{2c}\right)+2\sqrt{a-d}\sqrt{a+\left(x-\frac{-b+\sqrt{b^2-4cd}}{2c}\right)^2 c-d+\sqrt{b^2-4cd}\left(x-\frac{-b+\sqrt{b^2-4cd}}{2c}\right)}}{x-\frac{-b+\sqrt{b^2-4cd}}{2c}}\right)}{\sqrt{b^2-4cd}\,\sqrt{a-d}}+\frac{\ln\left(\frac{2a-2d-\sqrt{b^2-4cd}\left(x+\frac{b+\sqrt{b^2-4cd}}{2c}\right)+2\sqrt{a-d}\sqrt{a+\left(x+\frac{b+\sqrt{b^2-4cd}}{2c}\right)^2 c-d-\sqrt{b^2-4cd}\left(x+\frac{b+\sqrt{b^2-4cd}}{2c}\right)}}{x+\frac{b+\sqrt{b^2-4cd}}{2c}}\right)}{\sqrt{b^2-4cd}\,\sqrt{a-d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+b*x+d)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] 1/(b^2-4*c*d)^(1/2)/(a-d)^(1/2)*ln((2*a-2*d-(b^2-4*c*d)^(1/2)*(x+1/2*((b^2-
4*c*d)^(1/2)+b)/c)+2*(a-d)^(1/2)*((x+1/2*((b^2-4*c*d)^(1/2)+b)/c)^2*c-(b^2-
4*c*d)^(1/2)*(x+1/2*((b^2-4*c*d)^(1/2)+b)/c)+a-d)^(1/2))/(x+1/2*((b^2-4*c*d
)^(1/2)+b)/c))-1/(b^2-4*c*d)^(1/2)/(a-d)^(1/2)*ln((2*a-2*d+(b^2-4*c*d)^(1/2
)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)+2*(a-d)^(1/2)*((x-1/2*(-b+(b^2-4*c*d)^(1
/2))/c)^2*c+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)+a-d)^(1/2))/
(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c*d-b^2>0)', see `assume?` for mo
re details)Is 4*c*d-b^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{c\,x^2+b\,x+a}\,\left(c\,x^2+b\,x+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x + c*x^2)^(1/2)*(d + b*x + c*x^2)),x)
```

```
[Out] int(1/((a + b*x + c*x^2)^(1/2)*(d + b*x + c*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,\left(bx+cx^2+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*x + c*x**2)*(b*x + c*x**2 + d)), x)
```

3.4 $\int \frac{1}{\sqrt{a+bx+cx^2}\left(d+bx+cx^2\right)^2}\,dx$

Optimal. Leaf size=129

$$\frac{\left(4c(a-2d)+b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-d}\,(b+2cx)}{\sqrt{b^2-4cd}\,\sqrt{a+bx+cx^2}}\right)}{(a-d)^{3/2}\left(b^2-4cd\right)^{3/2}}-\frac{(b+2cx)\sqrt{a+bx+cx^2}}{(a-d)\left(b^2-4cd\right)\left(bx+cx^2+d\right)}$$

Rubi [A] time = 0.17, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {974, 12, 982, 208}

$$\frac{\left(4c(a-2d)+b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-d}\,(b+2cx)}{\sqrt{b^2-4cd}\,\sqrt{a+bx+cx^2}}\right)}{(a-d)^{3/2}\left(b^2-4cd\right)^{3/2}}-\frac{(b+2cx)\sqrt{a+bx+cx^2}}{(a-d)\left(b^2-4cd\right)\left(bx+cx^2+d\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x + c*x^2]*(d + b*x + c*x^2)^2),x]
```

```
[Out] -(((b + 2*c*x)*Sqrt[a + b*x + c*x^2])/((a - d)*(b^2 - 4*c*d)*(d + b*x + c*x
^2))) + ((b^2 + 4*c*(a - 2*d))*ArcTanh[(Sqrt[a - d]*(b + 2*c*x))/(Sqrt[b^2
- 4*c*d]*Sqrt[a + b*x + c*x^2])])/((a - d)^(3/2)*(b^2 - 4*c*d)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 974

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*
a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*(
d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
```

```
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q,
 0]
```

Rule 982

```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(
x_)^2]), x_Symbol] :> Dist[-2*e, Subst[Int[1/(e*(b*e - 4*a*f) - (b*d - a*e)
*x^2), x], x, (e + 2*f*x)/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0
]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx+cx^2}\left(d+bx+cx^2\right)^2}\,dx = -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{(a-d)\left(b^2-4cd\right)\left(d+bx+cx^2\right)} + \frac{\int -\frac{c^2\left(b^2+4c(a-2d)\right)(a-d)}{2\sqrt{a+bx+cx^2}\left(d+bx+cx^2\right)}\,dx}{c^2(a-d)^2\left(b^2-4cd\right)}$$

$$= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{(a-d)\left(b^2-4cd\right)\left(d+bx+cx^2\right)} - \frac{\left(b^2+4c(a-2d)\right)\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+bx\cdots}}{2(a-d)\left(b^2-4cd\right)}$$

$$= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{(a-d)\left(b^2-4cd\right)\left(d+bx+cx^2\right)} + \frac{\left(b\left(b^2+4c(a-2d)\right)\right)\mathrm{Subst}\left(\int \frac{\cdots}{b\left(b^{\cdots}\right.}\right.}{(a-d)\left(b^{\cdots}\right.}$$

$$= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{(a-d)\left(b^2-4cd\right)\left(d+bx+cx^2\right)} + \frac{\left(b^2+4c(a-2d)\right)\tanh^{-1}\left(\frac{\sqrt{a-d}\cdots}{\sqrt{b^2-4cd}\cdots}\right.}{(a-d)^{3/2}\left(b^2-4cd\right)^{3/2}}$$

Mathematica [B] time = 0.92, size = 296, normalized size = 2.29

$$\frac{1}{2}\left(-\frac{8c(b+2cx)\sqrt{a+x(b+cx)}}{(a-d)\left(4cd-b^2\right)\left(\sqrt{b^2-4cd}-b-2cx\right)\left(\sqrt{b^2-4cd}+b+2cx\right)} - \frac{\left(4c(a-2d)+b^2\right)\tanh^{-1}\left(\frac{4ac-2cx\sqrt{b^2-4cd}-b\left(\sqrt{b^2-4cd}+b\right)}{4c\sqrt{a-d}\sqrt{a+x(b+cx)}}\right)}{(a-d)^{3/2}\left(b^2-4cd\right)^{3/2}} - \frac{\left(4c(a-2d)+b^2\right)\tanh^{-1}\left(\frac{-2c\left(2a+x\sqrt{b^2-4cd}\right)-b\sqrt{b^2-4cd}+b^2}{4c\sqrt{a-d}\sqrt{a+x(b+cx)}}\right)}{(a-d)^{3/2}\left(b^2-4cd\right)^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x + c*x^2]*(d + b*x + c*x^2)^2),x]
```

```
[Out] ((-8*c*(b + 2*c*x)*Sqrt[a + x*(b + c*x)])/((a - d)*(-b^2 + 4*c*d)*(-b + Sqr
t[b^2 - 4*c*d] - 2*c*x)*(b + Sqrt[b^2 - 4*c*d] + 2*c*x)) - ((b^2 + 4*c*(a -
 2*d))*ArcTanh[(4*a*c - b*(b + Sqrt[b^2 - 4*c*d]) - 2*c*Sqrt[b^2 - 4*c*d]*x
)/(4*c*Sqrt[a - d]*Sqrt[a + x*(b + c*x)])]])/((a - d)^(3/2)*(b^2 - 4*c*d)^(3
/2)) - ((b^2 + 4*c*(a - 2*d))*ArcTanh[(b^2 - b*Sqrt[b^2 - 4*c*d] - 2*c*(2*a
 + Sqrt[b^2 - 4*c*d]*x))/(4*c*Sqrt[a - d]*Sqrt[a + x*(b + c*x)])]])/((a - d)
^(3/2)*(b^2 - 4*c*d)^(3/2)))/2
```

IntegrateAlgebraic [C] time = 0.86, size = 239, normalized size = 1.85

$$-\frac{\left(-4ac-b^2+8cd\right)\text{RootSum}\left[\#1^4c-2\#1^3b\sqrt{c}-2\#1^2ac+\#1^2b^2+4\#1^2cd+2\#1ab\sqrt{c}-4\#1b\sqrt{c}\,d+a^2c-ab^2+b^2d\&,\frac{\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}\,x\right)}{\#1^2\left(-\sqrt{c}\right)+\#1b+a\sqrt{c}-2\sqrt{c}\,d}\&\right]}{2(a-d)\left(4cd-b^2\right)}-\frac{(-b-2cx)\sqrt{a+bx+cx^2}}{(a-d)\left(4cd-b^2\right)\left(bx+cx^2+d\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[a + b*x + c*x^2]*(d + b*x + c*x^2)^2),x]
```

```
[Out] -(((-b - 2*c*x)*Sqrt[a + b*x + c*x^2])/((a - d)*(-b^2 + 4*c*d)*(d + b*x + c
*x^2))) - ((-b^2 - 4*a*c + 8*c*d)*RootSum[-(a*b^2) + a^2*c + b^2*d + 2*a*b*
Sqrt[c]*#1 - 4*b*Sqrt[c]*d*#1 + b^2*#1^2 - 2*a*c*#1^2 + 4*c*d*#1^2 - 2*b*Sq
rt[c]*#1^3 + c*#1^4 & , Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]/(a*S
qrt[c] - 2*Sqrt[c]*d + b*#1 - Sqrt[c]*#1^2) & ])/(2*(a - d)*(-b^2 + 4*c*d))
```

fricas [B] time = 0.84, size = 1544, normalized size = 11.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+d)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(a*b^2 + 4*c*d^2 - (b^2 + 4*a*c)*d)*(8*c*d^2 - (b^2*c + 4*a*c^2 -
 8*c^2*d)*x^2 - (b^2 + 4*a*c)*d - (b^3 + 4*a*b*c - 8*b*c*d)*x)*log((8*a^2*b
^4 + (b^4*c^2 + 24*a*b^2*c^3 + 16*a^2*c^4 + 128*c^4*d^2 - 32*(b^2*c^3 + 4*a
*c^4)*d)*x^4 + 2*(b^5*c + 24*a*b^3*c^2 + 16*a^2*b*c^3 + 128*b*c^3*d^2 - 32*
(b^3*c^2 + 4*a*b*c^3)*d)*x^3 + (b^4 + 24*a*b^2*c + 16*a^2*c^2)*d^2 + (b^6 +
 32*a*b^4*c + 48*a^2*b^2*c^2 + 32*(5*b^2*c^2 + 4*a*c^3)*d^2 - 2*(19*b^4*c +
 104*a*b^2*c^2 + 48*a^2*c^3)*d)*x^2 - 4*(2*a*b^3 + 2*(b^2*c^2 + 4*a*c^3 - 8
*c^3*d)*x^3 + 3*(b^3*c + 4*a*b*c^2 - 8*b*c^2*d)*x^2 - (b^3 + 4*a*b*c)*d + (
b^4 + 8*a*b^2*c - 2*(5*b^2*c + 4*a*c^2)*d)*x)*sqrt(a*b^2 + 4*c*d^2 - (b^2 +
 4*a*c)*d)*sqrt(c*x^2 + b*x + a) - 8*(a*b^4 + 4*a^2*b^2*c)*d + 2*(4*a*b^5 +
 16*a^2*b^3*c + 16*(b^3*c + 4*a*b*c^2)*d^2 - (3*b^5 + 40*a*b^3*c + 48*a^2*b
*c^2)*d)*x)/(c^2*x^4 + 2*b*c*x^3 + 2*b*d*x + (b^2 + 2*c*d)*x^2 + d^2)) - 4*
(a*b^3 + 4*b*c*d^2 - (b^3 + 4*a*b*c)*d + 2*(a*b^2*c + 4*c^2*d^2 - (b^2*c +
4*a*c^2)*d)*x)*sqrt(c*x^2 + b*x + a))/(a^2*b^4*d + 16*c^2*d^5 - 8*(b^2*c +
4*a*c^2)*d^4 + (b^4 + 16*a*b^2*c + 16*a^2*c^2)*d^3 - 2*(a*b^4 + 4*a^2*b^2*c
)*d^2 + (a^2*b^4*c + 16*c^3*d^4 - 8*(b^2*c^2 + 4*a*c^3)*d^3 + (b^4*c + 16*a
```

```
*b^2*c^2 + 16*a^2*c^3)*d^2 - 2*(a*b^4*c + 4*a^2*b^2*c^2)*d)*x^2 + (a^2*b^5
+ 16*b*c^2*d^4 - 8*(b^3*c + 4*a*b*c^2)*d^3 + (b^5 + 16*a*b^3*c + 16*a^2*b*c
^2)*d^2 - 2*(a*b^5 + 4*a^2*b^3*c)*d)*x), -1/2*(sqrt(-a*b^2 - 4*c*d^2 + (b^2
 + 4*a*c)*d)*(8*c*d^2 - (b^2*c + 4*a*c^2 - 8*c^2*d)*x^2 - (b^2 + 4*a*c)*d -
 (b^3 + 4*a*b*c - 8*b*c*d)*x)*arctan(-1/2*(2*a*b^2 + (b^2*c + 4*a*c^2 - 8*c
^2*d)*x^2 - (b^2 + 4*a*c)*d + (b^3 + 4*a*b*c - 8*b*c*d)*x)*sqrt(-a*b^2 - 4*
c*d^2 + (b^2 + 4*a*c)*d)*sqrt(c*x^2 + b*x + a)/(a^2*b^3 + 4*a*b*c*d^2 + 2*(
a*b^2*c^2 + 4*c^3*d^2 - (b^2*c^2 + 4*a*c^3)*d)*x^3 + 3*(a*b^3*c + 4*b*c^2*d
^2 - (b^3*c + 4*a*b*c^2)*d)*x^2 - (a*b^3 + 4*a^2*b*c)*d + (a*b^4 + 2*a^2*b^
2*c + 4*(b^2*c + 2*a*c^2)*d^2 - (b^4 + 6*a*b^2*c + 8*a^2*c^2)*d)*x)) + 2*(a
*b^3 + 4*b*c*d^2 - (b^3 + 4*a*b*c)*d + 2*(a*b^2*c + 4*c^2*d^2 - (b^2*c + 4*
a*c^2)*d)*x)*sqrt(c*x^2 + b*x + a))/(a^2*b^4*d + 16*c^2*d^5 - 8*(b^2*c + 4*
a*c^2)*d^4 + (b^4 + 16*a*b^2*c + 16*a^2*c^2)*d^3 - 2*(a*b^4 + 4*a^2*b^2*c)*
d^2 + (a^2*b^4*c + 16*c^3*d^4 - 8*(b^2*c^2 + 4*a*c^3)*d^3 + (b^4*c + 16*a*b
^2*c^2 + 16*a^2*c^3)*d^2 - 2*(a*b^4*c + 4*a^2*b^2*c^2)*d)*x^2 + (a^2*b^5 +
16*b*c^2*d^4 - 8*(b^3*c + 4*a*b*c^2)*d^3 + (b^5 + 16*a*b^3*c + 16*a^2*b*c^2
)*d^2 - 2*(a*b^5 + 4*a^2*b^3*c)*d)*x)]
```

giac [B] time = 2.27, size = 1166, normalized size = 9.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+d)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*((b^2 + 4*a*c - 8*c*d)*log(abs((sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b
^2*c + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*c^2 - 8*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^2*c^2*d + (sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^3*sqrt(c)
 + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b*c^(3/2) - 8*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))*b*c^(3/2)*d + 3*a*b^2*c + 4*sqrt(a*b^2 - b^2*d - 4*a*c*d
+ 4*c*d^2)*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*c^(3/2) - 4*a^2*c^2 - 2*b^
2*c*d + 4*sqrt(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2)*(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))*b*c + sqrt(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2)*b^2*sqrt(c)))/sqrt
(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2) - (b^2 + 4*a*c - 8*c*d)*log(abs((sqrt(c
)*x - sqrt(c*x^2 + b*x + a))^2*b^2*c + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a)
)^2*a*c^2 - 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*c^2*d + (sqrt(c)*x - sq
rt(c*x^2 + b*x + a))*b^3*sqrt(c) + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*
b*c^(3/2) - 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b*c^(3/2)*d + 3*a*b^2*c -
 4*sqrt(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2)*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))^2*c^(3/2) - 4*a^2*c^2 - 2*b^2*c*d - 4*sqrt(a*b^2 - b^2*d - 4*a*c*d + 4*
c*d^2)*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b*c - sqrt(a*b^2 - b^2*d - 4*a*c
*d + 4*c*d^2)*b^2*sqrt(c)))/sqrt(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2))/(a*b^2
 - b^2*d - 4*a*c*d + 4*c*d^2) + ((sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^2*
sqrt(c) + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*c^(3/2) - 8*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))^2*c^(3/2)*d + (sqrt(c)*x - sqrt(c*x^2 + b*x + a))*
```

```
b^3 + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b*c - 8*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))*b*c*d + 3*a*b^2*sqrt(c) - 4*a^2*c^(3/2) - 2*b^2*sqrt(c)*d)/(
((sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*c + 2*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))^3*b*sqrt(c) + (sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^2 - 2*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))^2*a*c + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*
c*d - 2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b*sqrt(c) + 4*(sqrt(c)*x - sq
rt(c*x^2 + b*x + a))*b*sqrt(c)*d - a*b^2 + a^2*c + b^2*d)*(a*b^2 - b^2*d -
4*a*c*d + 4*c*d^2))
```

maple [B] time = 0.03, size = 829, normalized size = 6.43

$$\frac{2c\ln\left(\frac{2a-2d+\sqrt{b^2-4cd}\left(x-\frac{-b+\sqrt{b^2-4cd}}{2c}\right)+2\sqrt{a-d}\sqrt{a+\left(x-\frac{-b+\sqrt{b^2-4cd}}{2c}\right)^2c-d+\sqrt{b^2-4cd}\left(x-\frac{-b+\sqrt{b^2-4cd}}{2c}\right)}}{x-\frac{-b+\sqrt{b^2-4cd}}{2c}}\right)}{\left(b^2-4cd\right)^{\frac{3}{2}}\sqrt{a-d}}-\frac{2c\ln\left(\frac{2a-2d-\sqrt{b^2-4cd}\left(x+\frac{b+\sqrt{b^2-4cd}}{2c}\right)+2\sqrt{a-d}\sqrt{a+\left(x+\frac{b+\sqrt{b^2-4cd}}{2c}\right)^2c-d-\sqrt{b^2-4cd}\left(x+\frac{b+\sqrt{b^2-4cd}}{2c}\right)}}{x+\frac{b+\sqrt{b^2-4cd}}{2c}}\right)}{\left(b^2-4cd\right)^{\frac{3}{2}}\sqrt{a-d}}+\frac{\ln\left(\frac{2a-2d+\sqrt{b^2-4cd}\left(x-\frac{-b+\sqrt{b^2-4cd}}{2c}\right)+2\sqrt{a-d}\sqrt{a+\left(x-\frac{-b+\sqrt{b^2-4cd}}{2c}\right)^2c-d+\sqrt{b^2-4cd}\left(x-\frac{-b+\sqrt{b^2-4cd}}{2c}\right)}}{x-\frac{-b+\sqrt{b^2-4cd}}{2c}}\right)}{2\sqrt{b^2-4cd}\,(a-d)^{\frac{3}{2}}}-\frac{\ln\left(\frac{2a-2d-\sqrt{b^2-4cd}\left(x+\frac{b+\sqrt{b^2-4cd}}{2c}\right)+2\sqrt{a-d}\sqrt{a+\left(x+\frac{b+\sqrt{b^2-4cd}}{2c}\right)^2c-d-\sqrt{b^2-4cd}\left(x+\frac{b+\sqrt{b^2-4cd}}{2c}\right)}}{x+\frac{b+\sqrt{b^2-4cd}}{2c}}\right)}{2\sqrt{b^2-4cd}\,(a-d)^{\frac{3}{2}}}-\frac{\sqrt{a+\left(x-\frac{-b+\sqrt{b^2-4cd}}{2c}\right)^2c-d+\sqrt{b^2-4cd}\left(x-\frac{-b+\sqrt{b^2-4cd}}{2c}\right)}}{\left(b^2-4cd\right)(a-d)\left(x+\frac{b}{2c}-\frac{\sqrt{b^2-4cd}}{2c}\right)}-\frac{\sqrt{a+\left(x+\frac{b+\sqrt{b^2-4cd}}{2c}\right)^2c-d-\sqrt{b^2-4cd}\left(x+\frac{b+\sqrt{b^2-4cd}}{2c}\right)}}{\left(b^2-4cd\right)(a-d)\left(x+\frac{b}{2c}+\frac{\sqrt{b^2-4cd}}{2c}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+b*x+d)^2/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] -2/(b^2-4*c*d)^(3/2)*c/(a-d)^(1/2)*ln((2*a-2*d-(b^2-4*c*d)^(1/2)*(x+1/2*(b+
(b^2-4*c*d)^(1/2))/c)+2*(a-d)^(1/2)*(a+(x+1/2*(b+(b^2-4*c*d)^(1/2))/c)^2*c-
d-(b^2-4*c*d)^(1/2)*(x+1/2*(b+(b^2-4*c*d)^(1/2))/c))^(1/2))/(x+1/2*(b+(b^2-
4*c*d)^(1/2))/c))+2/(b^2-4*c*d)^(3/2)*c/(a-d)^(1/2)*ln((2*a-2*d+(b^2-4*c*d)
^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)+2*(a-d)^(1/2)*(a+(x-1/2*(-b+(b^2-4*
c*d)^(1/2))/c)^2*c-d+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c))^(1
/2))/(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c))-1/(b^2-4*c*d)/(a-d)/(x-1/2/c*(b^2-4*
c*d)^(1/2)+1/2/c*b)*(a+(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)^2*c-d+(b^2-4*c*d)^(
1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c))^(1/2)+1/2/(b^2-4*c*d)^(1/2)/(a-d)^(3
/2)*ln((2*a-2*d+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)+2*(a-d)^
(1/2)*(a+(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)^2*c-d+(b^2-4*c*d)^(1/2)*(x-1/2*(-
b+(b^2-4*c*d)^(1/2))/c))^(1/2))/(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c))-1/(b^2-4*
c*d)/(a-d)/(x+1/2/c*(b^2-4*c*d)^(1/2)+1/2/c*b)*(a+(x+1/2*(b+(b^2-4*c*d)^(1/
2))/c)^2*c-d-(b^2-4*c*d)^(1/2)*(x+1/2*(b+(b^2-4*c*d)^(1/2))/c))^(1/2)-1/2/(
b^2-4*c*d)^(1/2)/(a-d)^(3/2)*ln((2*a-2*d-(b^2-4*c*d)^(1/2)*(x+1/2*(b+(b^2-4
*c*d)^(1/2))/c)+2*(a-d)^(1/2)*(a+(x+1/2*(b+(b^2-4*c*d)^(1/2))/c)^2*c-d-(b^2
-4*c*d)^(1/2)*(x+1/2*(b+(b^2-4*c*d)^(1/2))/c))^(1/2))/(x+1/2*(b+(b^2-4*c*d)
^(1/2))/c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^2+bx+a}\left(cx^2+bx+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+d)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*(c*x^2 + b*x + d)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{c\,x^2+b\,x+a}\,\left(c\,x^2+b\,x+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x + c*x^2)^(1/2)*(d + b*x + c*x^2)^2),x)
[Out] int(1/((a + b*x + c*x^2)^(1/2)*(d + b*x + c*x^2)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+bx+cx^2}\left(bx+cx^2+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x+d)**2/(c*x**2+b*x+a)**(1/2),x)
[Out] Integral(1/(sqrt(a + b*x + c*x**2)*(b*x + c*x**2 + d)**2), x)
```

3.5 $\int \frac{1}{\sqrt{a+bx+cx^2}\left(d+bx+cx^2\right)^3} dx$

Optimal. Leaf size=224

$$-\frac{\left(16c^2\left(3a^2-8ad+8d^2\right)+8b^2c(a-4d)+3b^4\right)\tanh^{-1}\left(\frac{\sqrt{a-d}\,(b+2cx)}{\sqrt{b^2-4cd}\,\sqrt{a+bx+cx^2}}\right)}{4(a-d)^{5/2}\left(b^2-4cd\right)^{5/2}}+\frac{3(b+2cx)\left(4c(a-2d)+b^2\right)\sqrt{a+b}}{4(a-d)^2\left(b^2-4cd\right)^2\left(bx+cx^2+\right.}$$

Rubi [A] time = 0.43, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {974, 1060, 12, 982, 208}

$$-\frac{\left(16c^2\left(3a^2-8ad+8d^2\right)+8b^2c(a-4d)+3b^4\right)\tanh^{-1}\left(\frac{\sqrt{a-d}\,(b+2cx)}{\sqrt{b^2-4cd}\,\sqrt{a+bx+cx^2}}\right)}{4(a-d)^{5/2}\left(b^2-4cd\right)^{5/2}}+\frac{3(b+2cx)\left(4c(a-2d)+b^2\right)\sqrt{a+bx+cx^2}}{4(a-d)^2\left(b^2-4cd\right)^2\left(bx+cx^2+d\right)}-\frac{(b+2cx)\sqrt{a+bx+cx^2}}{2(a-d)\left(b^2-4cd\right)\left(bx+cx^2+d\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x + c*x^2]*(d + b*x + c*x^2)^3),x]
```

```
[Out] -((b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(2*(a - d)*(b^2 - 4*c*d)*(d + b*x + c*
x^2)^2) + (3*(b^2 + 4*c*(a - 2*d))*(b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(4*(a
 - d)^2*(b^2 - 4*c*d)^2*(d + b*x + c*x^2)) - ((3*b^4 + 8*b^2*c*(a - 4*d) +
16*c^2*(3*a^2 - 8*a*d + 8*d^2))*ArcTanh[(Sqrt[a - d]*(b + 2*c*x))/(Sqrt[b^2
 - 4*c*d]*Sqrt[a + b*x + c*x^2])])/(4*(a - d)^(5/2)*(b^2 - 4*c*d)^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 974

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*
a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*(
d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
```

```
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q,
 0]
```

Rule 982

```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(
x_)^2]), x_Symbol] :> Dist[-2*e, Subst[Int[1/(e*(b*e - 4*a*f) - (b*d - a*e)
*x^2), x], x, (e + 2*f*x)/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0
]
```

Rule 1060

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x +
 c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
2*a*(c*d - a*f)))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !
IGtQ[q, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx+cx^2}\left(d+bx+cx^2\right)^3}\,dx = -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{2(a-d)\left(b^2-4cd\right)\left(d+bx+cx^2\right)^2} + \frac{\int \frac{-\frac{1}{2}c^2(a-d)\left(3b^2+12ac-16cd\right)-4bc^3(a-d)x}{\sqrt{a+bx+cx^2}\left(d+bx+cx^2\right)^2}}{2c^2(a-d)^2\left(b^2-4cd\right.}$$

$$= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{2(a-d)\left(b^2-4cd\right)\left(d+bx+cx^2\right)^2} + \frac{3\left(b^2+4c(a-2d)\right)(b+2cx)\sqrt{a+}}{4(a-d)^2\left(b^2-4cd\right)^2\left(d+bx\right.}$$

$$= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{2(a-d)\left(b^2-4cd\right)\left(d+bx+cx^2\right)^2} + \frac{3\left(b^2+4c(a-2d)\right)(b+2cx)\sqrt{a+}}{4(a-d)^2\left(b^2-4cd\right)^2\left(d+bx\right.}$$

$$= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{2(a-d)\left(b^2-4cd\right)\left(d+bx+cx^2\right)^2} + \frac{3\left(b^2+4c(a-2d)\right)(b+2cx)\sqrt{a+}}{4(a-d)^2\left(b^2-4cd\right)^2\left(d+bx\right.}$$

$$= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{2(a-d)\left(b^2-4cd\right)\left(d+bx+cx^2\right)^2} + \frac{3\left(b^2+4c(a-2d)\right)(b+2cx)\sqrt{a+}}{4(a-d)^2\left(b^2-4cd\right)^2\left(d+bx\right.}$$

Mathematica [B] time = 6.38, size = 1746, normalized size = 7.79

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x + c*x^2]*(d + b*x + c*x^2)^3),x]
```

```
[Out] (-2*c^2*(a + b*x + c*x^2))/((a - d)*(b^2 - 4*c*d)^(3/2)*(b - Sqrt[b^2 - 4*c
*d] + 2*c*x)^2*Sqrt[a + x*(b + c*x)]) + (6*c^2*(a + b*x + c*x^2))/((a - d)*
(b^2 - 4*c*d)^2*(b - Sqrt[b^2 - 4*c*d] + 2*c*x)*Sqrt[a + x*(b + c*x)]) + (2
*c^2*(a + b*x + c*x^2))/((a - d)*(b^2 - 4*c*d)^(3/2)*(b + Sqrt[b^2 - 4*c*d]
 + 2*c*x)^2*Sqrt[a + x*(b + c*x)]) + (6*c^2*(a + b*x + c*x^2))/((a - d)*(b^
2 - 4*c*d)^2*(b + Sqrt[b^2 - 4*c*d] + 2*c*x)*Sqrt[a + x*(b + c*x)]) + (6*c^
2*Sqrt[a + b*x + c*x^2]*ArcTanh[(b^2 - 4*a*c - b*Sqrt[b^2 - 4*c*d] - 2*c*Sq
rt[b^2 - 4*c*d]*x)/(4*c*Sqrt[a - d]*Sqrt[a + b*x + c*x^2])]])/(Sqrt[a - d]*(
b^2 - 4*c*d)^(5/2)*Sqrt[a + x*(b + c*x)]) + (3*c*Sqrt[a + b*x + c*x^2]*ArcT
anh[(b^2 - 4*a*c - b*Sqrt[b^2 - 4*c*d] - 2*c*Sqrt[b^2 - 4*c*d]*x)/(4*c*Sqrt
[a - d]*Sqrt[a + b*x + c*x^2])]])/(2*(a - d)^(3/2)*(b^2 - 4*c*d)^(3/2)*Sqrt[
a + x*(b + c*x)]) + (6*c^2*Sqrt[a + b*x + c*x^2]*ArcTanh[(4*a*c - b*(b + Sq
rt[b^2 - 4*c*d]) - 2*c*Sqrt[b^2 - 4*c*d]*x)/(4*c*Sqrt[a - d]*Sqrt[a + b*x +
 c*x^2])]])/(Sqrt[a - d]*(b^2 - 4*c*d)^(5/2)*Sqrt[a + x*(b + c*x)]) + (3*c*S
qrt[a + b*x + c*x^2]*ArcTanh[(4*a*c - b*(b + Sqrt[b^2 - 4*c*d]) - 2*c*Sqrt[
```

```
b^2 - 4*c*d]*x)/(4*c*Sqrt[a - d]*Sqrt[a + b*x + c*x^2])])/(2*(a - d)^(3/2)*
(b^2 - 4*c*d)^(3/2)*Sqrt[a + x*(b + c*x)]) + (4*c^3*Sqrt[a + b*x + c*x^2]*(
((-2*c^2*(-b + Sqrt[b^2 - 4*c*d]) - 2*c^2*(b + 2*Sqrt[b^2 - 4*c*d]))*Sqrt[a
 + b*x + c*x^2])/((4*a*c^2 + 2*b*c*(-b + Sqrt[b^2 - 4*c*d]) + c*(-b + Sqrt[
b^2 - 4*c*d])^2)*(-b + Sqrt[b^2 - 4*c*d] - 2*c*x)) + (4*c*Sqrt[a - d]*(b*(-
2*c^2*(-b + Sqrt[b^2 - 4*c*d]) + 2*c^2*(b + 2*Sqrt[b^2 - 4*c*d])) - 2*(4*a*
c^3 - c^2*(-b + Sqrt[b^2 - 4*c*d])*(b + 2*Sqrt[b^2 - 4*c*d])))*ArcTanh[(-4*
a*c - b*(-b + Sqrt[b^2 - 4*c*d]) - (2*b*c + 2*c*(-b + Sqrt[b^2 - 4*c*d]))*x
)/(4*c*Sqrt[a - d]*Sqrt[a + b*x + c*x^2])])/((4*a*c^2 + 2*b*c*(-b + Sqrt[b^
2 - 4*c*d]) + c*(-b + Sqrt[b^2 - 4*c*d])^2)*(16*a*c^2 + 8*b*c*(-b + Sqrt[b^
2 - 4*c*d]) + 4*c*(-b + Sqrt[b^2 - 4*c*d])^2))))/((b^2 - 4*c*d)^(3/2)*(4*a*
c^2 + 2*b*c*(-b + Sqrt[b^2 - 4*c*d]) + c*(-b + Sqrt[b^2 - 4*c*d])^2)*Sqrt[a
 + x*(b + c*x)]) + (4*c^3*Sqrt[a + b*x + c*x^2]*(((-2*c^2*(b - 2*Sqrt[b^2 -
 4*c*d]) + 2*c^2*(b + Sqrt[b^2 - 4*c*d]))*Sqrt[a + b*x + c*x^2])/((4*a*c^2
- 2*b*c*(b + Sqrt[b^2 - 4*c*d]) + c*(b + Sqrt[b^2 - 4*c*d])^2)*(b + Sqrt[b^
2 - 4*c*d] + 2*c*x)) + (4*c*Sqrt[a - d]*(b*(2*c^2*(b - 2*Sqrt[b^2 - 4*c*d])
 + 2*c^2*(b + Sqrt[b^2 - 4*c*d])) - 2*(4*a*c^3 + c^2*(b - 2*Sqrt[b^2 - 4*c*
d])*(b + Sqrt[b^2 - 4*c*d])))*ArcTanh[(4*a*c - b*(b + Sqrt[b^2 - 4*c*d]) -
(-2*b*c + 2*c*(b + Sqrt[b^2 - 4*c*d]))*x)/(4*c*Sqrt[a - d]*Sqrt[a + b*x + c
*x^2])])/((4*a*c^2 - 2*b*c*(b + Sqrt[b^2 - 4*c*d]) + c*(b + Sqrt[b^2 - 4*c*
d])^2)*(16*a*c^2 - 8*b*c*(b + Sqrt[b^2 - 4*c*d]) + 4*c*(b + Sqrt[b^2 - 4*c*
d])^2))))/((b^2 - 4*c*d)^(3/2)*(4*a*c^2 - 2*b*c*(b + Sqrt[b^2 - 4*c*d]) + c
*(b + Sqrt[b^2 - 4*c*d])^2)*Sqrt[a + x*(b + c*x)])
```

IntegrateAlgebraic [F] time = 180.01, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(Sqrt[a + b*x + c*x^2]*(d + b*x + c*x^2)^3),x]
```

```
[Out] $Aborted
```

fricas [B] time = 4.82, size = 3818, normalized size = 17.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+d)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*((128*c^2*d^4 + (3*b^4*c^2 + 8*a*b^2*c^3 + 48*a^2*c^4 + 128*c^4*d^2 -
 32*(b^2*c^3 + 4*a*c^4)*d)*x^4 - 32*(b^2*c + 4*a*c^2)*d^3 + 2*(3*b^5*c + 8*
a*b^3*c^2 + 48*a^2*b*c^3 + 128*b*c^3*d^2 - 32*(b^3*c^2 + 4*a*b*c^3)*d)*x^3
+ (3*b^4 + 8*a*b^2*c + 48*a^2*c^2)*d^2 + (3*b^6 + 8*a*b^4*c + 48*a^2*b^2*c^
2 + 256*c^3*d^3 + 64*(b^2*c^2 - 4*a*c^3)*d^2 - 2*(13*b^4*c + 56*a*b^2*c^2 -
 48*a^2*c^3)*d)*x^2 + 2*(128*b*c^2*d^3 - 32*(b^3*c + 4*a*b*c^2)*d^2 + (3*b^
```

```
5 + 8*a*b^3*c + 48*a^2*b*c^2)*d)*x)*sqrt(a*b^2 + 4*c*d^2 - (b^2 + 4*a*c)*d)
*log((8*a^2*b^4 + (b^4*c^2 + 24*a*b^2*c^3 + 16*a^2*c^4 + 128*c^4*d^2 - 32*(
b^2*c^3 + 4*a*c^4)*d)*x^4 + 2*(b^5*c + 24*a*b^3*c^2 + 16*a^2*b*c^3 + 128*b*
c^3*d^2 - 32*(b^3*c^2 + 4*a*b*c^3)*d)*x^3 + (b^4 + 24*a*b^2*c + 16*a^2*c^2)
*d^2 + (b^6 + 32*a*b^4*c + 48*a^2*b^2*c^2 + 32*(5*b^2*c^2 + 4*a*c^3)*d^2 -
2*(19*b^4*c + 104*a*b^2*c^2 + 48*a^2*c^3)*d)*x^2 - 4*(2*a*b^3 + 2*(b^2*c^2
+ 4*a*c^3 - 8*c^3*d)*x^3 + 3*(b^3*c + 4*a*b*c^2 - 8*b*c^2*d)*x^2 - (b^3 + 4
*a*b*c)*d + (b^4 + 8*a*b^2*c - 2*(5*b^2*c + 4*a*c^2)*d)*x)*sqrt(a*b^2 + 4*c
*d^2 - (b^2 + 4*a*c)*d)*sqrt(c*x^2 + b*x + a) - 8*(a*b^4 + 4*a^2*b^2*c)*d +
 2*(4*a*b^5 + 16*a^2*b^3*c + 16*(b^3*c + 4*a*b*c^2)*d^2 - (3*b^5 + 40*a*b^3
*c + 48*a^2*b*c^2)*d)*x)/(c^2*x^4 + 2*b*c*x^3 + 2*b*d*x + (b^2 + 2*c*d)*x^2
 + d^2)) - 4*(2*a^2*b^5 + 128*b*c^2*d^4 - 52*(b^3*c + 4*a*b*c^2)*d^3 - 6*(a
*b^4*c^2 + 4*a^2*b^2*c^3 - 32*c^4*d^3 + 12*(b^2*c^3 + 4*a*c^4)*d^2 - (b^4*c
^2 + 16*a*b^2*c^3 + 16*a^2*c^4)*d)*x^3 + 5*(b^5 + 16*a*b^3*c + 16*a^2*b*c^2
)*d^2 - 9*(a*b^5*c + 4*a^2*b^3*c^2 - 32*b*c^3*d^3 + 12*(b^3*c^2 + 4*a*b*c^3
)*d^2 - (b^5*c + 16*a*b^3*c^2 + 16*a^2*b*c^3)*d)*x^2 - 7*(a*b^5 + 4*a^2*b^3
*c)*d - (3*a*b^6 + 8*a^2*b^4*c - 256*c^3*d^4 + 8*(b^2*c^2 + 52*a*c^3)*d^3 +
 2*(13*b^4*c - 8*a*b^2*c^2 - 80*a^2*c^3)*d^2 - (3*b^6 + 34*a*b^4*c - 8*a^2*
b^2*c^2)*d)*x)*sqrt(c*x^2 + b*x + a))/(a^3*b^6*d^2 + 64*c^3*d^8 - 48*(b^2*c
^2 + 4*a*c^3)*d^7 + 12*(b^4*c + 12*a*b^2*c^2 + 16*a^2*c^3)*d^6 - (b^6 + 36*
a*b^4*c + 144*a^2*b^2*c^2 + 64*a^3*c^3)*d^5 + 3*(a*b^6 + 12*a^2*b^4*c + 16*
a^3*b^2*c^2)*d^4 + (a^3*b^6*c^2 + 64*c^5*d^6 - 48*(b^2*c^4 + 4*a*c^5)*d^5 +
 12*(b^4*c^3 + 12*a*b^2*c^4 + 16*a^2*c^5)*d^4 - (b^6*c^2 + 36*a*b^4*c^3 + 1
44*a^2*b^2*c^4 + 64*a^3*c^5)*d^3 + 3*(a*b^6*c^2 + 12*a^2*b^4*c^3 + 16*a^3*b
^2*c^4)*d^2 - 3*(a^2*b^6*c^2 + 4*a^3*b^4*c^3)*d)*x^4 - 3*(a^2*b^6 + 4*a^3*b
^4*c)*d^3 + 2*(a^3*b^7*c + 64*b*c^4*d^6 - 48*(b^3*c^3 + 4*a*b*c^4)*d^5 + 12
*(b^5*c^2 + 12*a*b^3*c^3 + 16*a^2*b*c^4)*d^4 - (b^7*c + 36*a*b^5*c^2 + 144*
a^2*b^3*c^3 + 64*a^3*b*c^4)*d^3 + 3*(a*b^7*c + 12*a^2*b^5*c^2 + 16*a^3*b^3*
c^3)*d^2 - 3*(a^2*b^7*c + 4*a^3*b^5*c^2)*d)*x^3 + (a^3*b^8 + 128*c^4*d^7 -
32*(b^2*c^3 + 12*a*c^4)*d^6 - 24*(b^4*c^2 - 4*a*b^2*c^3 - 16*a^2*c^4)*d^5 +
 2*(5*b^6*c + 36*a*b^4*c^2 - 48*a^2*b^2*c^3 - 64*a^3*c^4)*d^4 - (b^8 + 30*a
*b^6*c + 72*a^2*b^4*c^2 - 32*a^3*b^2*c^3)*d^3 + 3*(a*b^8 + 10*a^2*b^6*c + 8
*a^3*b^4*c^2)*d^2 - (3*a^2*b^8 + 10*a^3*b^6*c)*d)*x^2 + 2*(a^3*b^7*d + 64*b
*c^3*d^7 - 48*(b^3*c^2 + 4*a*b*c^3)*d^6 + 12*(b^5*c + 12*a*b^3*c^2 + 16*a^2
*b*c^3)*d^5 - (b^7 + 36*a*b^5*c + 144*a^2*b^3*c^2 + 64*a^3*b*c^3)*d^4 + 3*(
a*b^7 + 12*a^2*b^5*c + 16*a^3*b^3*c^2)*d^3 - 3*(a^2*b^7 + 4*a^3*b^5*c)*d^2)
*x), -1/8*((128*c^2*d^4 + (3*b^4*c^2 + 8*a*b^2*c^3 + 48*a^2*c^4 + 128*c^4*d
^2 - 32*(b^2*c^3 + 4*a*c^4)*d)*x^4 - 32*(b^2*c + 4*a*c^2)*d^3 + 2*(3*b^5*c
+ 8*a*b^3*c^2 + 48*a^2*b*c^3 + 128*b*c^3*d^2 - 32*(b^3*c^2 + 4*a*b*c^3)*d)*
x^3 + (3*b^4 + 8*a*b^2*c + 48*a^2*c^2)*d^2 + (3*b^6 + 8*a*b^4*c + 48*a^2*b^
2*c^2 + 256*c^3*d^3 + 64*(b^2*c^2 - 4*a*c^3)*d^2 - 2*(13*b^4*c + 56*a*b^2*c
^2 - 48*a^2*c^3)*d)*x^2 + 2*(128*b*c^2*d^3 - 32*(b^3*c + 4*a*b*c^2)*d^2 + (
3*b^5 + 8*a*b^3*c + 48*a^2*b*c^2)*d)*x)*sqrt(-a*b^2 - 4*c*d^2 + (b^2 + 4*a*
c)*d)*arctan(-1/2*(2*a*b^2 + (b^2*c + 4*a*c^2 - 8*c^2*d)*x^2 - (b^2 + 4*a*c
)*d + (b^3 + 4*a*b*c - 8*b*c*d)*x)*sqrt(-a*b^2 - 4*c*d^2 + (b^2 + 4*a*c)*d)
```

```
*sqrt(c*x^2 + b*x + a)/(a^2*b^3 + 4*a*b*c*d^2 + 2*(a*b^2*c^2 + 4*c^3*d^2 -
(b^2*c^2 + 4*a*c^3)*d)*x^3 + 3*(a*b^3*c + 4*b*c^2*d^2 - (b^3*c + 4*a*b*c^2)
*d)*x^2 - (a*b^3 + 4*a^2*b*c)*d + (a*b^4 + 2*a^2*b^2*c + 4*(b^2*c + 2*a*c^2
)*d^2 - (b^4 + 6*a*b^2*c + 8*a^2*c^2)*d)*x)) + 2*(2*a^2*b^5 + 128*b*c^2*d^4
 - 52*(b^3*c + 4*a*b*c^2)*d^3 - 6*(a*b^4*c^2 + 4*a^2*b^2*c^3 - 32*c^4*d^3 +
 12*(b^2*c^3 + 4*a*c^4)*d^2 - (b^4*c^2 + 16*a*b^2*c^3 + 16*a^2*c^4)*d)*x^3
+ 5*(b^5 + 16*a*b^3*c + 16*a^2*b*c^2)*d^2 - 9*(a*b^5*c + 4*a^2*b^3*c^2 - 32
*b*c^3*d^3 + 12*(b^3*c^2 + 4*a*b*c^3)*d^2 - (b^5*c + 16*a*b^3*c^2 + 16*a^2*
b*c^3)*d)*x^2 - 7*(a*b^5 + 4*a^2*b^3*c)*d - (3*a*b^6 + 8*a^2*b^4*c - 256*c^
3*d^4 + 8*(b^2*c^2 + 52*a*c^3)*d^3 + 2*(13*b^4*c - 8*a*b^2*c^2 - 80*a^2*c^3
)*d^2 - (3*b^6 + 34*a*b^4*c - 8*a^2*b^2*c^2)*d)*x)*sqrt(c*x^2 + b*x + a))/(
a^3*b^6*d^2 + 64*c^3*d^8 - 48*(b^2*c^2 + 4*a*c^3)*d^7 + 12*(b^4*c + 12*a*b^
2*c^2 + 16*a^2*c^3)*d^6 - (b^6 + 36*a*b^4*c + 144*a^2*b^2*c^2 + 64*a^3*c^3)
*d^5 + 3*(a*b^6 + 12*a^2*b^4*c + 16*a^3*b^2*c^2)*d^4 + (a^3*b^6*c^2 + 64*c^
5*d^6 - 48*(b^2*c^4 + 4*a*c^5)*d^5 + 12*(b^4*c^3 + 12*a*b^2*c^4 + 16*a^2*c^
5)*d^4 - (b^6*c^2 + 36*a*b^4*c^3 + 144*a^2*b^2*c^4 + 64*a^3*c^5)*d^3 + 3*(a
*b^6*c^2 + 12*a^2*b^4*c^3 + 16*a^3*b^2*c^4)*d^2 - 3*(a^2*b^6*c^2 + 4*a^3*b^
4*c^3)*d)*x^4 - 3*(a^2*b^6 + 4*a^3*b^4*c)*d^3 + 2*(a^3*b^7*c + 64*b*c^4*d^6
 - 48*(b^3*c^3 + 4*a*b*c^4)*d^5 + 12*(b^5*c^2 + 12*a*b^3*c^3 + 16*a^2*b*c^4
)*d^4 - (b^7*c + 36*a*b^5*c^2 + 144*a^2*b^3*c^3 + 64*a^3*b*c^4)*d^3 + 3*(a*
b^7*c + 12*a^2*b^5*c^2 + 16*a^3*b^3*c^3)*d^2 - 3*(a^2*b^7*c + 4*a^3*b^5*c^2
)*d)*x^3 + (a^3*b^8 + 128*c^4*d^7 - 32*(b^2*c^3 + 12*a*c^4)*d^6 - 24*(b^4*c
^2 - 4*a*b^2*c^3 - 16*a^2*c^4)*d^5 + 2*(5*b^6*c + 36*a*b^4*c^2 - 48*a^2*b^2
*c^3 - 64*a^3*c^4)*d^4 - (b^8 + 30*a*b^6*c + 72*a^2*b^4*c^2 - 32*a^3*b^2*c^
3)*d^3 + 3*(a*b^8 + 10*a^2*b^6*c + 8*a^3*b^4*c^2)*d^2 - (3*a^2*b^8 + 10*a^3
*b^6*c)*d)*x^2 + 2*(a^3*b^7*d + 64*b*c^3*d^7 - 48*(b^3*c^2 + 4*a*b*c^3)*d^6
 + 12*(b^5*c + 12*a*b^3*c^2 + 16*a^2*b*c^3)*d^5 - (b^7 + 36*a*b^5*c + 144*a
^2*b^3*c^2 + 64*a^3*b*c^3)*d^4 + 3*(a*b^7 + 12*a^2*b^5*c + 16*a^3*b^3*c^2)*
d^3 - 3*(a^2*b^7 + 4*a^3*b^5*c)*d^2)*x)]
```

giac [B] time = 4.87, size = 2986, normalized size = 13.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+d)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/8*((3*b^4 + 8*a*b^2*c + 48*a^2*c^2 - 32*b^2*c*d - 128*a*c^2*d + 128*c^2*
d^2)*log(abs(-(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^2*c - 4*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))^2*a*c^2 + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*c^
2*d - (sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^3*sqrt(c) - 4*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))*a*b*c^(3/2) + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b*c^
(3/2)*d - 3*a*b^2*c + 4*sqrt(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2)*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))^2*c^(3/2) + 4*a^2*c^2 + 2*b^2*c*d + 4*sqrt(a*b^2 -
 b^2*d - 4*a*c*d + 4*c*d^2)*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b*c + sqrt(
```

a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2)*b^2*sqrt(c)))/sqrt(a*b^2 - b^2*d - 4*a*c
*d + 4*c*d^2) - (3*b^4 + 8*a*b^2*c + 48*a^2*c^2 - 32*b^2*c*d - 128*a*c^2*d
+ 128*c^2*d^2)*log(abs(-(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^2*c - 4*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*c^2 + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))^2*c^2*d - (sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^3*sqrt(c) - 4*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))*a*b*c^(3/2) + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))*b*c^(3/2)*d - 3*a*b^2*c - 4*sqrt(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2)*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*c^(3/2) + 4*a^2*c^2 + 2*b^2*c*d - 4*sq
rt(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2)*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b
*c - sqrt(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2)*b^2*sqrt(c)))/sqrt(a*b^2 - b^2
*d - 4*a*c*d + 4*c*d^2))/(a^2*b^4 - 2*a*b^4*d - 8*a^2*b^2*c*d + b^4*d^2 + 1
6*a*b^2*c*d^2 + 16*a^2*c^2*d^2 - 8*b^2*c*d^3 - 32*a*c^2*d^3 + 16*c^2*d^4) -
1/4*(3*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*b^4*c^(3/2) + 8*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))^6*a*b^2*c^(5/2) + 48*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))^6*a^2*c^(7/2) - 32*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*b^2*c^(5/2)*d
- 128*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a*c^(7/2)*d + 128*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))^6*c^(7/2)*d^2 + 9*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))^5*b^5*c + 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b^3*c^2 + 144*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))^5*a^2*b*c^3 - 96*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))^5*b^3*c^2*d - 384*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b*c^3*d
+ 384*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b*c^3*d^2 + 9*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^4*b^6*sqrt(c) + 15*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4
*a*b^4*c^(3/2) + 120*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^2*b^2*c^(5/2)
- 144*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^3*c^(7/2) - 78*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^4*b^4*c^(3/2)*d - 240*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))^4*a*b^2*c^(5/2)*d + 672*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^2*c^(7
/2)*d + 192*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b^2*c^(5/2)*d^2 - 1152*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*c^(7/2)*d^2 + 768*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))^4*c^(7/2)*d^3 + 3*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^
7 - 10*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^5*c - 288*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))^3*a^3*b*c^3 + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b
^5*c*d + 160*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^3*c^2*d + 1344*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*b*c^3*d - 256*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^3*b^3*c^2*d^2 - 2304*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b*
c^3*d^2 + 1536*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b*c^3*d^3 - 14*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))^2*a*b^6*sqrt(c) - 71*(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))^2*a^2*b^4*c^(3/2) - 200*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^
3*b^2*c^(5/2) + 144*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^4*c^(7/2) + 23*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^6*sqrt(c)*d + 280*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^2*a*b^4*c^(3/2)*d + 1168*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))^2*a^2*b^2*c^(5/2)*d - 640*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^3*c^
(7/2)*d - 272*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^4*c^(3/2)*d^2 - 2048*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b^2*c^(5/2)*d^2 + 640*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))^2*a^2*c^(7/2)*d^2 + 1152*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))^2*b^2*c^(5/2)*d^3 - 5*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^7 -

```
47*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b^5*c - 56*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))*a^3*b^3*c^2 + 144*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^4*b*
c^3 + 5*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^7*d + 136*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))*a*b^5*c*d + 496*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b^
3*c^2*d - 640*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^3*b*c^3*d - 80*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))*b^5*c*d^2 - 896*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))*a*b^3*c^2*d^2 + 640*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b*c^3*d^2
+ 384*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^3*c^2*d^3 - 11*a^2*b^6*sqrt(c)
- 11*a^3*b^4*c^(3/2) + 72*a^4*b^2*c^(5/2) - 48*a^5*c^(7/2) + 17*a*b^6*sqrt(
c)*d + 118*a^2*b^4*c^(3/2)*d - 256*a^3*b^2*c^(5/2)*d + 96*a^4*c^(7/2)*d - 6
*b^6*sqrt(c)*d^2 - 152*a*b^4*c^(3/2)*d^2 + 160*a^2*b^2*c^(5/2)*d^2 + 48*b^4
*c^(3/2)*d^3)/((a^2*b^4 - 2*a*b^4*d - 8*a^2*b^2*c*d + b^4*d^2 + 16*a*b^2*c*
d^2 + 16*a^2*c^2*d^2 - 8*b^2*c*d^3 - 32*a*c^2*d^3 + 16*c^2*d^4)*((sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))^4*c + 2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b*s
qrt(c) + (sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^2 - 2*(sqrt(c)*x - sqrt(c*
x^2 + b*x + a))^2*a*c + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*c*d - 2*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))*a*b*sqrt(c) + 4*(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))*b*sqrt(c)*d - a*b^2 + a^2*c + b^2*d)^2)
```

maple [B] time = 0.03, size = 1884, normalized size = 8.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+b*x+d)^3/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] -1/2/(b^2-4*c*d)^(3/2)/(a-d)/(x+1/2*b/c-1/2*(b^2-4*c*d)^(1/2)/c)^2*(a+(x-1/
2*(-b+(b^2-4*c*d)^(1/2))/c)^2*c-d+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^
(1/2))/c))^(1/2)+3/4/(b^2-4*c*d)/(a-d)^2/(x+1/2*b/c-1/2*(b^2-4*c*d)^(1/2)/c
)*(a+(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)^2*c-d+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b
^2-4*c*d)^(1/2))/c))^(1/2)-3/8/(b^2-4*c*d)^(3/2)/(a-d)^(5/2)*ln((2*a-2*d+(b
^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)+2*(a-d)^(1/2)*(a+(x-1/2*(-
b+(b^2-4*c*d)^(1/2))/c)^2*c-d+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2
))/c))^(1/2))/(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c))*b^2+3/2/(b^2-4*c*d)^(3/2)/(
a-d)^(5/2)*ln((2*a-2*d+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)+2
*(a-d)^(1/2)*(a+(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)^2*c-d+(b^2-4*c*d)^(1/2)*(x
-1/2*(-b+(b^2-4*c*d)^(1/2))/c))^(1/2))/(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c))*c*
d-1/(b^2-4*c*d)^(3/2)*c/(a-d)^(3/2)*ln((2*a-2*d+(b^2-4*c*d)^(1/2)*(x-1/2*(-
b+(b^2-4*c*d)^(1/2))/c)+2*(a-d)^(1/2)*(a+(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)^2
*c-d+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c))^(1/2))/(x-1/2*(-b+
(b^2-4*c*d)^(1/2))/c))+1/2/(b^2-4*c*d)^(3/2)/(a-d)/(x+1/2*b/c+1/2*(b^2-4*c*
d)^(1/2)/c)^2*(a+(x+1/2*(b+(b^2-4*c*d)^(1/2))/c)^2*c-d-(b^2-4*c*d)^(1/2)*(x
+1/2*(b+(b^2-4*c*d)^(1/2))/c))^(1/2)+3/4/(b^2-4*c*d)/(a-d)^2/(x+1/2*b/c+1/2
*(b^2-4*c*d)^(1/2)/c)*(a+(x+1/2*(b+(b^2-4*c*d)^(1/2))/c)^2*c-d-(b^2-4*c*d)^
(1/2)*(x+1/2*(b+(b^2-4*c*d)^(1/2))/c))^(1/2)+3/8/(b^2-4*c*d)^(3/2)/(a-d)^(5
```

```
/2)*ln((2*a-2*d-(b^2-4*c*d)^(1/2)*(x+1/2*(b+(b^2-4*c*d)^(1/2))/c)+2*(a-d)^(
1/2)*(a+(x+1/2*(b+(b^2-4*c*d)^(1/2))/c)^2*c-d-(b^2-4*c*d)^(1/2)*(x+1/2*(b+(
b^2-4*c*d)^(1/2))/c))^(1/2))/(x+1/2*(b+(b^2-4*c*d)^(1/2))/c))*b^2-3/2/(b^2-
4*c*d)^(3/2)/(a-d)^(5/2)*ln((2*a-2*d-(b^2-4*c*d)^(1/2)*(x+1/2*(b+(b^2-4*c*d
)^(1/2))/c)+2*(a-d)^(1/2)*(a+(x+1/2*(b+(b^2-4*c*d)^(1/2))/c)^2*c-d-(b^2-4*c
*d)^(1/2)*(x+1/2*(b+(b^2-4*c*d)^(1/2))/c))^(1/2))/(x+1/2*(b+(b^2-4*c*d)^(1/
2))/c))*c*d+1/(b^2-4*c*d)^(3/2)*c/(a-d)^(3/2)*ln((2*a-2*d-(b^2-4*c*d)^(1/2)
*(x+1/2*(b+(b^2-4*c*d)^(1/2))/c)+2*(a-d)^(1/2)*(a+(x+1/2*(b+(b^2-4*c*d)^(1/
2))/c)^2*c-d-(b^2-4*c*d)^(1/2)*(x+1/2*(b+(b^2-4*c*d)^(1/2))/c))^(1/2))/(x+1
/2*(b+(b^2-4*c*d)^(1/2))/c))+6*c^2/(b^2-4*c*d)^(5/2)/(a-d)^(1/2)*ln((2*a-2*
d-(b^2-4*c*d)^(1/2)*(x+1/2*(b+(b^2-4*c*d)^(1/2))/c)+2*(a-d)^(1/2)*(a+(x+1/2
*(b+(b^2-4*c*d)^(1/2))/c)^2*c-d-(b^2-4*c*d)^(1/2)*(x+1/2*(b+(b^2-4*c*d)^(1/
2))/c))^(1/2))/(x+1/2*(b+(b^2-4*c*d)^(1/2))/c))-6*c^2/(b^2-4*c*d)^(5/2)/(a-
d)^(1/2)*ln((2*a-2*d+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)+2*(
a-d)^(1/2)*(a+(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)^2*c-d+(b^2-4*c*d)^(1/2)*(x-1
/2*(-b+(b^2-4*c*d)^(1/2))/c))^(1/2))/(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c))+3/(b
^2-4*c*d)^2*c/(a-d)/(x+1/2*b/c-1/2*(b^2-4*c*d)^(1/2)/c)*(a+(x-1/2*(-b+(b^2-
4*c*d)^(1/2))/c)^2*c-d+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c))^
(1/2)+3/(b^2-4*c*d)^2*c/(a-d)/(x+1/2*b/c+1/2*(b^2-4*c*d)^(1/2)/c)*(a+(x+1/2
*(b+(b^2-4*c*d)^(1/2))/c)^2*c-d-(b^2-4*c*d)^(1/2)*(x+1/2*(b+(b^2-4*c*d)^(1/
2))/c))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^2 + bx + a}\left(cx^2 + bx + d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+d)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*(c*x^2 + b*x + d)^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{c\,x^2 + b\,x + a}\,\left(c\,x^2 + b\,x + d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x + c*x^2)^(1/2)*(d + b*x + c*x^2)^3),x)
[Out] int(1/((a + b*x + c*x^2)^(1/2)*(d + b*x + c*x^2)^3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x+d)**3/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Timed out
```

3.6 $\int \frac{1}{\sqrt{a+bx+cx^2}\left(d+bx+cx^2\right)^4}\,dx$

Optimal. Leaf size=328

$$-\frac{(b+2cx)\left(16c^2\left(15a^2-44ad+44d^2\right)+8b^2c(7a-22d)+15b^4\right)\sqrt{a+bx+cx^2}}{24(a-d)^3\left(b^2-4cd\right)^3\left(bx+cx^2+d\right)}+\frac{\left(4c(a-2d)+b^2\right)\left(16c^2\left(5a^2-8\right.\right.}{}$$

Rubi [A] time = 0.97, antiderivative size = 328, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {974, 1060, 12, 982, 208}

$$-\frac{(b+2cx)\left(16c^2\left(15a^2-44ad+44d^2\right)+8b^2c(7a-22d)+15b^4\right)\sqrt{a+bx+cx^2}}{24(a-d)^3\left(b^2-4cd\right)^3\left(bx+cx^2+d\right)}+\frac{\left(4c(a-2d)+b^2\right)\left(16c^2\left(5a^2-8ad+8d^2\right)-8b^2c(a+4d)+5b^4\right)\tanh^{-1}\left(\frac{\sqrt{a-d}\,(b+2cx)}{\sqrt{b^2-4cd}\,\sqrt{a+bx+cx^2}}\right)}{8(a-d)^{7/2}\left(b^2-4cd\right)^{7/2}}+\frac{5(b+2cx)\left(4c(a-2d)+b^2\right)\sqrt{a+bx+cx^2}}{12(a-d)^2\left(b^2-4cd\right)^2\left(bx+cx^2+d\right)^2}-\frac{(b+2cx)\sqrt{a+bx+cx^2}}{3(a-d)\left(b^2-4cd\right)\left(bx+cx^2+d\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x + c*x^2]*(d + b*x + c*x^2)^4),x]
```

```
[Out] -((b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(3*(a - d)*(b^2 - 4*c*d)*(d + b*x + c*
x^2)^3) + (5*(b^2 + 4*c*(a - 2*d))*(b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(12*(
a - d)^2*(b^2 - 4*c*d)^2*(d + b*x + c*x^2)^2) - ((15*b^4 + 8*b^2*c*(7*a - 2
2*d) + 16*c^2*(15*a^2 - 44*a*d + 44*d^2))*(b + 2*c*x)*Sqrt[a + b*x + c*x^2]
)/(24*(a - d)^3*(b^2 - 4*c*d)^3*(d + b*x + c*x^2)) + ((b^2 + 4*c*(a - 2*d))
*(5*b^4 - 8*b^2*c*(a + 4*d) + 16*c^2*(5*a^2 - 8*a*d + 8*d^2))*ArcTanh[(Sqrt
[a - d]*(b + 2*c*x))/(Sqrt[b^2 - 4*c*d]*Sqrt[a + b*x + c*x^2])])/(8*(a - d)
^(7/2)*(b^2 - 4*c*d)^(7/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 974

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*
a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*(
d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
```

```
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q,
 0]
```

Rule 982

```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(
x_)^2]), x_Symbol] :> Dist[-2*e, Subst[Int[1/(e*(b*e - 4*a*f) - (b*d - a*e)
*x^2), x], x, (e + 2*f*x)/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0
]
```

Rule 1060

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x +
 c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
2*a*(c*d - a*f)))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !
IGtQ[q, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx+cx^2}\left(d+bx+cx^2\right)^4}\,dx = -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{3(a-d)\left(b^2-4cd\right)\left(d+bx+cx^2\right)^3} + \frac{\int \frac{-\frac{1}{2}c^2(a-d)\left(5b^2+20ac-24cd\right)-8bc^3(a-d)x}{\sqrt{a+bx+cx^2}\left(d+bx+cx^2\right)^3}}{3c^2(a-d)^2\left(b^2-4cd\right)}$$

$$= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{3(a-d)\left(b^2-4cd\right)\left(d+bx+cx^2\right)^3} + \frac{5\left(b^2+4c(a-2d)\right)(b+2cx)\sqrt{a+}}{12(a-d)^2\left(b^2-4cd\right)^2\left(d+bx\right.}$$

$$= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{3(a-d)\left(b^2-4cd\right)\left(d+bx+cx^2\right)^3} + \frac{5\left(b^2+4c(a-2d)\right)(b+2cx)\sqrt{a+}}{12(a-d)^2\left(b^2-4cd\right)^2\left(d+bx\right.}$$

$$= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{3(a-d)\left(b^2-4cd\right)\left(d+bx+cx^2\right)^3} + \frac{5\left(b^2+4c(a-2d)\right)(b+2cx)\sqrt{a+}}{12(a-d)^2\left(b^2-4cd\right)^2\left(d+bx\right.}$$

$$= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{3(a-d)\left(b^2-4cd\right)\left(d+bx+cx^2\right)^3} + \frac{5\left(b^2+4c(a-2d)\right)(b+2cx)\sqrt{a+}}{12(a-d)^2\left(b^2-4cd\right)^2\left(d+bx\right.}$$

$$= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{3(a-d)\left(b^2-4cd\right)\left(d+bx+cx^2\right)^3} + \frac{5\left(b^2+4c(a-2d)\right)(b+2cx)\sqrt{a+}}{12(a-d)^2\left(b^2-4cd\right)^2\left(d+bx\right.}$$

Mathematica [B] time = 6.61, size = 3382, normalized size = 10.31

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[a + b*x + c*x^2]*(d + b*x + c*x^2)^4),x]
```

```
[Out] (-8*c^3*(a + b*x + c*x^2))/(3*(a - d)*(b^2 - 4*c*d)^2*(b - Sqrt[b^2 - 4*c*d
] + 2*c*x)^3*Sqrt[a + x*(b + c*x)]) + (8*c^3*(a + b*x + c*x^2))/((a - d)*(b
^2 - 4*c*d)^(5/2)*(b - Sqrt[b^2 - 4*c*d] + 2*c*x)^2*Sqrt[a + x*(b + c*x)])
- (20*c^3*(a + b*x + c*x^2))/((a - d)*(b^2 - 4*c*d)^3*(b - Sqrt[b^2 - 4*c*d
] + 2*c*x)*Sqrt[a + x*(b + c*x)]) - (8*c^3*(a + b*x + c*x^2))/(3*(a - d)*(b
^2 - 4*c*d)^2*(b + Sqrt[b^2 - 4*c*d] + 2*c*x)^3*Sqrt[a + x*(b + c*x)]) - (8
*c^3*(a + b*x + c*x^2))/((a - d)*(b^2 - 4*c*d)^(5/2)*(b + Sqrt[b^2 - 4*c*d]
 + 2*c*x)^2*Sqrt[a + x*(b + c*x)]) - (20*c^3*(a + b*x + c*x^2))/((a - d)*(b
^2 - 4*c*d)^3*(b + Sqrt[b^2 - 4*c*d] + 2*c*x)*Sqrt[a + x*(b + c*x)]) - (20*
```

```
c^3*Sqrt[a + b*x + c*x^2]*ArcTanh[(b^2 - 4*a*c - b*Sqrt[b^2 - 4*c*d] - 2*c*
Sqrt[b^2 - 4*c*d]*x)/(4*c*Sqrt[a - d]*Sqrt[a + b*x + c*x^2])])/(Sqrt[a - d]
*(b^2 - 4*c*d)^(7/2)*Sqrt[a + x*(b + c*x)]) - (5*c^2*Sqrt[a + b*x + c*x^2]*
ArcTanh[(b^2 - 4*a*c - b*Sqrt[b^2 - 4*c*d] - 2*c*Sqrt[b^2 - 4*c*d]*x)/(4*c*
Sqrt[a - d]*Sqrt[a + b*x + c*x^2])])/((a - d)^(3/2)*(b^2 - 4*c*d)^(5/2)*Sqr
t[a + x*(b + c*x)]) - (20*c^3*Sqrt[a + b*x + c*x^2]*ArcTanh[(4*a*c - b*(b +
 Sqrt[b^2 - 4*c*d]) - 2*c*Sqrt[b^2 - 4*c*d]*x)/(4*c*Sqrt[a - d]*Sqrt[a + b*
x + c*x^2])])/(Sqrt[a - d]*(b^2 - 4*c*d)^(7/2)*Sqrt[a + x*(b + c*x)]) - (5*
c^2*Sqrt[a + b*x + c*x^2]*ArcTanh[(4*a*c - b*(b + Sqrt[b^2 - 4*c*d]) - 2*c*
Sqrt[b^2 - 4*c*d]*x)/(4*c*Sqrt[a - d]*Sqrt[a + b*x + c*x^2])])/((a - d)^(3/
2)*(b^2 - 4*c*d)^(5/2)*Sqrt[a + x*(b + c*x)]) - (16*c^4*Sqrt[a + b*x + c*x^
2]*(((-2*c^2*(-b + Sqrt[b^2 - 4*c*d]) - 2*c^2*(b + 2*Sqrt[b^2 - 4*c*d]))*Sq
rt[a + b*x + c*x^2])/((4*a*c^2 + 2*b*c*(-b + Sqrt[b^2 - 4*c*d]) + c*(-b + S
qrt[b^2 - 4*c*d])^2)*(-b + Sqrt[b^2 - 4*c*d] - 2*c*x)) + (4*c*Sqrt[a - d]*(
b*(-2*c^2*(-b + Sqrt[b^2 - 4*c*d]) + 2*c^2*(b + 2*Sqrt[b^2 - 4*c*d])) - 2*(
4*a*c^3 - c^2*(-b + Sqrt[b^2 - 4*c*d])*(b + 2*Sqrt[b^2 - 4*c*d])))*ArcTanh[
(-4*a*c - b*(-b + Sqrt[b^2 - 4*c*d]) - (2*b*c + 2*c*(-b + Sqrt[b^2 - 4*c*d]
))*x)/(4*c*Sqrt[a - d]*Sqrt[a + b*x + c*x^2])])/((4*a*c^2 + 2*b*c*(-b + Sqr
t[b^2 - 4*c*d]) + c*(-b + Sqrt[b^2 - 4*c*d])^2)*(16*a*c^2 + 8*b*c*(-b + Sqr
t[b^2 - 4*c*d]) + 4*c*(-b + Sqrt[b^2 - 4*c*d])^2))))/((b^2 - 4*c*d)^(5/2)*(
4*a*c^2 + 2*b*c*(-b + Sqrt[b^2 - 4*c*d]) + c*(-b + Sqrt[b^2 - 4*c*d])^2)*Sq
rt[a + x*(b + c*x)]) - (16*c^4*Sqrt[a + b*x + c*x^2]*(-1/2*((4*c^2*(-b + Sq
rt[b^2 - 4*c*d]) + 2*c^2*(2*b + 3*Sqrt[b^2 - 4*c*d]))*Sqrt[a + b*x + c*x^2]
)/((4*a*c^2 + 2*b*c*(-b + Sqrt[b^2 - 4*c*d]) + c*(-b + Sqrt[b^2 - 4*c*d])^2
)*(-b + Sqrt[b^2 - 4*c*d] - 2*c*x)^2) - (((10*c^3*Sqrt[b^2 - 4*c*d]*(-b + S
qrt[b^2 - 4*c*d]) + 2*c^3*(10*b^2 - 16*a*c - 24*c*d + 5*b*Sqrt[b^2 - 4*c*d]
))*Sqrt[a + b*x + c*x^2])/((4*a*c^2 + 2*b*c*(-b + Sqrt[b^2 - 4*c*d]) + c*(-
b + Sqrt[b^2 - 4*c*d])^2)*(-b + Sqrt[b^2 - 4*c*d] - 2*c*x)) + (4*c*Sqrt[a -
 d]*(b*(10*c^3*Sqrt[b^2 - 4*c*d]*(-b + Sqrt[b^2 - 4*c*d]) - 2*c^3*(10*b^2 -
 16*a*c - 24*c*d + 5*b*Sqrt[b^2 - 4*c*d])) - 2*(-20*a*c^4*Sqrt[b^2 - 4*c*d]
 + c^3*(-b + Sqrt[b^2 - 4*c*d])*(10*b^2 - 16*a*c - 24*c*d + 5*b*Sqrt[b^2 -
4*c*d])))*ArcTanh[(-4*a*c - b*(-b + Sqrt[b^2 - 4*c*d]) - (2*b*c + 2*c*(-b +
 Sqrt[b^2 - 4*c*d]))*x)/(4*c*Sqrt[a - d]*Sqrt[a + b*x + c*x^2])])/((4*a*c^2
 + 2*b*c*(-b + Sqrt[b^2 - 4*c*d]) + c*(-b + Sqrt[b^2 - 4*c*d])^2)*(16*a*c^2
 + 8*b*c*(-b + Sqrt[b^2 - 4*c*d]) + 4*c*(-b + Sqrt[b^2 - 4*c*d])^2)))/(2*(4
*a*c^2 + 2*b*c*(-b + Sqrt[b^2 - 4*c*d]) + c*(-b + Sqrt[b^2 - 4*c*d])^2))))/
(3*(b^2 - 4*c*d)^2*(4*a*c^2 + 2*b*c*(-b + Sqrt[b^2 - 4*c*d]) + c*(-b + Sqrt
[b^2 - 4*c*d])^2)*Sqrt[a + x*(b + c*x)]) - (16*c^4*Sqrt[a + b*x + c*x^2]*((
(-2*c^2*(b - 2*Sqrt[b^2 - 4*c*d]) + 2*c^2*(b + Sqrt[b^2 - 4*c*d]))*Sqrt[a +
 b*x + c*x^2])/((4*a*c^2 - 2*b*c*(b + Sqrt[b^2 - 4*c*d]) + c*(b + Sqrt[b^2
- 4*c*d])^2)*(b + Sqrt[b^2 - 4*c*d] + 2*c*x)) + (4*c*Sqrt[a - d]*(b*(2*c^2*
(b - 2*Sqrt[b^2 - 4*c*d]) + 2*c^2*(b + Sqrt[b^2 - 4*c*d])) - 2*(4*a*c^3 + c
^2*(b - 2*Sqrt[b^2 - 4*c*d])*(b + Sqrt[b^2 - 4*c*d])))*ArcTanh[(4*a*c - b*(
b + Sqrt[b^2 - 4*c*d]) - (-2*b*c + 2*c*(b + Sqrt[b^2 - 4*c*d]))*x)/(4*c*Sqr
t[a - d]*Sqrt[a + b*x + c*x^2])])/((4*a*c^2 - 2*b*c*(b + Sqrt[b^2 - 4*c*d])
```

```
 + c*(b + Sqrt[b^2 - 4*c*d])^2)*(16*a*c^2 - 8*b*c*(b + Sqrt[b^2 - 4*c*d]) +
 4*c*(b + Sqrt[b^2 - 4*c*d])^2))))/((b^2 - 4*c*d)^(5/2)*(4*a*c^2 - 2*b*c*(b
 + Sqrt[b^2 - 4*c*d]) + c*(b + Sqrt[b^2 - 4*c*d])^2)*Sqrt[a + x*(b + c*x)])
 - (16*c^4*Sqrt[a + b*x + c*x^2]*(-1/2*((2*c^2*(2*b - 3*Sqrt[b^2 - 4*c*d])
- 4*c^2*(b + Sqrt[b^2 - 4*c*d]))*Sqrt[a + b*x + c*x^2])/((4*a*c^2 - 2*b*c*(
b + Sqrt[b^2 - 4*c*d]) + c*(b + Sqrt[b^2 - 4*c*d])^2)*(b + Sqrt[b^2 - 4*c*d
] + 2*c*x)^2) - (((-10*c^3*Sqrt[b^2 - 4*c*d]*(b + Sqrt[b^2 - 4*c*d]) - 2*c^
3*(10*b^2 - 16*a*c - 24*c*d - 5*b*Sqrt[b^2 - 4*c*d]))*Sqrt[a + b*x + c*x^2]
)/((4*a*c^2 - 2*b*c*(b + Sqrt[b^2 - 4*c*d]) + c*(b + Sqrt[b^2 - 4*c*d])^2)*
(b + Sqrt[b^2 - 4*c*d] + 2*c*x)) + (4*c*Sqrt[a - d]*(b*(-10*c^3*Sqrt[b^2 -
4*c*d]*(b + Sqrt[b^2 - 4*c*d]) + 2*c^3*(10*b^2 - 16*a*c - 24*c*d - 5*b*Sqrt
[b^2 - 4*c*d])) - 2*(-20*a*c^4*Sqrt[b^2 - 4*c*d] + c^3*(b + Sqrt[b^2 - 4*c*
d])*(10*b^2 - 16*a*c - 24*c*d - 5*b*Sqrt[b^2 - 4*c*d])))*ArcTanh[(4*a*c - b
*(b + Sqrt[b^2 - 4*c*d]) - (-2*b*c + 2*c*(b + Sqrt[b^2 - 4*c*d]))*x)/(4*c*S
qrt[a - d]*Sqrt[a + b*x + c*x^2])]])/((4*a*c^2 - 2*b*c*(b + Sqrt[b^2 - 4*c*d
]) + c*(b + Sqrt[b^2 - 4*c*d])^2)*(16*a*c^2 - 8*b*c*(b + Sqrt[b^2 - 4*c*d])
 + 4*c*(b + Sqrt[b^2 - 4*c*d])^2)))/(2*(4*a*c^2 - 2*b*c*(b + Sqrt[b^2 - 4*c
*d]) + c*(b + Sqrt[b^2 - 4*c*d])^2))))/(3*(b^2 - 4*c*d)^2*(4*a*c^2 - 2*b*c*
(b + Sqrt[b^2 - 4*c*d]) + c*(b + Sqrt[b^2 - 4*c*d])^2)*Sqrt[a + x*(b + c*x)
])
```

IntegrateAlgebraic [F] time = 180.02, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(Sqrt[a + b*x + c*x^2]*(d + b*x + c*x^2)^4),x]
```

```
[Out] $Aborted
```

fricas [B] time = 20.45, size = 8134, normalized size = 24.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+d)^4/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/96*(3*(1024*c^3*d^6 - (5*b^6*c^3 + 12*a*b^4*c^4 + 48*a^2*b^2*c^5 + 320*a
^3*c^6 - 1024*c^6*d^3 + 384*(b^2*c^5 + 4*a*c^6)*d^2 - 24*(3*b^4*c^4 + 8*a*b
^2*c^5 + 48*a^2*c^6)*d)*x^6 - 384*(b^2*c^2 + 4*a*c^3)*d^5 - 3*(5*b^7*c^2 +
12*a*b^5*c^3 + 48*a^2*b^3*c^4 + 320*a^3*b*c^5 - 1024*b*c^5*d^3 + 384*(b^3*c
^4 + 4*a*b*c^5)*d^2 - 24*(3*b^5*c^3 + 8*a*b^3*c^4 + 48*a^2*b*c^5)*d)*x^5 +
24*(3*b^4*c + 8*a*b^2*c^2 + 48*a^2*c^3)*d^4 - 3*(5*b^8*c + 12*a*b^6*c^2 + 4
8*a^2*b^4*c^3 + 320*a^3*b^2*c^4 - 1024*c^5*d^4 - 128*(5*b^2*c^4 - 12*a*c^5)
*d^3 + 24*(13*b^4*c^3 + 56*a*b^2*c^4 - 48*a^2*c^5)*d^2 - (67*b^6*c^2 + 180*
a*b^4*c^3 + 1104*a^2*b^2*c^4 - 320*a^3*c^5)*d)*x^4 - (5*b^6 + 12*a*b^4*c +
```

```
48*a^2*b^2*c^2 + 320*a^3*c^3)*d^3 - (5*b^9 + 12*a*b^7*c + 48*a^2*b^5*c^2 +
320*a^3*b^3*c^3 - 6144*b*c^4*d^4 + 256*(5*b^3*c^3 + 36*a*b*c^4)*d^3 - 48*(b
^5*c^2 - 8*a*b^3*c^3 + 144*a^2*b*c^4)*d^2 - 6*(7*b^7*c + 20*a*b^5*c^2 + 144
*a^2*b^3*c^3 - 320*a^3*b*c^4)*d)*x^3 + 3*(1024*c^4*d^5 + 128*(5*b^2*c^3 - 1
2*a*c^4)*d^4 - 24*(13*b^4*c^2 + 56*a*b^2*c^3 - 48*a^2*c^4)*d^3 + (67*b^6*c
+ 180*a*b^4*c^2 + 1104*a^2*b^2*c^3 - 320*a^3*c^4)*d^2 - (5*b^8 + 12*a*b^6*c
 + 48*a^2*b^4*c^2 + 320*a^3*b^2*c^3)*d)*x^2 + 3*(1024*b*c^3*d^5 - 384*(b^3*
c^2 + 4*a*b*c^3)*d^4 + 24*(3*b^5*c + 8*a*b^3*c^2 + 48*a^2*b*c^3)*d^3 - (5*b
^7 + 12*a*b^5*c + 48*a^2*b^3*c^2 + 320*a^3*b*c^3)*d^2)*x)*sqrt(a*b^2 + 4*c*
d^2 - (b^2 + 4*a*c)*d)*log((8*a^2*b^4 + (b^4*c^2 + 24*a*b^2*c^3 + 16*a^2*c^
4 + 128*c^4*d^2 - 32*(b^2*c^3 + 4*a*c^4)*d)*x^4 + 2*(b^5*c + 24*a*b^3*c^2 +
 16*a^2*b*c^3 + 128*b*c^3*d^2 - 32*(b^3*c^2 + 4*a*b*c^3)*d)*x^3 + (b^4 + 24
*a*b^2*c + 16*a^2*c^2)*d^2 + (b^6 + 32*a*b^4*c + 48*a^2*b^2*c^2 + 32*(5*b^2
*c^2 + 4*a*c^3)*d^2 - 2*(19*b^4*c + 104*a*b^2*c^2 + 48*a^2*c^3)*d)*x^2 - 4*
(2*a*b^3 + 2*(b^2*c^2 + 4*a*c^3 - 8*c^3*d)*x^3 + 3*(b^3*c + 4*a*b*c^2 - 8*b
*c^2*d)*x^2 - (b^3 + 4*a*b*c)*d + (b^4 + 8*a*b^2*c - 2*(5*b^2*c + 4*a*c^2)*
d)*x)*sqrt(a*b^2 + 4*c*d^2 - (b^2 + 4*a*c)*d)*sqrt(c*x^2 + b*x + a) - 8*(a*
b^4 + 4*a^2*b^2*c)*d + 2*(4*a*b^5 + 16*a^2*b^3*c + 16*(b^3*c + 4*a*b*c^2)*d
^2 - (3*b^5 + 40*a*b^3*c + 48*a^2*b*c^2)*d)*x)/(c^2*x^4 + 2*b*c*x^3 + 2*b*d
*x + (b^2 + 2*c*d)*x^2 + d^2)) - 4*(8*a^3*b^7 + 4608*b*c^3*d^6 - 2592*(b^3*
c^2 + 4*a*b*c^3)*d^5 + 2*(15*a*b^6*c^3 + 56*a^2*b^4*c^4 + 240*a^3*b^2*c^5 +
 2816*c^6*d^4 - 1408*(b^2*c^5 + 4*a*c^6)*d^3 + 4*(59*b^4*c^4 + 584*a*b^2*c^
5 + 944*a^2*c^6)*d^2 - (15*b^6*c^3 + 292*a*b^4*c^4 + 1168*a^2*b^2*c^5 + 960
*a^3*c^6)*d)*x^5 + 4*(123*b^5*c + 1352*a*b^3*c^2 + 1968*a^2*b*c^3)*d^4 + 5*
(15*a*b^7*c^2 + 56*a^2*b^5*c^3 + 240*a^3*b^3*c^4 + 2816*b*c^5*d^4 - 1408*(b
^3*c^4 + 4*a*b*c^5)*d^3 + 4*(59*b^5*c^3 + 584*a*b^3*c^4 + 944*a^2*b*c^5)*d^
2 - (15*b^7*c^2 + 292*a*b^5*c^3 + 1168*a^2*b^3*c^4 + 960*a^3*b*c^5)*d)*x^4
- (33*b^7 + 940*a*b^5*c + 3760*a^2*b^3*c^2 + 2112*a^3*b*c^3)*d^3 + 4*(15*a*
b^8*c + 51*a^2*b^6*c^2 + 220*a^3*b^4*c^3 + 3456*c^5*d^5 + 16*(63*b^2*c^4 -
452*a*c^5)*d^4 - 4*(273*b^4*c^3 + 584*a*b^2*c^4 - 1264*a^2*c^5)*d^3 + 8*(27
*b^6*c^2 + 233*a*b^4*c^3 + 236*a^2*b^2*c^4 - 160*a^3*c^5)*d^2 - (15*b^8*c +
 267*a*b^6*c^2 + 992*a^2*b^4*c^3 + 560*a^3*b^2*c^4)*d)*x^3 + (59*a*b^7 + 58
4*a^2*b^5*c + 944*a^3*b^3*c^2)*d^2 + (15*a*b^9 + 26*a^2*b^7*c + 120*a^3*b^5
*c^2 + 20736*b*c^4*d^5 - 32*(251*b^3*c^3 + 1356*a*b*c^4)*d^4 + 8*(61*b^5*c^
2 + 1768*a*b^3*c^3 + 3792*a^2*b*c^4)*d^3 + 4*(29*b^7*c - 124*a*b^5*c^2 - 18
88*a^2*b^3*c^3 - 1920*a^3*b*c^4)*d^2 - (15*b^9 + 142*a*b^7*c + 112*a^2*b^5*
c^2 - 1440*a^3*b^3*c^3)*d)*x^2 - 34*(a^2*b^7 + 4*a^3*b^5*c)*d - 2*(5*a^2*b^
8 + 12*a^3*b^6*c - 4608*c^4*d^6 - 864*(b^2*c^3 - 12*a*c^4)*d^5 + 4*(329*b^4
*c^2 + 456*a*b^2*c^3 - 1968*a^2*c^4)*d^4 - (283*b^6*c + 2356*a*b^4*c^2 + 12
96*a^2*b^2*c^3 - 2112*a^3*c^4)*d^3 + (20*b^8 + 413*a*b^6*c + 1304*a^2*b^4*c
^2 + 336*a^3*b^2*c^3)*d^2 - (25*a*b^8 + 142*a^2*b^6*c + 264*a^3*b^4*c^2)*d)
*x)*sqrt(c*x^2 + b*x + a))/(a^4*b^8*d^3 + 256*c^4*d^11 - 256*(b^2*c^3 + 4*a
*c^4)*d^10 + 32*(3*b^4*c^2 + 32*a*b^2*c^3 + 48*a^2*c^4)*d^9 - 16*(b^6*c + 2
4*a*b^4*c^2 + 96*a^2*b^2*c^3 + 64*a^3*c^4)*d^8 + (b^8 + 64*a*b^6*c + 576*a^
2*b^4*c^2 + 1024*a^3*b^2*c^3 + 256*a^4*c^4)*d^7 - 4*(a*b^8 + 24*a^2*b^6*c +
```

```
 96*a^3*b^4*c^2 + 64*a^4*b^2*c^3)*d^6 + (a^4*b^8*c^3 + 256*c^7*d^8 - 256*(b
^2*c^6 + 4*a*c^7)*d^7 + 32*(3*b^4*c^5 + 32*a*b^2*c^6 + 48*a^2*c^7)*d^6 - 16
*(b^6*c^4 + 24*a*b^4*c^5 + 96*a^2*b^2*c^6 + 64*a^3*c^7)*d^5 + (b^8*c^3 + 64
*a*b^6*c^4 + 576*a^2*b^4*c^5 + 1024*a^3*b^2*c^6 + 256*a^4*c^7)*d^4 - 4*(a*b
^8*c^3 + 24*a^2*b^6*c^4 + 96*a^3*b^4*c^5 + 64*a^4*b^2*c^6)*d^3 + 2*(3*a^2*b
^8*c^3 + 32*a^3*b^6*c^4 + 48*a^4*b^4*c^5)*d^2 - 4*(a^3*b^8*c^3 + 4*a^4*b^6*
c^4)*d)*x^6 + 2*(3*a^2*b^8 + 32*a^3*b^6*c + 48*a^4*b^4*c^2)*d^5 + 3*(a^4*b^
9*c^2 + 256*b*c^6*d^8 - 256*(b^3*c^5 + 4*a*b*c^6)*d^7 + 32*(3*b^5*c^4 + 32*
a*b^3*c^5 + 48*a^2*b*c^6)*d^6 - 16*(b^7*c^3 + 24*a*b^5*c^4 + 96*a^2*b^3*c^5
 + 64*a^3*b*c^6)*d^5 + (b^9*c^2 + 64*a*b^7*c^3 + 576*a^2*b^5*c^4 + 1024*a^3
*b^3*c^5 + 256*a^4*b*c^6)*d^4 - 4*(a*b^9*c^2 + 24*a^2*b^7*c^3 + 96*a^3*b^5*
c^4 + 64*a^4*b^3*c^5)*d^3 + 2*(3*a^2*b^9*c^2 + 32*a^3*b^7*c^3 + 48*a^4*b^5*
c^4)*d^2 - 4*(a^3*b^9*c^2 + 4*a^4*b^7*c^3)*d)*x^5 - 4*(a^3*b^8 + 4*a^4*b^6*
c)*d^4 + 3*(a^4*b^10*c - 1024*a*c^6*d^8 + 256*c^6*d^9 - 32*(5*b^4*c^4 - 48*
a^2*c^6)*d^7 + 16*(5*b^6*c^3 + 40*a*b^4*c^4 - 64*a^3*c^6)*d^6 - (15*b^8*c^2
 + 320*a*b^6*c^3 + 960*a^2*b^4*c^4 - 256*a^4*c^6)*d^5 + (b^10*c + 60*a*b^8*
c^2 + 480*a^2*b^6*c^3 + 640*a^3*b^4*c^4)*d^4 - 2*(2*a*b^10*c + 45*a^2*b^8*c
^2 + 160*a^3*b^6*c^3 + 80*a^4*b^4*c^4)*d^3 + 2*(3*a^2*b^10*c + 30*a^3*b^8*c
^2 + 40*a^4*b^6*c^3)*d^2 - (4*a^3*b^10*c + 15*a^4*b^8*c^2)*d)*x^4 + (a^4*b^
11 + 1536*b*c^5*d^9 - 256*(5*b^3*c^4 + 24*a*b*c^5)*d^8 + 64*(5*b^5*c^3 + 80
*a*b^3*c^4 + 144*a^2*b*c^5)*d^7 - 256*(5*a*b^5*c^3 + 30*a^2*b^3*c^4 + 24*a^
3*b*c^5)*d^6 - 2*(5*b^9*c - 960*a^2*b^5*c^3 - 2560*a^3*b^3*c^4 - 768*a^4*b*
c^5)*d^5 + (b^11 + 40*a*b^9*c - 1280*a^3*b^5*c^3 - 1280*a^4*b^3*c^4)*d^4 -
4*(a*b^11 + 15*a^2*b^9*c - 80*a^4*b^5*c^3)*d^3 + 2*(3*a^2*b^11 + 20*a^3*b^9
*c)*d^2 - 2*(2*a^3*b^11 + 5*a^4*b^9*c)*d)*x^3 + 3*(a^4*b^10*d - 1024*a*c^5*
d^9 + 256*c^5*d^10 - 32*(5*b^4*c^3 - 48*a^2*c^5)*d^8 + 16*(5*b^6*c^2 + 40*a
*b^4*c^3 - 64*a^3*c^5)*d^7 - (15*b^8*c + 320*a*b^6*c^2 + 960*a^2*b^4*c^3 -
256*a^4*c^5)*d^6 + (b^10 + 60*a*b^8*c + 480*a^2*b^6*c^2 + 640*a^3*b^4*c^3)*
d^5 - 2*(2*a*b^10 + 45*a^2*b^8*c + 160*a^3*b^6*c^2 + 80*a^4*b^4*c^3)*d^4 +
2*(3*a^2*b^10 + 30*a^3*b^8*c + 40*a^4*b^6*c^2)*d^3 - (4*a^3*b^10 + 15*a^4*b
^8*c)*d^2)*x^2 + 3*(a^4*b^9*d^2 + 256*b*c^4*d^10 - 256*(b^3*c^3 + 4*a*b*c^4
)*d^9 + 32*(3*b^5*c^2 + 32*a*b^3*c^3 + 48*a^2*b*c^4)*d^8 - 16*(b^7*c + 24*a
*b^5*c^2 + 96*a^2*b^3*c^3 + 64*a^3*b*c^4)*d^7 + (b^9 + 64*a*b^7*c + 576*a^2
*b^5*c^2 + 1024*a^3*b^3*c^3 + 256*a^4*b*c^4)*d^6 - 4*(a*b^9 + 24*a^2*b^7*c
+ 96*a^3*b^5*c^2 + 64*a^4*b^3*c^3)*d^5 + 2*(3*a^2*b^9 + 32*a^3*b^7*c + 48*a
^4*b^5*c^2)*d^4 - 4*(a^3*b^9 + 4*a^4*b^7*c)*d^3)*x), -1/48*(3*(1024*c^3*d^6
 - (5*b^6*c^3 + 12*a*b^4*c^4 + 48*a^2*b^2*c^5 + 320*a^3*c^6 - 1024*c^6*d^3
+ 384*(b^2*c^5 + 4*a*c^6)*d^2 - 24*(3*b^4*c^4 + 8*a*b^2*c^5 + 48*a^2*c^6)*d
)*x^6 - 384*(b^2*c^2 + 4*a*c^3)*d^5 - 3*(5*b^7*c^2 + 12*a*b^5*c^3 + 48*a^2*
b^3*c^4 + 320*a^3*b*c^5 - 1024*b*c^5*d^3 + 384*(b^3*c^4 + 4*a*b*c^5)*d^2 -
24*(3*b^5*c^3 + 8*a*b^3*c^4 + 48*a^2*b*c^5)*d)*x^5 + 24*(3*b^4*c + 8*a*b^2*
c^2 + 48*a^2*c^3)*d^4 - 3*(5*b^8*c + 12*a*b^6*c^2 + 48*a^2*b^4*c^3 + 320*a^
3*b^2*c^4 - 1024*c^5*d^4 - 128*(5*b^2*c^4 - 12*a*c^5)*d^3 + 24*(13*b^4*c^3
+ 56*a*b^2*c^4 - 48*a^2*c^5)*d^2 - (67*b^6*c^2 + 180*a*b^4*c^3 + 1104*a^2*b
^2*c^4 - 320*a^3*c^5)*d)*x^4 - (5*b^6 + 12*a*b^4*c + 48*a^2*b^2*c^2 + 320*a
```

^3*c^3)*d^3 - (5*b^9 + 12*a*b^7*c + 48*a^2*b^5*c^2 + 320*a^3*b^3*c^3 - 6144
*b*c^4*d^4 + 256*(5*b^3*c^3 + 36*a*b*c^4)*d^3 - 48*(b^5*c^2 - 8*a*b^3*c^3 +
144*a^2*b*c^4)*d^2 - 6*(7*b^7*c + 20*a*b^5*c^2 + 144*a^2*b^3*c^3 - 320*a^3
*b*c^4)*d)*x^3 + 3*(1024*c^4*d^5 + 128*(5*b^2*c^3 - 12*a*c^4)*d^4 - 24*(13*
b^4*c^2 + 56*a*b^2*c^3 - 48*a^2*c^4)*d^3 + (67*b^6*c + 180*a*b^4*c^2 + 1104
*a^2*b^2*c^3 - 320*a^3*c^4)*d^2 - (5*b^8 + 12*a*b^6*c + 48*a^2*b^4*c^2 + 32
0*a^3*b^2*c^3)*d)*x^2 + 3*(1024*b*c^3*d^5 - 384*(b^3*c^2 + 4*a*b*c^3)*d^4 +
24*(3*b^5*c + 8*a*b^3*c^2 + 48*a^2*b*c^3)*d^3 - (5*b^7 + 12*a*b^5*c + 48*a
^2*b^3*c^2 + 320*a^3*b*c^3)*d^2)*x)*sqrt(-a*b^2 - 4*c*d^2 + (b^2 + 4*a*c)*d
)*arctan(-1/2*(2*a*b^2 + (b^2*c + 4*a*c^2 - 8*c^2*d)*x^2 - (b^2 + 4*a*c)*d
+ (b^3 + 4*a*b*c - 8*b*c*d)*x)*sqrt(-a*b^2 - 4*c*d^2 + (b^2 + 4*a*c)*d)*sqr
t(c*x^2 + b*x + a)/(a^2*b^3 + 4*a*b*c*d^2 + 2*(a*b^2*c^2 + 4*c^3*d^2 - (b^2
*c^2 + 4*a*c^3)*d)*x^3 + 3*(a*b^3*c + 4*b*c^2*d^2 - (b^3*c + 4*a*b*c^2)*d)*
x^2 - (a*b^3 + 4*a^2*b*c)*d + (a*b^4 + 2*a^2*b^2*c + 4*(b^2*c + 2*a*c^2)*d^
2 - (b^4 + 6*a*b^2*c + 8*a^2*c^2)*d)*x)) + 2*(8*a^3*b^7 + 4608*b*c^3*d^6 -
2592*(b^3*c^2 + 4*a*b*c^3)*d^5 + 2*(15*a*b^6*c^3 + 56*a^2*b^4*c^4 + 240*a^3
*b^2*c^5 + 2816*c^6*d^4 - 1408*(b^2*c^5 + 4*a*c^6)*d^3 + 4*(59*b^4*c^4 + 58
4*a*b^2*c^5 + 944*a^2*c^6)*d^2 - (15*b^6*c^3 + 292*a*b^4*c^4 + 1168*a^2*b^2
*c^5 + 960*a^3*c^6)*d)*x^5 + 4*(123*b^5*c + 1352*a*b^3*c^2 + 1968*a^2*b*c^3
)*d^4 + 5*(15*a*b^7*c^2 + 56*a^2*b^5*c^3 + 240*a^3*b^3*c^4 + 2816*b*c^5*d^4
- 1408*(b^3*c^4 + 4*a*b*c^5)*d^3 + 4*(59*b^5*c^3 + 584*a*b^3*c^4 + 944*a^2
*b*c^5)*d^2 - (15*b^7*c^2 + 292*a*b^5*c^3 + 1168*a^2*b^3*c^4 + 960*a^3*b*c^
5)*d)*x^4 - (33*b^7 + 940*a*b^5*c + 3760*a^2*b^3*c^2 + 2112*a^3*b*c^3)*d^3
+ 4*(15*a*b^8*c + 51*a^2*b^6*c^2 + 220*a^3*b^4*c^3 + 3456*c^5*d^5 + 16*(63*
b^2*c^4 - 452*a*c^5)*d^4 - 4*(273*b^4*c^3 + 584*a*b^2*c^4 - 1264*a^2*c^5)*d
^3 + 8*(27*b^6*c^2 + 233*a*b^4*c^3 + 236*a^2*b^2*c^4 - 160*a^3*c^5)*d^2 - (
15*b^8*c + 267*a*b^6*c^2 + 992*a^2*b^4*c^3 + 560*a^3*b^2*c^4)*d)*x^3 + (59*
a*b^7 + 584*a^2*b^5*c + 944*a^3*b^3*c^2)*d^2 + (15*a*b^9 + 26*a^2*b^7*c + 1
20*a^3*b^5*c^2 + 20736*b*c^4*d^5 - 32*(251*b^3*c^3 + 1356*a*b*c^4)*d^4 + 8*
(61*b^5*c^2 + 1768*a*b^3*c^3 + 3792*a^2*b*c^4)*d^3 + 4*(29*b^7*c - 124*a*b^
5*c^2 - 1888*a^2*b^3*c^3 - 1920*a^3*b*c^4)*d^2 - (15*b^9 + 142*a*b^7*c + 11
2*a^2*b^5*c^2 - 1440*a^3*b^3*c^3)*d)*x^2 - 34*(a^2*b^7 + 4*a^3*b^5*c)*d - 2
*(5*a^2*b^8 + 12*a^3*b^6*c - 4608*c^4*d^6 - 864*(b^2*c^3 - 12*a*c^4)*d^5 +
4*(329*b^4*c^2 + 456*a*b^2*c^3 - 1968*a^2*c^4)*d^4 - (283*b^6*c + 2356*a*b^
4*c^2 + 1296*a^2*b^2*c^3 - 2112*a^3*c^4)*d^3 + (20*b^8 + 413*a*b^6*c + 1304
*a^2*b^4*c^2 + 336*a^3*b^2*c^3)*d^2 - (25*a*b^8 + 142*a^2*b^6*c + 264*a^3*b
^4*c^2)*d)*x)*sqrt(c*x^2 + b*x + a))/(a^4*b^8*d^3 + 256*c^4*d^11 - 256*(b^2
*c^3 + 4*a*c^4)*d^10 + 32*(3*b^4*c^2 + 32*a*b^2*c^3 + 48*a^2*c^4)*d^9 - 16*
(b^6*c + 24*a*b^4*c^2 + 96*a^2*b^2*c^3 + 64*a^3*c^4)*d^8 + (b^8 + 64*a*b^6*
c + 576*a^2*b^4*c^2 + 1024*a^3*b^2*c^3 + 256*a^4*c^4)*d^7 - 4*(a*b^8 + 24*a
^2*b^6*c + 96*a^3*b^4*c^2 + 64*a^4*b^2*c^3)*d^6 + (a^4*b^8*c^3 + 256*c^7*d^
8 - 256*(b^2*c^6 + 4*a*c^7)*d^7 + 32*(3*b^4*c^5 + 32*a*b^2*c^6 + 48*a^2*c^7
)*d^6 - 16*(b^6*c^4 + 24*a*b^4*c^5 + 96*a^2*b^2*c^6 + 64*a^3*c^7)*d^5 + (b^
8*c^3 + 64*a*b^6*c^4 + 576*a^2*b^4*c^5 + 1024*a^3*b^2*c^6 + 256*a^4*c^7)*d^
4 - 4*(a*b^8*c^3 + 24*a^2*b^6*c^4 + 96*a^3*b^4*c^5 + 64*a^4*b^2*c^6)*d^3 +

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2*(3*a^2*b^8*c^3 + 32*a^3*b^6*c^4 + 48*a^4*b^4*c^5)*d^2 - 4*(a^3*b^8*c^3 +
4*a^4*b^6*c^4)*d)*x^6 + 2*(3*a^2*b^8 + 32*a^3*b^6*c + 48*a^4*b^4*c^2)*d^5 +
 3*(a^4*b^9*c^2 + 256*b*c^6*d^8 - 256*(b^3*c^5 + 4*a*b*c^6)*d^7 + 32*(3*b^5
*c^4 + 32*a*b^3*c^5 + 48*a^2*b*c^6)*d^6 - 16*(b^7*c^3 + 24*a*b^5*c^4 + 96*a
^2*b^3*c^5 + 64*a^3*b*c^6)*d^5 + (b^9*c^2 + 64*a*b^7*c^3 + 576*a^2*b^5*c^4
+ 1024*a^3*b^3*c^5 + 256*a^4*b*c^6)*d^4 - 4*(a*b^9*c^2 + 24*a^2*b^7*c^3 + 9
6*a^3*b^5*c^4 + 64*a^4*b^3*c^5)*d^3 + 2*(3*a^2*b^9*c^2 + 32*a^3*b^7*c^3 + 4
8*a^4*b^5*c^4)*d^2 - 4*(a^3*b^9*c^2 + 4*a^4*b^7*c^3)*d)*x^5 - 4*(a^3*b^8 +
4*a^4*b^6*c)*d^4 + 3*(a^4*b^10*c - 1024*a*c^6*d^8 + 256*c^6*d^9 - 32*(5*b^4
*c^4 - 48*a^2*c^6)*d^7 + 16*(5*b^6*c^3 + 40*a*b^4*c^4 - 64*a^3*c^6)*d^6 - (
15*b^8*c^2 + 320*a*b^6*c^3 + 960*a^2*b^4*c^4 - 256*a^4*c^6)*d^5 + (b^10*c +
 60*a*b^8*c^2 + 480*a^2*b^6*c^3 + 640*a^3*b^4*c^4)*d^4 - 2*(2*a*b^10*c + 45
*a^2*b^8*c^2 + 160*a^3*b^6*c^3 + 80*a^4*b^4*c^4)*d^3 + 2*(3*a^2*b^10*c + 30
*a^3*b^8*c^2 + 40*a^4*b^6*c^3)*d^2 - (4*a^3*b^10*c + 15*a^4*b^8*c^2)*d)*x^4
 + (a^4*b^11 + 1536*b*c^5*d^9 - 256*(5*b^3*c^4 + 24*a*b*c^5)*d^8 + 64*(5*b^
5*c^3 + 80*a*b^3*c^4 + 144*a^2*b*c^5)*d^7 - 256*(5*a*b^5*c^3 + 30*a^2*b^3*c
^4 + 24*a^3*b*c^5)*d^6 - 2*(5*b^9*c - 960*a^2*b^5*c^3 - 2560*a^3*b^3*c^4 -
768*a^4*b*c^5)*d^5 + (b^11 + 40*a*b^9*c - 1280*a^3*b^5*c^3 - 1280*a^4*b^3*c
^4)*d^4 - 4*(a*b^11 + 15*a^2*b^9*c - 80*a^4*b^5*c^3)*d^3 + 2*(3*a^2*b^11 +
20*a^3*b^9*c)*d^2 - 2*(2*a^3*b^11 + 5*a^4*b^9*c)*d)*x^3 + 3*(a^4*b^10*d - 1
024*a*c^5*d^9 + 256*c^5*d^10 - 32*(5*b^4*c^3 - 48*a^2*c^5)*d^8 + 16*(5*b^6*
c^2 + 40*a*b^4*c^3 - 64*a^3*c^5)*d^7 - (15*b^8*c + 320*a*b^6*c^2 + 960*a^2*
b^4*c^3 - 256*a^4*c^5)*d^6 + (b^10 + 60*a*b^8*c + 480*a^2*b^6*c^2 + 640*a^3
*b^4*c^3)*d^5 - 2*(2*a*b^10 + 45*a^2*b^8*c + 160*a^3*b^6*c^2 + 80*a^4*b^4*c
^3)*d^4 + 2*(3*a^2*b^10 + 30*a^3*b^8*c + 40*a^4*b^6*c^2)*d^3 - (4*a^3*b^10
+ 15*a^4*b^8*c)*d^2)*x^2 + 3*(a^4*b^9*d^2 + 256*b*c^4*d^10 - 256*(b^3*c^3 +
 4*a*b*c^4)*d^9 + 32*(3*b^5*c^2 + 32*a*b^3*c^3 + 48*a^2*b*c^4)*d^8 - 16*(b^
7*c + 24*a*b^5*c^2 + 96*a^2*b^3*c^3 + 64*a^3*b*c^4)*d^7 + (b^9 + 64*a*b^7*c
 + 576*a^2*b^5*c^2 + 1024*a^3*b^3*c^3 + 256*a^4*b*c^4)*d^6 - 4*(a*b^9 + 24*
a^2*b^7*c + 96*a^3*b^5*c^2 + 64*a^4*b^3*c^3)*d^5 + 2*(3*a^2*b^9 + 32*a^3*b^
7*c + 48*a^4*b^5*c^2)*d^4 - 4*(a^3*b^9 + 4*a^4*b^7*c)*d^3)*x)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+d)^4/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.04, size = 3695, normalized size = 11.27

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+b*x+d)^4/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] 5/2/(b^2-4*c*d)^2/(a-d)^3/(x+1/2*b/c+1/2*(b^2-4*c*d)^(1/2)/c)*(a+(x+1/2*(b+
(b^2-4*c*d)^(1/2))/c)^2*c-d-(b^2-4*c*d)^(1/2)*(x+1/2*(b+(b^2-4*c*d)^(1/2))/
c))^(1/2)*c*d+5/2/(b^2-4*c*d)^2/(a-d)^3/(x+1/2*b/c-1/2*(b^2-4*c*d)^(1/2)/c)
*(a+(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)^2*c-d+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^
2-4*c*d)^(1/2))/c))^(1/2)*c*d-3/4/(b^2-4*c*d)^(3/2)/(a-d)^(5/2)*c*ln((2*a-2
*d+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)+2*(a-d)^(1/2)*(a+(x-1
/2*(-b+(b^2-4*c*d)^(1/2))/c)^2*c-d+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)
^(1/2))/c))^(1/2))/(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c))-1/3/(b^2-4*c*d)^2/(a-d
)/(x+1/2*b/c+1/2*(b^2-4*c*d)^(1/2)/c)^3*(a+(x+1/2*(b+(b^2-4*c*d)^(1/2))/c)^
2*c-d-(b^2-4*c*d)^(1/2)*(x+1/2*(b+(b^2-4*c*d)^(1/2))/c))^(1/2)-3/(b^2-4*c*d
)^(5/2)*c^2/(a-d)^(3/2)*ln((2*a-2*d-(b^2-4*c*d)^(1/2)*(x+1/2*(b+(b^2-4*c*d)
^(1/2))/c)+2*(a-d)^(1/2)*(a+(x+1/2*(b+(b^2-4*c*d)^(1/2))/c)^2*c-d-(b^2-4*c*
d)^(1/2)*(x+1/2*(b+(b^2-4*c*d)^(1/2))/c))^(1/2))/(x+1/2*(b+(b^2-4*c*d)^(1/2
))/c))-1/3/(b^2-4*c*d)^2/(a-d)/(x+1/2*b/c-1/2*(b^2-4*c*d)^(1/2)/c)^3*(a+(x-
1/2*(-b+(b^2-4*c*d)^(1/2))/c)^2*c-d+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d
)^(1/2))/c))^(1/2)+3/(b^2-4*c*d)^(5/2)*c^2/(a-d)^(3/2)*ln((2*a-2*d+(b^2-4*c
*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)+2*(a-d)^(1/2)*(a+(x-1/2*(-b+(b^2
-4*c*d)^(1/2))/c)^2*c-d+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c))
^(1/2))/(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c))+20*c^3/(b^2-4*c*d)^(7/2)/(a-d)^(1
/2)*ln((2*a-2*d+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)+2*(a-d)^
(1/2)*(a+(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)^2*c-d+(b^2-4*c*d)^(1/2)*(x-1/2*(-
b+(b^2-4*c*d)^(1/2))/c))^(1/2))/(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c))-20*c^3/(b
^2-4*c*d)^(7/2)/(a-d)^(1/2)*ln((2*a-2*d-(b^2-4*c*d)^(1/2)*(x+1/2*(b+(b^2-4*
c*d)^(1/2))/c)+2*(a-d)^(1/2)*(a+(x+1/2*(b+(b^2-4*c*d)^(1/2))/c)^2*c-d-(b^2-
4*c*d)^(1/2)*(x+1/2*(b+(b^2-4*c*d)^(1/2))/c))^(1/2))/(x+1/2*(b+(b^2-4*c*d)^
(1/2))/c))-5/12/(b^2-4*c*d)^(3/2)/(a-d)^2/(x+1/2*b/c+1/2*(b^2-4*c*d)^(1/2)/
c)^2*(a+(x+1/2*(b+(b^2-4*c*d)^(1/2))/c)^2*c-d-(b^2-4*c*d)^(1/2)*(x+1/2*(b+(
b^2-4*c*d)^(1/2))/c))^(1/2)-5/16/(b^2-4*c*d)^(3/2)/(a-d)^(7/2)*ln((2*a-2*d-
(b^2-4*c*d)^(1/2)*(x+1/2*(b+(b^2-4*c*d)^(1/2))/c)+2*(a-d)^(1/2)*(a+(x+1/2*(
b+(b^2-4*c*d)^(1/2))/c)^2*c-d-(b^2-4*c*d)^(1/2)*(x+1/2*(b+(b^2-4*c*d)^(1/2)
)/c))^(1/2))/(x+1/2*(b+(b^2-4*c*d)^(1/2))/c))*b^2+3/4/(b^2-4*c*d)^(3/2)/(a-
d)^(5/2)*c*ln((2*a-2*d-(b^2-4*c*d)^(1/2)*(x+1/2*(b+(b^2-4*c*d)^(1/2))/c)+2*
(a-d)^(1/2)*(a+(x+1/2*(b+(b^2-4*c*d)^(1/2))/c)^2*c-d-(b^2-4*c*d)^(1/2)*(x+1
/2*(b+(b^2-4*c*d)^(1/2))/c))^(1/2))/(x+1/2*(b+(b^2-4*c*d)^(1/2))/c))+5/12/(
b^2-4*c*d)^(3/2)/(a-d)^2/(x+1/2*b/c-1/2*(b^2-4*c*d)^(1/2)/c)^2*(a+(x-1/2*(-
b+(b^2-4*c*d)^(1/2))/c)^2*c-d+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2
))/c))^(1/2)+5/16/(b^2-4*c*d)^(3/2)/(a-d)^(7/2)*ln((2*a-2*d+(b^2-4*c*d)^(1/
2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)+2*(a-d)^(1/2)*(a+(x-1/2*(-b+(b^2-4*c*d)
^(1/2))/c)^2*c-d+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c))^(1/2))
/(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c))*b^2-5/8/(b^2-4*c*d)^2/(a-d)^3/(x+1/2*b/c
+1/2*(b^2-4*c*d)^(1/2)/c)*(a+(x+1/2*(b+(b^2-4*c*d)^(1/2))/c)^2*c-d-(b^2-4*c
```

```
*d)^(1/2)*(x+1/2*(b+(b^2-4*c*d)^(1/2))/c))^(1/2)*b^2-10/(b^2-4*c*d)^3*c^2/(
a-d)/(x+1/2*b/c+1/2*(b^2-4*c*d)^(1/2)/c)*(a+(x+1/2*(b+(b^2-4*c*d)^(1/2))/c)
^2*c-d-(b^2-4*c*d)^(1/2)*(x+1/2*(b+(b^2-4*c*d)^(1/2))/c))^(1/2)+2/(b^2-4*c*
d)^(5/2)*c/(a-d)/(x+1/2*b/c-1/2*(b^2-4*c*d)^(1/2)/c)^2*(a+(x-1/2*(-b+(b^2-4
*c*d)^(1/2))/c)^2*c-d+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c))^(
1/2)+3/2/(b^2-4*c*d)^(5/2)*c/(a-d)^(5/2)*ln((2*a-2*d+(b^2-4*c*d)^(1/2)*(x-1
/2*(-b+(b^2-4*c*d)^(1/2))/c)+2*(a-d)^(1/2)*(a+(x-1/2*(-b+(b^2-4*c*d)^(1/2))
/c)^2*c-d+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c))^(1/2))/(x-1/2
*(-b+(b^2-4*c*d)^(1/2))/c))*b^2-6/(b^2-4*c*d)^(5/2)*c^2/(a-d)^(5/2)*ln((2*a
-2*d+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)+2*(a-d)^(1/2)*(a+(x
-1/2*(-b+(b^2-4*c*d)^(1/2))/c)^2*c-d+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*
d)^(1/2))/c))^(1/2))/(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c))*d-2/(b^2-4*c*d)^(5/2
)*c/(a-d)/(x+1/2*b/c+1/2*(b^2-4*c*d)^(1/2)/c)^2*(a+(x+1/2*(b+(b^2-4*c*d)^(1
/2))/c)^2*c-d-(b^2-4*c*d)^(1/2)*(x+1/2*(b+(b^2-4*c*d)^(1/2))/c))^(1/2)-7/3/
(b^2-4*c*d)^2*c/(a-d)^2/(x+1/2*b/c+1/2*(b^2-4*c*d)^(1/2)/c)*(a+(x+1/2*(b+(b
^2-4*c*d)^(1/2))/c)^2*c-d-(b^2-4*c*d)^(1/2)*(x+1/2*(b+(b^2-4*c*d)^(1/2))/c)
)^(1/2)-3/2/(b^2-4*c*d)^(5/2)*c/(a-d)^(5/2)*ln((2*a-2*d-(b^2-4*c*d)^(1/2)*(
x+1/2*(b+(b^2-4*c*d)^(1/2))/c)+2*(a-d)^(1/2)*(a+(x+1/2*(b+(b^2-4*c*d)^(1/2)
)/c)^2*c-d-(b^2-4*c*d)^(1/2)*(x+1/2*(b+(b^2-4*c*d)^(1/2))/c))^(1/2))/(x+1/2
*(b+(b^2-4*c*d)^(1/2))/c))*b^2+6/(b^2-4*c*d)^(5/2)*c^2/(a-d)^(5/2)*ln((2*a-
2*d-(b^2-4*c*d)^(1/2)*(x+1/2*(b+(b^2-4*c*d)^(1/2))/c)+2*(a-d)^(1/2)*(a+(x+1
/2*(b+(b^2-4*c*d)^(1/2))/c)^2*c-d-(b^2-4*c*d)^(1/2)*(x+1/2*(b+(b^2-4*c*d)^(
1/2))/c))^(1/2))/(x+1/2*(b+(b^2-4*c*d)^(1/2))/c))*d-10/(b^2-4*c*d)^3*c^2/(a
-d)/(x+1/2*b/c-1/2*(b^2-4*c*d)^(1/2)/c)*(a+(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)
^2*c-d+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c))^(1/2)-5/8/(b^2-4
*c*d)^2/(a-d)^3/(x+1/2*b/c-1/2*(b^2-4*c*d)^(1/2)/c)*(a+(x-1/2*(-b+(b^2-4*c*
d)^(1/2))/c)^2*c-d+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c))^(1/2
)*b^2-7/3/(b^2-4*c*d)^2*c/(a-d)^2/(x+1/2*b/c-1/2*(b^2-4*c*d)^(1/2)/c)*(a+(x
-1/2*(-b+(b^2-4*c*d)^(1/2))/c)^2*c-d+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*
d)^(1/2))/c))^(1/2)+5/4/(b^2-4*c*d)^(3/2)/(a-d)^(7/2)*ln((2*a-2*d-(b^2-4*c*
d)^(1/2)*(x+1/2*(b+(b^2-4*c*d)^(1/2))/c)+2*(a-d)^(1/2)*(a+(x+1/2*(b+(b^2-4*
c*d)^(1/2))/c)^2*c-d-(b^2-4*c*d)^(1/2)*(x+1/2*(b+(b^2-4*c*d)^(1/2))/c))^(1/
2))/(x+1/2*(b+(b^2-4*c*d)^(1/2))/c))*c*d-5/4/(b^2-4*c*d)^(3/2)/(a-d)^(7/2)*
ln((2*a-2*d+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)+2*(a-d)^(1/2
)*(a+(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)^2*c-d+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b
^2-4*c*d)^(1/2))/c))^(1/2))/(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c))*c*d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^2 + bx + a}\left(cx^2 + bx + d\right)^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+d)^4/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*(c*x^2 + b*x + d)^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{c\,x^2+b\,x+a}\,{\left(c\,x^2+b\,x+d\right)}^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x + c*x^2)^(1/2)*(d + b*x + c*x^2)^4),x)
[Out] int(1/((a + b*x + c*x^2)^(1/2)*(d + b*x + c*x^2)^4), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x+d)**4/(c*x**2+b*x+a)**(1/2),x)
[Out] Timed out
```

3.7 $\int \frac{1}{\sqrt{d+ex+fx^2}\left(ae+bex+bfx^2\right)^2}\,dx$

Optimal. Leaf size=162

$$-\frac{\left(8aef-b\left(4df+e^2\right)\right)\tanh^{-1}\left(\frac{(e+2fx)\sqrt{bd-ae}}{\sqrt{e}\sqrt{be-4af}\sqrt{d+ex+fx^2}}\right)}{e^{3/2}(bd-ae)^{3/2}(be-4af)^{3/2}}-\frac{b(e+2fx)\sqrt{d+ex+fx^2}}{e(bd-ae)(be-4af)\left(ae+bex+bfx^2\right)}$$

Rubi [A] time = 0.31, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {974, 12, 982, 208}

$$-\frac{\left(8aef-b\left(4df+e^2\right)\right)\tanh^{-1}\left(\frac{(e+2fx)\sqrt{bd-ae}}{\sqrt{e}\sqrt{be-4af}\sqrt{d+ex+fx^2}}\right)}{e^{3/2}(bd-ae)^{3/2}(be-4af)^{3/2}}-\frac{b(e+2fx)\sqrt{d+ex+fx^2}}{e(bd-ae)(be-4af)\left(ae+bex+bfx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d + e*x + f*x^2]*(a*e + b*e*x + b*f*x^2)^2),x]
```

```
[Out] -((b*(e + 2*f*x)*Sqrt[d + e*x + f*x^2])/(e*(b*d - a*e)*(b*e - 4*a*f)*(a*e +
 b*e*x + b*f*x^2))) - ((8*a*e*f - b*(e^2 + 4*d*f))*ArcTanh[(Sqrt[b*d - a*e]
*(e + 2*f*x))/(Sqrt[e]*Sqrt[b*e - 4*a*f]*Sqrt[d + e*x + f*x^2])])/(e^(3/2)*
(b*d - a*e)^(3/2)*(b*e - 4*a*f)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 974

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*
a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*(
d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
```

```
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q,
 0]
```

Rule 982

```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(
x_)^2]), x_Symbol] :> Dist[-2*e, Subst[Int[1/(e*(b*e - 4*a*f) - (b*d - a*e)
*x^2), x], x, (e + 2*f*x)/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0
]
```

Rubi steps

$$\int \frac{1}{\sqrt{d+ex+fx^2}\,\left(ae+bex+bfx^2\right)^2}\,dx = -\frac{b(e+2fx)\sqrt{d+ex+fx^2}}{e(bd-ae)(be-4af)\left(ae+bex+bfx^2\right)} + \frac{\int \frac{b(bd-ae)f^2\left(8aef-b\left(e^2+4df\right)\right.}{2\sqrt{d+ex+fx^2}\,\left(ae+bex+bfx^2\right.}}{be(bd-ae)^2f^2(be-4a}$$

$$= -\frac{b(e+2fx)\sqrt{d+ex+fx^2}}{e(bd-ae)(be-4af)\left(ae+bex+bfx^2\right)} + \frac{\left(8aef-b\left(e^2+4df\right)\right)\int}{2e(bd-ae}$$

$$= -\frac{b(e+2fx)\sqrt{d+ex+fx^2}}{e(bd-ae)(be-4af)\left(ae+bex+bfx^2\right)} - \frac{\left(8aef-b\left(e^2+4df\right)\right)\text{Su}}{}$$

$$= -\frac{b(e+2fx)\sqrt{d+ex+fx^2}}{e(bd-ae)(be-4af)\left(ae+bex+bfx^2\right)} - \frac{\left(8aef-b\left(e^2+4df\right)\right)\text{ta}}{e^{3/2}(bd-ae)}$$

Mathematica [B] time = 1.96, size = 490, normalized size = 3.02

$$\frac{2f\left(-\frac{\left(b\left(4df+e^2\right)-8aef\right)\tanh^{-1}\left(\frac{-\sqrt{e}(e+2fx)\sqrt{be-4af}-\sqrt{b}\left(e^2-4df\right)}{4f\sqrt{bd-ae}\sqrt{d+x(e+fx)}}\right)}{4f(bd-ae)^{3/2}(be-4af)^{3/2}}-\frac{e\tanh^{-1}\left(\frac{\sqrt{b}\left(e^2-4df\right)-\sqrt{e}(e+2fx)\sqrt{be-4af}}{4f\sqrt{bd-ae}\sqrt{d+x(e+fx)}}\right)}{4f(bd-ae)^{3/2}\sqrt{be-4af}}+\frac{\tanh^{-1}\left(\frac{\sqrt{e}(e+2fx)\sqrt{be-4af}-\sqrt{b}\left(e^2-4df\right)}{4f\sqrt{bd-ae}\sqrt{d+x(e+fx)}}\right)}{\sqrt{bd-ae}(be-4af)^{3/2}}-\frac{\sqrt{b}\sqrt{e}\sqrt{d+x(e+fx)}}{(bd-ae)(be-4af)\left(\sqrt{b}(e+2fx)-\sqrt{e}\sqrt{be-4af}\right)}-\frac{\sqrt{b}\sqrt{e}\sqrt{d+x(e+fx)}}{(bd-ae)(be-4af)\left(\sqrt{e}\sqrt{be-4af}+\sqrt{b}(e+2fx)\right)}\right)}{e^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d + e*x + f*x^2]*(a*e + b*e*x + b*f*x^2)^2),x]
```

```
[Out] (2*f*(-((Sqrt[b]*Sqrt[e]*Sqrt[d + x*(e + f*x)])/((b*d - a*e)*(b*e - 4*a*f)*
(-(Sqrt[e]*Sqrt[b*e - 4*a*f]) + Sqrt[b]*(e + 2*f*x)))) - (Sqrt[b]*Sqrt[e]*S
qrt[d + x*(e + f*x)])/((b*d - a*e)*(b*e - 4*a*f)*(Sqrt[e]*Sqrt[b*e - 4*a*f]
 + Sqrt[b]*(e + 2*f*x))) - ((-8*a*e*f + b*(e^2 + 4*d*f))*ArcTanh[(-(Sqrt[b]
*(e^2 - 4*d*f)) - Sqrt[e]*Sqrt[b*e - 4*a*f]*(e + 2*f*x))/(4*Sqrt[b*d - a*e]
*f*Sqrt[d + x*(e + f*x)])])/(4*(b*d - a*e)^(3/2)*f*(b*e - 4*a*f)^(3/2)) - (
e*ArcTanh[(Sqrt[b]*(e^2 - 4*d*f) - Sqrt[e]*Sqrt[b*e - 4*a*f]*(e + 2*f*x))/(
4*Sqrt[b*d - a*e]*f*Sqrt[d + x*(e + f*x)])])/(4*(b*d - a*e)^(3/2)*f*Sqrt[b*
e - 4*a*f]) + ArcTanh[(-(Sqrt[b]*(e^2 - 4*d*f)) + Sqrt[e]*Sqrt[b*e - 4*a*f]
*(e + 2*f*x))/(4*Sqrt[b*d - a*e]*f*Sqrt[d + x*(e + f*x)])]/(Sqrt[b*d - a*e]
*(b*e - 4*a*f)^(3/2))))/e^(3/2)
```

IntegrateAlgebraic [C] time = 1.34, size = 266, normalized size = 1.64

$$\frac{b(e+2fx)\sqrt{d+ex+fx^2}}{e(ae-bd)(be-4af)\left(ae+bex+bfx^2\right)}-\frac{\left(8aef-4bdf-be^2\right)\text{RootSum}\left[\#1^4bf-2\#1^3be\sqrt{f}+4\#1^2aef-2\#1^2bdf+\#1^2be^2-4\#1ae^2\sqrt{f}+2\#1bde\sqrt{f}+ae^3+bd^2f-bde^2\&,\frac{\log\left(-\#1+\sqrt{d+ex+fx^2}-\sqrt{f}x\right)}{\#1^2(-b)\sqrt{f}+\#1be-2ae\sqrt{f}+bd\sqrt{f}}\&\right]}{2e(ae-bd)(be-4af)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[d + e*x + f*x^2]*(a*e + b*e*x + b*f*x^2)^2),x]
```

```
[Out] (b*(e + 2*f*x)*Sqrt[d + e*x + f*x^2])/(e*(-(b*d) + a*e)*(b*e - 4*a*f)*(a*e
+ b*e*x + b*f*x^2)) - ((-(b*e^2) - 4*b*d*f + 8*a*e*f)*RootSum[-(b*d*e^2) +
a*e^3 + b*d^2*f + 2*b*d*e*Sqrt[f]*#1 - 4*a*e^2*Sqrt[f]*#1 + b*e^2*#1^2 - 2*
b*d*f*#1^2 + 4*a*e*f*#1^2 - 2*b*e*Sqrt[f]*#1^3 + b*f*#1^4 & , Log[-(Sqrt[f]
*x) + Sqrt[d + e*x + f*x^2] - #1]/(b*d*Sqrt[f] - 2*a*e*Sqrt[f] + b*e*#1 - b
*Sqrt[f]*#1^2) & ])/(2*e*(-(b*d) + a*e)*(b*e - 4*a*f))
```

fricas [B] time = 1.57, size = 2005, normalized size = 12.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*f*x^2+b*e*x+a*e)^2/(f*x^2+e*x+d)^(1/2),x, algorithm="fricas"
)
```

```
[Out] [-1/4*(sqrt(b^2*d*e^2 - a*b*e^3 - 4*(a*b*d*e - a^2*e^2)*f)*(a*b*e^3 + (b^2*
e^2*f + 4*(b^2*d - 2*a*b*e)*f^2)*x^2 + 4*(a*b*d*e - 2*a^2*e^2)*f + (b^2*e^3
 + 4*(b^2*d*e - 2*a*b*e^2)*f)*x)*log((8*b^2*d^2*e^4 - 8*a*b*d*e^5 + a^2*e^6
 + 16*a^2*d^2*e^2*f^2 + (b^2*e^4*f^2 + 16*(b^2*d^2 - 8*a*b*d*e + 8*a^2*e^2)
*f^4 + 8*(3*b^2*d*e^2 - 4*a*b*e^3)*f^3)*x^4 + 2*(b^2*e^5*f + 16*(b^2*d^2*e
- 8*a*b*d*e^2 + 8*a^2*e^3)*f^3 + 8*(3*b^2*d*e^3 - 4*a*b*e^4)*f^2)*x^3 + (b^
2*e^6 - 32*(3*a*b*d^2*e - 4*a^2*d*e^2)*f^3 + 16*(3*b^2*d^2*e^2 - 13*a*b*d*e
^3 + 10*a^2*e^4)*f^2 + 2*(16*b^2*d*e^4 - 19*a*b*e^5)*f)*x^2 - 4*sqrt(b^2*d*
e^2 - a*b*e^3 - 4*(a*b*d*e - a^2*e^2)*f)*(2*b*d*e^3 - a*e^4 - 4*a*d*e^2*f +
```

```
 2*(b*e^2*f^2 + 4*(b*d - 2*a*e)*f^3)*x^3 + 3*(b*e^3*f + 4*(b*d*e - 2*a*e^2)
*f^2)*x^2 + (b*e^4 - 8*a*d*e*f^2 + 2*(4*b*d*e^2 - 5*a*e^3)*f)*x)*sqrt(f*x^2
 + e*x + d) - 8*(4*a*b*d^2*e^3 - 3*a^2*d*e^4)*f + 2*(4*b^2*d*e^5 - 3*a*b*e^
6 - 16*(3*a*b*d^2*e^2 - 4*a^2*d*e^3)*f^2 + 8*(2*b^2*d^2*e^3 - 5*a*b*d*e^4 +
 2*a^2*e^5)*f)*x)/(b^2*f^2*x^4 + 2*b^2*e*f*x^3 + 2*a*b*e^2*x + a^2*e^2 + (b
^2*e^2 + 2*a*b*e*f)*x^2)) + 4*(b^3*d*e^3 - a*b^2*e^4 - 4*(a*b^2*d*e^2 - a^2
*b*e^3)*f - 2*(4*(a*b^2*d*e - a^2*b*e^2)*f^2 - (b^3*d*e^2 - a*b^2*e^3)*f)*x
)*sqrt(f*x^2 + e*x + d))/(a*b^4*d^2*e^5 - 2*a^2*b^3*d*e^6 + a^3*b^2*e^7 + 1
6*(a^3*b^2*d^2*e^3 - 2*a^4*b*d*e^4 + a^5*e^5)*f^2 + (16*(a^2*b^3*d^2*e^2 -
2*a^3*b^2*d*e^3 + a^4*b*e^4)*f^3 - 8*(a*b^4*d^2*e^3 - 2*a^2*b^3*d*e^4 + a^3
*b^2*e^5)*f^2 + (b^5*d^2*e^4 - 2*a*b^4*d*e^5 + a^2*b^3*e^6)*f)*x^2 - 8*(a^2
*b^3*d^2*e^4 - 2*a^3*b^2*d*e^5 + a^4*b*e^6)*f + (b^5*d^2*e^5 - 2*a*b^4*d*e^
6 + a^2*b^3*e^7 + 16*(a^2*b^3*d^2*e^3 - 2*a^3*b^2*d*e^4 + a^4*b*e^5)*f^2 -
8*(a*b^4*d^2*e^4 - 2*a^2*b^3*d*e^5 + a^3*b^2*e^6)*f)*x), 1/2*(sqrt(-b^2*d*e
^2 + a*b*e^3 + 4*(a*b*d*e - a^2*e^2)*f)*(a*b*e^3 + (b^2*e^2*f + 4*(b^2*d -
2*a*b*e)*f^2)*x^2 + 4*(a*b*d*e - 2*a^2*e^2)*f + (b^2*e^3 + 4*(b^2*d*e - 2*a
*b*e^2)*f)*x)*arctan(-1/2*sqrt(-b^2*d*e^2 + a*b*e^3 + 4*(a*b*d*e - a^2*e^2)
*f)*(2*b*d*e^2 - a*e^3 - 4*a*d*e*f + (b*e^2*f + 4*(b*d - 2*a*e)*f^2)*x^2 +
(b*e^3 + 4*(b*d*e - 2*a*e^2)*f)*x)*sqrt(f*x^2 + e*x + d)/(b^2*d^2*e^3 - a*b
*d*e^4 - 2*(4*(a*b*d*e - a^2*e^2)*f^3 - (b^2*d*e^2 - a*b*e^3)*f^2)*x^3 - 3*
(4*(a*b*d*e^2 - a^2*e^3)*f^2 - (b^2*d*e^3 - a*b*e^4)*f)*x^2 - 4*(a*b*d^2*e^
2 - a^2*d*e^3)*f + (b^2*d*e^4 - a*b*e^5 - 8*(a*b*d^2*e - a^2*d*e^2)*f^2 + 2
*(b^2*d^2*e^2 - 3*a*b*d*e^3 + 2*a^2*e^4)*f)*x)) - 2*(b^3*d*e^3 - a*b^2*e^4
- 4*(a*b^2*d*e^2 - a^2*b*e^3)*f - 2*(4*(a*b^2*d*e - a^2*b*e^2)*f^2 - (b^3*d
*e^2 - a*b^2*e^3)*f)*x)*sqrt(f*x^2 + e*x + d))/(a*b^4*d^2*e^5 - 2*a^2*b^3*d
*e^6 + a^3*b^2*e^7 + 16*(a^3*b^2*d^2*e^3 - 2*a^4*b*d*e^4 + a^5*e^5)*f^2 + (
16*(a^2*b^3*d^2*e^2 - 2*a^3*b^2*d*e^3 + a^4*b*e^4)*f^3 - 8*(a*b^4*d^2*e^3 -
 2*a^2*b^3*d*e^4 + a^3*b^2*e^5)*f^2 + (b^5*d^2*e^4 - 2*a*b^4*d*e^5 + a^2*b^
3*e^6)*f)*x^2 - 8*(a^2*b^3*d^2*e^4 - 2*a^3*b^2*d*e^5 + a^4*b*e^6)*f + (b^5*
d^2*e^5 - 2*a*b^4*d*e^6 + a^2*b^3*e^7 + 16*(a^2*b^3*d^2*e^3 - 2*a^3*b^2*d*e
^4 + a^4*b*e^5)*f^2 - 8*(a*b^4*d^2*e^4 - 2*a^2*b^3*d*e^5 + a^3*b^2*e^6)*f)*
x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*f*x^2+b*e*x+a*e)^2/(f*x^2+e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{%%%{1,[2]%%%},[8,2,0,0,0]%%%}+%%%{%%{[%%%{-4,[1]%%%},0]:[1,0,%%%{-1,
[1]%%%}]%%},[7,2,1,0,0]%%%}+%%%{%%%{6,[1]%%%},[6,2,2,0,0]%%%}+%%%{%%%{-4,[2
]%%%},[6,2,0,0,1]%%%}+%%%{%%%{8,[2]%%%},[6,1,1,1,0]%%%}+%%%{%%{[-4,0]:[1,0,
```

```
%%%{-1,[1]%%%}]%%},[5,2,3,0,0]%%%}+%%%{%%{[%%%{12,[1]%%%},0]:[1,0,%%%{-1,[1
]%%%}]%%},[5,2,1,0,1]%%%}+%%%{%%{[%%%{-24,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%
},[5,1,2,1,0]%%%}+%%%{1,[4,2,4,0,0]%%%}+%%%{%%%{-14,[1]%%%},[4,2,2,0,1]%%%}
+%%%{%%%{6,[2]%%%},[4,2,0,0,2]%%%}+%%%{%%%{26,[1]%%%},[4,1,3,1,0]%%%}+%%%{%
%%{-16,[2]%%%},[4,1,1,1,1]%%%}+%%%{%%%{16,[2]%%%},[4,0,2,2,0]%%%}+%%%{%%{[8
,0]:[1,0,%%%{-1,[1]%%%}]%%},[3,2,3,0,1]%%%}+%%%{%%{[%%%{-12,[1]%%%},0]:[1,0
,%%%{-1,[1]%%%}]%%},[3,2,1,0,2]%%%}+%%%{%%{[-12,0]:[1,0,%%%{-1,[1]%%%}]%%},
[3,1,4,1,0]%%%}+%%%{%%{[%%%{32,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[3,1,2,1,
1]%%%}+%%%{%%{[%%%{-32,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[3,0,3,2,0]%%%}+%
%%{-2,[2,2,4,0,1]%%%}+%%%{%%%{10,[1]%%%},[2,2,2,0,2]%%%}+%%%{%%%{-4,[2]%%%}
,[2,2,0,0,3]%%%}+%%%{2,[2,1,5,1,0]%%%}+%%%{%%%{-28,[1]%%%},[2,1,3,1,1]%%%}+
%%%{%%%{8,[2]%%%},[2,1,1,1,2]%%%}+%%%{%%%{24,[1]%%%},[2,0,4,2,0]%%%}+%%%{%%
{[-4,0]:[1,0,%%%{-1,[1]%%%}]%%},[1,2,3,0,2]%%%}+%%%{%%{[%%%{4,[1]%%%},0]:[1
,0,%%%{-1,[1]%%%}]%%},[1,2,1,0,3]%%%}+%%%{%%{[12,0]:[1,0,%%%{-1,[1]%%%}]%%}
,[1,1,4,1,1]%%%}+%%%{%%{[%%%{-8,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[1,1,2,1
,2]%%%}+%%%{%%{[-8,0]:[1,0,%%%{-1,[1]%%%}]%%},[1,0,5,2,0]%%%}+%%%{1,[0,2,4,
0,2]%%%}+%%%{%%%{-2,[1]%%%},[0,2,2,0,3]%%%}+%%%{%%%{1,[2]%%%},[0,2,0,0,4]%%
%}+%%%{-2,[0,1,5,1,1]%%%}+%%%{%%%{2,[1]%%%},[0,1,3,1,2]%%%}+%%%{1,[0,0,6,2,
0]%%%} / %%%{%%%{1,[3]%%%},[8,2,0,0,0]%%%}+%%%{%%{poly1[%%%{-4,[2]%%%},0]:[
1,0,%%%{-1,[1]%%%}]%%},[7,2,1,0,0]%%%}+%%%{%%%{6,[2]%%%},[6,2,2,0,0]%%%}+%%
%{%%%{-4,[3]%%%},[6,2,0,0,1]%%%}+%%%{%%%{8,[3]%%%},[6,1,1,1,0]%%%}+%%%{%%{p
oly1[%%%{-4,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[5,2,3,0,0]%%%}+%%%{%%{poly1
[%%%{12,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[5,2,1,0,1]%%%}+%%%{%%{poly1[%%%
{-24,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[5,1,2,1,0]%%%}+%%%{%%%{1,[1]%%%},[
4,2,4,0,0]%%%}+%%%{%%%{-14,[2]%%%},[4,2,2,0,1]%%%}+%%%{%%%{6,[3]%%%},[4,2,0
,0,2]%%%}+%%%{%%%{26,[2]%%%},[4,1,3,1,0]%%%}+%%%{%%%{-16,[3]%%%},[4,1,1,1,1
]%%%}+%%%{%%%{16,[3]%%%},[4,0,2,2,0]%%%}+%%%{%%{poly1[%%%{8,[1]%%%},0]:[1,0
,%%%{-1,[1]%%%}]%%},[3,2,3,0,1]%%%}+%%%{%%{poly1[%%%{-12,[2]%%%},0]:[1,0,%%
%{-1,[1]%%%}]%%},[3,2,1,0,2]%%%}+%%%{%%{poly1[%%%{-12,[1]%%%},0]:[1,0,%%%{-
1,[1]%%%}]%%},[3,1,4,1,0]%%%}+%%%{%%{poly1[%%%{32,[2]%%%},0]:[1,0,%%%{-1,[1
]%%%}]%%},[3,1,2,1,1]%%%}+%%%{%%{poly1[%%%{-32,[2]%%%},0]:[1,0,%%%{-1,[1]%%
%}]%%},[3,0,3,2,0]%%%}+%%%{%%%{-2,[1]%%%},[2,2,4,0,1]%%%}+%%%{%%%{10,[2]%%%
},[2,2,2,0,2]%%%}+%%%{%%%{-4,[3]%%%},[2,2,0,0,3]%%%}+%%%{%%%{2,[1]%%%},[2,1
,5,1,0]%%%}+%%%{%%%{-28,[2]%%%},[2,1,3,1,1]%%%}+%%%{%%%{8,[3]%%%},[2,1,1,1,
2]%%%}+%%%{%%%{24,[2]%%%},[2,0,4,2,0]%%%}+%%%{%%{poly1[%%%{-4,[1]%%%},0]:[1
,0,%%%{-1,[1]%%%}]%%},[1,2,3,0,2]%%%}+%%%{%%{poly1[%%%{4,[2]%%%},0]:[1,0,%%
%{-1,[1]%%%}]%%},[1,2,1,0,3]%%%}+%%%{%%{poly1[%%%{12,[1]%%%},0]:[1,0,%%%{-1
,[1]%%%}]%%},[1,1,4,1,1]%%%}+%%%{%%{poly1[%%%{-8,[2]%%%},0]:[1,0,%%%{-1,[1]
%%%}]%%},[1,1,2,1,2]%%%}+%%%{%%{poly1[%%%{-8,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}
]%%},[1,0,5,2,0]%%%}+%%%{%%%{1,[1]%%%},[0,2,4,0,2]%%%}+%%%{%%%{-2,[2]%%%},[
0,2,2,0,3]%%%}+%%%{%%%{1,[3]%%%},[0,2,0,0,4]%%%}+%%%{%%%{-2,[1]%%%},[0,1,5,
1,1]%%%}+%%%{%%%{2,[2]%%%},[0,1,3,1,2]%%%}+%%%{%%%{1,[1]%%%},[0,0,6,2,0]%%%
} Error: Bad Argument Value
```

maple [B] time = 0.03, size = 1377, normalized size = 8.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*f*x^2+b*e*x+a*e)^2/(f*x^2+e*x+d)^(1/2),x)
```

```
[Out] -2/(4*a*f-b*e)/e*f/(-(4*a*f-b*e)*b*e)^(1/2)/(-(a*e-b*d)/b)^(1/2)*ln((-2*(a*
e-b*d)/b+(-(4*a*f-b*e)*b*e)^(1/2)*(x-1/2*(-b*e+(-(4*a*f-b*e)*b*e)^(1/2))/b/
f)/b+2*(-(a*e-b*d)/b)^(1/2)*((x-1/2*(-b*e+(-(4*a*f-b*e)*b*e)^(1/2))/b/f)^2*
f+(-(4*a*f-b*e)*b*e)^(1/2)*(x-1/2*(-b*e+(-(4*a*f-b*e)*b*e)^(1/2))/b/f)/b-(a
*e-b*d)/b)^(1/2))/(x-1/2*(-b*e+(-(4*a*f-b*e)*b*e)^(1/2))/b/f))-1/(4*a*f-b*e
)/e/(a*e-b*d)/(x+1/2/f*e-1/2/b/f*(-(4*a*f-b*e)*b*e)^(1/2))*((x-1/2*(-b*e+(-
(4*a*f-b*e)*b*e)^(1/2))/b/f)^2*f+(-(4*a*f-b*e)*b*e)^(1/2)*(x-1/2*(-b*e+(-(4
*a*f-b*e)*b*e)^(1/2))/b/f)/b-(a*e-b*d)/b)^(1/2)+1/2/(4*a*f-b*e)/b/e*(-(4*a*
f-b*e)*b*e)^(1/2)/(a*e-b*d)/(-(a*e-b*d)/b)^(1/2)*ln((-2*(a*e-b*d)/b+(-(4*a*
f-b*e)*b*e)^(1/2)*(x-1/2*(-b*e+(-(4*a*f-b*e)*b*e)^(1/2))/b/f)/b+2*(-(a*e-b*
d)/b)^(1/2)*((x-1/2*(-b*e+(-(4*a*f-b*e)*b*e)^(1/2))/b/f)^2*f+(-(4*a*f-b*e)*
b*e)^(1/2)*(x-1/2*(-b*e+(-(4*a*f-b*e)*b*e)^(1/2))/b/f)/b-(a*e-b*d)/b)^(1/2)
)/(x-1/2*(-b*e+(-(4*a*f-b*e)*b*e)^(1/2))/b/f))-1/(4*a*f-b*e)/e/(a*e-b*d)/(x
+1/2/f*e+1/2/b/f*(-(4*a*f-b*e)*b*e)^(1/2))*((x+1/2*(b*e+(-(4*a*f-b*e)*b*e)^
(1/2))/b/f)^2*f-(-(4*a*f-b*e)*b*e)^(1/2)*(x+1/2*(b*e+(-(4*a*f-b*e)*b*e)^(1/
2))/b/f)/b-(a*e-b*d)/b)^(1/2)-1/2/(4*a*f-b*e)/b/e*(-(4*a*f-b*e)*b*e)^(1/2)/
(a*e-b*d)/(-(a*e-b*d)/b)^(1/2)*ln((-2*(a*e-b*d)/b-(-(4*a*f-b*e)*b*e)^(1/2)*
(x+1/2*(b*e+(-(4*a*f-b*e)*b*e)^(1/2))/b/f)/b+2*(-(a*e-b*d)/b)^(1/2)*((x+1/2
*(b*e+(-(4*a*f-b*e)*b*e)^(1/2))/b/f)^2*f-(-(4*a*f-b*e)*b*e)^(1/2)*(x+1/2*(b
*e+(-(4*a*f-b*e)*b*e)^(1/2))/b/f)/b-(a*e-b*d)/b)^(1/2))/(x+1/2*(b*e+(-(4*a*
f-b*e)*b*e)^(1/2))/b/f))+2/(4*a*f-b*e)/e*f/(-(4*a*f-b*e)*b*e)^(1/2)/(-(a*e-
b*d)/b)^(1/2)*ln((-2*(a*e-b*d)/b-(-(4*a*f-b*e)*b*e)^(1/2)*(x+1/2*(b*e+(-(4*
a*f-b*e)*b*e)^(1/2))/b/f)/b+2*(-(a*e-b*d)/b)^(1/2)*((x+1/2*(b*e+(-(4*a*f-b*
e)*b*e)^(1/2))/b/f)^2*f-(-(4*a*f-b*e)*b*e)^(1/2)*(x+1/2*(b*e+(-(4*a*f-b*e)*
b*e)^(1/2))/b/f)/b-(a*e-b*d)/b)^(1/2))/(x+1/2*(b*e+(-(4*a*f-b*e)*b*e)^(1/2)
)/b/f))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bfx^2 + bex + ae\right)^2 \sqrt{fx^2 + ex + d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*f*x^2+b*e*x+a*e)^2/(f*x^2+e*x+d)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((b*f*x^2 + b*e*x + a*e)^2*sqrt(f*x^2 + e*x + d)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(b\,f\,x^2+b\,e\,x+a\,e\right)^2\,\sqrt{f\,x^2+e\,x+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*e + b*e*x + b*f*x^2)^2*(d + e*x + f*x^2)^(1/2)),x)
[Out] int(1/((a*e + b*e*x + b*f*x^2)^2*(d + e*x + f*x^2)^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*f*x**2+b*e*x+a*e)**2/(f*x**2+e*x+d)**(1/2),x)
[Out] Timed out
```

3.8 $\int \frac{1}{(4+2x+x^2)\sqrt{5+2x+x^2}}\,dx$

Optimal. Leaf size=28

$$\frac{\tan^{-1}\left(\frac{x+1}{\sqrt{3}\sqrt{x^2+2x+5}}\right)}{\sqrt{3}}$$

Rubi [A] time = 0.02, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {982, 204}

$$\frac{\tan^{-1}\left(\frac{x+1}{\sqrt{3}\sqrt{x^2+2x+5}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((4 + 2*x + x^2)*Sqrt[5 + 2*x + x^2]),x]
```

```
[Out] ArcTan[(1 + x)/(Sqrt[3]*Sqrt[5 + 2*x + x^2])]/Sqrt[3]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 982

```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(
x_)^2]), x_Symbol] :> Dist[-2*e, Subst[Int[1/(e*(b*e - 4*a*f) - (b*d - a*e)
*x^2), x], x, (e + 2*f*x)/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0
]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(4+2x+x^2)\sqrt{5+2x+x^2}}\,dx &= -\left(4\,\text{Subst}\left(\int \frac{1}{-24-2x^2}\,dx, x, \frac{2+2x}{\sqrt{5+2x+x^2}}\right)\right)\\ &= \frac{\tan^{-1}\left(\frac{1+x}{\sqrt{3}\sqrt{5+2x+x^2}}\right)}{\sqrt{3}}\end{aligned}$$

Mathematica [C] time = 0.07, size = 84, normalized size = 3.00

$$-\frac{i\left(\tanh^{-1}\left(\frac{-i\sqrt{3}\,x-i\sqrt{3}+4}{\sqrt{x^2+2x+5}}\right)-\tanh^{-1}\left(\frac{i\sqrt{3}\,x+i\sqrt{3}+4}{\sqrt{x^2+2x+5}}\right)\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((4 + 2*x + x^2)*Sqrt[5 + 2*x + x^2]),x]
```

```
[Out] ((-1/2*I)*(ArcTanh[(4 - I*Sqrt[3] - I*Sqrt[3]*x)/Sqrt[5 + 2*x + x^2]] - Arc
Tanh[(4 + I*Sqrt[3] + I*Sqrt[3]*x)/Sqrt[5 + 2*x + x^2]]))/Sqrt[3]
```

IntegrateAlgebraic [A] time = 0.28, size = 55, normalized size = 1.96

$$-\frac{\tan^{-1}\left(\frac{x^2}{\sqrt{3}}-\frac{(x+1)\sqrt{x^2+2x+5}}{\sqrt{3}}+\frac{2x}{\sqrt{3}}+\frac{4}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((4 + 2*x + x^2)*Sqrt[5 + 2*x + x^2]),x]
```

```
[Out] -(ArcTan[4/Sqrt[3] + (2*x)/Sqrt[3] + x^2/Sqrt[3] - ((1 + x)*Sqrt[5 + 2*x +
x^2])/Sqrt[3]]/Sqrt[3])
```

fricas [A] time = 0.40, size = 38, normalized size = 1.36

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\sqrt{x^2+2x+5}\,(x+1)-\frac{1}{3}\sqrt{3}\left(x^2+2x+4\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+2*x+4)/(x^2+2*x+5)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*sqrt(x^2 + 2*x + 5)*(x + 1) - 1/3*sqrt(3)*(x
^2 + 2*x + 4))
```

giac [B] time = 0.22, size = 52, normalized size = 1.86

$$-\frac{1}{3}\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}\left(x-\sqrt{x^2+2x+5}+2\right)\right)+\frac{1}{3}\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}\left(x-\sqrt{x^2+2x+5}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+2*x+4)/(x^2+2*x+5)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*arctan(-1/3*sqrt(3)*(x - sqrt(x^2 + 2*x + 5) + 2)) + 1/3*sqrt(
3)*arctan(-1/3*sqrt(3)*(x - sqrt(x^2 + 2*x + 5)))
```

maple [A] time = 0.02, size = 27, normalized size = 0.96

$$\frac{\sqrt{3}\ \arctan\left(\frac{\sqrt{3}\ (2x+2)}{6\sqrt{x^2+2x+5}}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+2*x+4)/(x^2+2*x+5)^(1/2),x)
```

```
[Out] 1/3*3^(1/2)*arctan(1/6*3^(1/2)/(x^2+2*x+5)^(1/2)*(2*x+2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^2+2\,x+5}\left(x^2+2\,x+4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+2*x+4)/(x^2+2*x+5)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^2 + 2*x + 5)*(x^2 + 2*x + 4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\left(x^2+2\,x+4\right)\,\sqrt{x^2+2\,x+5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*x + x^2 + 4)*(2*x + x^2 + 5)^(1/2)),x)
```

```
[Out] int(1/((2*x + x^2 + 4)*(2*x + x^2 + 5)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^2+2x+4\right)\sqrt{x^2+2x+5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+2*x+4)/(x**2+2*x+5)**(1/2),x)
```

```
[Out] Integral(1/((x**2 + 2*x + 4)*sqrt(x**2 + 2*x + 5)), x)
```

3.9 $\int \frac{\sqrt{1+2x+x^2}}{\sqrt{1+x^2}} \, dx$

Optimal. Leaf size=48

$$\frac{\sqrt{x^2+1}\sqrt{x^2+2x+1}}{x+1}+\frac{\sqrt{x^2+2x+1}\sinh^{-1}(x)}{x+1}$$

Rubi [A] time = 0.02, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {970, 641, 215}

$$\frac{\sqrt{x^2+1}\sqrt{x^2+2x+1}}{x+1}+\frac{\sqrt{x^2+2x+1}\sinh^{-1}(x)}{x+1}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + 2*x + x^2]/Sqrt[1 + x^2],x]
```

```
[Out] (Sqrt[1 + x^2]*Sqrt[1 + 2*x + x^2])/(1 + x) + (Sqrt[1 + 2*x + x^2]*ArcSinh[
x])/(1 + x)
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 970

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q_.), x_
Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((4*c)^IntPart[p]*(b + 2*c*x)
^(2*FracPart[p])), Int[(b + 2*c*x)^(2*p)*(d + f*x^2)^q, x], x] /; FreeQ[{a,
 b, c, d, f, p, q}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\sqrt{1+2x+x^2}}{\sqrt{1+x^2}}\,dx = \frac{\sqrt{1+2x+x^2}\int \frac{2+2x}{\sqrt{1+x^2}}\,dx}{2+2x}$$
$$= \frac{\sqrt{1+x^2}\sqrt{1+2x+x^2}}{1+x} + \frac{\left(2\sqrt{1+2x+x^2}\right)\int \frac{1}{\sqrt{1+x^2}}\,dx}{2+2x}$$
$$= \frac{\sqrt{1+x^2}\sqrt{1+2x+x^2}}{1+x} + \frac{\sqrt{1+2x+x^2}\sinh^{-1}(x)}{1+x}$$

Mathematica [A] time = 0.02, size = 27, normalized size = 0.56

$$\frac{\sqrt{(x+1)^2}\left(\sqrt{x^2+1}+\sinh^{-1}(x)\right)}{x+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + 2*x + x^2]/Sqrt[1 + x^2],x]
[Out] (Sqrt[(1 + x)^2]*(Sqrt[1 + x^2] + ArcSinh[x]))/(1 + x)
```

IntegrateAlgebraic [A] time = 0.14, size = 41, normalized size = 0.85

$$\frac{\sqrt{(x+1)^2}\left(\sqrt{x^2+1}-\log\left(\sqrt{x^2+1}-x\right)\right)}{x+1}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 + 2*x + x^2]/Sqrt[1 + x^2],x]
[Out] (Sqrt[(1 + x)^2]*(Sqrt[1 + x^2] - Log[-x + Sqrt[1 + x^2]]))/(1 + x)
```

fricas [A] time = 0.40, size = 22, normalized size = 0.46

$$\sqrt{x^2+1}-\log\left(-x+\sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)^2)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")
[Out] sqrt(x^2 + 1) - log(-x + sqrt(x^2 + 1))
```

giac [A] time = 0.23, size = 49, normalized size = 1.02

$$-\left(\sqrt{2}-\log\left(\sqrt{2}+1\right)\right)\operatorname{sgn}\left(x+1\right)-\log\left(-x+\sqrt{x^2+1}\right)\operatorname{sgn}\left(x+1\right)+\sqrt{x^2+1}\operatorname{sgn}\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)^2)^(1/2)/(x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -(sqrt(2) - log(sqrt(2) + 1))*sgn(x + 1) - log(-x + sqrt(x^2 + 1))*sgn(x +
1) + sqrt(x^2 + 1)*sgn(x + 1)
```

maple [C] time = 0.05, size = 16, normalized size = 0.33

$$\left(\operatorname{arcsinh}(x)+\sqrt{x^2+1}\right)\operatorname{csgn}\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+x)^2)^(1/2)/(x^2+1)^(1/2),x)
```

```
[Out] csgn(1+x)*(arcsinh(x)+(x^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{(x+1)^2}}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)^2)^(1/2)/(x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt((x + 1)^2)/sqrt(x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{\sqrt{(x+1)^2}}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x + 1)^2)^(1/2)/(x^2 + 1)^(1/2),x)
```

```
[Out] int(((x + 1)^2)^(1/2)/(x^2 + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(x+1)^2}}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)**2)**(1/2)/(x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt((x + 1)**2)/sqrt(x**2 + 1), x)
```

3.10 $\int \frac{1}{\left(-1+x^2\right)^2 \sqrt{-1+x+x^2}} \, dx$

Optimal. Leaf size=70

$$\frac{\sqrt{x^2+x-1}}{2\left(1-x^2\right)}-\frac{1}{8}\tan^{-1}\left(\frac{x+3}{2\sqrt{x^2+x-1}}\right)-\frac{5}{8}\tanh^{-1}\left(\frac{1-3x}{2\sqrt{x^2+x-1}}\right)$$

Rubi [A] time = 0.05, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {976, 1033, 724, 206, 204}

$$\frac{\sqrt{x^2+x-1}}{2\left(1-x^2\right)}-\frac{1}{8}\tan^{-1}\left(\frac{x+3}{2\sqrt{x^2+x-1}}\right)-\frac{5}{8}\tanh^{-1}\left(\frac{1-3x}{2\sqrt{x^2+x-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/((-1 + x^2)^2*Sqrt[-1 + x + x^2]),x]
```

```
[Out] Sqrt[-1 + x + x^2]/(2*(1 - x^2)) - ArcTan[(3 + x)/(2*Sqrt[-1 + x + x^2])]/8
 - (5*ArcTanh[(1 - 3*x)/(2*Sqrt[-1 + x + x^2])])/8
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 976

```
Int[((a_.) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[((2*a*c^2*e + c*(2*c^2*d - c*(2*a*f))*x)*(a + c*x^2)^(p +
```

```
1)*(d + e*x + f*x^2)^(q + 1))/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1)),
 x] - Dist[1/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1)), Int[(a + c*x^2)^
(p + 1)*(d + e*x + f*x^2)^q*Simp[2*c*((c*d - a*f)^2 - (-(a*e))*(c*e))*(p +
1) - (2*c^2*d - c*(2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(-2*a*c^2*e)*(p
+ q + 2) + (2*f*(2*a*c^2*e)*(p + q + 2) - (2*c^2*d - c*(2*a*f))*(-(c*e*(2*p
 + q + 4))))*x + c*f*(2*c^2*d - c*(2*a*f))*(2*p + 2*q + 5)*x^2, x], x], x]
/; FreeQ[{a, c, d, e, f, q}, x] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ
[a*c*e^2 + (c*d - a*f)^2, 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[
q, 0]
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rubi steps

$$\int \frac{1}{\left(-1+x^2\right)^2\sqrt{-1+x+x^2}}\,dx = \frac{\sqrt{-1+x+x^2}}{2\left(1-x^2\right)} - \frac{1}{4}\int \frac{3+2x}{\left(-1+x^2\right)\sqrt{-1+x+x^2}}\,dx$$

$$= \frac{\sqrt{-1+x+x^2}}{2\left(1-x^2\right)} + \frac{1}{8}\int \frac{1}{(1+x)\sqrt{-1+x+x^2}}\,dx - \frac{5}{8}\int \frac{1}{(-1+x)\sqrt{-1+x+x^2}}$$

$$= \frac{\sqrt{-1+x+x^2}}{2\left(1-x^2\right)} - \frac{1}{4}\,\mathrm{Subst}\left(\int \frac{1}{-4-x^2}\,dx, x, \frac{-3-x}{\sqrt{-1+x+x^2}}\right) + \frac{5}{4}\,\mathrm{Subst}\left(\int \frac{}{4}\right.$$

$$= \frac{\sqrt{-1+x+x^2}}{2\left(1-x^2\right)} - \frac{1}{8}\tan^{-1}\left(\frac{3+x}{2\sqrt{-1+x+x^2}}\right) - \frac{5}{8}\tanh^{-1}\left(\frac{1-3x}{2\sqrt{-1+x+x^2}}\right)$$

Mathematica [A] time = 0.09, size = 66, normalized size = 0.94

$$\frac{1}{8}\left(-\frac{4\sqrt{x^2+x-1}}{x^2-1} - \tan^{-1}\left(\frac{x+3}{2\sqrt{x^2+x-1}}\right) - 5\tanh^{-1}\left(\frac{1-3x}{2\sqrt{x^2+x-1}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((-1 + x^2)^2*Sqrt[-1 + x + x^2]),x]
```

```
[Out] ((-4*Sqrt[-1 + x + x^2])/(-1 + x^2) - ArcTan[(3 + x)/(2*Sqrt[-1 + x + x^2])
] - 5*ArcTanh[(1 - 3*x)/(2*Sqrt[-1 + x + x^2])])/8
```

IntegrateAlgebraic [A] time = 0.28, size = 62, normalized size = 0.89

$$-\frac{\sqrt{x^2+x-1}}{2\left(x^2-1\right)}-\frac{1}{4}\tan^{-1}\left(-\sqrt{x^2+x-1}+x+1\right)+\frac{5}{4}\tanh^{-1}\left(\sqrt{x^2+x-1}-x+1\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((-1 + x^2)^2*Sqrt[-1 + x + x^2]),x]
```

```
[Out] -1/2*Sqrt[-1 + x + x^2]/(-1 + x^2) - ArcTan[1 + x - Sqrt[-1 + x + x^2]]/4 +
 (5*ArcTanh[1 - x + Sqrt[-1 + x + x^2]])/4
```

fricas [A] time = 0.41, size = 82, normalized size = 1.17

$$\frac{2\left(x^2-1\right)\arctan\left(-x+\sqrt{x^2+x-1}-1\right)+5\left(x^2-1\right)\log\left(-x+\sqrt{x^2+x-1}+2\right)-5\left(x^2-1\right)\log\left(-x+\sqrt{x^2+x-1}\right)-4\sqrt{x^2+x-1}}{8\left(x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^2/(x^2+x-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*(2*(x^2 - 1)*arctan(-x + sqrt(x^2 + x - 1) - 1) + 5*(x^2 - 1)*log(-x +
sqrt(x^2 + x - 1) + 2) - 5*(x^2 - 1)*log(-x + sqrt(x^2 + x - 1)) - 4*sqrt(x
^2 + x - 1))/(x^2 - 1)
```

giac [B] time = 0.26, size = 143, normalized size = 2.04

$$\frac{2\left(x-\sqrt{x^2+x-1}\right)^3+3\left(x-\sqrt{x^2+x-1}\right)^2-x+\sqrt{x^2+x-1}-1}{2\left(\left(x-\sqrt{x^2+x-1}\right)^4-2\left(x-\sqrt{x^2+x-1}\right)^2-4x+4\sqrt{x^2+x-1}\right)}+\frac{1}{4}\arctan\left(-x+\sqrt{x^2+x-1}-1\right)+\frac{5}{8}\log\left(\left|-x+\sqrt{x^2+x-1}+2\right|\right)-\frac{5}{8}\log\left(\left|-x+\sqrt{x^2+x-1}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^2/(x^2+x-1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(2*(x - sqrt(x^2 + x - 1))^3 + 3*(x - sqrt(x^2 + x - 1))^2 - x + sqrt(x
^2 + x - 1) - 1)/((x - sqrt(x^2 + x - 1))^4 - 2*(x - sqrt(x^2 + x - 1))^2 -
 4*x + 4*sqrt(x^2 + x - 1)) + 1/4*arctan(-x + sqrt(x^2 + x - 1) - 1) + 5/8*
log(abs(-x + sqrt(x^2 + x - 1) + 2)) - 5/8*log(abs(-x + sqrt(x^2 + x - 1)))
```

maple [A] time = 0.02, size = 84, normalized size = 1.20

$$\frac{5\,\mathrm{arctanh}\left(\frac{3x-1}{2\sqrt{3x+(x-1)^2-2}}\right)}{8}+\frac{\arctan\left(\frac{-x-3}{2\sqrt{-x+(x+1)^2-2}}\right)}{8}+\frac{\sqrt{-x+(x+1)^2-2}}{4x+4}-\frac{\sqrt{3x+(x-1)^2-2}}{4\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2-1)^2/(x^2+x-1)^(1/2),x)
```

```
[Out] 1/4/(x+1)*((x+1)^2-2-x)^(1/2)+1/8*arctan(1/2*(-3-x)/((x+1)^2-2-x)^(1/2))+5/
8*arctanh(1/2*(-1+3*x)/((x-1)^2+3*x-2)^(1/2))-1/4/(x-1)*((x-1)^2+3*x-2)^(1/
2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^2+x-1}\left(x^2-1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^2/(x^2+x-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^2 + x - 1)*(x^2 - 1)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(x^2-1\right)^2\sqrt{x^2+x-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 - 1)^2*(x + x^2 - 1)^(1/2)),x)
```

```
[Out] int(1/((x^2 - 1)^2*(x + x^2 - 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x-1)^2\,(x+1)^2\,\sqrt{x^2+x-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2-1)**2/(x**2+x-1)**(1/2),x)
```

```
[Out] Integral(1/((x - 1)**2*(x + 1)**2*sqrt(x**2 + x - 1)), x)
```

3.11 $\int \frac{\sqrt{-3-4x-x^2}}{3+4x+2x^2}\,dx$

Optimal. Leaf size=98

$$-\frac{\tan^{-1}\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right)}{\sqrt{2}}+\frac{\tan^{-1}\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)-\frac{1}{2}\sin^{-1}(x+2)$$

Rubi [A] time = 0.21, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {989, 619, 216, 1028, 986, 12, 1026, 1161, 618, 204, 1027, 206}

$$-\frac{\tan^{-1}\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right)}{\sqrt{2}}+\frac{\tan^{-1}\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)-\frac{1}{2}\sin^{-1}(x+2)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-3 - 4*x - x^2]/(3 + 4*x + 2*x^2),x]
```

```
[Out] -ArcSin[2 + x]/2 - ArcTan[(1 - (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]]/Sqrt[
2] + ArcTan[(1 + (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]]/Sqrt[2] - ArcTanh[x
/Sqrt[-3 - 4*x - x^2]]/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 986

```
Int[1/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*
(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f)
, 2]}, Dist[1/(2*q), Int[(c*d - a*f + q + (c*e - b*f)*x)/((a + b*x + c*x^2)
*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[(c*d - a*f - q + (c*e -
 b*f)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b,
 c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[c*e -
 b*f, 0] && NegQ[b^2 - 4*a*c]
```

Rule 989

```
Int[Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]/((d_) + (e_.)*(x_) + (f_.)*(x_)^
2), x_Symbol] :> Dist[c/f, Int[1/Sqrt[a + b*x + c*x^2], x], x] - Dist[1/f,
Int[(c*d - a*f + (c*e - b*f)*x)/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 -
4*d*f, 0]
```

Rule 1026

```
Int[(x_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_.)*(x_) + (f_.)
*(x_)^2]), x_Symbol] :> Dist[-2*e, Subst[Int[(1 - d*x^2)/(c*e - b*f - e*(2*
c*d - b*e + 2*a*f)*x^2 + d^2*(c*e - b*f)*x^4), x], x, (1 + ((e + Sqrt[e^2 -
 4*d*f])*x)/(2*d))/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f},
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0]
```

Rule 1027

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_
.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[g, Subst[Int[1/(a + (c*d - a*f)
```

```
*x^2), x], x, x/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h
}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0] &&
 EqQ[2*h*d - g*e, 0]
```

Rule 1028

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_
.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> -Dist[(2*h*d - g*e)/e, Int[1/((a + b
*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/e, Int[(2*d + e*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, g
, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0]
 && NeQ[2*h*d - g*e, 0]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rubi steps

$$\int \frac{\sqrt{-3-4x-x^2}}{3+4x+2x^2}\,dx = -\left(\frac{1}{2}\int \frac{1}{\sqrt{-3-4x-x^2}}\,dx\right) - \frac{1}{2}\int \frac{3+4x}{\sqrt{-3-4x-x^2}\,\left(3+4x+2x^2\right)}\,dx$$

$$= \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{4}}}\,dx, x, -4-2x\right) + \frac{1}{2}\int \frac{-6-4x}{\sqrt{-3-4x-x^2}\,\left(3+4x+2x^2\right)}\,dx + \frac{3}{2}\int$$

$$= -\frac{1}{2}\sin^{-1}(2+x) - \frac{1}{4}\int \frac{-6-4x}{\sqrt{-3-4x-x^2}\,\left(3+4x+2x^2\right)}\,dx + \frac{1}{4}\int -\frac{4x}{\sqrt{-3-4x-x^2}\,\left(3+\right.}$$

$$= -\frac{1}{2}\sin^{-1}(2+x) - \tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) + \frac{3}{2}\,\text{Subst}\left(\int \frac{1}{3-3x^2}\,dx, x, \frac{x}{\sqrt{-3-4x-x}}\right.$$

$$= -\frac{1}{2}\sin^{-1}(2+x) - \frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) - 8\,\text{Subst}\left(\int \frac{1+3x^2}{-4-8x^2-36x^4}\,dx, x, \right.$$

$$= -\frac{1}{2}\sin^{-1}(2+x) - \frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) + \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{\frac{1}{3}-\frac{2x}{3}+x^2}\,dx, x, \frac{1-}{\sqrt{-3-}}\right.$$

$$= -\frac{1}{2}\sin^{-1}(2+x) - \frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) - \frac{2}{3}\,\text{Subst}\left(\int \frac{1}{-\frac{8}{9}-x^2}\,dx, x, \frac{2}{3}\left(-1+\right.\right.$$

$$= -\frac{1}{2}\sin^{-1}(2+x) - \frac{\tan^{-1}\left(\frac{1-\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right)}{\sqrt{2}} + \frac{\tan^{-1}\left(\frac{1+\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right)}{\sqrt{2}} - \frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-}}\right.$$

Mathematica [C] time = 0.39, size = 159, normalized size = 1.62

$$\frac{1}{4}\left(-i\sqrt{1-2i\sqrt{2}}\,\tanh^{-1}\left(\frac{i\sqrt{2}\,x+2x+2i\sqrt{2}+2}{\sqrt{2-4i\sqrt{2}}\,\sqrt{-x^2-4x-3}}\right) + i\sqrt{1+2i\sqrt{2}}\,\tanh^{-1}\left(\frac{\left(2-i\sqrt{2}\right)x-2i\sqrt{2}+2}{\sqrt{2+4i\sqrt{2}}\,\sqrt{-x^2-4x-3}}\right) - 2\sin^{-1}(x+2)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-3 - 4*x - x^2]/(3 + 4*x + 2*x^2),x]
```

```
[Out] (-2*ArcSin[2 + x] - I*Sqrt[1 - (2*I)*Sqrt[2]]*ArcTanh[(2 + (2*I)*Sqrt[2] +
2*x + I*Sqrt[2]*x)/(Sqrt[2 - (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2])] + I*Sqrt
[1 + (2*I)*Sqrt[2]]*ArcTanh[(2 - (2*I)*Sqrt[2] + (2 - I*Sqrt[2])*x)/(Sqrt[2
 + (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2])])/4
```

IntegrateAlgebraic [A] time = 0.29, size = 80, normalized size = 0.82

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\,x+\frac{3}{\sqrt{2}}}{\sqrt{-x^2-4x-3}}\right)}{\sqrt{2}}+\tan^{-1}\left(\frac{\sqrt{-x^2-4x-3}}{x+3}\right)-\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[-3 - 4*x - x^2]/(3 + 4*x + 2*x^2),x]
```

```
[Out] ArcTan[(3/Sqrt[2] + Sqrt[2]*x)/Sqrt[-3 - 4*x - x^2]]/Sqrt[2] + ArcTan[Sqrt[
-3 - 4*x - x^2]/(3 + x)] - ArcTanh[x/Sqrt[-3 - 4*x - x^2]]/2
```

fricas [A] time = 0.42, size = 161, normalized size = 1.64

$$-\frac{1}{4}\sqrt{2}\arctan\left(\frac{\sqrt{2}\,x+3\sqrt{2}\sqrt{-x^2-4x-3}}{2(2x+3)}\right)-\frac{1}{4}\sqrt{2}\arctan\left(-\frac{\sqrt{2}\,x-3\sqrt{2}\sqrt{-x^2-4x-3}}{2(2x+3)}\right)+\frac{1}{2}\arctan\left(\frac{\sqrt{-x^2-4x-3}\,(x+2)}{x^2+4x+3}\right)+\frac{1}{8}\log\left(-\frac{2\sqrt{-x^2-4x-3}\,x+4x+3}{x^2}\right)-\frac{1}{8}\log\left(\frac{2\sqrt{-x^2-4x-3}\,x-4x-3}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2-4*x-3)^(1/2)/(2*x^2+4*x+3),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(2)*arctan(1/2*(sqrt(2)*x + 3*sqrt(2)*sqrt(-x^2 - 4*x - 3))/(2*x +
 3)) - 1/4*sqrt(2)*arctan(-1/2*(sqrt(2)*x - 3*sqrt(2)*sqrt(-x^2 - 4*x - 3))
/(2*x + 3)) + 1/2*arctan(sqrt(-x^2 - 4*x - 3)*(x + 2)/(x^2 + 4*x + 3)) + 1/
8*log(-(2*sqrt(-x^2 - 4*x - 3)*x + 4*x + 3)/x^2) - 1/8*log((2*sqrt(-x^2 - 4
*x - 3)*x - 4*x - 3)/x^2)
```

giac [B] time = 0.21, size = 171, normalized size = 1.74

$$-\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{3\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+1\right)\right)-\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{\sqrt{-x^2-4x-3}-1}{x+2}+1\right)\right)-\frac{1}{2}\arcsin(x+2)-\frac{1}{4}\log\left(\frac{2\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+\frac{3\left(\sqrt{-x^2-4x-3}-1\right)^2}{(x+2)^2}+1\right)+\frac{1}{4}\log\left(\frac{2\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+\frac{\left(\sqrt{-x^2-4x-3}-1\right)^2}{(x+2)^2}+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2-4*x-3)^(1/2)/(2*x^2+4*x+3),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*arctan(1/2*sqrt(2)*(3*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1))
 - 1/2*sqrt(2)*arctan(1/2*sqrt(2)*((sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1))
 - 1/2*arcsin(x + 2) - 1/4*log(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 3*(sq
rt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 1) + 1/4*log(2*(sqrt(-x^2 - 4*x - 3)
- 1)/(x + 2) + (sqrt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 3)
```

maple [B] time = 0.03, size = 341, normalized size = 3.48

$$-\frac{\arcsin(x+2)}{2}-\frac{\sqrt{3}\,\sqrt{4}\,\sqrt{\frac{3x^2}{\left(-x-\frac{3}{2}\right)^2}-12}\,\sqrt{2}\,\arctan\left(\frac{\sqrt{\frac{3x^2}{\left(-x-\frac{3}{2}\right)^2}-12}\,\sqrt{2}}{6}\right)}{3\sqrt{\frac{\frac{x^2}{\left(-x-\frac{3}{2}\right)^2}-4}{\left(\frac{x}{-x-\frac{3}{2}}+1\right)^2}}\left(\frac{x}{-x-\frac{3}{2}}+1\right)}+\frac{\sqrt{3}\,\sqrt{4}\,\sqrt{\frac{3x^2}{\left(-x-\frac{3}{2}\right)^2}-12}\left(-\operatorname{arctanh}\left(\frac{3x}{\left(-x-\frac{3}{2}\right)\sqrt{\frac{3x^2}{\left(-x-\frac{3}{2}\right)^2}-12}}\right)+\sqrt{2}\,\arctan\left(\frac{\sqrt{\frac{3x^2}{\left(-x-\frac{3}{2}\right)^2}-12}\,\sqrt{2}}{6}\right)\right)}{12\sqrt{\frac{\frac{x^2}{\left(-x-\frac{3}{2}\right)^2}-4}{\left(\frac{x}{-x-\frac{3}{2}}+1\right)^2}}\left(\frac{x}{-x-\frac{3}{2}}+1\right)}+\frac{\sqrt{3}\,\sqrt{4}\,\sqrt{\frac{3x^2}{\left(-x-\frac{3}{2}\right)^2}-12}\left(\operatorname{arctanh}\left(\frac{3x}{\left(-x-\frac{3}{2}\right)\sqrt{\frac{3x^2}{\left(-x-\frac{3}{2}\right)^2}-12}}\right)+\sqrt{2}\,\arctan\left(\frac{\sqrt{\frac{3x^2}{\left(-x-\frac{3}{2}\right)^2}-12}\,\sqrt{2}}{6}\right)\right)}{6\sqrt{\frac{\frac{x^2}{\left(-x-\frac{3}{2}\right)^2}-4}{\left(\frac{x}{-x-\frac{3}{2}}+1\right)^2}}\left(\frac{x}{-x-\frac{3}{2}}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2-4*x-3)^(1/2)/(2*x^2+4*x+3),x)
```

```
[Out] -1/2*arcsin(2+x)+1/12*3^(1/2)*4^(1/2)*(3*x^2/(-3/2-x)^2-12)^(1/2)*(2^(1/2)*
arctan(1/6*(3*x^2/(-3/2-x)^2-12)^(1/2)*2^(1/2))-arctanh(3*x/(-3/2-x)/(3*x^2
/(-3/2-x)^2-12)^(1/2)))/((x^2/(-3/2-x)^2-4)/(1+x/(-3/2-x))^2)^(1/2)/(1+x/(-
3/2-x))-1/3*3^(1/2)*4^(1/2)/((x^2/(-3/2-x)^2-4)/(1+x/(-3/2-x))^2)^(1/2)/(1+
x/(-3/2-x))*(3*x^2/(-3/2-x)^2-12)^(1/2)*2^(1/2)*arctan(1/6*(3*x^2/(-3/2-x)^
2-12)^(1/2)*2^(1/2))+1/6*3^(1/2)*4^(1/2)*(3*x^2/(-3/2-x)^2-12)^(1/2)*(2^(1/
2)*arctan(1/6*(3*x^2/(-3/2-x)^2-12)^(1/2)*2^(1/2))+arctanh(3*x/(-3/2-x)/(3*
x^2/(-3/2-x)^2-12)^(1/2)))/((x^2/(-3/2-x)^2-4)/(1+x/(-3/2-x))^2)^(1/2)/(1+x
/(-3/2-x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-x^2-4\,x-3}}{2\,x^2+4\,x+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2-4*x-3)^(1/2)/(2*x^2+4*x+3),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^2 - 4*x - 3)/(2*x^2 + 4*x + 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{-x^2-4\,x-3}}{2\,x^2+4\,x+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((- 4*x - x^2 - 3)^(1/2)/(4*x + 2*x^2 + 3),x)
```

```
[Out] int((- 4*x - x^2 - 3)^(1/2)/(4*x + 2*x^2 + 3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(x+1)(x+3)}}{2x^2+4x+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2-4*x-3)**(1/2)/(2*x**2+4*x+3),x)
```

```
[Out] Integral(sqrt(-(x + 1)*(x + 3))/(2*x**2 + 4*x + 3), x)
```

3.12 $\int \left(3-x+2x^2\right)\left(2+3x+5x^2\right)^4\,dx$

Optimal. Leaf size=68

$$\frac{1250x^{11}}{11}+\frac{475x^{10}}{2}+\frac{5075x^9}{9}+\frac{3415x^8}{4}+1176x^7+\frac{2377x^6}{2}+\frac{5099x^5}{5}+656x^4+\frac{1064x^3}{3}+136x^2+48x$$

Rubi [A] time = 0.05, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1657}

$$\frac{1250x^{11}}{11}+\frac{475x^{10}}{2}+\frac{5075x^9}{9}+\frac{3415x^8}{4}+1176x^7+\frac{2377x^6}{2}+\frac{5099x^5}{5}+656x^4+\frac{1064x^3}{3}+136x^2+48x$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)*(2 + 3*x + 5*x^2)^4,x]
```

```
[Out] 48*x + 136*x^2 + (1064*x^3)/3 + 656*x^4 + (5099*x^5)/5 + (2377*x^6)/2 + 117
6*x^7 + (3415*x^8)/4 + (5075*x^9)/9 + (475*x^10)/2 + (1250*x^11)/11
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \left(3-x+2x^2\right)\left(2+3x+5x^2\right)^4\,dx = \int \left(48+272x+1064x^2+2624x^3+5099x^4+7131x^5+8232x^6+6830x^7\right.$$

$$= 48x+136x^2+\frac{1064x^3}{3}+656x^4+\frac{5099x^5}{5}+\frac{2377x^6}{2}+1176x^7+\frac{3415x^8}{4}$$

Mathematica [A] time = 0.00, size = 68, normalized size = 1.00

$$\frac{1250x^{11}}{11}+\frac{475x^{10}}{2}+\frac{5075x^9}{9}+\frac{3415x^8}{4}+1176x^7+\frac{2377x^6}{2}+\frac{5099x^5}{5}+656x^4+\frac{1064x^3}{3}+136x^2+48x$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)*(2 + 3*x + 5*x^2)^4,x]
```

```
[Out] 48*x + 136*x^2 + (1064*x^3)/3 + 656*x^4 + (5099*x^5)/5 + (2377*x^6)/2 + 117
6*x^7 + (3415*x^8)/4 + (5075*x^9)/9 + (475*x^10)/2 + (1250*x^11)/11
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(3-x+2x^2\right)\left(2+3x+5x^2\right)^4\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(3 - x + 2*x^2)*(2 + 3*x + 5*x^2)^4,x]
```

```
[Out] IntegrateAlgebraic[(3 - x + 2*x^2)*(2 + 3*x + 5*x^2)^4, x]
```

fricas [A] time = 0.34, size = 54, normalized size = 0.79

$$\frac{1250}{11}x^{11}+\frac{475}{2}x^{10}+\frac{5075}{9}x^9+\frac{3415}{4}x^8+1176x^7+\frac{2377}{2}x^6+\frac{5099}{5}x^5+656x^4+\frac{1064}{3}x^3+136x^2+48x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)*(5*x^2+3*x+2)^4,x, algorithm="fricas")
```

```
[Out] 1250/11*x^11 + 475/2*x^10 + 5075/9*x^9 + 3415/4*x^8 + 1176*x^7 + 2377/2*x^6
 + 5099/5*x^5 + 656*x^4 + 1064/3*x^3 + 136*x^2 + 48*x
```

giac [A] time = 0.21, size = 54, normalized size = 0.79

$$\frac{1250}{11}\,x^{11}+\frac{475}{2}\,x^{10}+\frac{5075}{9}\,x^9+\frac{3415}{4}\,x^8+1176\,x^7+\frac{2377}{2}\,x^6+\frac{5099}{5}\,x^5+656\,x^4+\frac{1064}{3}\,x^3+136\,x^2+48\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)*(5*x^2+3*x+2)^4,x, algorithm="giac")
```

```
[Out] 1250/11*x^11 + 475/2*x^10 + 5075/9*x^9 + 3415/4*x^8 + 1176*x^7 + 2377/2*x^6
 + 5099/5*x^5 + 656*x^4 + 1064/3*x^3 + 136*x^2 + 48*x
```

maple [A] time = 0.00, size = 55, normalized size = 0.81

$$\frac{1250}{11}x^{11}+\frac{475}{2}x^{10}+\frac{5075}{9}x^9+\frac{3415}{4}x^8+1176x^7+\frac{2377}{2}x^6+\frac{5099}{5}x^5+656x^4+\frac{1064}{3}x^3+136x^2+48x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)*(5*x^2+3*x+2)^4,x)
```

```
[Out] 48*x+136*x^2+1064/3*x^3+656*x^4+5099/5*x^5+2377/2*x^6+1176*x^7+3415/4*x^8+5
075/9*x^9+475/2*x^10+1250/11*x^11
```

maxima [A] time = 0.46, size = 54, normalized size = 0.79

$$\frac{1250}{11}\,x^{11}+\frac{475}{2}\,x^{10}+\frac{5075}{9}\,x^{9}+\frac{3415}{4}\,x^{8}+1176\,x^{7}+\frac{2377}{2}\,x^{6}+\frac{5099}{5}\,x^{5}+656\,x^{4}+\frac{1064}{3}\,x^{3}+136\,x^{2}+48\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)*(5*x^2+3*x+2)^4,x, algorithm="maxima")
```

```
[Out] 1250/11*x^11 + 475/2*x^10 + 5075/9*x^9 + 3415/4*x^8 + 1176*x^7 + 2377/2*x^6
 + 5099/5*x^5 + 656*x^4 + 1064/3*x^3 + 136*x^2 + 48*x
```

mupad [B] time = 0.06, size = 54, normalized size = 0.79

$$\frac{1250\,x^{11}}{11}+\frac{475\,x^{10}}{2}+\frac{5075\,x^{9}}{9}+\frac{3415\,x^{8}}{4}+1176\,x^{7}+\frac{2377\,x^{6}}{2}+\frac{5099\,x^{5}}{5}+656\,x^{4}+\frac{1064\,x^{3}}{3}+136\,x^{2}+48\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)*(3*x + 5*x^2 + 2)^4,x)
```

```
[Out] 48*x + 136*x^2 + (1064*x^3)/3 + 656*x^4 + (5099*x^5)/5 + (2377*x^6)/2 + 117
6*x^7 + (3415*x^8)/4 + (5075*x^9)/9 + (475*x^10)/2 + (1250*x^11)/11
```

sympy [A] time = 0.09, size = 65, normalized size = 0.96

$$\frac{1250x^{11}}{11}+\frac{475x^{10}}{2}+\frac{5075x^{9}}{9}+\frac{3415x^{8}}{4}+1176x^{7}+\frac{2377x^{6}}{2}+\frac{5099x^{5}}{5}+656x^{4}+\frac{1064x^{3}}{3}+136x^{2}+48x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)*(5*x**2+3*x+2)**4,x)
```

```
[Out] 1250*x**11/11 + 475*x**10/2 + 5075*x**9/9 + 3415*x**8/4 + 1176*x**7 + 2377*
x**6/2 + 5099*x**5/5 + 656*x**4 + 1064*x**3/3 + 136*x**2 + 48*x
```

3.13 $\int \left(3-x+2x^2\right)\left(2+3x+5x^2\right)^3\,dx$

Optimal. Leaf size=56

$$\frac{250x^9}{9}+\frac{325x^8}{8}+\frac{720x^7}{7}+134x^6+\frac{876x^5}{5}+\frac{579x^4}{4}+\frac{322x^3}{3}+50x^2+24x$$

Rubi [A] time = 0.04, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1657}

$$\frac{250x^9}{9}+\frac{325x^8}{8}+\frac{720x^7}{7}+134x^6+\frac{876x^5}{5}+\frac{579x^4}{4}+\frac{322x^3}{3}+50x^2+24x$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)*(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] 24*x + 50*x^2 + (322*x^3)/3 + (579*x^4)/4 + (876*x^5)/5 + 134*x^6 + (720*x^
7)/7 + (325*x^8)/8 + (250*x^9)/9
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \left(3-x+2x^2\right)\left(2+3x+5x^2\right)^3\,dx &= \int \left(24+100x+322x^2+579x^3+876x^4+804x^5+720x^6+325x^7+250\right.\\ &= 24x+50x^2+\frac{322x^3}{3}+\frac{579x^4}{4}+\frac{876x^5}{5}+134x^6+\frac{720x^7}{7}+\frac{325x^8}{8}+\frac{250}{9}\end{aligned}$$

Mathematica [A] time = 0.00, size = 56, normalized size = 1.00

$$\frac{250x^9}{9}+\frac{325x^8}{8}+\frac{720x^7}{7}+134x^6+\frac{876x^5}{5}+\frac{579x^4}{4}+\frac{322x^3}{3}+50x^2+24x$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)*(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] 24*x + 50*x^2 + (322*x^3)/3 + (579*x^4)/4 + (876*x^5)/5 + 134*x^6 + (720*x^
7)/7 + (325*x^8)/8 + (250*x^9)/9
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(3-x+2x^2\right)\left(2+3x+5x^2\right)^3\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(3 - x + 2*x^2)*(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] IntegrateAlgebraic[(3 - x + 2*x^2)*(2 + 3*x + 5*x^2)^3, x]
```

fricas [A] time = 0.34, size = 44, normalized size = 0.79

$$\frac{250}{9}x^9+\frac{325}{8}x^8+\frac{720}{7}x^7+134x^6+\frac{876}{5}x^5+\frac{579}{4}x^4+\frac{322}{3}x^3+50x^2+24x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)*(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

```
[Out] 250/9*x^9 + 325/8*x^8 + 720/7*x^7 + 134*x^6 + 876/5*x^5 + 579/4*x^4 + 322/3
*x^3 + 50*x^2 + 24*x
```

giac [A] time = 0.18, size = 44, normalized size = 0.79

$$\frac{250}{9}\,x^9+\frac{325}{8}\,x^8+\frac{720}{7}\,x^7+134\,x^6+\frac{876}{5}\,x^5+\frac{579}{4}\,x^4+\frac{322}{3}\,x^3+50\,x^2+24\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)*(5*x^2+3*x+2)^3,x, algorithm="giac")
```

```
[Out] 250/9*x^9 + 325/8*x^8 + 720/7*x^7 + 134*x^6 + 876/5*x^5 + 579/4*x^4 + 322/3
*x^3 + 50*x^2 + 24*x
```

maple [A] time = 0.00, size = 45, normalized size = 0.80

$$\frac{250}{9}x^9+\frac{325}{8}x^8+\frac{720}{7}x^7+134x^6+\frac{876}{5}x^5+\frac{579}{4}x^4+\frac{322}{3}x^3+50x^2+24x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)*(5*x^2+3*x+2)^3,x)
```

```
[Out] 24*x+50*x^2+322/3*x^3+579/4*x^4+876/5*x^5+134*x^6+720/7*x^7+325/8*x^8+250/9
*x^9
```

maxima [A] time = 0.45, size = 44, normalized size = 0.79

$$\frac{250}{9}\,x^{9}+\frac{325}{8}\,x^{8}+\frac{720}{7}\,x^{7}+134\,x^{6}+\frac{876}{5}\,x^{5}+\frac{579}{4}\,x^{4}+\frac{322}{3}\,x^{3}+50\,x^{2}+24\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)*(5*x^2+3*x+2)^3,x, algorithm="maxima")
```

```
[Out] 250/9*x^9 + 325/8*x^8 + 720/7*x^7 + 134*x^6 + 876/5*x^5 + 579/4*x^4 + 322/3
*x^3 + 50*x^2 + 24*x
```

mupad [B] time = 0.03, size = 44, normalized size = 0.79

$$\frac{250\,x^{9}}{9}+\frac{325\,x^{8}}{8}+\frac{720\,x^{7}}{7}+134\,x^{6}+\frac{876\,x^{5}}{5}+\frac{579\,x^{4}}{4}+\frac{322\,x^{3}}{3}+50\,x^{2}+24\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)*(3*x + 5*x^2 + 2)^3,x)
```

```
[Out] 24*x + 50*x^2 + (322*x^3)/3 + (579*x^4)/4 + (876*x^5)/5 + 134*x^6 + (720*x^
7)/7 + (325*x^8)/8 + (250*x^9)/9
```

sympy [A] time = 0.08, size = 53, normalized size = 0.95

$$\frac{250x^{9}}{9}+\frac{325x^{8}}{8}+\frac{720x^{7}}{7}+134x^{6}+\frac{876x^{5}}{5}+\frac{579x^{4}}{4}+\frac{322x^{3}}{3}+50x^{2}+24x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)*(5*x**2+3*x+2)**3,x)
```

```
[Out] 250*x**9/9 + 325*x**8/8 + 720*x**7/7 + 134*x**6 + 876*x**5/5 + 579*x**4/4 +
 322*x**3/3 + 50*x**2 + 24*x
```

3.14 $\int \left(3-x+2x^2\right)\left(2+3x+5x^2\right)^2\,dx$

Optimal. Leaf size=44

$$\frac{50x^7}{7}+\frac{35x^6}{6}+\frac{103x^5}{5}+\frac{85x^4}{4}+\frac{83x^3}{3}+16x^2+12x$$

Rubi [A] time = 0.03, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1657}

$$\frac{50x^7}{7}+\frac{35x^6}{6}+\frac{103x^5}{5}+\frac{85x^4}{4}+\frac{83x^3}{3}+16x^2+12x$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)*(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] 12*x + 16*x^2 + (83*x^3)/3 + (85*x^4)/4 + (103*x^5)/5 + (35*x^6)/6 + (50*x^
7)/7
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \left(3-x+2x^2\right)\left(2+3x+5x^2\right)^2\,dx &= \int \left(12+32x+83x^2+85x^3+103x^4+35x^5+50x^6\right)dx\\ &= 12x+16x^2+\frac{83x^3}{3}+\frac{85x^4}{4}+\frac{103x^5}{5}+\frac{35x^6}{6}+\frac{50x^7}{7}\end{aligned}$$

Mathematica [A] time = 0.00, size = 44, normalized size = 1.00

$$\frac{50x^7}{7}+\frac{35x^6}{6}+\frac{103x^5}{5}+\frac{85x^4}{4}+\frac{83x^3}{3}+16x^2+12x$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)*(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] 12*x + 16*x^2 + (83*x^3)/3 + (85*x^4)/4 + (103*x^5)/5 + (35*x^6)/6 + (50*x^
7)/7
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(3-x+2x^2\right)\left(2+3x+5x^2\right)^2\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(3 - x + 2*x^2)*(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] IntegrateAlgebraic[(3 - x + 2*x^2)*(2 + 3*x + 5*x^2)^2, x]
```

fricas [A] time = 0.34, size = 34, normalized size = 0.77

$$\frac{50}{7}x^7+\frac{35}{6}x^6+\frac{103}{5}x^5+\frac{85}{4}x^4+\frac{83}{3}x^3+16x^2+12x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)*(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

```
[Out] 50/7*x^7 + 35/6*x^6 + 103/5*x^5 + 85/4*x^4 + 83/3*x^3 + 16*x^2 + 12*x
```

giac [A] time = 0.21, size = 34, normalized size = 0.77

$$\frac{50}{7}\,x^7+\frac{35}{6}\,x^6+\frac{103}{5}\,x^5+\frac{85}{4}\,x^4+\frac{83}{3}\,x^3+16\,x^2+12\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)*(5*x^2+3*x+2)^2,x, algorithm="giac")
```

```
[Out] 50/7*x^7 + 35/6*x^6 + 103/5*x^5 + 85/4*x^4 + 83/3*x^3 + 16*x^2 + 12*x
```

maple [A] time = 0.00, size = 35, normalized size = 0.80

$$\frac{50}{7}x^7+\frac{35}{6}x^6+\frac{103}{5}x^5+\frac{85}{4}x^4+\frac{83}{3}x^3+16x^2+12x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)*(5*x^2+3*x+2)^2,x)
```

```
[Out] 12*x+16*x^2+83/3*x^3+85/4*x^4+103/5*x^5+35/6*x^6+50/7*x^7
```

maxima [A] time = 0.43, size = 34, normalized size = 0.77

$$\frac{50}{7}\,x^7+\frac{35}{6}\,x^6+\frac{103}{5}\,x^5+\frac{85}{4}\,x^4+\frac{83}{3}\,x^3+16\,x^2+12\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)*(5*x^2+3*x+2)^2,x, algorithm="maxima")
```

```
[Out] 50/7*x^7 + 35/6*x^6 + 103/5*x^5 + 85/4*x^4 + 83/3*x^3 + 16*x^2 + 12*x
```

mupad [B] time = 0.03, size = 34, normalized size = 0.77

$$\frac{50\,x^7}{7}+\frac{35\,x^6}{6}+\frac{103\,x^5}{5}+\frac{85\,x^4}{4}+\frac{83\,x^3}{3}+16\,x^2+12\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)*(3*x + 5*x^2 + 2)^2,x)
```

```
[Out] 12*x + 16*x^2 + (83*x^3)/3 + (85*x^4)/4 + (103*x^5)/5 + (35*x^6)/6 + (50*x^
7)/7
```

sympy [A] time = 0.08, size = 41, normalized size = 0.93

$$\frac{50x^7}{7}+\frac{35x^6}{6}+\frac{103x^5}{5}+\frac{85x^4}{4}+\frac{83x^3}{3}+16x^2+12x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)*(5*x**2+3*x+2)**2,x)
```

```
[Out] 50*x**7/7 + 35*x**6/6 + 103*x**5/5 + 85*x**4/4 + 83*x**3/3 + 16*x**2 + 12*x
```

3.15 $\int \left(3-x+2x^2\right)\left(2+3x+5x^2\right)\,dx$

Optimal. Leaf size=30

$$2x^5+\frac{x^4}{4}+\frac{16x^3}{3}+\frac{7x^2}{2}+6x$$

Rubi [A] time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {1657}

$$2x^5+\frac{x^4}{4}+\frac{16x^3}{3}+\frac{7x^2}{2}+6x$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)*(2 + 3*x + 5*x^2),x]
```

```
[Out] 6*x + (7*x^2)/2 + (16*x^3)/3 + x^4/4 + 2*x^5
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \left(3-x+2x^2\right)\left(2+3x+5x^2\right)\,dx &= \int \left(6+7x+16x^2+x^3+10x^4\right)\,dx\\ &= 6x+\frac{7x^2}{2}+\frac{16x^3}{3}+\frac{x^4}{4}+2x^5\end{aligned}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.00

$$2x^5+\frac{x^4}{4}+\frac{16x^3}{3}+\frac{7x^2}{2}+6x$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)*(2 + 3*x + 5*x^2),x]
```

```
[Out] 6*x + (7*x^2)/2 + (16*x^3)/3 + x^4/4 + 2*x^5
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(3 - x + 2x^2\right)\left(2 + 3x + 5x^2\right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(3 - x + 2*x^2)*(2 + 3*x + 5*x^2),x]
[Out] IntegrateAlgebraic[(3 - x + 2*x^2)*(2 + 3*x + 5*x^2), x]
```

fricas [A] time = 0.34, size = 24, normalized size = 0.80

$$2x^5 + \frac{1}{4}x^4 + \frac{16}{3}x^3 + \frac{7}{2}x^2 + 6x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)*(5*x^2+3*x+2),x, algorithm="fricas")
[Out] 2*x^5 + 1/4*x^4 + 16/3*x^3 + 7/2*x^2 + 6*x
```

giac [A] time = 0.18, size = 24, normalized size = 0.80

$$2\,x^5 + \frac{1}{4}\,x^4 + \frac{16}{3}\,x^3 + \frac{7}{2}\,x^2 + 6\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)*(5*x^2+3*x+2),x, algorithm="giac")
[Out] 2*x^5 + 1/4*x^4 + 16/3*x^3 + 7/2*x^2 + 6*x
```

maple [A] time = 0.00, size = 25, normalized size = 0.83

$$2x^5 + \frac{1}{4}x^4 + \frac{16}{3}x^3 + \frac{7}{2}x^2 + 6x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)*(5*x^2+3*x+2),x)
[Out] 6*x+7/2*x^2+16/3*x^3+1/4*x^4+2*x^5
```

maxima [A] time = 0.43, size = 24, normalized size = 0.80

$$2\,x^5 + \frac{1}{4}\,x^4 + \frac{16}{3}\,x^3 + \frac{7}{2}\,x^2 + 6\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)*(5*x^2+3*x+2),x, algorithm="maxima")
```

```
[Out] 2*x^5 + 1/4*x^4 + 16/3*x^3 + 7/2*x^2 + 6*x
```

mupad [B] time = 0.02, size = 24, normalized size = 0.80

$$2x^5 + \frac{x^4}{4} + \frac{16x^3}{3} + \frac{7x^2}{2} + 6x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)*(3*x + 5*x^2 + 2),x)
```

```
[Out] 6*x + (7*x^2)/2 + (16*x^3)/3 + x^4/4 + 2*x^5
```

sympy [A] time = 0.06, size = 26, normalized size = 0.87

$$2x^5 + \frac{x^4}{4} + \frac{16x^3}{3} + \frac{7x^2}{2} + 6x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)*(5*x**2+3*x+2),x)
```

```
[Out] 2*x**5 + x**4/4 + 16*x**3/3 + 7*x**2/2 + 6*x
```

3.16 $\int \frac{3-x+2x^2}{2+3x+5x^2}\,dx$

Optimal. Leaf size=42

$$-\frac{11}{50}\log\left(5x^2+3x+2\right)+\frac{2x}{5}+\frac{143\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{25\sqrt{31}}$$

Rubi [A] time = 0.04, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {1657, 634, 618, 204, 628}

$$-\frac{11}{50}\log\left(5x^2+3x+2\right)+\frac{2x}{5}+\frac{143\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{25\sqrt{31}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)/(2 + 3*x + 5*x^2),x]
```

```
[Out] (2*x)/5 + (143*ArcTan[(3 + 10*x)/Sqrt[31]])/(25*Sqrt[31]) - (11*Log[2 + 3*x
 + 5*x^2])/50
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{3-x+2x^2}{2+3x+5x^2}\,dx &= \int \left(\frac{2}{5}+\frac{11(1-x)}{5\left(2+3x+5x^2\right)}\right)dx\\ &= \frac{2x}{5}+\frac{11}{5}\int \frac{1-x}{2+3x+5x^2}\,dx\\ &= \frac{2x}{5}-\frac{11}{50}\int \frac{3+10x}{2+3x+5x^2}\,dx+\frac{143}{50}\int \frac{1}{2+3x+5x^2}\,dx\\ &= \frac{2x}{5}-\frac{11}{50}\log\left(2+3x+5x^2\right)-\frac{143}{25}\,\text{Subst}\left(\int \frac{1}{-31-x^2}\,dx,x,3+10x\right)\\ &= \frac{2x}{5}+\frac{143\tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)}{25\sqrt{31}}-\frac{11}{50}\log\left(2+3x+5x^2\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 42, normalized size = 1.00

$$-\frac{11}{50}\log\left(5x^2+3x+2\right)+\frac{2x}{5}+\frac{143\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{25\sqrt{31}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)/(2 + 3*x + 5*x^2),x]
```

```
[Out] (2*x)/5 + (143*ArcTan[(3 + 10*x)/Sqrt[31]])/(25*Sqrt[31]) - (11*Log[2 + 3*x
 + 5*x^2])/50
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3-x+2x^2}{2+3x+5x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(3 - x + 2*x^2)/(2 + 3*x + 5*x^2),x]
```

```
[Out] IntegrateAlgebraic[(3 - x + 2*x^2)/(2 + 3*x + 5*x^2), x]
```

fricas [A] time = 0.39, size = 33, normalized size = 0.79

$$\frac{143}{775}\,\sqrt{31}\,\arctan\left(\frac{1}{31}\,\sqrt{31}\,(10\,x+3)\right)+\frac{2}{5}\,x-\frac{11}{50}\,\log\left(5\,x^2+3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)/(5*x^2+3*x+2),x, algorithm="fricas")
```

```
[Out] 143/775*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 2/5*x - 11/50*log(5*x^2
 + 3*x + 2)
```

giac [A] time = 0.18, size = 33, normalized size = 0.79

$$\frac{143}{775}\,\sqrt{31}\,\arctan\left(\frac{1}{31}\,\sqrt{31}\,(10\,x+3)\right)+\frac{2}{5}\,x-\frac{11}{50}\,\log\left(5\,x^2+3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)/(5*x^2+3*x+2),x, algorithm="giac")
```

```
[Out] 143/775*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 2/5*x - 11/50*log(5*x^2
 + 3*x + 2)
```

maple [A] time = 0.00, size = 34, normalized size = 0.81

$$\frac{2x}{5}+\frac{143\sqrt{31}\,\arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)}{775}-\frac{11\ln\left(5x^2+3x+2\right)}{50}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)/(5*x^2+3*x+2),x)
```

```
[Out] 2/5*x-11/50*ln(5*x^2+3*x+2)+143/775*arctan(1/31*(3+10*x)*31^(1/2))*31^(1/2)
```

maxima [A] time = 0.97, size = 33, normalized size = 0.79

$$\frac{143}{775}\,\sqrt{31}\,\arctan\left(\frac{1}{31}\,\sqrt{31}\,(10\,x+3)\right)+\frac{2}{5}\,x-\frac{11}{50}\,\log\left(5\,x^2+3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)/(5*x^2+3*x+2),x, algorithm="maxima")
```

```
[Out] 143/775*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 2/5*x - 11/50*log(5*x^2
 + 3*x + 2)
```

mupad [B] time = 3.39, size = 35, normalized size = 0.83

$$\frac{2\,x}{5}-\frac{11\,\ln\left(5\,x^2+3\,x+2\right)}{50}+\frac{143\,\sqrt{31}\,\mathrm{atan}\left(\frac{10\,\sqrt{31}\,x}{31}+\frac{3\,\sqrt{31}}{31}\right)}{775}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)/(3*x + 5*x^2 + 2),x)
```

```
[Out] (2*x)/5 - (11*log(3*x + 5*x^2 + 2))/50 + (143*31^(1/2)*atan((10*31^(1/2)*x)
/31 + (3*31^(1/2))/31))/775
```

sympy [A] time = 0.14, size = 49, normalized size = 1.17

$$\frac{2x}{5}-\frac{11\log\left(x^2+\frac{3x}{5}+\frac{2}{5}\right)}{50}+\frac{143\sqrt{31}\,\mathrm{atan}\left(\frac{10\sqrt{31}\,x}{31}+\frac{3\sqrt{31}}{31}\right)}{775}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)/(5*x**2+3*x+2),x)
```

```
[Out] 2*x/5 - 11*log(x**2 + 3*x/5 + 2/5)/50 + 143*sqrt(31)*atan(10*sqrt(31)*x/31
+ 3*sqrt(31)/31)/775
```

3.17 $\int \frac{3-x+2x^2}{\left(2+3x+5x^2\right)^2} dx$

Optimal. Leaf size=43

$$\frac{11(13x+7)}{155\left(5x^2+3x+2\right)}+\frac{82\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{31\sqrt{31}}$$

Rubi [A] time = 0.03, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {1660, 12, 618, 204}

$$\frac{11(13x+7)}{155\left(5x^2+3x+2\right)}+\frac{82\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{31\sqrt{31}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)/(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] (11*(7 + 13*x))/(155*(2 + 3*x + 5*x^2)) + (82*ArcTan[(3 + 10*x)/Sqrt[31]])/
(31*Sqrt[31])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
```

```
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{3-x+2x^2}{\left(2+3x+5x^2\right)^2}\,dx &= \frac{11(7+13x)}{155\left(2+3x+5x^2\right)}+\frac{1}{31}\int \frac{41}{2+3x+5x^2}\,dx \\
&= \frac{11(7+13x)}{155\left(2+3x+5x^2\right)}+\frac{41}{31}\int \frac{1}{2+3x+5x^2}\,dx \\
&= \frac{11(7+13x)}{155\left(2+3x+5x^2\right)}-\frac{82}{31}\,\mathrm{Subst}\left(\int \frac{1}{-31-x^2}\,dx,x,3+10x\right) \\
&= \frac{11(7+13x)}{155\left(2+3x+5x^2\right)}+\frac{82\tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)}{31\sqrt{31}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 43, normalized size = 1.00

$$\frac{11(13x+7)}{155\left(5x^2+3x+2\right)}+\frac{82\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{31\sqrt{31}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)/(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] (11*(7 + 13*x))/(155*(2 + 3*x + 5*x^2)) + (82*ArcTan[(3 + 10*x)/Sqrt[31]])/
(31*Sqrt[31])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3-x+2x^2}{\left(2+3x+5x^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(3 - x + 2*x^2)/(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] IntegrateAlgebraic[(3 - x + 2*x^2)/(2 + 3*x + 5*x^2)^2, x]
```

fricas [A] time = 0.39, size = 45, normalized size = 1.05

$$\frac{410 \sqrt{31} \left(5 x^2 + 3 x + 2\right) \arctan\left(\frac{1}{31} \sqrt{31} (10 x + 3)\right) + 4433 x + 2387}{4805 \left(5 x^2 + 3 x + 2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)/(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

```
[Out] 1/4805*(410*sqrt(31)*(5*x^2 + 3*x + 2)*arctan(1/31*sqrt(31)*(10*x + 3)) + 4
433*x + 2387)/(5*x^2 + 3*x + 2)
```

giac [A] time = 0.21, size = 36, normalized size = 0.84

$$\frac{82}{961} \sqrt{31} \arctan\left(\frac{1}{31} \sqrt{31} (10 x + 3)\right) + \frac{11 (13 x + 7)}{155 \left(5 x^2 + 3 x + 2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)/(5*x^2+3*x+2)^2,x, algorithm="giac")
```

```
[Out] 82/961*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 11/155*(13*x + 7)/(5*x^2
 + 3*x + 2)
```

maple [A] time = 0.00, size = 34, normalized size = 0.79

$$\frac{82\sqrt{31}\ \arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)}{961} + \frac{\frac{143x}{775} + \frac{77}{775}}{x^2 + \frac{3}{5}x + \frac{2}{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)/(5*x^2+3*x+2)^2,x)
```

```
[Out] (143/775*x+77/775)/(x^2+3/5*x+2/5)+82/961*31^(1/2)*arctan(1/31*(10*x+3)*31^
(1/2))
```

maxima [A] time = 0.97, size = 36, normalized size = 0.84

$$\frac{82}{961} \sqrt{31} \arctan\left(\frac{1}{31} \sqrt{31} (10 x + 3)\right) + \frac{11 (13 x + 7)}{155 \left(5 x^2 + 3 x + 2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)/(5*x^2+3*x+2)^2,x, algorithm="maxima")
```

```
[Out] 82/961*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 11/155*(13*x + 7)/(5*x^2
 + 3*x + 2)
```

mupad [B] time = 0.04, size = 35, normalized size = 0.81

$$\frac{\frac{143\,x}{775}+\frac{77}{775}}{x^2+\frac{3\,x}{5}+\frac{2}{5}}+\frac{82\,\sqrt{31}\,\mathrm{atan}\left(\frac{10\,\sqrt{31}\,x}{31}+\frac{3\,\sqrt{31}}{31}\right)}{961}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)/(3*x + 5*x^2 + 2)^2,x)
```

```
[Out] ((143*x)/775 + 77/775)/((3*x)/5 + x^2 + 2/5) + (82*31^(1/2)*atan((10*31^(1/
2)*x)/31 + (3*31^(1/2))/31))/961
```

sympy [A] time = 0.16, size = 42, normalized size = 0.98

$$\frac{143x+77}{775x^2+465x+310}+\frac{82\sqrt{31}\,\mathrm{atan}\left(\frac{10\sqrt{31}\,x}{31}+\frac{3\sqrt{31}}{31}\right)}{961}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)/(5*x**2+3*x+2)**2,x)
```

```
[Out] (143*x + 77)/(775*x**2 + 465*x + 310) + 82*sqrt(31)*atan(10*sqrt(31)*x/31 +
 3*sqrt(31)/31)/961
```

3.18 $\int \frac{3-x+2x^2}{\left(2+3x+5x^2\right)^3}\,dx$

Optimal. Leaf size=64

$$\frac{553(10x+3)}{9610\left(5x^2+3x+2\right)}+\frac{11(13x+7)}{310\left(5x^2+3x+2\right)^2}+\frac{1106\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{961\sqrt{31}}$$

Rubi [A] time = 0.04, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {1660, 12, 614, 618, 204}

$$\frac{553(10x+3)}{9610\left(5x^2+3x+2\right)}+\frac{11(13x+7)}{310\left(5x^2+3x+2\right)^2}+\frac{1106\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{961\sqrt{31}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)/(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] (11*(7 + 13*x))/(310*(2 + 3*x + 5*x^2)^2) + (553*(3 + 10*x))/(9610*(2 + 3*x
 + 5*x^2)) + (1106*ArcTan[(3 + 10*x)/Sqrt[31]])/(961*Sqrt[31])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{3-x+2x^2}{\left(2+3x+5x^2\right)^3}\,dx &= \frac{11(7+13x)}{310\left(2+3x+5x^2\right)^2}+\frac{1}{62}\int \frac{553}{5\left(2+3x+5x^2\right)^2}\,dx \\
&= \frac{11(7+13x)}{310\left(2+3x+5x^2\right)^2}+\frac{553}{310}\int \frac{1}{\left(2+3x+5x^2\right)^2}\,dx \\
&= \frac{11(7+13x)}{310\left(2+3x+5x^2\right)^2}+\frac{553(3+10x)}{9610\left(2+3x+5x^2\right)}+\frac{553}{961}\int \frac{1}{2+3x+5x^2}\,dx \\
&= \frac{11(7+13x)}{310\left(2+3x+5x^2\right)^2}+\frac{553(3+10x)}{9610\left(2+3x+5x^2\right)}-\frac{1106}{961}\text{Subst}\left(\int \frac{1}{-31-x^2}\,dx,x,3+10x\right) \\
&= \frac{11(7+13x)}{310\left(2+3x+5x^2\right)^2}+\frac{553(3+10x)}{9610\left(2+3x+5x^2\right)}+\frac{1106\tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)}{961\sqrt{31}}
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 53, normalized size = 0.83

$$
\frac{\frac{31\left(5530x^3+4977x^2+4094x+1141\right)}{\left(5x^2+3x+2\right)^2}+2212\sqrt{31}\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{59582}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)/(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] ((31*(1141 + 4094*x + 4977*x^2 + 5530*x^3))/(2 + 3*x + 5*x^2)^2 + 2212*Sqrt
[31]*ArcTan[(3 + 10*x)/Sqrt[31]])/59582
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3-x+2x^2}{(2+3x+5x^2)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(3 - x + 2*x^2)/(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] IntegrateAlgebraic[(3 - x + 2*x^2)/(2 + 3*x + 5*x^2)^3, x]
```

fricas [A] time = 0.39, size = 75, normalized size = 1.17

$$\frac{171430\,x^3+2212\,\sqrt{31}\left(25\,x^4+30\,x^3+29\,x^2+12\,x+4\right)\arctan\left(\frac{1}{31}\,\sqrt{31}\,(10\,x+3)\right)+154287\,x^2+126914\,x+35371}{59582\left(25\,x^4+30\,x^3+29\,x^2+12\,x+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)/(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

```
[Out] 1/59582*(171430*x^3 + 2212*sqrt(31)*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)*a
rctan(1/31*sqrt(31)*(10*x + 3)) + 154287*x^2 + 126914*x + 35371)/(25*x^4 +
30*x^3 + 29*x^2 + 12*x + 4)
```

giac [A] time = 0.17, size = 46, normalized size = 0.72

$$\frac{1106}{29791}\,\sqrt{31}\,\arctan\left(\frac{1}{31}\,\sqrt{31}\,(10\,x+3)\right)+\frac{5530\,x^3+4977\,x^2+4094\,x+1141}{1922\left(5\,x^2+3\,x+2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)/(5*x^2+3*x+2)^3,x, algorithm="giac")
```

```
[Out] 1106/29791*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 1/1922*(5530*x^3 + 4
977*x^2 + 4094*x + 1141)/(5*x^2 + 3*x + 2)^2
```

maple [A] time = 0.00, size = 47, normalized size = 0.73

$$\frac{1106\sqrt{31}\,\arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)}{29791}+\frac{\frac{2765}{961}x^3+\frac{4977}{1922}x^2+\frac{2047}{961}x+\frac{1141}{1922}}{\left(5x^2+3x+2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)/(5*x^2+3*x+2)^3,x)
```

```
[Out] 25*(553/4805*x^3+4977/48050*x^2+2047/24025*x+1141/48050)/(5*x^2+3*x+2)^2+11
06/29791*31^(1/2)*arctan(1/31*(10*x+3)*31^(1/2))
```

maxima [A] time = 0.96, size = 56, normalized size = 0.88

$$\frac{1106}{29791}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}\,(10\,x+3)\right)+\frac{5530\,x^3+4977\,x^2+4094\,x+1141}{1922\left(25\,x^4+30\,x^3+29\,x^2+12\,x+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)/(5*x^2+3*x+2)^3,x, algorithm="maxima")
```

```
[Out] 1106/29791*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 1/1922*(5530*x^3 + 4
977*x^2 + 4094*x + 1141)/(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)
```

mupad [B] time = 0.05, size = 55, normalized size = 0.86

$$\frac{1106\sqrt{31}\,\mathrm{atan}\left(\frac{10\sqrt{31}\,x}{31}+\frac{3\sqrt{31}}{31}\right)}{29791}+\frac{\frac{553\,x^3}{4805}+\frac{4977\,x^2}{48050}+\frac{2047\,x}{24025}+\frac{1141}{48050}}{x^4+\frac{6\,x^3}{5}+\frac{29\,x^2}{25}+\frac{12\,x}{25}+\frac{4}{25}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)/(3*x + 5*x^2 + 2)^3,x)
```

```
[Out] (1106*31^(1/2)*atan((10*31^(1/2)*x)/31 + (3*31^(1/2))/31))/29791 + ((2047*x
)/24025 + (4977*x^2)/48050 + (553*x^3)/4805 + 1141/48050)/((12*x)/25 + (29*
x^2)/25 + (6*x^3)/5 + x^4 + 4/25)
```

sympy [A] time = 0.18, size = 63, normalized size = 0.98

$$\frac{5530x^3+4977x^2+4094x+1141}{48050x^4+57660x^3+55738x^2+23064x+7688}+\frac{1106\sqrt{31}\,\mathrm{atan}\left(\frac{10\sqrt{31}\,x}{31}+\frac{3\sqrt{31}}{31}\right)}{29791}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)/(5*x**2+3*x+2)**3,x)
```

```
[Out] (5530*x**3 + 4977*x**2 + 4094*x + 1141)/(48050*x**4 + 57660*x**3 + 55738*x*
*2 + 23064*x + 7688) + 1106*sqrt(31)*atan(10*sqrt(31)*x/31 + 3*sqrt(31)/31)
/29791
```

3.19 $\int \left(3-x+2x^2\right)^2 \left(2+3x+5x^2\right)^4 \, dx$

Optimal. Leaf size=80

$$\frac{2500x^{13}}{13}+\frac{875x^{12}}{3}+\frac{11525x^{11}}{11}+1571x^{10}+\frac{24859x^9}{9}+3315x^8+\frac{27763x^7}{7}+\frac{10771x^6}{3}+\frac{14801x^5}{5}+1838x^4+\frac{3016x^3}{3}+384$$

Rubi [A] time = 0.06, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {1657}

$$\frac{2500x^{13}}{13}+\frac{875x^{12}}{3}+\frac{11525x^{11}}{11}+1571x^{10}+\frac{24859x^9}{9}+3315x^8+\frac{27763x^7}{7}+\frac{10771x^6}{3}+\frac{14801x^5}{5}+1838x^4+\frac{3016x^3}{3}+384x^2+144x$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)^4,x]
```

```
[Out] 144*x + 384*x^2 + (3016*x^3)/3 + 1838*x^4 + (14801*x^5)/5 + (10771*x^6)/3 +
 (27763*x^7)/7 + 3315*x^8 + (24859*x^9)/9 + 1571*x^10 + (11525*x^11)/11 + (
875*x^12)/3 + (2500*x^13)/13
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \left(3-x+2x^2\right)^2 \left(2+3x+5x^2\right)^4 \, dx = \int \left(144+768x+3016x^2+7352x^3+14801x^4+21542x^5+27763x^6+2\right.$$

$$= 144x+384x^2+\frac{3016x^3}{3}+1838x^4+\frac{14801x^5}{5}+\frac{10771x^6}{3}+\frac{27763x^7}{7}+3$$

Mathematica [A] time = 0.00, size = 80, normalized size = 1.00

$$\frac{2500x^{13}}{13}+\frac{875x^{12}}{3}+\frac{11525x^{11}}{11}+1571x^{10}+\frac{24859x^9}{9}+3315x^8+\frac{27763x^7}{7}+\frac{10771x^6}{3}+\frac{14801x^5}{5}+1838x^4+\frac{3016x^3}{3}+384x^2+144x$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)^4,x]
```

```
[Out] 144*x + 384*x^2 + (3016*x^3)/3 + 1838*x^4 + (14801*x^5)/5 + (10771*x^6)/3 +
 (27763*x^7)/7 + 3315*x^8 + (24859*x^9)/9 + 1571*x^10 + (11525*x^11)/11 + (
875*x^12)/3 + (2500*x^13)/13
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (3 - x + 2x^2)^2 (2 + 3x + 5x^2)^4 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)^4,x]
```

```
[Out] IntegrateAlgebraic[(3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)^4, x]
```

fricas [A] time = 0.35, size = 64, normalized size = 0.80

$$\frac{2500}{13}x^{13} + \frac{875}{3}x^{12} + \frac{11525}{11}x^{11} + 1571x^{10} + \frac{24859}{9}x^9 + 3315x^8 + \frac{27763}{7}x^7 + \frac{10771}{3}x^6 + \frac{14801}{5}x^5 + 1838x^4 + \frac{3016}{3}x^3 + 384x^2 + 144x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^2*(5*x^2+3*x+2)^4,x, algorithm="fricas")
```

```
[Out] 2500/13*x^13 + 875/3*x^12 + 11525/11*x^11 + 1571*x^10 + 24859/9*x^9 + 3315*
x^8 + 27763/7*x^7 + 10771/3*x^6 + 14801/5*x^5 + 1838*x^4 + 3016/3*x^3 + 384
*x^2 + 144*x
```

giac [A] time = 0.20, size = 64, normalized size = 0.80

$$\frac{2500}{13}x^{13} + \frac{875}{3}x^{12} + \frac{11525}{11}x^{11} + 1571x^{10} + \frac{24859}{9}x^9 + 3315x^8 + \frac{27763}{7}x^7 + \frac{10771}{3}x^6 + \frac{14801}{5}x^5 + 1838x^4 + \frac{3016}{3}x^3 + 384x^2 + 144x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^2*(5*x^2+3*x+2)^4,x, algorithm="giac")
```

```
[Out] 2500/13*x^13 + 875/3*x^12 + 11525/11*x^11 + 1571*x^10 + 24859/9*x^9 + 3315*
x^8 + 27763/7*x^7 + 10771/3*x^6 + 14801/5*x^5 + 1838*x^4 + 3016/3*x^3 + 384
*x^2 + 144*x
```

maple [A] time = 0.00, size = 65, normalized size = 0.81

$$\frac{2500}{13}x^{13} + \frac{875}{3}x^{12} + \frac{11525}{11}x^{11} + 1571x^{10} + \frac{24859}{9}x^9 + 3315x^8 + \frac{27763}{7}x^7 + \frac{10771}{3}x^6 + \frac{14801}{5}x^5 + 1838x^4 + \frac{3016}{3}x^3 + 384x^2 + 144x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^2*(5*x^2+3*x+2)^4,x)
```

```
[Out] 144*x+384*x^2+3016/3*x^3+1838*x^4+14801/5*x^5+10771/3*x^6+27763/7*x^7+3315*
x^8+24859/9*x^9+1571*x^10+11525/11*x^11+875/3*x^12+2500/13*x^13
```

maxima [A] time = 0.45, size = 64, normalized size = 0.80

$$\frac{2500}{13}\,x^{13}+\frac{875}{3}\,x^{12}+\frac{11525}{11}\,x^{11}+1571\,x^{10}+\frac{24859}{9}\,x^{9}+3315\,x^{8}+\frac{27763}{7}\,x^{7}+\frac{10771}{3}\,x^{6}+\frac{14801}{5}\,x^{5}+1838\,x^{4}+\frac{3016}{3}\,x^{3}+384\,x^{2}+144\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^2*(5*x^2+3*x+2)^4,x, algorithm="maxima")
```

```
[Out] 2500/13*x^13 + 875/3*x^12 + 11525/11*x^11 + 1571*x^10 + 24859/9*x^9 + 3315*
x^8 + 27763/7*x^7 + 10771/3*x^6 + 14801/5*x^5 + 1838*x^4 + 3016/3*x^3 + 384
*x^2 + 144*x
```

mupad [B] time = 0.08, size = 64, normalized size = 0.80

$$\frac{2500\,x^{13}}{13}+\frac{875\,x^{12}}{3}+\frac{11525\,x^{11}}{11}+1571\,x^{10}+\frac{24859\,x^{9}}{9}+3315\,x^{8}+\frac{27763\,x^{7}}{7}+\frac{10771\,x^{6}}{3}+\frac{14801\,x^{5}}{5}+1838\,x^{4}+\frac{3016\,x^{3}}{3}+384\,x^{2}+144\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^2*(3*x + 5*x^2 + 2)^4,x)
```

```
[Out] 144*x + 384*x^2 + (3016*x^3)/3 + 1838*x^4 + (14801*x^5)/5 + (10771*x^6)/3 +
 (27763*x^7)/7 + 3315*x^8 + (24859*x^9)/9 + 1571*x^10 + (11525*x^11)/11 + (
875*x^12)/3 + (2500*x^13)/13
```

sympy [A] time = 0.10, size = 76, normalized size = 0.95

$$\frac{2500x^{13}}{13}+\frac{875x^{12}}{3}+\frac{11525x^{11}}{11}+1571x^{10}+\frac{24859x^{9}}{9}+3315x^{8}+\frac{27763x^{7}}{7}+\frac{10771x^{6}}{3}+\frac{14801x^{5}}{5}+1838x^{4}+\frac{3016x^{3}}{3}+384x^{2}+144x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**2*(5*x**2+3*x+2)**4,x)
```

```
[Out] 2500*x**13/13 + 875*x**12/3 + 11525*x**11/11 + 1571*x**10 + 24859*x**9/9 +
3315*x**8 + 27763*x**7/7 + 10771*x**6/3 + 14801*x**5/5 + 1838*x**4 + 3016*x
**3/3 + 384*x**2 + 144*x
```

3.20 $\int \left(3-x+2x^2\right)^2 \left(2+3x+5x^2\right)^3 \, dx$

Optimal. Leaf size=66

$$\frac{500x^{11}}{11}+40x^{10}+\frac{1865x^9}{9}+\frac{1863x^8}{8}+444x^7+449x^6+\frac{2693x^5}{5}+\frac{1615x^4}{4}+\frac{914x^3}{3}+138x^2+72x$$

Rubi [A] time = 0.05, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {1657}

$$\frac{500x^{11}}{11}+40x^{10}+\frac{1865x^9}{9}+\frac{1863x^8}{8}+444x^7+449x^6+\frac{2693x^5}{5}+\frac{1615x^4}{4}+\frac{914x^3}{3}+138x^2+72x$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] 72*x + 138*x^2 + (914*x^3)/3 + (1615*x^4)/4 + (2693*x^5)/5 + 449*x^6 + 444*
x^7 + (1863*x^8)/8 + (1865*x^9)/9 + 40*x^10 + (500*x^11)/11
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \left(3-x+2x^2\right)^2 \left(2+3x+5x^2\right)^3 \, dx = \int \left(72+276x+914x^2+1615x^3+2693x^4+2694x^5+3108x^6+1863x\right.$$

$$= 72x+138x^2+\frac{914x^3}{3}+\frac{1615x^4}{4}+\frac{2693x^5}{5}+449x^6+444x^7+\frac{1863x^8}{8}$$

Mathematica [A] time = 0.00, size = 66, normalized size = 1.00

$$\frac{500x^{11}}{11}+40x^{10}+\frac{1865x^9}{9}+\frac{1863x^8}{8}+444x^7+449x^6+\frac{2693x^5}{5}+\frac{1615x^4}{4}+\frac{914x^3}{3}+138x^2+72x$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] 72*x + 138*x^2 + (914*x^3)/3 + (1615*x^4)/4 + (2693*x^5)/5 + 449*x^6 + 444*
x^7 + (1863*x^8)/8 + (1865*x^9)/9 + 40*x^10 + (500*x^11)/11
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(3 - x + 2x^2\right)^2 \left(2 + 3x + 5x^2\right)^3 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] IntegrateAlgebraic[(3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)^3, x]
```

fricas [A] time = 0.34, size = 54, normalized size = 0.82

$$\frac{500}{11}x^{11} + 40x^{10} + \frac{1865}{9}x^9 + \frac{1863}{8}x^8 + 444x^7 + 449x^6 + \frac{2693}{5}x^5 + \frac{1615}{4}x^4 + \frac{914}{3}x^3 + 138x^2 + 72x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^2*(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

```
[Out] 500/11*x^11 + 40*x^10 + 1865/9*x^9 + 1863/8*x^8 + 444*x^7 + 449*x^6 + 2693/
5*x^5 + 1615/4*x^4 + 914/3*x^3 + 138*x^2 + 72*x
```

giac [A] time = 0.21, size = 54, normalized size = 0.82

$$\frac{500}{11}\,x^{11} + 40\,x^{10} + \frac{1865}{9}\,x^9 + \frac{1863}{8}\,x^8 + 444\,x^7 + 449\,x^6 + \frac{2693}{5}\,x^5 + \frac{1615}{4}\,x^4 + \frac{914}{3}\,x^3 + 138\,x^2 + 72\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^2*(5*x^2+3*x+2)^3,x, algorithm="giac")
```

```
[Out] 500/11*x^11 + 40*x^10 + 1865/9*x^9 + 1863/8*x^8 + 444*x^7 + 449*x^6 + 2693/
5*x^5 + 1615/4*x^4 + 914/3*x^3 + 138*x^2 + 72*x
```

maple [A] time = 0.00, size = 55, normalized size = 0.83

$$\frac{500}{11}x^{11} + 40x^{10} + \frac{1865}{9}x^9 + \frac{1863}{8}x^8 + 444x^7 + 449x^6 + \frac{2693}{5}x^5 + \frac{1615}{4}x^4 + \frac{914}{3}x^3 + 138x^2 + 72x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^2*(5*x^2+3*x+2)^3,x)
```

```
[Out] 72*x+138*x^2+914/3*x^3+1615/4*x^4+2693/5*x^5+449*x^6+444*x^7+1863/8*x^8+186
5/9*x^9+40*x^10+500/11*x^11
```

maxima [A] time = 0.44, size = 54, normalized size = 0.82

$$\frac{500}{11}\,x^{11} + 40\,x^{10} + \frac{1865}{9}\,x^{9} + \frac{1863}{8}\,x^{8} + 444\,x^{7} + 449\,x^{6} + \frac{2693}{5}\,x^{5} + \frac{1615}{4}\,x^{4} + \frac{914}{3}\,x^{3} + 138\,x^{2} + 72\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^2*(5*x^2+3*x+2)^3,x, algorithm="maxima")
```

```
[Out] 500/11*x^11 + 40*x^10 + 1865/9*x^9 + 1863/8*x^8 + 444*x^7 + 449*x^6 + 2693/
5*x^5 + 1615/4*x^4 + 914/3*x^3 + 138*x^2 + 72*x
```

mupad [B] time = 0.05, size = 54, normalized size = 0.82

$$\frac{500\,x^{11}}{11} + 40\,x^{10} + \frac{1865\,x^{9}}{9} + \frac{1863\,x^{8}}{8} + 444\,x^{7} + 449\,x^{6} + \frac{2693\,x^{5}}{5} + \frac{1615\,x^{4}}{4} + \frac{914\,x^{3}}{3} + 138\,x^{2} + 72\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^2*(3*x + 5*x^2 + 2)^3,x)
```

```
[Out] 72*x + 138*x^2 + (914*x^3)/3 + (1615*x^4)/4 + (2693*x^5)/5 + 449*x^6 + 444*
x^7 + (1863*x^8)/8 + (1865*x^9)/9 + 40*x^10 + (500*x^11)/11
```

sympy [A] time = 0.09, size = 63, normalized size = 0.95

$$\frac{500x^{11}}{11} + 40x^{10} + \frac{1865x^{9}}{9} + \frac{1863x^{8}}{8} + 444x^{7} + 449x^{6} + \frac{2693x^{5}}{5} + \frac{1615x^{4}}{4} + \frac{914x^{3}}{3} + 138x^{2} + 72x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**2*(5*x**2+3*x+2)**3,x)
```

```
[Out] 500*x**11/11 + 40*x**10 + 1865*x**9/9 + 1863*x**8/8 + 444*x**7 + 449*x**6 +
 2693*x**5/5 + 1615*x**4/4 + 914*x**3/3 + 138*x**2 + 72*x
```

3.21 $\int \left(3-x+2x^2\right)^2 \left(2+3x+5x^2\right)^2 \, dx$

Optimal. Leaf size=54

$$\frac{100x^9}{9}+\frac{5x^8}{2}+\frac{321x^7}{7}+\frac{86x^6}{3}+78x^5+59x^4+\frac{241x^3}{3}+42x^2+36x$$

Rubi [A] time = 0.04, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {1657}

$$\frac{100x^9}{9}+\frac{5x^8}{2}+\frac{321x^7}{7}+\frac{86x^6}{3}+78x^5+59x^4+\frac{241x^3}{3}+42x^2+36x$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] 36*x + 42*x^2 + (241*x^3)/3 + 59*x^4 + 78*x^5 + (86*x^6)/3 + (321*x^7)/7 +
(5*x^8)/2 + (100*x^9)/9
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \left(3-x+2x^2\right)^2 \left(2+3x+5x^2\right)^2 \, dx &= \int \left(36+84x+241x^2+236x^3+390x^4+172x^5+321x^6+20x^7+100x^8\right) \\ &= 36x+42x^2+\frac{241x^3}{3}+59x^4+78x^5+\frac{86x^6}{3}+\frac{321x^7}{7}+\frac{5x^8}{2}+\frac{100x^9}{9}\end{aligned}$$

Mathematica [A] time = 0.00, size = 54, normalized size = 1.00

$$\frac{100x^9}{9}+\frac{5x^8}{2}+\frac{321x^7}{7}+\frac{86x^6}{3}+78x^5+59x^4+\frac{241x^3}{3}+42x^2+36x$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] 36*x + 42*x^2 + (241*x^3)/3 + 59*x^4 + 78*x^5 + (86*x^6)/3 + (321*x^7)/7 +
(5*x^8)/2 + (100*x^9)/9
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(3 - x + 2x^2\right)^2 \left(2 + 3x + 5x^2\right)^2 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] IntegrateAlgebraic[(3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)^2, x]
```

fricas [A] time = 0.33, size = 44, normalized size = 0.81

$$\frac{100}{9}x^9 + \frac{5}{2}x^8 + \frac{321}{7}x^7 + \frac{86}{3}x^6 + 78x^5 + 59x^4 + \frac{241}{3}x^3 + 42x^2 + 36x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^2*(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

```
[Out] 100/9*x^9 + 5/2*x^8 + 321/7*x^7 + 86/3*x^6 + 78*x^5 + 59*x^4 + 241/3*x^3 +
42*x^2 + 36*x
```

giac [A] time = 0.20, size = 44, normalized size = 0.81

$$\frac{100}{9}\,x^9 + \frac{5}{2}\,x^8 + \frac{321}{7}\,x^7 + \frac{86}{3}\,x^6 + 78\,x^5 + 59\,x^4 + \frac{241}{3}\,x^3 + 42\,x^2 + 36\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^2*(5*x^2+3*x+2)^2,x, algorithm="giac")
```

```
[Out] 100/9*x^9 + 5/2*x^8 + 321/7*x^7 + 86/3*x^6 + 78*x^5 + 59*x^4 + 241/3*x^3 +
42*x^2 + 36*x
```

maple [A] time = 0.00, size = 45, normalized size = 0.83

$$\frac{100}{9}x^9 + \frac{5}{2}x^8 + \frac{321}{7}x^7 + \frac{86}{3}x^6 + 78x^5 + 59x^4 + \frac{241}{3}x^3 + 42x^2 + 36x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^2*(5*x^2+3*x+2)^2,x)
```

```
[Out] 36*x+42*x^2+241/3*x^3+59*x^4+78*x^5+86/3*x^6+321/7*x^7+5/2*x^8+100/9*x^9
```

maxima [A] time = 0.44, size = 44, normalized size = 0.81

$$\frac{100}{9}x^9 + \frac{5}{2}x^8 + \frac{321}{7}x^7 + \frac{86}{3}x^6 + 78x^5 + 59x^4 + \frac{241}{3}x^3 + 42x^2 + 36x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^2*(5*x^2+3*x+2)^2,x, algorithm="maxima")
```

```
[Out] 100/9*x^9 + 5/2*x^8 + 321/7*x^7 + 86/3*x^6 + 78*x^5 + 59*x^4 + 241/3*x^3 +
42*x^2 + 36*x
```

mupad [B] time = 0.03, size = 44, normalized size = 0.81

$$\frac{100x^9}{9} + \frac{5x^8}{2} + \frac{321x^7}{7} + \frac{86x^6}{3} + 78x^5 + 59x^4 + \frac{241x^3}{3} + 42x^2 + 36x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^2*(3*x + 5*x^2 + 2)^2,x)
```

```
[Out] 36*x + 42*x^2 + (241*x^3)/3 + 59*x^4 + 78*x^5 + (86*x^6)/3 + (321*x^7)/7 +
(5*x^8)/2 + (100*x^9)/9
```

sympy [A] time = 0.08, size = 51, normalized size = 0.94

$$\frac{100x^9}{9} + \frac{5x^8}{2} + \frac{321x^7}{7} + \frac{86x^6}{3} + 78x^5 + 59x^4 + \frac{241x^3}{3} + 42x^2 + 36x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**2*(5*x**2+3*x+2)**2,x)
```

```
[Out] 100*x**9/9 + 5*x**8/2 + 321*x**7/7 + 86*x**6/3 + 78*x**5 + 59*x**4 + 241*x*
*3/3 + 42*x**2 + 36*x
```

3.22 $\int \left(3-x+2x^2\right)^2 \left(2+3x+5x^2\right) dx$

Optimal. Leaf size=46

$$\frac{20x^7}{7}-\frac{4x^6}{3}+\frac{61x^5}{5}+\frac{x^4}{4}+\frac{53x^3}{3}+\frac{15x^2}{2}+18x$$

Rubi [A] time = 0.03, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1657}

$$\frac{20x^7}{7}-\frac{4x^6}{3}+\frac{61x^5}{5}+\frac{x^4}{4}+\frac{53x^3}{3}+\frac{15x^2}{2}+18x$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2),x]
```

```
[Out] 18*x + (15*x^2)/2 + (53*x^3)/3 + x^4/4 + (61*x^5)/5 - (4*x^6)/3 + (20*x^7)/
7
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \left(3-x+2x^2\right)^2 \left(2+3x+5x^2\right) dx &= \int \left(18+15x+53x^2+x^3+61x^4-8x^5+20x^6\right) dx\\ &= 18x+\frac{15x^2}{2}+\frac{53x^3}{3}+\frac{x^4}{4}+\frac{61x^5}{5}-\frac{4x^6}{3}+\frac{20x^7}{7}\end{aligned}$$

Mathematica [A] time = 0.00, size = 46, normalized size = 1.00

$$\frac{20x^7}{7}-\frac{4x^6}{3}+\frac{61x^5}{5}+\frac{x^4}{4}+\frac{53x^3}{3}+\frac{15x^2}{2}+18x$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2),x]
```

```
[Out] 18*x + (15*x^2)/2 + (53*x^3)/3 + x^4/4 + (61*x^5)/5 - (4*x^6)/3 + (20*x^7)/
7
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(3-x+2x^2\right)^2 \left(2+3x+5x^2\right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2),x]
[Out] IntegrateAlgebraic[(3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2), x]
```

fricas [A] time = 0.34, size = 34, normalized size = 0.74

$$\frac{20}{7}x^7 - \frac{4}{3}x^6 + \frac{61}{5}x^5 + \frac{1}{4}x^4 + \frac{53}{3}x^3 + \frac{15}{2}x^2 + 18x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^2*(5*x^2+3*x+2),x, algorithm="fricas")
[Out] 20/7*x^7 - 4/3*x^6 + 61/5*x^5 + 1/4*x^4 + 53/3*x^3 + 15/2*x^2 + 18*x
```

giac [A] time = 0.19, size = 34, normalized size = 0.74

$$\frac{20}{7}\,x^7 - \frac{4}{3}\,x^6 + \frac{61}{5}\,x^5 + \frac{1}{4}\,x^4 + \frac{53}{3}\,x^3 + \frac{15}{2}\,x^2 + 18\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^2*(5*x^2+3*x+2),x, algorithm="giac")
[Out] 20/7*x^7 - 4/3*x^6 + 61/5*x^5 + 1/4*x^4 + 53/3*x^3 + 15/2*x^2 + 18*x
```

maple [A] time = 0.00, size = 35, normalized size = 0.76

$$\frac{20}{7}x^7 - \frac{4}{3}x^6 + \frac{61}{5}x^5 + \frac{1}{4}x^4 + \frac{53}{3}x^3 + \frac{15}{2}x^2 + 18x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^2*(5*x^2+3*x+2),x)
[Out] 18*x+15/2*x^2+53/3*x^3+1/4*x^4+61/5*x^5-4/3*x^6+20/7*x^7
```

maxima [A] time = 0.44, size = 34, normalized size = 0.74

$$\frac{20}{7}\,x^7 - \frac{4}{3}\,x^6 + \frac{61}{5}\,x^5 + \frac{1}{4}\,x^4 + \frac{53}{3}\,x^3 + \frac{15}{2}\,x^2 + 18\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^2*(5*x^2+3*x+2),x, algorithm="maxima")
```

```
[Out] 20/7*x^7 - 4/3*x^6 + 61/5*x^5 + 1/4*x^4 + 53/3*x^3 + 15/2*x^2 + 18*x
```

mupad [B] time = 0.03, size = 34, normalized size = 0.74

$$\frac{20\,x^7}{7}-\frac{4\,x^6}{3}+\frac{61\,x^5}{5}+\frac{x^4}{4}+\frac{53\,x^3}{3}+\frac{15\,x^2}{2}+18\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^2*(3*x + 5*x^2 + 2),x)
```

```
[Out] 18*x + (15*x^2)/2 + (53*x^3)/3 + x^4/4 + (61*x^5)/5 - (4*x^6)/3 + (20*x^7)/
7
```

sympy [A] time = 0.07, size = 41, normalized size = 0.89

$$\frac{20x^7}{7}-\frac{4x^6}{3}+\frac{61x^5}{5}+\frac{x^4}{4}+\frac{53x^3}{3}+\frac{15x^2}{2}+18x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**2*(5*x**2+3*x+2),x)
```

```
[Out] 20*x**7/7 - 4*x**6/3 + 61*x**5/5 + x**4/4 + 53*x**3/3 + 15*x**2/2 + 18*x
```

3.23 $\int \frac{(3-x+2x^2)^2}{2+3x+5x^2}\,dx$

Optimal. Leaf size=56

$$\frac{4x^3}{15}-\frac{16x^2}{25}-\frac{1573\log(5x^2+3x+2)}{1250}+\frac{381x}{125}+\frac{8349\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{625\sqrt{31}}$$

Rubi [A] time = 0.05, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1657, 634, 618, 204, 628}

$$\frac{4x^3}{15}-\frac{16x^2}{25}-\frac{1573\log(5x^2+3x+2)}{1250}+\frac{381x}{125}+\frac{8349\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{625\sqrt{31}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^2/(2 + 3*x + 5*x^2),x]
```

```
[Out] (381*x)/125 - (16*x^2)/25 + (4*x^3)/15 + (8349*ArcTan[(3 + 10*x)/Sqrt[31]])
/(625*Sqrt[31]) - (1573*Log[2 + 3*x + 5*x^2])/1250
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(3-x+2x^2\right)^2}{2+3x+5x^2}\,dx &= \int \left(\frac{381}{125}-\frac{32x}{25}+\frac{4x^2}{5}+\frac{121(3-13x)}{125\left(2+3x+5x^2\right)}\right)dx\\
&= \frac{381x}{125}-\frac{16x^2}{25}+\frac{4x^3}{15}+\frac{121}{125}\int \frac{3-13x}{2+3x+5x^2}\,dx\\
&= \frac{381x}{125}-\frac{16x^2}{25}+\frac{4x^3}{15}-\frac{1573\int \frac{3+10x}{2+3x+5x^2}\,dx}{1250}+\frac{8349\int \frac{1}{2+3x+5x^2}\,dx}{1250}\\
&= \frac{381x}{125}-\frac{16x^2}{25}+\frac{4x^3}{15}-\frac{1573\log\left(2+3x+5x^2\right)}{1250}-\frac{8349}{625}\operatorname{Subst}\left(\int \frac{1}{-31-x^2}\,dx,x,3+10\right.\\
&= \frac{381x}{125}-\frac{16x^2}{25}+\frac{4x^3}{15}+\frac{8349\tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)}{625\sqrt{31}}-\frac{1573\log\left(2+3x+5x^2\right)}{1250}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 53, normalized size = 0.95

$$\frac{10x\left(100x^2-240x+1143\right)-4719\log\left(5x^2+3x+2\right)}{3750}+\frac{8349\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{625\sqrt{31}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^2/(2 + 3*x + 5*x^2),x]
```

```
[Out] (8349*ArcTan[(3 + 10*x)/Sqrt[31]])/(625*Sqrt[31]) + (10*x*(1143 - 240*x + 1
00*x^2) - 4719*Log[2 + 3*x + 5*x^2])/3750
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3-x+2x^2\right)^2}{2+3x+5x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(3 - x + 2*x^2)^2/(2 + 3*x + 5*x^2),x]
```

```
[Out] IntegrateAlgebraic[(3 - x + 2*x^2)^2/(2 + 3*x + 5*x^2), x]
```

fricas [A] time = 0.39, size = 43, normalized size = 0.77

$$\frac{4}{15}\,x^3 - \frac{16}{25}\,x^2 + \frac{8349}{19375}\,\sqrt{31}\,\arctan\left(\frac{1}{31}\,\sqrt{31}\,(10\,x+3)\right) + \frac{381}{125}\,x - \frac{1573}{1250}\,\log\left(5\,x^2+3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^2/(5*x^2+3*x+2),x, algorithm="fricas")
```

```
[Out] 4/15*x^3 - 16/25*x^2 + 8349/19375*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3))
 + 381/125*x - 1573/1250*log(5*x^2 + 3*x + 2)
```

giac [A] time = 0.21, size = 43, normalized size = 0.77

$$\frac{4}{15}\,x^3 - \frac{16}{25}\,x^2 + \frac{8349}{19375}\,\sqrt{31}\,\arctan\left(\frac{1}{31}\,\sqrt{31}\,(10\,x+3)\right) + \frac{381}{125}\,x - \frac{1573}{1250}\,\log\left(5\,x^2+3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^2/(5*x^2+3*x+2),x, algorithm="giac")
```

```
[Out] 4/15*x^3 - 16/25*x^2 + 8349/19375*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3))
 + 381/125*x - 1573/1250*log(5*x^2 + 3*x + 2)
```

maple [A] time = 0.00, size = 44, normalized size = 0.79

$$\frac{4x^3}{15} - \frac{16x^2}{25} + \frac{381x}{125} + \frac{8349\sqrt{31}\,\arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)}{19375} - \frac{1573\ln\left(5x^2+3x+2\right)}{1250}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^2/(5*x^2+3*x+2),x)
```

```
[Out] 381/125*x-16/25*x^2+4/15*x^3-1573/1250*ln(5*x^2+3*x+2)+8349/19375*31^(1/2)*
arctan(1/31*(10*x+3)*31^(1/2))
```

maxima [A] time = 0.97, size = 43, normalized size = 0.77

$$\frac{4}{15}\,x^3 - \frac{16}{25}\,x^2 + \frac{8349}{19375}\,\sqrt{31}\,\arctan\left(\frac{1}{31}\,\sqrt{31}\,(10\,x+3)\right) + \frac{381}{125}\,x - \frac{1573}{1250}\,\log\left(5\,x^2+3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^2/(5*x^2+3*x+2),x, algorithm="maxima")
```

```
[Out] 4/15*x^3 - 16/25*x^2 + 8349/19375*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3))
 + 381/125*x - 1573/1250*log(5*x^2 + 3*x + 2)
```

mupad [B] time = 3.45, size = 45, normalized size = 0.80

$$\frac{381\,x}{125}-\frac{1573\,\ln\left(5\,x^2+3\,x+2\right)}{1250}+\frac{8349\,\sqrt{31}\,\mathrm{atan}\left(\frac{10\,\sqrt{31}\,x}{31}+\frac{3\,\sqrt{31}}{31}\right)}{19375}-\frac{16\,x^2}{25}+\frac{4\,x^3}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^2/(3*x + 5*x^2 + 2),x)
```

```
[Out] (381*x)/125 - (1573*log(3*x + 5*x^2 + 2))/1250 + (8349*31^(1/2)*atan((10*31
^(1/2)*x)/31 + (3*31^(1/2))/31))/19375 - (16*x^2)/25 + (4*x^3)/15
```

sympy [A] time = 0.15, size = 63, normalized size = 1.12

$$\frac{4x^3}{15}-\frac{16x^2}{25}+\frac{381x}{125}-\frac{1573\log\left(x^2+\frac{3x}{5}+\frac{2}{5}\right)}{1250}+\frac{8349\sqrt{31}\,\mathrm{atan}\left(\frac{10\sqrt{31}\,x}{31}+\frac{3\sqrt{31}}{31}\right)}{19375}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**2/(5*x**2+3*x+2),x)
```

```
[Out] 4*x**3/15 - 16*x**2/25 + 381*x/125 - 1573*log(x**2 + 3*x/5 + 2/5)/1250 + 83
49*sqrt(31)*atan(10*sqrt(31)*x/31 + 3*sqrt(31)/31)/19375
```

3.24 $\int \frac{(3-x+2x^2)^2}{(2+3x+5x^2)^2}\,dx$

Optimal. Leaf size=63

$$\frac{121(69x+61)}{3875\left(5x^2+3x+2\right)}-\frac{22}{125}\log\left(5x^2+3x+2\right)+\frac{4x}{25}+\frac{41932\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{3875\sqrt{31}}$$

Rubi [A] time = 0.06, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1660, 1657, 634, 618, 204, 628}

$$\frac{121(69x+61)}{3875\left(5x^2+3x+2\right)}-\frac{22}{125}\log\left(5x^2+3x+2\right)+\frac{4x}{25}+\frac{41932\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{3875\sqrt{31}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^2/(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] (4*x)/25 + (121*(61 + 69*x))/(3875*(2 + 3*x + 5*x^2)) + (41932*ArcTan[(3 +
10*x)/Sqrt[31]])/(3875*Sqrt[31]) - (22*Log[2 + 3*x + 5*x^2])/125
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{\left(3-x+2x^2\right)^2}{\left(2+3x+5x^2\right)^2}\,dx = \frac{121(61+69x)}{3875\left(2+3x+5x^2\right)}+\frac{1}{31}\int \frac{\frac{4032}{25}-\frac{992x}{25}+\frac{124x^2}{5}}{2+3x+5x^2}\,dx$$

$$= \frac{121(61+69x)}{3875\left(2+3x+5x^2\right)}+\frac{1}{31}\int \left(\frac{124}{25}+\frac{44(86-31x)}{25\left(2+3x+5x^2\right)}\right)dx$$

$$= \frac{4x}{25}+\frac{121(61+69x)}{3875\left(2+3x+5x^2\right)}+\frac{44}{775}\int \frac{86-31x}{2+3x+5x^2}\,dx$$

$$= \frac{4x}{25}+\frac{121(61+69x)}{3875\left(2+3x+5x^2\right)}-\frac{22}{125}\int \frac{3+10x}{2+3x+5x^2}\,dx+\frac{20966\int \frac{1}{2+3x+5x^2}\,dx}{3875}$$

$$= \frac{4x}{25}+\frac{121(61+69x)}{3875\left(2+3x+5x^2\right)}-\frac{22}{125}\log\left(2+3x+5x^2\right)-\frac{41932\,\text{Subst}\left(\int \frac{1}{-31-x^2}\,dx,x,3+\right.}{3875}$$

$$= \frac{4x}{25}+\frac{121(61+69x)}{3875\left(2+3x+5x^2\right)}+\frac{41932\tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)}{3875\sqrt{31}}-\frac{22}{125}\log\left(2+3x+5x^2\right)$$

Mathematica [A] time = 0.03, size = 59, normalized size = 0.94

$$\frac{\frac{3751(69x+61)}{5x^2+3x+2} - 21142\log\left(5x^2+3x+2\right) + 19220x + 41932\sqrt{31}\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{120125}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^2/(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] (19220*x + (3751*(61 + 69*x))/(2 + 3*x + 5*x^2) + 41932*Sqrt[31]*ArcTan[(3
+ 10*x)/Sqrt[31]] - 21142*Log[2 + 3*x + 5*x^2])/120125
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3-x+2x^2\right)^2}{\left(2+3x+5x^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(3 - x + 2*x^2)^2/(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] IntegrateAlgebraic[(3 - x + 2*x^2)^2/(2 + 3*x + 5*x^2)^2, x]
```

fricas [A] time = 0.40, size = 78, normalized size = 1.24

$$\frac{96100\,x^3 + 41932\,\sqrt{31}\left(5\,x^2+3\,x+2\right)\arctan\left(\frac{1}{31}\,\sqrt{31}\,(10\,x+3)\right) + 57660\,x^2 - 21142\left(5\,x^2+3\,x+2\right)\log\left(5\,x^2+3\,x+2\right) + 297259\,x + 228811}{120125\left(5\,x^2+3\,x+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^2/(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

```
[Out] 1/120125*(96100*x^3 + 41932*sqrt(31)*(5*x^2 + 3*x + 2)*arctan(1/31*sqrt(31)
*(10*x + 3)) + 57660*x^2 - 21142*(5*x^2 + 3*x + 2)*log(5*x^2 + 3*x + 2) + 2
97259*x + 228811)/(5*x^2 + 3*x + 2)
```

giac [A] time = 0.17, size = 52, normalized size = 0.83

$$\frac{41932}{120125}\,\sqrt{31}\arctan\left(\frac{1}{31}\,\sqrt{31}\,(10\,x+3)\right) + \frac{4}{25}\,x + \frac{121\,(69\,x+61)}{3875\left(5\,x^2+3\,x+2\right)} - \frac{22}{125}\,\log\left(5\,x^2+3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^2/(5*x^2+3*x+2)^2,x, algorithm="giac")
```

```
[Out] 41932/120125*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 4/25*x + 121/3875*
(69*x + 61)/(5*x^2 + 3*x + 2) - 22/125*log(5*x^2 + 3*x + 2)
```

maple [A] time = 0.01, size = 51, normalized size = 0.81

$$\frac{4x}{25}+\frac{41932\sqrt{31}\ \arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)}{120125}-\frac{22\ln\left(5x^2+3x+2\right)}{125}-\frac{11\left(-\frac{759x}{775}-\frac{671}{775}\right)}{25\left(x^2+\frac{3}{5}x+\frac{2}{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^2/(5*x^2+3*x+2)^2,x)
```

```
[Out] 4/25*x-11/25*(-759/775*x-671/775)/(x^2+3/5*x+2/5)-22/125*ln(5*x^2+3*x+2)+41
932/120125*31^(1/2)*arctan(1/31*(10*x+3)*31^(1/2))
```

maxima [A] time = 0.96, size = 52, normalized size = 0.83

$$\frac{41932}{120125}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}\,(10\,x+3)\right)+\frac{4}{25}\,x+\frac{121\,(69\,x+61)}{3875\,\left(5\,x^2+3\,x+2\right)}-\frac{22}{125}\,\log\left(5\,x^2+3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^2/(5*x^2+3*x+2)^2,x, algorithm="maxima")
```

```
[Out] 41932/120125*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 4/25*x + 121/3875*
(69*x + 61)/(5*x^2 + 3*x + 2) - 22/125*log(5*x^2 + 3*x + 2)
```

mupad [B] time = 0.05, size = 51, normalized size = 0.81

$$\frac{4\,x}{25}-\frac{22\,\ln\left(5\,x^2+3\,x+2\right)}{125}+\frac{\frac{8349\,x}{19375}+\frac{7381}{19375}}{x^2+\frac{3\,x}{5}+\frac{2}{5}}+\frac{41932\,\sqrt{31}\,\mathrm{atan}\left(\frac{10\,\sqrt{31}\,x}{31}+\frac{3\,\sqrt{31}}{31}\right)}{120125}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^2/(3*x + 5*x^2 + 2)^2,x)
```

```
[Out] (4*x)/25 - (22*log(3*x + 5*x^2 + 2))/125 + ((8349*x)/19375 + 7381/19375)/((
3*x)/5 + x^2 + 2/5) + (41932*31^(1/2)*atan((10*31^(1/2)*x)/31 + (3*31^(1/2)
)/31))/120125
```

sympy [A] time = 0.20, size = 65, normalized size = 1.03

$$\frac{4x}{25}+\frac{8349x+7381}{19375x^2+11625x+7750}-\frac{22\log\left(x^2+\frac{3x}{5}+\frac{2}{5}\right)}{125}+\frac{41932\sqrt{31}\ \mathrm{atan}\left(\frac{10\sqrt{31}\,x}{31}+\frac{3\sqrt{31}}{31}\right)}{120125}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**2/(5*x**2+3*x+2)**2,x)
```

```
[Out] 4*x/25 + (8349*x + 7381)/(19375*x**2 + 11625*x + 7750) - 22*log(x**2 + 3*x/
5 + 2/5)/125 + 41932*sqrt(31)*atan(10*sqrt(31)*x/31 + 3*sqrt(31)/31)/120125
```

3.25 $\int \frac{(3-x+2x^2)^2}{(2+3x+5x^2)^3}\,dx$

Optimal. Leaf size=64

$$\frac{121(69x+61)}{7750\left(5x^2+3x+2\right)^2}+\frac{11(45710x+17557)}{240250\left(5x^2+3x+2\right)}+\frac{4330\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{961\sqrt{31}}$$

Rubi [A] time = 0.05, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1660, 12, 618, 204}

$$\frac{121(69x+61)}{7750\left(5x^2+3x+2\right)^2}+\frac{11(45710x+17557)}{240250\left(5x^2+3x+2\right)}+\frac{4330\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{961\sqrt{31}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^2/(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] (121*(61 + 69*x))/(7750*(2 + 3*x + 5*x^2)^2) + (11*(17557 + 45710*x))/(2402
50*(2 + 3*x + 5*x^2)) + (4330*ArcTan[(3 + 10*x)/Sqrt[31]])/(961*Sqrt[31])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
```

```
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(3-x+2x^2)^2}{(2+3x+5x^2)^3}\,dx &= \frac{121(61+69x)}{7750(2+3x+5x^2)^2}+\frac{1}{62}\int \frac{\frac{48669}{125}-\frac{1984x}{25}+\frac{248x^2}{5}}{(2+3x+5x^2)^2}\,dx\\
&= \frac{121(61+69x)}{7750(2+3x+5x^2)^2}+\frac{11(17557+45710x)}{240250(2+3x+5x^2)}+\frac{\int \frac{4330}{2+3x+5x^2}\,dx}{1922}\\
&= \frac{121(61+69x)}{7750(2+3x+5x^2)^2}+\frac{11(17557+45710x)}{240250(2+3x+5x^2)}+\frac{2165}{961}\int \frac{1}{2+3x+5x^2}\,dx\\
&= \frac{121(61+69x)}{7750(2+3x+5x^2)^2}+\frac{11(17557+45710x)}{240250(2+3x+5x^2)}-\frac{4330}{961}\text{Subst}\left(\int \frac{1}{-31-x^2}\,dx,x,3+10\right.\\
&= \frac{121(61+69x)}{7750(2+3x+5x^2)^2}+\frac{11(17557+45710x)}{240250(2+3x+5x^2)}+\frac{4330\tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)}{961\sqrt{31}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 53, normalized size = 0.83

$$\frac{11\left(45710x^3+44983x^2+33524x+11183\right)}{48050\left(5x^2+3x+2\right)^2}+\frac{4330\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{961\sqrt{31}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^2/(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] (11*(11183 + 33524*x + 44983*x^2 + 45710*x^3))/(48050*(2 + 3*x + 5*x^2)^2)
+ (4330*ArcTan[(3 + 10*x)/Sqrt[31]])/(961*Sqrt[31])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(3-x+2x^2)^2}{(2+3x+5x^2)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(3 - x + 2*x^2)^2/(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] IntegrateAlgebraic[(3 - x + 2*x^2)^2/(2 + 3*x + 5*x^2)^3, x]
```

fricas [A] time = 0.38, size = 75, normalized size = 1.17

$$\frac{15587110\,x^3+216500\,\sqrt{31}\left(25\,x^4+30\,x^3+29\,x^2+12\,x+4\right)\arctan\left(\frac{1}{31}\,\sqrt{31}\,(10\,x+3)\right)+15339203\,x^2+11431684\,x+3813403}{1489550\left(25\,x^4+30\,x^3+29\,x^2+12\,x+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^2/(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

```
[Out] 1/1489550*(15587110*x^3 + 216500*sqrt(31)*(25*x^4 + 30*x^3 + 29*x^2 + 12*x
+ 4)*arctan(1/31*sqrt(31)*(10*x + 3)) + 15339203*x^2 + 11431684*x + 3813403
)/(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)
```

giac [A] time = 0.18, size = 46, normalized size = 0.72

$$\frac{4330}{29791}\,\sqrt{31}\,\arctan\left(\frac{1}{31}\,\sqrt{31}\,(10\,x+3)\right)+\frac{11\left(45710\,x^3+44983\,x^2+33524\,x+11183\right)}{48050\left(5\,x^2+3\,x+2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^2/(5*x^2+3*x+2)^3,x, algorithm="giac")
```

```
[Out] 4330/29791*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 11/48050*(45710*x^3
+ 44983*x^2 + 33524*x + 11183)/(5*x^2 + 3*x + 2)^2
```

maple [A] time = 0.01, size = 47, normalized size = 0.73

$$\frac{4330\sqrt{31}\;\arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)}{29791}+\frac{\frac{50281}{4805}x^3+\frac{494813}{48050}x^2+\frac{184382}{24025}x+\frac{123013}{48050}}{\left(5x^2+3x+2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^2/(5*x^2+3*x+2)^3,x)
```

```
[Out] 25*(50281/120125*x^3+494813/1201250*x^2+184382/600625*x+123013/1201250)/(5*
x^2+3*x+2)^2+4330/29791*31^(1/2)*arctan(1/31*(10*x+3)*31^(1/2))
```

maxima [A] time = 0.97, size = 56, normalized size = 0.88

$$\frac{4330}{29791}\,\sqrt{31}\,\arctan\left(\frac{1}{31}\,\sqrt{31}\,(10\,x+3)\right)+\frac{11\left(45710\,x^3+44983\,x^2+33524\,x+11183\right)}{48050\left(25\,x^4+30\,x^3+29\,x^2+12\,x+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^2/(5*x^2+3*x+2)^3,x, algorithm="maxima")
```

```
[Out] 4330/29791*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 11/48050*(45710*x^3
+ 44983*x^2 + 33524*x + 11183)/(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)
```

mupad [B] time = 3.44, size = 55, normalized size = 0.86

$$\frac{4330\,\sqrt{31}\,\mathrm{atan}\left(\frac{10\,\sqrt{31}\,x}{31}+\frac{3\,\sqrt{31}}{31}\right)}{29791}+\frac{\frac{50281\,x^3}{120125}+\frac{494813\,x^2}{1201250}+\frac{184382\,x}{600625}+\frac{123013}{1201250}}{x^4+\frac{6\,x^3}{5}+\frac{29\,x^2}{25}+\frac{12\,x}{25}+\frac{4}{25}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^2/(3*x + 5*x^2 + 2)^3,x)
```

```
[Out] (4330*31^(1/2)*atan((10*31^(1/2)*x)/31 + (3*31^(1/2))/31))/29791 + ((184382
*x)/600625 + (494813*x^2)/1201250 + (50281*x^3)/120125 + 123013/1201250)/((
12*x)/25 + (29*x^2)/25 + (6*x^3)/5 + x^4 + 4/25)
```

sympy [A] time = 0.20, size = 63, normalized size = 0.98

$$\frac{502810x^3+494813x^2+368764x+123013}{1201250x^4+1441500x^3+1393450x^2+576600x+192200}+\frac{4330\sqrt{31}\,\mathrm{atan}\left(\frac{10\sqrt{31}\,x}{31}+\frac{3\sqrt{31}}{31}\right)}{29791}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**2/(5*x**2+3*x+2)**3,x)
```

```
[Out] (502810*x**3 + 494813*x**2 + 368764*x + 123013)/(1201250*x**4 + 1441500*x**
3 + 1393450*x**2 + 576600*x + 192200) + 4330*sqrt(31)*atan(10*sqrt(31)*x/31
 + 3*sqrt(31)/31)/29791
```

3.26 $\int \frac{(3-x+2x^2)^2}{(2+3x+5x^2)^4}\,dx$

Optimal. Leaf size=85

$$\frac{16688(10x+3)}{148955\left(5x^2+3x+2\right)}+\frac{11(12060x+4579)}{120125\left(5x^2+3x+2\right)^2}+\frac{121(69x+61)}{11625\left(5x^2+3x+2\right)^3}+\frac{66752\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{29791\sqrt{31}}$$

Rubi [A] time = 0.06, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1660, 12, 614, 618, 204}

$$\frac{16688(10x+3)}{148955\left(5x^2+3x+2\right)}+\frac{11(12060x+4579)}{120125\left(5x^2+3x+2\right)^2}+\frac{121(69x+61)}{11625\left(5x^2+3x+2\right)^3}+\frac{66752\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{29791\sqrt{31}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^2/(2 + 3*x + 5*x^2)^4,x]
```

```
[Out] (121*(61 + 69*x))/(11625*(2 + 3*x + 5*x^2)^3) + (11*(4579 + 12060*x))/(1201
25*(2 + 3*x + 5*x^2)^2) + (16688*(3 + 10*x))/(148955*(2 + 3*x + 5*x^2)) + (
66752*ArcTan[(3 + 10*x)/Sqrt[31]])/(29791*Sqrt[31])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{\left(3-x+2x^2\right)^2}{\left(2+3x+5x^2\right)^4}\,dx = \frac{121(61+69x)}{11625\left(2+3x+5x^2\right)^3}+\frac{1}{93}\int \frac{\frac{77178}{125}-\frac{2976x}{25}+\frac{372x^2}{5}}{\left(2+3x+5x^2\right)^3}\,dx$$

$$= \frac{121(61+69x)}{11625\left(2+3x+5x^2\right)^3}+\frac{11(4579+12060x)}{120125\left(2+3x+5x^2\right)^2}+\frac{\int \frac{100128}{5\left(2+3x+5x^2\right)^2}\,dx}{5766}$$

$$= \frac{121(61+69x)}{11625\left(2+3x+5x^2\right)^3}+\frac{11(4579+12060x)}{120125\left(2+3x+5x^2\right)^2}+\frac{16688\int \frac{1}{\left(2+3x+5x^2\right)^2}\,dx}{4805}$$

$$= \frac{121(61+69x)}{11625\left(2+3x+5x^2\right)^3}+\frac{11(4579+12060x)}{120125\left(2+3x+5x^2\right)^2}+\frac{16688(3+10x)}{148955\left(2+3x+5x^2\right)}+\frac{33376\int \ldots}{29\ldots}$$

$$= \frac{121(61+69x)}{11625\left(2+3x+5x^2\right)^3}+\frac{11(4579+12060x)}{120125\left(2+3x+5x^2\right)^2}+\frac{16688(3+10x)}{148955\left(2+3x+5x^2\right)}-\frac{66752\,\mathrm{Su}\ldots}{\ldots}$$

$$= \frac{121(61+69x)}{11625\left(2+3x+5x^2\right)^3}+\frac{11(4579+12060x)}{120125\left(2+3x+5x^2\right)^2}+\frac{16688(3+10x)}{148955\left(2+3x+5x^2\right)}+\frac{66752\tan\ldots}{2979\ldots}$$

Mathematica [A] time = 0.04, size = 63, normalized size = 0.74

$$\frac{12516000x^5+18774000x^4+21491796x^3+12780597x^2+5674908x+1259239}{446865\left(5x^2+3x+2\right)^3}+\frac{66752\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{29791\sqrt{31}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^2/(2 + 3*x + 5*x^2)^4,x]
```

```
[Out] (1259239 + 5674908*x + 12780597*x^2 + 21491796*x^3 + 18774000*x^4 + 1251600
0*x^5)/(446865*(2 + 3*x + 5*x^2)^3) + (66752*ArcTan[(3 + 10*x)/Sqrt[31]])/(
29791*Sqrt[31])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3-x+2x^2\right)^2}{\left(2+3x+5x^2\right)^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(3 - x + 2*x^2)^2/(2 + 3*x + 5*x^2)^4,x]
```

```
[Out] IntegrateAlgebraic[(3 - x + 2*x^2)^2/(2 + 3*x + 5*x^2)^4, x]
```

fricas [A] time = 0.38, size = 105, normalized size = 1.24

$$\frac{387996000\,x^5+581994000\,x^4+666245676\,x^3+1001280\,\sqrt{31}\left(125\,x^6+225\,x^5+285\,x^4+207\,x^3+114\,x^2+36\,x+8\right)\arctan\left(\frac{1}{31}\,\sqrt{31}\,(10\,x+3)\right)+396198507\,x^2+175922148\,x+39036409}{13852815\left(125\,x^6+225\,x^5+285\,x^4+207\,x^3+114\,x^2+36\,x+8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^2/(5*x^2+3*x+2)^4,x, algorithm="fricas")
```

```
[Out] 1/13852815*(387996000*x^5 + 581994000*x^4 + 666245676*x^3 + 1001280*sqrt(31
)*(125*x^6 + 225*x^5 + 285*x^4 + 207*x^3 + 114*x^2 + 36*x + 8)*arctan(1/31*
sqrt(31)*(10*x + 3)) + 396198507*x^2 + 175922148*x + 39036409)/(125*x^6 + 2
25*x^5 + 285*x^4 + 207*x^3 + 114*x^2 + 36*x + 8)
```

giac [A] time = 0.18, size = 56, normalized size = 0.66

$$\frac{66752}{923521}\,\sqrt{31}\,\arctan\left(\frac{1}{31}\,\sqrt{31}\,(10\,x+3)\right)+\frac{12516000\,x^5+18774000\,x^4+21491796\,x^3+12780597\,x^2+5674908\,x+1259239}{446865\left(5\,x^2+3\,x+2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^2/(5*x^2+3*x+2)^4,x, algorithm="giac")
```

```
[Out] 66752/923521*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 1/446865*(12516000
*x^5 + 18774000*x^4 + 21491796*x^3 + 12780597*x^2 + 5674908*x + 1259239)/(5
*x^2 + 3*x + 2)^3
```

maple [A] time = 0.01, size = 57, normalized size = 0.67

$$\frac{66752\sqrt{31}\ \arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)}{923521}+\frac{\frac{834400}{29791}x^5+\frac{1251600}{29791}x^4+\frac{7163932}{148955}x^3+\frac{4260199}{148955}x^2+\frac{1891636}{148955}x+\frac{1259239}{446865}}{\left(5x^2+3x+2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^2/(5*x^2+3*x+2)^4,x)
```

```
[Out] 125*(33376/148955*x^5+50064/148955*x^4+7163932/18619375*x^3+4260199/1861937
5*x^2+1891636/18619375*x+1259239/55858125)/(5*x^2+3*x+2)^3+66752/923521*31^
(1/2)*arctan(1/31*(10*x+3)*31^(1/2))
```

maxima [A] time = 0.96, size = 76, normalized size = 0.89

$$\frac{66752}{923521}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}\left(10x+3\right)\right)+\frac{12516000x^5+18774000x^4+21491796x^3+12780597x^2+5674908x+1259239}{446865\left(125x^6+225x^5+285x^4+207x^3+114x^2+36x+8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^2/(5*x^2+3*x+2)^4,x, algorithm="maxima")
```

```
[Out] 66752/923521*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 1/446865*(12516000
*x^5 + 18774000*x^4 + 21491796*x^3 + 12780597*x^2 + 5674908*x + 1259239)/(1
25*x^6 + 225*x^5 + 285*x^4 + 207*x^3 + 114*x^2 + 36*x + 8)
```

mupad [B] time = 3.47, size = 75, normalized size = 0.88

$$\frac{66752\sqrt{31}\ \mathrm{atan}\left(\frac{10\sqrt{31}\,x}{31}+\frac{3\sqrt{31}}{31}\right)}{923521}+\frac{\frac{33376x^5}{148955}+\frac{50064x^4}{148955}+\frac{7163932x^3}{18619375}+\frac{4260199x^2}{18619375}+\frac{1891636x}{18619375}+\frac{1259239}{55858125}}{x^6+\frac{9x^5}{5}+\frac{57x^4}{25}+\frac{207x^3}{125}+\frac{114x^2}{125}+\frac{36x}{125}+\frac{8}{125}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^2/(3*x + 5*x^2 + 2)^4,x)
```

```
[Out] (66752*31^(1/2)*atan((10*31^(1/2)*x)/31 + (3*31^(1/2))/31))/923521 + ((1891
636*x)/18619375 + (4260199*x^2)/18619375 + (7163932*x^3)/18619375 + (50064*
x^4)/148955 + (33376*x^5)/148955 + 1259239/55858125)/((36*x)/125 + (114*x^2
)/125 + (207*x^3)/125 + (57*x^4)/25 + (9*x^5)/5 + x^6 + 8/125)
```

sympy [A] time = 0.23, size = 83, normalized size = 0.98

$$\frac{12516000x^5 + 18774000x^4 + 21491796x^3 + 12780597x^2 + 5674908x + 1259239}{55858125x^6 + 100544625x^5 + 127356525x^4 + 92501055x^3 + 50942610x^2 + 16087140x + 3574920} + \frac{66752\sqrt{31}\,\operatorname{atan}\left(\frac{10\sqrt{31}\,x}{31} + \frac{3\sqrt{31}}{31}\right)}{923521}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**2/(5*x**2+3*x+2)**4,x)
```

```
[Out] (12516000*x**5 + 18774000*x**4 + 21491796*x**3 + 12780597*x**2 + 5674908*x
+ 1259239)/(55858125*x**6 + 100544625*x**5 + 127356525*x**4 + 92501055*x**3
 + 50942610*x**2 + 16087140*x + 3574920) + 66752*sqrt(31)*atan(10*sqrt(31)*
x/31 + 3*sqrt(31)/31)/923521
```

3.27 $\int \left(3-x+2x^2\right)^3 \left(2+3x+5x^2\right)^4 \, dx$

Optimal. Leaf size=96

$$\frac{1000x^{15}}{3}+\frac{2250x^{14}}{7}+\frac{27050x^{13}}{13}+\frac{30395x^{12}}{12}+\frac{68583x^{11}}{11}+\frac{75311x^{10}}{10}+\frac{103583x^9}{9}+\frac{94881x^8}{8}+\frac{91349x^7}{7}+\frac{64529x^6}{6}+43$$

Rubi [A] time = 0.07, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {1657}

$$\frac{1000x^{15}}{3}+\frac{2250x^{14}}{7}+\frac{27050x^{13}}{13}+\frac{30395x^{12}}{12}+\frac{68583x^{11}}{11}+\frac{75311x^{10}}{10}+\frac{103583x^9}{9}+\frac{94881x^8}{8}+\frac{91349x^7}{7}+\frac{64529x^6}{6}+\frac{43083x^5}{5}+5144x^4+2856x^3+1080x^2+432x$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2)^4,x]
```

```
[Out] 432*x + 1080*x^2 + 2856*x^3 + 5144*x^4 + (43083*x^5)/5 + (64529*x^6)/6 + (9
1349*x^7)/7 + (94881*x^8)/8 + (103583*x^9)/9 + (75311*x^10)/10 + (68583*x^1
1)/11 + (30395*x^12)/12 + (27050*x^13)/13 + (2250*x^14)/7 + (1000*x^15)/3
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \left(3-x+2x^2\right)^3 \left(2+3x+5x^2\right)^4 \, dx = \int \left(432+2160x+8568x^2+20576x^3+43083x^4+64529x^5+91349x^6+\right.$$
$$= 432x+1080x^2+2856x^3+5144x^4+\frac{43083x^5}{5}+\frac{64529x^6}{6}+\frac{91349x^7}{7}+$$

Mathematica [A] time = 0.00, size = 96, normalized size = 1.00

$$\frac{1000x^{15}}{3}+\frac{2250x^{14}}{7}+\frac{27050x^{13}}{13}+\frac{30395x^{12}}{12}+\frac{68583x^{11}}{11}+\frac{75311x^{10}}{10}+\frac{103583x^9}{9}+\frac{94881x^8}{8}+\frac{91349x^7}{7}+\frac{64529x^6}{6}+\frac{43083x^5}{5}+5144x^4+2856x^3+1080x^2+432x$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2)^4,x]
```

[Out] 432*x + 1080*x^2 + 2856*x^3 + 5144*x^4 + (43083*x^5)/5 + (64529*x^6)/6 + (9
1349*x^7)/7 + (94881*x^8)/8 + (103583*x^9)/9 + (75311*x^10)/10 + (68583*x^1
1)/11 + (30395*x^12)/12 + (27050*x^13)/13 + (2250*x^14)/7 + (1000*x^15)/3

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(3-x+2x^2\right)^3 \left(2+3x+5x^2\right)^4 \, dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2)^4,x]

[Out] IntegrateAlgebraic[(3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2)^4, x]

fricas [A] time = 0.34, size = 74, normalized size = 0.77

$$\frac{1000}{3}x^{15} + \frac{2250}{7}x^{14} + \frac{27050}{13}x^{13} + \frac{30395}{12}x^{12} + \frac{68583}{11}x^{11} + \frac{75311}{10}x^{10} + \frac{103583}{9}x^9 + \frac{94881}{8}x^8 + \frac{91349}{7}x^7 + \frac{64529}{6}x^6 + \frac{43083}{5}x^5 + 5144x^4 + 2856x^3 + 1080x^2 + 432x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-x+3)^3*(5*x^2+3*x+2)^4,x, algorithm="fricas")

[Out] 1000/3*x^15 + 2250/7*x^14 + 27050/13*x^13 + 30395/12*x^12 + 68583/11*x^11 +
 75311/10*x^10 + 103583/9*x^9 + 94881/8*x^8 + 91349/7*x^7 + 64529/6*x^6 + 4
3083/5*x^5 + 5144*x^4 + 2856*x^3 + 1080*x^2 + 432*x

giac [A] time = 0.18, size = 74, normalized size = 0.77

$$\frac{1000}{3}x^{15} + \frac{2250}{7}x^{14} + \frac{27050}{13}x^{13} + \frac{30395}{12}x^{12} + \frac{68583}{11}x^{11} + \frac{75311}{10}x^{10} + \frac{103583}{9}x^9 + \frac{94881}{8}x^8 + \frac{91349}{7}x^7 + \frac{64529}{6}x^6 + \frac{43083}{5}x^5 + 5144x^4 + 2856x^3 + 1080x^2 + 432x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-x+3)^3*(5*x^2+3*x+2)^4,x, algorithm="giac")

[Out] 1000/3*x^15 + 2250/7*x^14 + 27050/13*x^13 + 30395/12*x^12 + 68583/11*x^11 +
 75311/10*x^10 + 103583/9*x^9 + 94881/8*x^8 + 91349/7*x^7 + 64529/6*x^6 + 4
3083/5*x^5 + 5144*x^4 + 2856*x^3 + 1080*x^2 + 432*x

maple [A] time = 0.00, size = 75, normalized size = 0.78

$$\frac{1000}{3}x^{15} + \frac{2250}{7}x^{14} + \frac{27050}{13}x^{13} + \frac{30395}{12}x^{12} + \frac{68583}{11}x^{11} + \frac{75311}{10}x^{10} + \frac{103583}{9}x^9 + \frac{94881}{8}x^8 + \frac{91349}{7}x^7 + \frac{64529}{6}x^6 + \frac{43083}{5}x^5 + 5144x^4 + 2856x^3 + 1080x^2 + 432x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^2-x+3)^3*(5*x^2+3*x+2)^4,x)

```
[Out] 432*x+1080*x^2+2856*x^3+5144*x^4+43083/5*x^5+64529/6*x^6+91349/7*x^7+94881/
8*x^8+103583/9*x^9+75311/10*x^10+68583/11*x^11+30395/12*x^12+27050/13*x^13+
2250/7*x^14+1000/3*x^15
```

maxima [A] time = 0.43, size = 74, normalized size = 0.77

$$\frac{1000}{3}\,x^{15}+\frac{2250}{7}\,x^{14}+\frac{27050}{13}\,x^{13}+\frac{30395}{12}\,x^{12}+\frac{68583}{11}\,x^{11}+\frac{75311}{10}\,x^{10}+\frac{103583}{9}\,x^{9}+\frac{94881}{8}\,x^{8}+\frac{91349}{7}\,x^{7}+\frac{64529}{6}\,x^{6}+\frac{43083}{5}\,x^{5}+5144\,x^{4}+2856\,x^{3}+1080\,x^{2}+432\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^3*(5*x^2+3*x+2)^4,x, algorithm="maxima")
```

```
[Out] 1000/3*x^15 + 2250/7*x^14 + 27050/13*x^13 + 30395/12*x^12 + 68583/11*x^11 +
 75311/10*x^10 + 103583/9*x^9 + 94881/8*x^8 + 91349/7*x^7 + 64529/6*x^6 + 4
3083/5*x^5 + 5144*x^4 + 2856*x^3 + 1080*x^2 + 432*x
```

mupad [B] time = 0.12, size = 74, normalized size = 0.77

$$\frac{1000\,x^{15}}{3}+\frac{2250\,x^{14}}{7}+\frac{27050\,x^{13}}{13}+\frac{30395\,x^{12}}{12}+\frac{68583\,x^{11}}{11}+\frac{75311\,x^{10}}{10}+\frac{103583\,x^{9}}{9}+\frac{94881\,x^{8}}{8}+\frac{91349\,x^{7}}{7}+\frac{64529\,x^{6}}{6}+\frac{43083\,x^{5}}{5}+5144\,x^{4}+2856\,x^{3}+1080\,x^{2}+432\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^3*(3*x + 5*x^2 + 2)^4,x)
```

```
[Out] 432*x + 1080*x^2 + 2856*x^3 + 5144*x^4 + (43083*x^5)/5 + (64529*x^6)/6 + (9
1349*x^7)/7 + (94881*x^8)/8 + (103583*x^9)/9 + (75311*x^10)/10 + (68583*x^1
1)/11 + (30395*x^12)/12 + (27050*x^13)/13 + (2250*x^14)/7 + (1000*x^15)/3
```

sympy [A] time = 0.10, size = 92, normalized size = 0.96

$$\frac{1000x^{15}}{3}+\frac{2250x^{14}}{7}+\frac{27050x^{13}}{13}+\frac{30395x^{12}}{12}+\frac{68583x^{11}}{11}+\frac{75311x^{10}}{10}+\frac{103583x^{9}}{9}+\frac{94881x^{8}}{8}+\frac{91349x^{7}}{7}+\frac{64529x^{6}}{6}+\frac{43083x^{5}}{5}+5144x^{4}+2856x^{3}+1080x^{2}+432x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**3*(5*x**2+3*x+2)**4,x)
```

```
[Out] 1000*x**15/3 + 2250*x**14/7 + 27050*x**13/13 + 30395*x**12/12 + 68583*x**11
/11 + 75311*x**10/10 + 103583*x**9/9 + 94881*x**8/8 + 91349*x**7/7 + 64529*
x**6/6 + 43083*x**5/5 + 5144*x**4 + 2856*x**3 + 1080*x**2 + 432*x
```

3.28 $\int \left(3-x+2x^2\right)^3 \left(2+3x+5x^2\right)^3 \, dx$

Optimal. Leaf size=82

$$\frac{1000x^{13}}{13}+25x^{12}+\frac{4830x^{11}}{11}+\frac{3061x^{10}}{10}+\frac{3316x^9}{3}+\frac{7869x^8}{8}+\frac{12016x^7}{7}+\frac{2873x^6}{2}+\frac{8292x^5}{5}+\frac{4483x^4}{4}+870x^3+378x^2+2$$

Rubi [A] time = 0.06, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {1657}

$$\frac{1000x^{13}}{13}+25x^{12}+\frac{4830x^{11}}{11}+\frac{3061x^{10}}{10}+\frac{3316x^9}{3}+\frac{7869x^8}{8}+\frac{12016x^7}{7}+\frac{2873x^6}{2}+\frac{8292x^5}{5}+\frac{4483x^4}{4}+870x^3+378x^2+216x$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] 216*x + 378*x^2 + 870*x^3 + (4483*x^4)/4 + (8292*x^5)/5 + (2873*x^6)/2 + (1
2016*x^7)/7 + (7869*x^8)/8 + (3316*x^9)/3 + (3061*x^10)/10 + (4830*x^11)/11
 + 25*x^12 + (1000*x^13)/13
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \left(3-x+2x^2\right)^3 \left(2+3x+5x^2\right)^3 \, dx = \int \left(216+756x+2610x^2+4483x^3+8292x^4+8619x^5+12016x^6+78\right.$$

$$= 216x+378x^2+870x^3+\frac{4483x^4}{4}+\frac{8292x^5}{5}+\frac{2873x^6}{2}+\frac{12016x^7}{7}+\frac{786}{}$$

Mathematica [A] time = 0.00, size = 82, normalized size = 1.00

$$\frac{1000x^{13}}{13}+25x^{12}+\frac{4830x^{11}}{11}+\frac{3061x^{10}}{10}+\frac{3316x^9}{3}+\frac{7869x^8}{8}+\frac{12016x^7}{7}+\frac{2873x^6}{2}+\frac{8292x^5}{5}+\frac{4483x^4}{4}+870x^3+378x^2+216x$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] 216*x + 378*x^2 + 870*x^3 + (4483*x^4)/4 + (8292*x^5)/5 + (2873*x^6)/2 + (1
2016*x^7)/7 + (7869*x^8)/8 + (3316*x^9)/3 + (3061*x^10)/10 + (4830*x^11)/11
 + 25*x^12 + (1000*x^13)/13
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(3-x+2x^2\right)^3 \left(2+3x+5x^2\right)^3 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] IntegrateAlgebraic[(3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2)^3, x]
```

fricas [A] time = 0.33, size = 64, normalized size = 0.78

$$\frac{1000}{13}x^{13}+25x^{12}+\frac{4830}{11}x^{11}+\frac{3061}{10}x^{10}+\frac{3316}{3}x^9+\frac{7869}{8}x^8+\frac{12016}{7}x^7+\frac{2873}{2}x^6+\frac{8292}{5}x^5+\frac{4483}{4}x^4+870x^3+378x^2+216x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^3*(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

```
[Out] 1000/13*x^13 + 25*x^12 + 4830/11*x^11 + 3061/10*x^10 + 3316/3*x^9 + 7869/8*
x^8 + 12016/7*x^7 + 2873/2*x^6 + 8292/5*x^5 + 4483/4*x^4 + 870*x^3 + 378*x^
2 + 216*x
```

giac [A] time = 0.21, size = 64, normalized size = 0.78

$$\frac{1000}{13}x^{13}+25x^{12}+\frac{4830}{11}x^{11}+\frac{3061}{10}x^{10}+\frac{3316}{3}x^9+\frac{7869}{8}x^8+\frac{12016}{7}x^7+\frac{2873}{2}x^6+\frac{8292}{5}x^5+\frac{4483}{4}x^4+870x^3+378x^2+216x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^3*(5*x^2+3*x+2)^3,x, algorithm="giac")
```

```
[Out] 1000/13*x^13 + 25*x^12 + 4830/11*x^11 + 3061/10*x^10 + 3316/3*x^9 + 7869/8*
x^8 + 12016/7*x^7 + 2873/2*x^6 + 8292/5*x^5 + 4483/4*x^4 + 870*x^3 + 378*x^
2 + 216*x
```

maple [A] time = 0.00, size = 65, normalized size = 0.79

$$\frac{1000}{13}x^{13}+25x^{12}+\frac{4830}{11}x^{11}+\frac{3061}{10}x^{10}+\frac{3316}{3}x^9+\frac{7869}{8}x^8+\frac{12016}{7}x^7+\frac{2873}{2}x^6+\frac{8292}{5}x^5+\frac{4483}{4}x^4+870x^3+378x^2+216x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^3*(5*x^2+3*x+2)^3,x)
```

```
[Out] 216*x+378*x^2+870*x^3+4483/4*x^4+8292/5*x^5+2873/2*x^6+12016/7*x^7+7869/8*x
^8+3316/3*x^9+3061/10*x^10+4830/11*x^11+25*x^12+1000/13*x^13
```

maxima [A] time = 0.43, size = 64, normalized size = 0.78

$$\frac{1000}{13}\,x^{13} + 25\,x^{12} + \frac{4830}{11}\,x^{11} + \frac{3061}{10}\,x^{10} + \frac{3316}{3}\,x^{9} + \frac{7869}{8}\,x^{8} + \frac{12016}{7}\,x^{7} + \frac{2873}{2}\,x^{6} + \frac{8292}{5}\,x^{5} + \frac{4483}{4}\,x^{4} + 870\,x^{3} + 378\,x^{2} + 216\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^3*(5*x^2+3*x+2)^3,x, algorithm="maxima")
```

```
[Out] 1000/13*x^13 + 25*x^12 + 4830/11*x^11 + 3061/10*x^10 + 3316/3*x^9 + 7869/8*
x^8 + 12016/7*x^7 + 2873/2*x^6 + 8292/5*x^5 + 4483/4*x^4 + 870*x^3 + 378*x^
2 + 216*x
```

mupad [B] time = 0.08, size = 64, normalized size = 0.78

$$\frac{1000\,x^{13}}{13} + 25\,x^{12} + \frac{4830\,x^{11}}{11} + \frac{3061\,x^{10}}{10} + \frac{3316\,x^{9}}{3} + \frac{7869\,x^{8}}{8} + \frac{12016\,x^{7}}{7} + \frac{2873\,x^{6}}{2} + \frac{8292\,x^{5}}{5} + \frac{4483\,x^{4}}{4} + 870\,x^{3} + 378\,x^{2} + 216\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^3*(3*x + 5*x^2 + 2)^3,x)
```

```
[Out] 216*x + 378*x^2 + 870*x^3 + (4483*x^4)/4 + (8292*x^5)/5 + (2873*x^6)/2 + (1
2016*x^7)/7 + (7869*x^8)/8 + (3316*x^9)/3 + (3061*x^10)/10 + (4830*x^11)/11
 + 25*x^12 + (1000*x^13)/13
```

sympy [A] time = 0.09, size = 78, normalized size = 0.95

$$\frac{1000x^{13}}{13} + 25x^{12} + \frac{4830x^{11}}{11} + \frac{3061x^{10}}{10} + \frac{3316x^{9}}{3} + \frac{7869x^{8}}{8} + \frac{12016x^{7}}{7} + \frac{2873x^{6}}{2} + \frac{8292x^{5}}{5} + \frac{4483x^{4}}{4} + 870x^{3} + 378x^{2} + 216x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**3*(5*x**2+3*x+2)**3,x)
```

```
[Out] 1000*x**13/13 + 25*x**12 + 4830*x**11/11 + 3061*x**10/10 + 3316*x**9/3 + 78
69*x**8/8 + 12016*x**7/7 + 2873*x**6/2 + 8292*x**5/5 + 4483*x**4/4 + 870*x*
*3 + 378*x**2 + 216*x
```

3.29 $\int \left(3-x+2x^2\right)^3 \left(2+3x+5x^2\right)^2 \, dx$

Optimal. Leaf size=68

$$\frac{200x^{11}}{11}-6x^{10}+\frac{922x^9}{9}+\frac{83x^8}{8}+\frac{1571x^7}{7}+\frac{299x^6}{3}+\frac{1416x^5}{5}+\frac{635x^4}{4}+237x^3+108x^2+108x$$

Rubi [A] time = 0.05, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {1657}

$$\frac{200x^{11}}{11}-6x^{10}+\frac{922x^9}{9}+\frac{83x^8}{8}+\frac{1571x^7}{7}+\frac{299x^6}{3}+\frac{1416x^5}{5}+\frac{635x^4}{4}+237x^3+108x^2+108x$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] 108*x + 108*x^2 + 237*x^3 + (635*x^4)/4 + (1416*x^5)/5 + (299*x^6)/3 + (157
1*x^7)/7 + (83*x^8)/8 + (922*x^9)/9 - 6*x^10 + (200*x^11)/11
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \left(3-x+2x^2\right)^3 \left(2+3x+5x^2\right)^2 \, dx &= \int \left(108+216x+711x^2+635x^3+1416x^4+598x^5+1571x^6+83x^7+92\right.\\ &= 108x+108x^2+237x^3+\frac{635x^4}{4}+\frac{1416x^5}{5}+\frac{299x^6}{3}+\frac{1571x^7}{7}+\frac{83x^8}{8}+\end{aligned}$$

Mathematica [A] time = 0.00, size = 68, normalized size = 1.00

$$\frac{200x^{11}}{11}-6x^{10}+\frac{922x^9}{9}+\frac{83x^8}{8}+\frac{1571x^7}{7}+\frac{299x^6}{3}+\frac{1416x^5}{5}+\frac{635x^4}{4}+237x^3+108x^2+108x$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] 108*x + 108*x^2 + 237*x^3 + (635*x^4)/4 + (1416*x^5)/5 + (299*x^6)/3 + (157
1*x^7)/7 + (83*x^8)/8 + (922*x^9)/9 - 6*x^10 + (200*x^11)/11
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(3-x+2x^2\right)^3 \left(2+3x+5x^2\right)^2 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] IntegrateAlgebraic[(3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2)^2, x]
```

fricas [A] time = 0.34, size = 54, normalized size = 0.79

$$\frac{200}{11}x^{11} - 6x^{10} + \frac{922}{9}x^9 + \frac{83}{8}x^8 + \frac{1571}{7}x^7 + \frac{299}{3}x^6 + \frac{1416}{5}x^5 + \frac{635}{4}x^4 + 237x^3 + 108x^2 + 108x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^3*(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

```
[Out] 200/11*x^11 - 6*x^10 + 922/9*x^9 + 83/8*x^8 + 1571/7*x^7 + 299/3*x^6 + 1416
/5*x^5 + 635/4*x^4 + 237*x^3 + 108*x^2 + 108*x
```

giac [A] time = 0.20, size = 54, normalized size = 0.79

$$\frac{200}{11}\,x^{11} - 6\,x^{10} + \frac{922}{9}\,x^9 + \frac{83}{8}\,x^8 + \frac{1571}{7}\,x^7 + \frac{299}{3}\,x^6 + \frac{1416}{5}\,x^5 + \frac{635}{4}\,x^4 + 237\,x^3 + 108\,x^2 + 108\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^3*(5*x^2+3*x+2)^2,x, algorithm="giac")
```

```
[Out] 200/11*x^11 - 6*x^10 + 922/9*x^9 + 83/8*x^8 + 1571/7*x^7 + 299/3*x^6 + 1416
/5*x^5 + 635/4*x^4 + 237*x^3 + 108*x^2 + 108*x
```

maple [A] time = 0.00, size = 55, normalized size = 0.81

$$\frac{200}{11}x^{11} - 6x^{10} + \frac{922}{9}x^9 + \frac{83}{8}x^8 + \frac{1571}{7}x^7 + \frac{299}{3}x^6 + \frac{1416}{5}x^5 + \frac{635}{4}x^4 + 237x^3 + 108x^2 + 108x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^3*(5*x^2+3*x+2)^2,x)
```

```
[Out] 108*x+108*x^2+237*x^3+635/4*x^4+1416/5*x^5+299/3*x^6+1571/7*x^7+83/8*x^8+92
2/9*x^9-6*x^10+200/11*x^11
```

maxima [A] time = 0.43, size = 54, normalized size = 0.79

$$\frac{200}{11}\,x^{11} - 6\,x^{10} + \frac{922}{9}\,x^{9} + \frac{83}{8}\,x^{8} + \frac{1571}{7}\,x^{7} + \frac{299}{3}\,x^{6} + \frac{1416}{5}\,x^{5} + \frac{635}{4}\,x^{4} + 237\,x^{3} + 108\,x^{2} + 108\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^3*(5*x^2+3*x+2)^2,x, algorithm="maxima")
```

```
[Out] 200/11*x^11 - 6*x^10 + 922/9*x^9 + 83/8*x^8 + 1571/7*x^7 + 299/3*x^6 + 1416
/5*x^5 + 635/4*x^4 + 237*x^3 + 108*x^2 + 108*x
```

mupad [B] time = 0.05, size = 54, normalized size = 0.79

$$\frac{200\,x^{11}}{11} - 6\,x^{10} + \frac{922\,x^{9}}{9} + \frac{83\,x^{8}}{8} + \frac{1571\,x^{7}}{7} + \frac{299\,x^{6}}{3} + \frac{1416\,x^{5}}{5} + \frac{635\,x^{4}}{4} + 237\,x^{3} + 108\,x^{2} + 108\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^3*(3*x + 5*x^2 + 2)^2,x)
```

```
[Out] 108*x + 108*x^2 + 237*x^3 + (635*x^4)/4 + (1416*x^5)/5 + (299*x^6)/3 + (157
1*x^7)/7 + (83*x^8)/8 + (922*x^9)/9 - 6*x^10 + (200*x^11)/11
```

sympy [A] time = 0.08, size = 65, normalized size = 0.96

$$\frac{200x^{11}}{11} - 6x^{10} + \frac{922x^{9}}{9} + \frac{83x^{8}}{8} + \frac{1571x^{7}}{7} + \frac{299x^{6}}{3} + \frac{1416x^{5}}{5} + \frac{635x^{4}}{4} + 237x^{3} + 108x^{2} + 108x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**3*(5*x**2+3*x+2)**2,x)
```

```
[Out] 200*x**11/11 - 6*x**10 + 922*x**9/9 + 83*x**8/8 + 1571*x**7/7 + 299*x**6/3
+ 1416*x**5/5 + 635*x**4/4 + 237*x**3 + 108*x**2 + 108*x
```

3.30 $\int \left(3-x+2x^2\right)^3 \left(2+3x+5x^2\right)\,dx$

Optimal. Leaf size=56

$$\frac{40x^9}{9}-\frac{9x^8}{2}+\frac{190x^7}{7}-\frac{83x^6}{6}+\frac{288x^5}{5}-5x^4+60x^3+\frac{27x^2}{2}+54x$$

Rubi [A] time = 0.03, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1657}

$$\frac{40x^9}{9}-\frac{9x^8}{2}+\frac{190x^7}{7}-\frac{83x^6}{6}+\frac{288x^5}{5}-5x^4+60x^3+\frac{27x^2}{2}+54x$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2),x]
```

```
[Out] 54*x + (27*x^2)/2 + 60*x^3 - 5*x^4 + (288*x^5)/5 - (83*x^6)/6 + (190*x^7)/7
 - (9*x^8)/2 + (40*x^9)/9
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \left(3-x+2x^2\right)^3 \left(2+3x+5x^2\right)\,dx &= \int \left(54+27x+180x^2-20x^3+288x^4-83x^5+190x^6-36x^7+40x^8\right)\,dx\\ &= 54x+\frac{27x^2}{2}+60x^3-5x^4+\frac{288x^5}{5}-\frac{83x^6}{6}+\frac{190x^7}{7}-\frac{9x^8}{2}+\frac{40x^9}{9}\end{aligned}$$

Mathematica [A] time = 0.00, size = 56, normalized size = 1.00

$$\frac{40x^9}{9}-\frac{9x^8}{2}+\frac{190x^7}{7}-\frac{83x^6}{6}+\frac{288x^5}{5}-5x^4+60x^3+\frac{27x^2}{2}+54x$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2),x]
```

```
[Out] 54*x + (27*x^2)/2 + 60*x^3 - 5*x^4 + (288*x^5)/5 - (83*x^6)/6 + (190*x^7)/7
 - (9*x^8)/2 + (40*x^9)/9
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(3-x+2x^2\right)^3 \left(2+3x+5x^2\right)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2),x]
```

```
[Out] IntegrateAlgebraic[(3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2), x]
```

fricas [A] time = 0.33, size = 44, normalized size = 0.79

$$\frac{40}{9}x^9-\frac{9}{2}x^8+\frac{190}{7}x^7-\frac{83}{6}x^6+\frac{288}{5}x^5-5x^4+60x^3+\frac{27}{2}x^2+54x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^3*(5*x^2+3*x+2),x, algorithm="fricas")
```

```
[Out] 40/9*x^9 - 9/2*x^8 + 190/7*x^7 - 83/6*x^6 + 288/5*x^5 - 5*x^4 + 60*x^3 + 27
/2*x^2 + 54*x
```

giac [A] time = 0.20, size = 44, normalized size = 0.79

$$\frac{40}{9}\,x^9-\frac{9}{2}\,x^8+\frac{190}{7}\,x^7-\frac{83}{6}\,x^6+\frac{288}{5}\,x^5-5\,x^4+60\,x^3+\frac{27}{2}\,x^2+54\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^3*(5*x^2+3*x+2),x, algorithm="giac")
```

```
[Out] 40/9*x^9 - 9/2*x^8 + 190/7*x^7 - 83/6*x^6 + 288/5*x^5 - 5*x^4 + 60*x^3 + 27
/2*x^2 + 54*x
```

maple [A] time = 0.00, size = 45, normalized size = 0.80

$$\frac{40}{9}x^9-\frac{9}{2}x^8+\frac{190}{7}x^7-\frac{83}{6}x^6+\frac{288}{5}x^5-5x^4+60x^3+\frac{27}{2}x^2+54x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^3*(5*x^2+3*x+2),x)
```

```
[Out] 54*x+27/2*x^2+60*x^3-5*x^4+288/5*x^5-83/6*x^6+190/7*x^7-9/2*x^8+40/9*x^9
```

maxima [A] time = 0.43, size = 44, normalized size = 0.79

$$\frac{40}{9}\,x^{9} - \frac{9}{2}\,x^{8} + \frac{190}{7}\,x^{7} - \frac{83}{6}\,x^{6} + \frac{288}{5}\,x^{5} - 5\,x^{4} + 60\,x^{3} + \frac{27}{2}\,x^{2} + 54\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^3*(5*x^2+3*x+2),x, algorithm="maxima")
```

```
[Out] 40/9*x^9 - 9/2*x^8 + 190/7*x^7 - 83/6*x^6 + 288/5*x^5 - 5*x^4 + 60*x^3 + 27
/2*x^2 + 54*x
```

mupad [B] time = 0.03, size = 44, normalized size = 0.79

$$\frac{40\,x^{9}}{9} - \frac{9\,x^{8}}{2} + \frac{190\,x^{7}}{7} - \frac{83\,x^{6}}{6} + \frac{288\,x^{5}}{5} - 5\,x^{4} + 60\,x^{3} + \frac{27\,x^{2}}{2} + 54\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^3*(3*x + 5*x^2 + 2),x)
```

```
[Out] 54*x + (27*x^2)/2 + 60*x^3 - 5*x^4 + (288*x^5)/5 - (83*x^6)/6 + (190*x^7)/7
 - (9*x^8)/2 + (40*x^9)/9
```

sympy [A] time = 0.08, size = 53, normalized size = 0.95

$$\frac{40x^{9}}{9} - \frac{9x^{8}}{2} + \frac{190x^{7}}{7} - \frac{83x^{6}}{6} + \frac{288x^{5}}{5} - 5x^{4} + 60x^{3} + \frac{27x^{2}}{2} + 54x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**3*(5*x**2+3*x+2),x)
```

```
[Out] 40*x**9/9 - 9*x**8/2 + 190*x**7/7 - 83*x**6/6 + 288*x**5/5 - 5*x**4 + 60*x*
*3 + 27*x**2/2 + 54*x
```

3.31 $\int \frac{(3-x+2x^2)^3}{2+3x+5x^2}\,dx$

Optimal. Leaf size=70

$$\frac{8x^5}{25}-\frac{21x^4}{25}+\frac{1222x^3}{375}-\frac{7451x^2}{1250}-\frac{158389\log\left(5x^2+3x+2\right)}{31250}+\frac{49508x}{3125}+\frac{328757\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{15625\sqrt{31}}$$

Rubi [A] time = 0.05, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1657, 634, 618, 204, 628}

$$\frac{8x^5}{25}-\frac{21x^4}{25}+\frac{1222x^3}{375}-\frac{7451x^2}{1250}-\frac{158389\log\left(5x^2+3x+2\right)}{31250}+\frac{49508x}{3125}+\frac{328757\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{15625\sqrt{31}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^3/(2 + 3*x + 5*x^2),x]
```

```
[Out] (49508*x)/3125 - (7451*x^2)/1250 + (1222*x^3)/375 - (21*x^4)/25 + (8*x^5)/2
5 + (328757*ArcTan[(3 + 10*x)/Sqrt[31]])/(15625*Sqrt[31]) - (158389*Log[2 +
 3*x + 5*x^2])/31250
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{\left(3-x+2x^2\right)^3}{2+3x+5x^2}\,dx = \int \left(\frac{49508}{3125}-\frac{7451x}{625}+\frac{1222x^2}{125}-\frac{84x^3}{25}+\frac{8x^4}{5}-\frac{1331(11+119x)}{3125\left(2+3x+5x^2\right)}\right)dx$$

$$= \frac{49508x}{3125}-\frac{7451x^2}{1250}+\frac{1222x^3}{375}-\frac{21x^4}{25}+\frac{8x^5}{25}-\frac{1331\int \frac{11+119x}{2+3x+5x^2}\,dx}{3125}$$

$$= \frac{49508x}{3125}-\frac{7451x^2}{1250}+\frac{1222x^3}{375}-\frac{21x^4}{25}+\frac{8x^5}{25}-\frac{158389\int \frac{3+10x}{2+3x+5x^2}\,dx}{31250}+\frac{328757\int \frac{1}{2+3x+5x}$$

$$= \frac{49508x}{3125}-\frac{7451x^2}{1250}+\frac{1222x^3}{375}-\frac{21x^4}{25}+\frac{8x^5}{25}-\frac{158389\log\left(2+3x+5x^2\right)}{31250}-\frac{328757\,\mathrm{Sub}}{}$$

$$= \frac{49508x}{3125}-\frac{7451x^2}{1250}+\frac{1222x^3}{375}-\frac{21x^4}{25}+\frac{8x^5}{25}+\frac{328757\tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)}{15625\sqrt{31}}-\frac{158389\log\left(2+\right.}{31250}$$

Mathematica [A] time = 0.02, size = 63, normalized size = 0.90

$$\frac{31\left(5x\left(6000x^4-15750x^3+61100x^2-111765x+297048\right)-475167\log\left(5x^2+3x+2\right)\right)+1972542\sqrt{31}\,\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{2906250}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^3/(2 + 3*x + 5*x^2),x]
```

```
[Out] (1972542*Sqrt[31]*ArcTan[(3 + 10*x)/Sqrt[31]] + 31*(5*x*(297048 - 111765*x
+ 61100*x^2 - 15750*x^3 + 6000*x^4) - 475167*Log[2 + 3*x + 5*x^2]))/2906250
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(3-x+2x^2)^3}{2+3x+5x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(3 - x + 2*x^2)^3/(2 + 3*x + 5*x^2),x]
```

```
[Out] IntegrateAlgebraic[(3 - x + 2*x^2)^3/(2 + 3*x + 5*x^2), x]
```

fricas [A] time = 0.41, size = 53, normalized size = 0.76

$$\frac{8}{25}\,x^5-\frac{21}{25}\,x^4+\frac{1222}{375}\,x^3-\frac{7451}{1250}\,x^2+\frac{328757}{484375}\,\sqrt{31}\,\arctan\left(\frac{1}{31}\,\sqrt{31}\,(10\,x+3)\right)+\frac{49508}{3125}\,x-\frac{158389}{31250}\,\log\left(5\,x^2+3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^3/(5*x^2+3*x+2),x, algorithm="fricas")
```

```
[Out] 8/25*x^5 - 21/25*x^4 + 1222/375*x^3 - 7451/1250*x^2 + 328757/484375*sqrt(31
)*arctan(1/31*sqrt(31)*(10*x + 3)) + 49508/3125*x - 158389/31250*log(5*x^2
+ 3*x + 2)
```

giac [A] time = 0.21, size = 53, normalized size = 0.76

$$\frac{8}{25}\,x^5-\frac{21}{25}\,x^4+\frac{1222}{375}\,x^3-\frac{7451}{1250}\,x^2+\frac{328757}{484375}\,\sqrt{31}\,\arctan\left(\frac{1}{31}\,\sqrt{31}\,(10\,x+3)\right)+\frac{49508}{3125}\,x-\frac{158389}{31250}\,\log\left(5\,x^2+3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^3/(5*x^2+3*x+2),x, algorithm="giac")
```

```
[Out] 8/25*x^5 - 21/25*x^4 + 1222/375*x^3 - 7451/1250*x^2 + 328757/484375*sqrt(31
)*arctan(1/31*sqrt(31)*(10*x + 3)) + 49508/3125*x - 158389/31250*log(5*x^2
+ 3*x + 2)
```

maple [A] time = 0.00, size = 54, normalized size = 0.77

$$\frac{8x^5}{25}-\frac{21x^4}{25}+\frac{1222x^3}{375}-\frac{7451x^2}{1250}+\frac{49508x}{3125}+\frac{328757\sqrt{31}\,\arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)}{484375}-\frac{158389\ln\left(5x^2+3x+2\right)}{31250}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^3/(5*x^2+3*x+2),x)
```

```
[Out] 49508/3125*x-7451/1250*x^2+1222/375*x^3-21/25*x^4+8/25*x^5-158389/31250*ln(
5*x^2+3*x+2)+328757/484375*31^(1/2)*arctan(1/31*(10*x+3)*31^(1/2))
```

maxima [A] time = 0.96, size = 53, normalized size = 0.76

$$\frac{8}{25}\,x^{5} - \frac{21}{25}\,x^{4} + \frac{1222}{375}\,x^{3} - \frac{7451}{1250}\,x^{2} + \frac{328757}{484375}\,\sqrt{31}\,\arctan\left(\frac{1}{31}\,\sqrt{31}\,(10\,x+3)\right) + \frac{49508}{3125}\,x - \frac{158389}{31250}\,\log\left(5\,x^{2}+3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^3/(5*x^2+3*x+2),x, algorithm="maxima")
```

```
[Out] 8/25*x^5 - 21/25*x^4 + 1222/375*x^3 - 7451/1250*x^2 + 328757/484375*sqrt(31
)*arctan(1/31*sqrt(31)*(10*x + 3)) + 49508/3125*x - 158389/31250*log(5*x^2
+ 3*x + 2)
```

mupad [B] time = 0.04, size = 55, normalized size = 0.79

$$\frac{49508\,x}{3125} - \frac{158389\,\ln\left(5\,x^{2}+3\,x+2\right)}{31250} + \frac{328757\,\sqrt{31}\,\mathrm{atan}\left(\frac{10\,\sqrt{31}\,x}{31}+\frac{3\,\sqrt{31}}{31}\right)}{484375} - \frac{7451\,x^{2}}{1250} + \frac{1222\,x^{3}}{375} - \frac{21\,x^{4}}{25} + \frac{8\,x^{5}}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^3/(3*x + 5*x^2 + 2),x)
```

```
[Out] (49508*x)/3125 - (158389*log(3*x + 5*x^2 + 2))/31250 + (328757*31^(1/2)*ata
n((10*31^(1/2)*x)/31 + (3*31^(1/2))/31))/484375 - (7451*x^2)/1250 + (1222*x
^3)/375 - (21*x^4)/25 + (8*x^5)/25
```

sympy [A] time = 0.16, size = 76, normalized size = 1.09

$$\frac{8x^{5}}{25} - \frac{21x^{4}}{25} + \frac{1222x^{3}}{375} - \frac{7451x^{2}}{1250} + \frac{49508x}{3125} - \frac{158389\log\left(x^{2}+\frac{3x}{5}+\frac{2}{5}\right)}{31250} + \frac{328757\sqrt{31}\,\mathrm{atan}\left(\frac{10\sqrt{31}\,x}{31}+\frac{3\sqrt{31}}{31}\right)}{484375}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**3/(5*x**2+3*x+2),x)
```

```
[Out] 8*x**5/25 - 21*x**4/25 + 1222*x**3/375 - 7451*x**2/1250 + 49508*x/3125 - 15
8389*log(x**2 + 3*x/5 + 2/5)/31250 + 328757*sqrt(31)*atan(10*sqrt(31)*x/31
+ 3*sqrt(31)/31)/484375
```

3.32 $\int \frac{(3-x+2x^2)^3}{(2+3x+5x^2)^2}\,dx$

Optimal. Leaf size=77

$$\frac{8x^3}{75}-\frac{54x^2}{125}+\frac{1331(247x+443)}{96875(5x^2+3x+2)}-\frac{10769\log(5x^2+3x+2)}{6250}+\frac{1466x}{625}+\frac{3819607\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{96875\sqrt{31}}$$

Rubi [A] time = 0.07, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1660, 1657, 634, 618, 204, 628}

$$\frac{8x^3}{75}-\frac{54x^2}{125}+\frac{1331(247x+443)}{96875(5x^2+3x+2)}-\frac{10769\log(5x^2+3x+2)}{6250}+\frac{1466x}{625}+\frac{3819607\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{96875\sqrt{31}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^3/(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] (1466*x)/625 - (54*x^2)/125 + (8*x^3)/75 + (1331*(443 + 247*x))/(96875*(2 +
 3*x + 5*x^2)) + (3819607*ArcTan[(3 + 10*x)/Sqrt[31]])/(96875*Sqrt[31]) - (
10769*Log[2 + 3*x + 5*x^2])/6250
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{\left(3-x+2x^2\right)^3}{\left(2+3x+5x^2\right)^2}\,dx = \frac{1331(443+247x)}{96875\left(2+3x+5x^2\right)} + \frac{1}{31}\int \frac{\frac{372701}{625}-\frac{230981x}{625}+\frac{37882x^2}{125}-\frac{2604x^3}{25}+\frac{248x^4}{5}}{2+3x+5x^2}\,dx$$

$$= \frac{1331(443+247x)}{96875\left(2+3x+5x^2\right)} + \frac{1}{31}\int \left(\frac{45446}{625}-\frac{3348x}{125}+\frac{248x^2}{25}+\frac{121(2329-2759x)}{625\left(2+3x+5x^2\right)}\right)dx$$

$$= \frac{1466x}{625}-\frac{54x^2}{125}+\frac{8x^3}{75}+\frac{1331(443+247x)}{96875\left(2+3x+5x^2\right)}+\frac{121\int \frac{2329-2759x}{2+3x+5x^2}\,dx}{19375}$$

$$= \frac{1466x}{625}-\frac{54x^2}{125}+\frac{8x^3}{75}+\frac{1331(443+247x)}{96875\left(2+3x+5x^2\right)}-\frac{10769\int \frac{3+10x}{2+3x+5x^2}\,dx}{6250}+\frac{3819607\int \frac{1}{2+3x\cdots}}{193750}$$

$$= \frac{1466x}{625}-\frac{54x^2}{125}+\frac{8x^3}{75}+\frac{1331(443+247x)}{96875\left(2+3x+5x^2\right)}-\frac{10769\log\left(2+3x+5x^2\right)}{6250}-\frac{3819607\,\mathrm{S}\cdots}{\cdots}$$

$$= \frac{1466x}{625}-\frac{54x^2}{125}+\frac{8x^3}{75}+\frac{1331(443+247x)}{96875\left(2+3x+5x^2\right)}+\frac{3819607\tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)}{96875\sqrt{31}}-\frac{10769\log\left(2\cdots\right.}{625\cdots}$$

Mathematica [A] time = 0.03, size = 77, normalized size = 1.00

$$\frac{8x^3}{75}-\frac{54x^2}{125}+\frac{1331(247x+443)}{96875\left(5x^2+3x+2\right)}-\frac{10769\log\left(5x^2+3x+2\right)}{6250}+\frac{1466x}{625}+\frac{3819607\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{96875\sqrt{31}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^3/(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] (1466*x)/625 - (54*x^2)/125 + (8*x^3)/75 + (1331*(443 + 247*x))/(96875*(2 +
 3*x + 5*x^2)) + (3819607*ArcTan[(3 + 10*x)/Sqrt[31]])/(96875*Sqrt[31]) - (
10769*Log[2 + 3*x + 5*x^2])/6250
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3-x+2x^2\right)^3}{\left(2+3x+5x^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(3 - x + 2*x^2)^3/(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] IntegrateAlgebraic[(3 - x + 2*x^2)^3/(2 + 3*x + 5*x^2)^2, x]
```

fricas [A] time = 0.41, size = 88, normalized size = 1.14

$$\frac{9610000\,x^{5}-33154500\,x^{4}+191815600\,x^{3}+22917642\,\sqrt{31}\left(5\,x^{2}+3\,x+2\right)\arctan\left(\frac{1}{31}\,\sqrt{31}\,(10\,x+3)\right)+111226140\,x^{2}-31047027\left(5\,x^{2}+3\,x+2\right)\log\left(5\,x^{2}+3\,x+2\right)+145678362\,x+109671738}{18018750\left(5\,x^{2}+3\,x+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^3/(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

```
[Out] 1/18018750*(9610000*x^5 - 33154500*x^4 + 191815600*x^3 + 22917642*sqrt(31)*
(5*x^2 + 3*x + 2)*arctan(1/31*sqrt(31)*(10*x + 3)) + 111226140*x^2 - 310470
27*(5*x^2 + 3*x + 2)*log(5*x^2 + 3*x + 2) + 145678362*x + 109671738)/(5*x^2
 + 3*x + 2)
```

giac [A] time = 0.21, size = 62, normalized size = 0.81

$$\frac{8}{75}\,x^{3}-\frac{54}{125}\,x^{2}+\frac{3819607}{3003125}\,\sqrt{31}\,\arctan\left(\frac{1}{31}\,\sqrt{31}\,(10\,x+3)\right)+\frac{1466}{625}\,x+\frac{1331\,(247\,x+443)}{96875\left(5\,x^{2}+3\,x+2\right)}-\frac{10769}{6250}\,\log\left(5\,x^{2}+3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^3/(5*x^2+3*x+2)^2,x, algorithm="giac")
```

```
[Out] 8/75*x^3 - 54/125*x^2 + 3819607/3003125*sqrt(31)*arctan(1/31*sqrt(31)*(10*x
 + 3)) + 1466/625*x + 1331/96875*(247*x + 443)/(5*x^2 + 3*x + 2) - 10769/62
50*log(5*x^2 + 3*x + 2)
```

maple [A] time = 0.01, size = 61, normalized size = 0.79

$$\frac{8x^{3}}{75}-\frac{54x^{2}}{125}+\frac{1466x}{625}+\frac{3819607\sqrt{31}\,\arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)}{3003125}-\frac{10769\ln\left(5x^{2}+3x+2\right)}{6250}-\frac{121\left(-\frac{2717x}{775}-\frac{4873}{775}\right)}{625\left(x^{2}+\frac{3}{5}x+\frac{2}{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^3/(5*x^2+3*x+2)^2,x)
```

```
[Out] 8/75*x^3-54/125*x^2+1466/625*x-121/625*(-2717/775*x-4873/775)/(x^2+3/5*x+2/
5)-10769/6250*ln(5*x^2+3*x+2)+3819607/3003125*31^(1/2)*arctan(1/31*(10*x+3)
*31^(1/2))
```

maxima [A] time = 0.96, size = 62, normalized size = 0.81

$$\frac{8}{75}\,x^{3}-\frac{54}{125}\,x^{2}+\frac{3819607}{3003125}\,\sqrt{31}\,\arctan\left(\frac{1}{31}\,\sqrt{31}\,(10\,x+3)\right)+\frac{1466}{625}\,x+\frac{1331\,(247\,x+443)}{96875\left(5\,x^{2}+3\,x+2\right)}-\frac{10769}{6250}\,\log\left(5\,x^{2}+3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^3/(5*x^2+3*x+2)^2,x, algorithm="maxima")
```

```
[Out] 8/75*x^3 - 54/125*x^2 + 3819607/3003125*sqrt(31)*arctan(1/31*sqrt(31)*(10*x
 + 3)) + 1466/625*x + 1331/96875*(247*x + 443)/(5*x^2 + 3*x + 2) - 10769/62
50*log(5*x^2 + 3*x + 2)
```

mupad [B] time = 3.43, size = 61, normalized size = 0.79

$$\frac{1466\,x}{625}-\frac{10769\,\ln\left(5\,x^2+3\,x+2\right)}{6250}+\frac{\frac{328757\,x}{484375}+\frac{589633}{484375}}{x^2+\frac{3\,x}{5}+\frac{2}{5}}+\frac{3819607\,\sqrt{31}\,\mathrm{atan}\left(\frac{10\,\sqrt{31}\,x}{31}+\frac{3\,\sqrt{31}}{31}\right)}{3003125}-\frac{54\,x^2}{125}+\frac{8\,x^3}{75}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^3/(3*x + 5*x^2 + 2)^2,x)
```

```
[Out] (1466*x)/625 - (10769*log(3*x + 5*x^2 + 2))/6250 + ((328757*x)/484375 + 589
633/484375)/((3*x)/5 + x^2 + 2/5) + (3819607*31^(1/2)*atan((10*31^(1/2)*x)/
31 + (3*31^(1/2))/31))/3003125 - (54*x^2)/125 + (8*x^3)/75
```

sympy [A] time = 0.19, size = 78, normalized size = 1.01

$$\frac{8x^3}{75}-\frac{54x^2}{125}+\frac{1466x}{625}+\frac{328757x+589633}{484375x^2+290625x+193750}-\frac{10769\log\left(x^2+\frac{3x}{5}+\frac{2}{5}\right)}{6250}+\frac{3819607\sqrt{31}\,\mathrm{atan}\left(\frac{10\sqrt{31}\,x}{31}+\frac{3\sqrt{31}}{31}\right)}{3003125}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**3/(5*x**2+3*x+2)**2,x)
```

```
[Out] 8*x**3/75 - 54*x**2/125 + 1466*x/625 + (328757*x + 589633)/(484375*x**2 + 2
90625*x + 193750) - 10769*log(x**2 + 3*x/5 + 2/5)/6250 + 3819607*sqrt(31)*a
tan(10*sqrt(31)*x/31 + 3*sqrt(31)/31)/3003125
```

3.33 $\int \frac{(3-x+2x^2)^3}{(2+3x+5x^2)^3}\,dx$

Optimal. Leaf size=84

$$\frac{121(342840x+188381)}{6006250(5x^2+3x+2)}+\frac{1331(247x+443)}{193750(5x^2+3x+2)^2}-\frac{66}{625}\log\left(5x^2+3x+2\right)+\frac{8x}{125}+\frac{11341176\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{600625\sqrt{31}}$$

Rubi [A] time = 0.09, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1660, 1657, 634, 618, 204, 628}

$$\frac{121(342840x+188381)}{6006250(5x^2+3x+2)}+\frac{1331(247x+443)}{193750(5x^2+3x+2)^2}-\frac{66}{625}\log\left(5x^2+3x+2\right)+\frac{8x}{125}+\frac{11341176\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{600625\sqrt{31}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^3/(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] (8*x)/125 + (1331*(443 + 247*x))/(193750*(2 + 3*x + 5*x^2)^2) + (121*(18838
1 + 342840*x))/(6006250*(2 + 3*x + 5*x^2)) + (11341176*ArcTan[(3 + 10*x)/Sq
rt[31]])/(600625*Sqrt[31]) - (66*Log[2 + 3*x + 5*x^2])/625
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{(3-x+2x^2)^3}{(2+3x+5x^2)^3}\,dx = \frac{1331(443+247x)}{193750\,(2+3x+5x^2)^2} + \frac{1}{62}\int \frac{\frac{4055767}{3125}-\frac{461962x}{625}+\frac{75764x^2}{125}-\frac{5208x^3}{25}+\frac{496x^4}{5}}{(2+3x+5x^2)^2}\,dx$$

$$= \frac{1331(443+247x)}{193750\,(2+3x+5x^2)^2} + \frac{121(188381+342840x)}{6006250\,(2+3x+5x^2)} + \frac{\int \frac{\frac{2222876}{125}-\frac{207576x}{125}+\frac{15376x^2}{25}}{2+3x+5x^2}\,dx}{1922}$$

$$= \frac{1331(443+247x)}{193750\,(2+3x+5x^2)^2} + \frac{121(188381+342840x)}{6006250\,(2+3x+5x^2)} + \frac{\int \left(\frac{15376}{125}+\frac{132(16607-1922x)}{125(2+3x+5x^2)}\right)dx}{1922}$$

$$= \frac{8x}{125} + \frac{1331(443+247x)}{193750\,(2+3x+5x^2)^2} + \frac{121(188381+342840x)}{6006250\,(2+3x+5x^2)} + \frac{66\int \frac{16607-1922x}{2+3x+5x^2}\,dx}{120125}$$

$$= \frac{8x}{125} + \frac{1331(443+247x)}{193750\,(2+3x+5x^2)^2} + \frac{121(188381+342840x)}{6006250\,(2+3x+5x^2)} - \frac{66}{625}\int \frac{3+10x}{2+3x+5x^2}\,dx +$$

$$= \frac{8x}{125} + \frac{1331(443+247x)}{193750\,(2+3x+5x^2)^2} + \frac{121(188381+342840x)}{6006250\,(2+3x+5x^2)} - \frac{66}{625}\log\left(2+3x+5x^2\right) -$$

$$= \frac{8x}{125} + \frac{1331(443+247x)}{193750\,(2+3x+5x^2)^2} + \frac{121(188381+342840x)}{6006250\,(2+3x+5x^2)} + \frac{11341176\tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)}{600625\sqrt{31}} -$$

Mathematica [A] time = 0.04, size = 78, normalized size = 0.93

$$\frac{\frac{3751(342840x+188381)}{5x^2+3x+2}+\frac{1279091(247x+443)}{(5x^2+3x+2)^2}-19662060\log\left(5x^2+3x+2\right)+11916400x+113411760\sqrt{31}\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{186193750}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^3/(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] (11916400*x + (1279091*(443 + 247*x))/(2 + 3*x + 5*x^2)^2 + (3751*(188381 +
 342840*x))/(2 + 3*x + 5*x^2) + 113411760*Sqrt[31]*ArcTan[(3 + 10*x)/Sqrt[3
1]] - 19662060*Log[2 + 3*x + 5*x^2])/186193750
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(3-x+2x^2)^3}{(2+3x+5x^2)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(3 - x + 2*x^2)^3/(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] IntegrateAlgebraic[(3 - x + 2*x^2)^3/(2 + 3*x + 5*x^2)^3, x]
```

fricas [A] time = 0.39, size = 118, normalized size = 1.40

$$\frac{59582000\,x^5+71498400\,x^4+1355107960\,x^3+22682352\,\sqrt{31}\left(25\,x^4+30\,x^3+29\,x^2+12\,x+4\right)\arctan\left(\frac{1}{31}\,\sqrt{31}\,(10\,x+3)\right)+1506812195\,x^2-3932412\left(25\,x^4+30\,x^3+29\,x^2+12\,x+4\right)\log\left(5\,x^2+3\,x+2\right)+1011087630\,x+395974315}{37238750\left(25\,x^4+30\,x^3+29\,x^2+12\,x+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^3/(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

```
[Out] 1/37238750*(59582000*x^5 + 71498400*x^4 + 1355107960*x^3 + 22682352*sqrt(31
)*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)*arctan(1/31*sqrt(31)*(10*x + 3)) +
1506812195*x^2 - 3932412*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)*log(5*x^2 +
3*x + 2) + 1011087630*x + 395974315)/(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)
```

giac [A] time = 0.19, size = 62, normalized size = 0.74

$$\frac{11341176}{18619375}\,\sqrt{31}\,\arctan\left(\frac{1}{31}\,\sqrt{31}\,(10\,x+3)\right)+\frac{8}{125}\,x+\frac{121\left(68568\,x^3+78817\,x^2+53402\,x+21113\right)}{240250\left(5\,x^2+3\,x+2\right)^2}-\frac{66}{625}\,\log\left(5\,x^2+3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^3/(5*x^2+3*x+2)^3,x, algorithm="giac")
```

```
[Out] 11341176/18619375*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 8/125*x + 121
/240250*(68568*x^3 + 78817*x^2 + 53402*x + 21113)/(5*x^2 + 3*x + 2)^2 - 66/
625*log(5*x^2 + 3*x + 2)
```

maple [A] time = 0.01, size = 63, normalized size = 0.75

$$\frac{8x}{125}+\frac{11341176\sqrt{31}\,\arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)}{18619375}-\frac{66\ln\left(5x^2+3x+2\right)}{625}-\frac{11\left(-\frac{377124}{24025}x^3-\frac{866987}{48050}x^2-\frac{293711}{24025}x-\frac{232243}{48050}\right)}{5\left(5x^2+3x+2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^3/(5*x^2+3*x+2)^3,x)
```

```
[Out] 8/125*x-11/5*(-377124/24025*x^3-866987/48050*x^2-293711/24025*x-232243/4805
0)/(5*x^2+3*x+2)^2-66/625*ln(5*x^2+3*x+2)+11341176/18619375*31^(1/2)*arctan
(1/31*(10*x+3)*31^(1/2))
```

maxima [A] time = 0.97, size = 72, normalized size = 0.86

$$\frac{11341176}{18619375}\,\sqrt{31}\,\arctan\left(\frac{1}{31}\,\sqrt{31}\,(10\,x+3)\right)+\frac{8}{125}\,x+\frac{121\,\left(68568\,x^3+78817\,x^2+53402\,x+21113\right)}{240250\,\left(25\,x^4+30\,x^3+29\,x^2+12\,x+4\right)}-\frac{66}{625}\,\log\left(5\,x^2+3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^3/(5*x^2+3*x+2)^3,x, algorithm="maxima")
```

```
[Out] 11341176/18619375*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 8/125*x + 121
/240250*(68568*x^3 + 78817*x^2 + 53402*x + 21113)/(25*x^4 + 30*x^3 + 29*x^2
 + 12*x + 4) - 66/625*log(5*x^2 + 3*x + 2)
```

mupad [B] time = 3.43, size = 71, normalized size = 0.85

$$\frac{8\,x}{125}-\frac{66\,\ln\left(5\,x^2+3\,x+2\right)}{625}+\frac{11341176\,\sqrt{31}\,\mathrm{atan}\left(\frac{10\,\sqrt{31}\,x}{31}+\frac{3\,\sqrt{31}}{31}\right)}{18619375}+\frac{\frac{4148364\,x^3}{3003125}+\frac{9536857\,x^2}{6006250}+\frac{3230821\,x}{3003125}+\frac{2554673}{6006250}}{x^4+\frac{6\,x^3}{5}+\frac{29\,x^2}{25}+\frac{12\,x}{25}+\frac{4}{25}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^3/(3*x + 5*x^2 + 2)^3,x)
```

```
[Out] (8*x)/125 - (66*log(3*x + 5*x^2 + 2))/625 + (11341176*31^(1/2)*atan((10*31^
(1/2)*x)/31 + (3*31^(1/2))/31))/18619375 + ((3230821*x)/3003125 + (9536857*
x^2)/6006250 + (4148364*x^3)/3003125 + 2554673/6006250)/((12*x)/25 + (29*x^
2)/25 + (6*x^3)/5 + x^4 + 4/25)
```

sympy [A] time = 0.23, size = 85, normalized size = 1.01

$$\frac{8x}{125}+\frac{8296728x^3+9536857x^2+6461642x+2554673}{6006250x^4+7207500x^3+6967250x^2+2883000x+961000}-\frac{66\log\left(x^2+\frac{3x}{5}+\frac{2}{5}\right)}{625}+\frac{11341176\sqrt{31}\,\mathrm{atan}\left(\frac{10\sqrt{31}\,x}{31}+\frac{3\sqrt{31}}{31}\right)}{18619375}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**3/(5*x**2+3*x+2)**3,x)
```

```
[Out] 8*x/125 + (8296728*x**3 + 9536857*x**2 + 6461642*x + 2554673)/(6006250*x**4
 + 7207500*x**3 + 6967250*x**2 + 2883000*x + 961000) - 66*log(x**2 + 3*x/5
+ 2/5)/625 + 11341176*sqrt(31)*atan(10*sqrt(31)*x/31 + 3*sqrt(31)/31)/18619
375
```

3.34 $\int \frac{(2+3x+5x^2)^4}{3-x+2x^2} \, dx$

Optimal. Leaf size=84

$$\frac{625x^7}{14}+\frac{3625x^6}{24}+\frac{1855x^5}{8}+\frac{6245x^4}{64}-\frac{21229x^3}{96}-\frac{28747x^2}{128}+\frac{307461}{512}\log\left(2x^2-x+3\right)+\frac{122691x}{128}+\frac{1156639\tan^{-1}\left(\frac{1-}{\sqrt{}}\right.}{256\sqrt{23}}$$

Rubi [A] time = 0.06, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1657, 634, 618, 204, 628}

$$\frac{625x^7}{14}+\frac{3625x^6}{24}+\frac{1855x^5}{8}+\frac{6245x^4}{64}-\frac{21229x^3}{96}-\frac{28747x^2}{128}+\frac{307461}{512}\log\left(2x^2-x+3\right)+\frac{122691x}{128}+\frac{1156639\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{256\sqrt{23}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + 5*x^2)^4/(3 - x + 2*x^2),x]
```

```
[Out] (122691*x)/128 - (28747*x^2)/128 - (21229*x^3)/96 + (6245*x^4)/64 + (1855*x
^5)/8 + (3625*x^6)/24 + (625*x^7)/14 + (1156639*ArcTan[(1 - 4*x)/Sqrt[23]])
/(256*Sqrt[23]) + (307461*Log[3 - x + 2*x^2])/512
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{\left(2+3x+5x^2\right)^4}{3-x+2x^2}\,dx = \int \left(\frac{122691}{128} - \frac{28747x}{64} - \frac{21229x^2}{32} + \frac{6245x^3}{16} + \frac{9275x^4}{8} + \frac{3625x^5}{4} + \frac{625x^6}{2} - \frac{14641(2\cdots}{128\left(3-\cdots\right.}\right.$$

$$= \frac{122691x}{128} - \frac{28747x^2}{128} - \frac{21229x^3}{96} + \frac{6245x^4}{64} + \frac{1855x^5}{8} + \frac{3625x^6}{24} + \frac{625x^7}{14} - \frac{14641}{128}\int \cdots$$

$$= \frac{122691x}{128} - \frac{28747x^2}{128} - \frac{21229x^3}{96} + \frac{6245x^4}{64} + \frac{1855x^5}{8} + \frac{3625x^6}{24} + \frac{625x^7}{14} + \frac{307461}{512}\int \cdots$$

$$= \frac{122691x}{128} - \frac{28747x^2}{128} - \frac{21229x^3}{96} + \frac{6245x^4}{64} + \frac{1855x^5}{8} + \frac{3625x^6}{24} + \frac{625x^7}{14} + \frac{307461}{512}\log\cdots$$

$$= \frac{122691x}{128} - \frac{28747x^2}{128} - \frac{21229x^3}{96} + \frac{6245x^4}{64} + \frac{1855x^5}{8} + \frac{3625x^6}{24} + \frac{625x^7}{14} + \frac{1156639\tan\cdots}{256\cdots}$$

Mathematica [A] time = 0.03, size = 72, normalized size = 0.86

$$\frac{307461}{512}\log\left(2x^2-x+3\right) + \frac{x\left(120000x^6+406000x^5+623280x^4+262290x^3-594412x^2-603687x+2576511\right)}{2688} - \frac{1156639\tan^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{256\sqrt{23}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)^4/(3 - x + 2*x^2),x]
```

```
[Out] (x*(2576511 - 603687*x - 594412*x^2 + 262290*x^3 + 623280*x^4 + 406000*x^5
+ 120000*x^6))/2688 - (1156639*ArcTan[(-1 + 4*x)/Sqrt[23]])/(256*Sqrt[23])
+ (307461*Log[3 - x + 2*x^2])/512
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2+3x+5x^2\right)^4}{3-x+2x^2}\,dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(2 + 3*x + 5*x^2)^4/(3 - x + 2*x^2),x]

[Out] IntegrateAlgebraic[(2 + 3*x + 5*x^2)^4/(3 - x + 2*x^2), x]

fricas [A] time = 0.40, size = 63, normalized size = 0.75

$$\frac{625}{14}\,x^{7} + \frac{3625}{24}\,x^{6} + \frac{1855}{8}\,x^{5} + \frac{6245}{64}\,x^{4} - \frac{21229}{96}\,x^{3} - \frac{28747}{128}\,x^{2} - \frac{1156639}{5888}\,\sqrt{23}\arctan\left(\frac{1}{23}\,\sqrt{23}\,(4\,x - 1)\right) + \frac{122691}{128}\,x + \frac{307461}{512}\,\log\left(2\,x^{2} - x + 3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+3*x+2)^4/(2*x^2-x+3),x, algorithm="fricas")

[Out] 625/14*x^7 + 3625/24*x^6 + 1855/8*x^5 + 6245/64*x^4 - 21229/96*x^3 - 28747/128*x^2 - 1156639/5888*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 122691/128*x + 307461/512*log(2*x^2 - x + 3)

giac [A] time = 0.21, size = 63, normalized size = 0.75

$$\frac{625}{14}\,x^{7} + \frac{3625}{24}\,x^{6} + \frac{1855}{8}\,x^{5} + \frac{6245}{64}\,x^{4} - \frac{21229}{96}\,x^{3} - \frac{28747}{128}\,x^{2} - \frac{1156639}{5888}\,\sqrt{23}\arctan\left(\frac{1}{23}\,\sqrt{23}\,(4\,x - 1)\right) + \frac{122691}{128}\,x + \frac{307461}{512}\,\log\left(2\,x^{2} - x + 3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+3*x+2)^4/(2*x^2-x+3),x, algorithm="giac")

[Out] 625/14*x^7 + 3625/24*x^6 + 1855/8*x^5 + 6245/64*x^4 - 21229/96*x^3 - 28747/128*x^2 - 1156639/5888*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 122691/128*x + 307461/512*log(2*x^2 - x + 3)

maple [A] time = 0.01, size = 64, normalized size = 0.76

$$\frac{625x^{7}}{14} + \frac{3625x^{6}}{24} + \frac{1855x^{5}}{8} + \frac{6245x^{4}}{64} - \frac{21229x^{3}}{96} - \frac{28747x^{2}}{128} + \frac{122691x}{128} - \frac{1156639\sqrt{23}\,\arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{5888} + \frac{307461\ln\left(2x^{2} - x + 3\right)}{512}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5*x^2+3*x+2)^4/(2*x^2-x+3),x)

[Out] 625/14*x^7+3625/24*x^6+1855/8*x^5+6245/64*x^4-21229/96*x^3-28747/128*x^2+122691/128*x+307461/512*ln(2*x^2-x+3)-1156639/5888*23^(1/2)*arctan(1/23*(4*x-1)*23^(1/2))

maxima [A] time = 0.98, size = 63, normalized size = 0.75

$$\frac{625}{14}\,x^{7} + \frac{3625}{24}\,x^{6} + \frac{1855}{8}\,x^{5} + \frac{6245}{64}\,x^{4} - \frac{21229}{96}\,x^{3} - \frac{28747}{128}\,x^{2} - \frac{1156639}{5888}\,\sqrt{23}\arctan\left(\frac{1}{23}\,\sqrt{23}\,(4\,x - 1)\right) + \frac{122691}{128}\,x + \frac{307461}{512}\,\log\left(2\,x^{2} - x + 3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^4/(2*x^2-x+3),x, algorithm="maxima")
```

```
[Out] 625/14*x^7 + 3625/24*x^6 + 1855/8*x^5 + 6245/64*x^4 - 21229/96*x^3 - 28747/
128*x^2 - 1156639/5888*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 122691/12
8*x + 307461/512*log(2*x^2 - x + 3)
```

mupad [B] time = 3.44, size = 65, normalized size = 0.77

$$\frac{122691\,x}{128}+\frac{307461\,\ln\left(2\,x^2-x+3\right)}{512}-\frac{1156639\,\sqrt{23}\,\mathrm{atan}\left(\frac{4\,\sqrt{23}\,x}{23}-\frac{\sqrt{23}}{23}\right)}{5888}-\frac{28747\,x^2}{128}-\frac{21229\,x^3}{96}+\frac{6245\,x^4}{64}+\frac{1855\,x^5}{8}+\frac{3625\,x^6}{24}+\frac{625\,x^7}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 5*x^2 + 2)^4/(2*x^2 - x + 3),x)
```

```
[Out] (122691*x)/128 + (307461*log(2*x^2 - x + 3))/512 - (1156639*23^(1/2)*atan((
4*23^(1/2)*x)/23 - 23^(1/2)/23))/5888 - (28747*x^2)/128 - (21229*x^3)/96 +
(6245*x^4)/64 + (1855*x^5)/8 + (3625*x^6)/24 + (625*x^7)/14
```

sympy [A] time = 0.17, size = 87, normalized size = 1.04

$$\frac{625x^7}{14}+\frac{3625x^6}{24}+\frac{1855x^5}{8}+\frac{6245x^4}{64}-\frac{21229x^3}{96}-\frac{28747x^2}{128}+\frac{122691x}{128}+\frac{307461\log\left(x^2-\frac{x}{2}+\frac{3}{2}\right)}{512}-\frac{1156639\sqrt{23}\,\mathrm{atan}\left(\frac{4\sqrt{23}x}{23}-\frac{\sqrt{23}}{23}\right)}{5888}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)**4/(2*x**2-x+3),x)
```

```
[Out] 625*x**7/14 + 3625*x**6/24 + 1855*x**5/8 + 6245*x**4/64 - 21229*x**3/96 - 2
8747*x**2/128 + 122691*x/128 + 307461*log(x**2 - x/2 + 3/2)/512 - 1156639*s
qrt(23)*atan(4*sqrt(23)*x/23 - sqrt(23)/23)/5888
```

3.35 $\int \frac{(2+3x+5x^2)^3}{3-x+2x^2}\,dx$

Optimal. Leaf size=70

$$\frac{25x^5}{2}+\frac{575x^4}{16}+\frac{965x^3}{24}-\frac{829x^2}{32}+\frac{1331}{128}\log\left(2x^2-x+3\right)-\frac{4795x}{32}-\frac{59895\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{64\sqrt{23}}$$

Rubi [A] time = 0.06, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1657, 634, 618, 204, 628}

$$\frac{25x^5}{2}+\frac{575x^4}{16}+\frac{965x^3}{24}-\frac{829x^2}{32}+\frac{1331}{128}\log\left(2x^2-x+3\right)-\frac{4795x}{32}-\frac{59895\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{64\sqrt{23}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + 5*x^2)^3/(3 - x + 2*x^2),x]
```

```
[Out] (-4795*x)/32 - (829*x^2)/32 + (965*x^3)/24 + (575*x^4)/16 + (25*x^5)/2 - (5
9895*ArcTan[(1 - 4*x)/Sqrt[23]])/(64*Sqrt[23]) + (1331*Log[3 - x + 2*x^2])/
128
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(2+3x+5x^2)^3}{3-x+2x^2}\,dx = \int \left(-\frac{4795}{32}-\frac{829x}{16}+\frac{965x^2}{8}+\frac{575x^3}{4}+\frac{125x^4}{2}+\frac{1331(11+x)}{32(3-x+2x^2)}\right)dx$$

$$= -\frac{4795x}{32}-\frac{829x^2}{32}+\frac{965x^3}{24}+\frac{575x^4}{16}+\frac{25x^5}{2}+\frac{1331}{32}\int \frac{11+x}{3-x+2x^2}\,dx$$

$$= -\frac{4795x}{32}-\frac{829x^2}{32}+\frac{965x^3}{24}+\frac{575x^4}{16}+\frac{25x^5}{2}+\frac{1331}{128}\int \frac{-1+4x}{3-x+2x^2}\,dx+\frac{59895}{128}\int \frac{}{3-}$$

$$= -\frac{4795x}{32}-\frac{829x^2}{32}+\frac{965x^3}{24}+\frac{575x^4}{16}+\frac{25x^5}{2}+\frac{1331}{128}\log(3-x+2x^2)-\frac{59895}{64}\,\mathrm{Subst}($$

$$= -\frac{4795x}{32}-\frac{829x^2}{32}+\frac{965x^3}{24}+\frac{575x^4}{16}+\frac{25x^5}{2}-\frac{59895\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{64\sqrt{23}}+\frac{1331}{128}\log(3-x+$$

Mathematica [A] time = 0.02, size = 63, normalized size = 0.90

$$\frac{1}{384}\left(3993\log(2x^2-x+3)+4x(1200x^4+3450x^3+3860x^2-2487x-14385)\right)+\frac{59895\tan^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{64\sqrt{23}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)^3/(3 - x + 2*x^2),x]
[Out] (59895*ArcTan[(-1 + 4*x)/Sqrt[23]])/(64*Sqrt[23]) + (4*x*(-14385 - 2487*x +
 3860*x^2 + 3450*x^3 + 1200*x^4) + 3993*Log[3 - x + 2*x^2])/384
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2+3x+5x^2)^3}{3-x+2x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(2 + 3*x + 5*x^2)^3/(3 - x + 2*x^2),x]
```

```
[Out] IntegrateAlgebraic[(2 + 3*x + 5*x^2)^3/(3 - x + 2*x^2), x]
```

fricas [A] time = 0.41, size = 53, normalized size = 0.76

$$\frac{25}{2}\,x^{5} + \frac{575}{16}\,x^{4} + \frac{965}{24}\,x^{3} - \frac{829}{32}\,x^{2} + \frac{59895}{1472}\,\sqrt{23}\,\arctan\left(\frac{1}{23}\,\sqrt{23}\,(4\,x-1)\right) - \frac{4795}{32}\,x + \frac{1331}{128}\,\log\left(2\,x^{2}-x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^3/(2*x^2-x+3),x, algorithm="fricas")
```

```
[Out] 25/2*x^5 + 575/16*x^4 + 965/24*x^3 - 829/32*x^2 + 59895/1472*sqrt(23)*arcta
n(1/23*sqrt(23)*(4*x - 1)) - 4795/32*x + 1331/128*log(2*x^2 - x + 3)
```

giac [A] time = 0.21, size = 53, normalized size = 0.76

$$\frac{25}{2}\,x^{5} + \frac{575}{16}\,x^{4} + \frac{965}{24}\,x^{3} - \frac{829}{32}\,x^{2} + \frac{59895}{1472}\,\sqrt{23}\,\arctan\left(\frac{1}{23}\,\sqrt{23}\,(4\,x-1)\right) - \frac{4795}{32}\,x + \frac{1331}{128}\,\log\left(2\,x^{2}-x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^3/(2*x^2-x+3),x, algorithm="giac")
```

```
[Out] 25/2*x^5 + 575/16*x^4 + 965/24*x^3 - 829/32*x^2 + 59895/1472*sqrt(23)*arcta
n(1/23*sqrt(23)*(4*x - 1)) - 4795/32*x + 1331/128*log(2*x^2 - x + 3)
```

maple [A] time = 0.00, size = 54, normalized size = 0.77

$$\frac{25x^{5}}{2} + \frac{575x^{4}}{16} + \frac{965x^{3}}{24} - \frac{829x^{2}}{32} - \frac{4795x}{32} + \frac{59895\sqrt{23}\;\arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{1472} + \frac{1331\ln\left(2x^{2}-x+3\right)}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)^3/(2*x^2-x+3),x)
```

```
[Out] 25/2*x^5+575/16*x^4+965/24*x^3-829/32*x^2-4795/32*x+1331/128*ln(2*x^2-x+3)+
59895/1472*23^(1/2)*arctan(1/23*(4*x-1)*23^(1/2))
```

maxima [A] time = 0.97, size = 53, normalized size = 0.76

$$\frac{25}{2}\,x^{5} + \frac{575}{16}\,x^{4} + \frac{965}{24}\,x^{3} - \frac{829}{32}\,x^{2} + \frac{59895}{1472}\,\sqrt{23}\,\arctan\left(\frac{1}{23}\,\sqrt{23}\,(4\,x-1)\right) - \frac{4795}{32}\,x + \frac{1331}{128}\,\log\left(2\,x^{2}-x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^3/(2*x^2-x+3),x, algorithm="maxima")
```

```
[Out] 25/2*x^5 + 575/16*x^4 + 965/24*x^3 - 829/32*x^2 + 59895/1472*sqrt(23)*arcta
n(1/23*sqrt(23)*(4*x - 1)) - 4795/32*x + 1331/128*log(2*x^2 - x + 3)
```

mupad [B] time = 0.04, size = 55, normalized size = 0.79

$$\frac{1331\,\ln\left(2\,x^2-x+3\right)}{128}-\frac{4795\,x}{32}+\frac{59895\,\sqrt{23}\,\mathrm{atan}\left(\frac{4\,\sqrt{23}\,x}{23}-\frac{\sqrt{23}}{23}\right)}{1472}-\frac{829\,x^2}{32}+\frac{965\,x^3}{24}+\frac{575\,x^4}{16}+\frac{25\,x^5}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 5*x^2 + 2)^3/(2*x^2 - x + 3),x)
```

```
[Out] (1331*log(2*x^2 - x + 3))/128 - (4795*x)/32 + (59895*23^(1/2)*atan((4*23^(1
/2)*x)/23 - 23^(1/2)/23))/1472 - (829*x^2)/32 + (965*x^3)/24 + (575*x^4)/16
 + (25*x^5)/2
```

sympy [A] time = 0.15, size = 73, normalized size = 1.04

$$\frac{25x^5}{2}+\frac{575x^4}{16}+\frac{965x^3}{24}-\frac{829x^2}{32}-\frac{4795x}{32}+\frac{1331\log\left(x^2-\frac{x}{2}+\frac{3}{2}\right)}{128}+\frac{59895\sqrt{23}\,\mathrm{atan}\left(\frac{4\sqrt{23}\,x}{23}-\frac{\sqrt{23}}{23}\right)}{1472}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)**3/(2*x**2-x+3),x)
```

```
[Out] 25*x**5/2 + 575*x**4/16 + 965*x**3/24 - 829*x**2/32 - 4795*x/32 + 1331*log(
x**2 - x/2 + 3/2)/128 + 59895*sqrt(23)*atan(4*sqrt(23)*x/23 - sqrt(23)/23)/
1472
```

3.36 $\int \frac{(2+3x+5x^2)^2}{3-x+2x^2}\,dx$

Optimal. Leaf size=56

$$\frac{25x^3}{6}+\frac{85x^2}{8}-\frac{363}{32}\log\left(2x^2-x+3\right)+\frac{51x}{8}+\frac{847\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{16\sqrt{23}}$$

Rubi [A] time = 0.05, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1657, 634, 618, 204, 628}

$$\frac{25x^3}{6}+\frac{85x^2}{8}-\frac{363}{32}\log\left(2x^2-x+3\right)+\frac{51x}{8}+\frac{847\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{16\sqrt{23}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + 5*x^2)^2/(3 - x + 2*x^2),x]
```

```
[Out] (51*x)/8 + (85*x^2)/8 + (25*x^3)/6 + (847*ArcTan[(1 - 4*x)/Sqrt[23]])/(16*S
qrt[23]) - (363*Log[3 - x + 2*x^2])/32
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(2+3x+5x^2\right)^2}{3-x+2x^2}\,dx &= \int \left(\frac{51}{8}+\frac{85x}{4}+\frac{25x^2}{2}-\frac{121(1+3x)}{8\left(3-x+2x^2\right)}\right)dx \\
&= \frac{51x}{8}+\frac{85x^2}{8}+\frac{25x^3}{6}-\frac{121}{8}\int \frac{1+3x}{3-x+2x^2}\,dx \\
&= \frac{51x}{8}+\frac{85x^2}{8}+\frac{25x^3}{6}-\frac{363}{32}\int \frac{-1+4x}{3-x+2x^2}\,dx-\frac{847}{32}\int \frac{1}{3-x+2x^2}\,dx \\
&= \frac{51x}{8}+\frac{85x^2}{8}+\frac{25x^3}{6}-\frac{363}{32}\log\left(3-x+2x^2\right)+\frac{847}{16}\,\text{Subst}\left(\int \frac{1}{-23-x^2}\,dx,x,-1+4x\right) \\
&= \frac{51x}{8}+\frac{85x^2}{8}+\frac{25x^3}{6}+\frac{847\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{16\sqrt{23}}-\frac{363}{32}\log\left(3-x+2x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 52, normalized size = 0.93

$$\frac{1}{24}x\left(100x^2+255x+153\right)-\frac{363}{32}\log\left(2x^2-x+3\right)-\frac{847\tan^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{16\sqrt{23}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)^2/(3 - x + 2*x^2),x]
```

```
[Out] (x*(153 + 255*x + 100*x^2))/24 - (847*ArcTan[(-1 + 4*x)/Sqrt[23]])/(16*Sqrt
[23]) - (363*Log[3 - x + 2*x^2])/32
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2+3x+5x^2\right)^2}{3-x+2x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(2 + 3*x + 5*x^2)^2/(3 - x + 2*x^2),x]
```

```
[Out] IntegrateAlgebraic[(2 + 3*x + 5*x^2)^2/(3 - x + 2*x^2), x]
```

fricas [A] time = 0.43, size = 43, normalized size = 0.77

$$\frac{25}{6}\,x^3+\frac{85}{8}\,x^2-\frac{847}{368}\,\sqrt{23}\,\arctan\left(\frac{1}{23}\,\sqrt{23}\,(4\,x-1)\right)+\frac{51}{8}\,x-\frac{363}{32}\,\log\left(2\,x^2-x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^2/(2*x^2-x+3),x, algorithm="fricas")
```

```
[Out] 25/6*x^3 + 85/8*x^2 - 847/368*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 51
/8*x - 363/32*log(2*x^2 - x + 3)
```

giac [A] time = 0.18, size = 43, normalized size = 0.77

$$\frac{25}{6}\,x^3+\frac{85}{8}\,x^2-\frac{847}{368}\,\sqrt{23}\,\arctan\left(\frac{1}{23}\,\sqrt{23}\,(4\,x-1)\right)+\frac{51}{8}\,x-\frac{363}{32}\,\log\left(2\,x^2-x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^2/(2*x^2-x+3),x, algorithm="giac")
```

```
[Out] 25/6*x^3 + 85/8*x^2 - 847/368*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 51
/8*x - 363/32*log(2*x^2 - x + 3)
```

maple [A] time = 0.00, size = 44, normalized size = 0.79

$$\frac{25x^3}{6}+\frac{85x^2}{8}+\frac{51x}{8}-\frac{847\sqrt{23}\,\arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{368}-\frac{363\ln\left(2x^2-x+3\right)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)^2/(2*x^2-x+3),x)
```

```
[Out] 25/6*x^3+85/8*x^2+51/8*x-363/32*ln(2*x^2-x+3)-847/368*23^(1/2)*arctan(1/23*
(4*x-1)*23^(1/2))
```

maxima [A] time = 0.96, size = 43, normalized size = 0.77

$$\frac{25}{6}\,x^3+\frac{85}{8}\,x^2-\frac{847}{368}\,\sqrt{23}\,\arctan\left(\frac{1}{23}\,\sqrt{23}\,(4\,x-1)\right)+\frac{51}{8}\,x-\frac{363}{32}\,\log\left(2\,x^2-x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^2/(2*x^2-x+3),x, algorithm="maxima")
```

```
[Out] 25/6*x^3 + 85/8*x^2 - 847/368*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 51
/8*x - 363/32*log(2*x^2 - x + 3)
```

mupad [B] time = 3.44, size = 45, normalized size = 0.80

$$\frac{51\,x}{8}-\frac{363\,\ln\left(2\,x^2-x+3\right)}{32}-\frac{847\,\sqrt{23}\,\mathrm{atan}\left(\frac{4\,\sqrt{23}\,x}{23}-\frac{\sqrt{23}}{23}\right)}{368}+\frac{85\,x^2}{8}+\frac{25\,x^3}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 5*x^2 + 2)^2/(2*x^2 - x + 3),x)
```

```
[Out] (51*x)/8 - (363*log(2*x^2 - x + 3))/32 - (847*23^(1/2)*atan((4*23^(1/2)*x)/
23 - 23^(1/2)/23))/368 + (85*x^2)/8 + (25*x^3)/6
```

sympy [A] time = 0.15, size = 60, normalized size = 1.07

$$\frac{25x^3}{6}+\frac{85x^2}{8}+\frac{51x}{8}-\frac{363\log\left(x^2-\frac{x}{2}+\frac{3}{2}\right)}{32}-\frac{847\sqrt{23}\,\mathrm{atan}\left(\frac{4\sqrt{23}\,x}{23}-\frac{\sqrt{23}}{23}\right)}{368}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)**2/(2*x**2-x+3),x)
```

```
[Out] 25*x**3/6 + 85*x**2/8 + 51*x/8 - 363*log(x**2 - x/2 + 3/2)/32 - 847*sqrt(23
)*atan(4*sqrt(23)*x/23 - sqrt(23)/23)/368
```

3.37 $\int \frac{2+3x+5x^2}{3-x+2x^2} \, dx$

Optimal. Leaf size=42

$$\frac{11}{8}\log\left(2x^2-x+3\right)+\frac{5x}{2}+\frac{33\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{4\sqrt{23}}$$

Rubi [A] time = 0.03, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {1657, 634, 618, 204, 628}

$$\frac{11}{8}\log\left(2x^2-x+3\right)+\frac{5x}{2}+\frac{33\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{4\sqrt{23}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + 5*x^2)/(3 - x + 2*x^2),x]
```

```
[Out] (5*x)/2 + (33*ArcTan[(1 - 4*x)/Sqrt[23]])/(4*Sqrt[23]) + (11*Log[3 - x + 2*
x^2])/8
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{2+3x+5x^2}{3-x+2x^2}\,dx &= \int \left(\frac{5}{2}-\frac{11(1-x)}{2\left(3-x+2x^2\right)}\right)dx \\
&= \frac{5x}{2}-\frac{11}{2}\int \frac{1-x}{3-x+2x^2}\,dx \\
&= \frac{5x}{2}+\frac{11}{8}\int \frac{-1+4x}{3-x+2x^2}\,dx-\frac{33}{8}\int \frac{1}{3-x+2x^2}\,dx \\
&= \frac{5x}{2}+\frac{11}{8}\log\left(3-x+2x^2\right)+\frac{33}{4}\,\mathrm{Subst}\left(\int \frac{1}{-23-x^2}\,dx,x,-1+4x\right) \\
&= \frac{5x}{2}+\frac{33\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{4\sqrt{23}}+\frac{11}{8}\log\left(3-x+2x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 42, normalized size = 1.00

$$\frac{11}{8}\log\left(2x^2-x+3\right)+\frac{5x}{2}-\frac{33\tan^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{4\sqrt{23}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)/(3 - x + 2*x^2),x]
```

```
[Out] (5*x)/2 - (33*ArcTan[(-1 + 4*x)/Sqrt[23]])/(4*Sqrt[23]) + (11*Log[3 - x + 2
*x^2])/8
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2+3x+5x^2}{3-x+2x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(2 + 3*x + 5*x^2)/(3 - x + 2*x^2),x]
[Out] IntegrateAlgebraic[(2 + 3*x + 5*x^2)/(3 - x + 2*x^2), x]
```

fricas [A] time = 0.39, size = 33, normalized size = 0.79

$$-\frac{33}{92}\,\sqrt{23}\,\arctan\left(\frac{1}{23}\,\sqrt{23}\,(4\,x-1)\right)+\frac{5}{2}\,x+\frac{11}{8}\,\log\left(2\,x^2-x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)/(2*x^2-x+3),x, algorithm="fricas")
[Out] -33/92*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 5/2*x + 11/8*log(2*x^2 -
x + 3)
```

giac [A] time = 0.21, size = 33, normalized size = 0.79

$$-\frac{33}{92}\,\sqrt{23}\,\arctan\left(\frac{1}{23}\,\sqrt{23}\,(4\,x-1)\right)+\frac{5}{2}\,x+\frac{11}{8}\,\log\left(2\,x^2-x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)/(2*x^2-x+3),x, algorithm="giac")
[Out] -33/92*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 5/2*x + 11/8*log(2*x^2 -
x + 3)
```

maple [A] time = 0.00, size = 34, normalized size = 0.81

$$\frac{5x}{2}-\frac{33\sqrt{23}\,\arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{92}+\frac{11\ln\left(2x^2-x+3\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)/(2*x^2-x+3),x)
[Out] 5/2*x+11/8*ln(2*x^2-x+3)-33/92*23^(1/2)*arctan(1/23*(4*x-1)*23^(1/2))
```

maxima [A] time = 0.95, size = 33, normalized size = 0.79

$$-\frac{33}{92}\,\sqrt{23}\,\arctan\left(\frac{1}{23}\,\sqrt{23}\,(4\,x-1)\right)+\frac{5}{2}\,x+\frac{11}{8}\,\log\left(2\,x^2-x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)/(2*x^2-x+3),x, algorithm="maxima")
```

```
[Out] -33/92*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 5/2*x + 11/8*log(2*x^2 -
x + 3)
```

mupad [B] time = 0.04, size = 35, normalized size = 0.83

$$\frac{5\,x}{2}+\frac{11\,\ln\left(2\,x^2-x+3\right)}{8}-\frac{33\,\sqrt{23}\,\mathrm{atan}\left(\frac{4\,\sqrt{23}\,x}{23}-\frac{\sqrt{23}}{23}\right)}{92}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 5*x^2 + 2)/(2*x^2 - x + 3),x)
```

```
[Out] (5*x)/2 + (11*log(2*x^2 - x + 3))/8 - (33*23^(1/2)*atan((4*23^(1/2)*x)/23 -
 23^(1/2)/23))/92
```

sympy [A] time = 0.14, size = 46, normalized size = 1.10

$$\frac{5x}{2}+\frac{11\log\left(x^2-\frac{x}{2}+\frac{3}{2}\right)}{8}-\frac{33\sqrt{23}\,\mathrm{atan}\left(\frac{4\sqrt{23}\,x}{23}-\frac{\sqrt{23}}{23}\right)}{92}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)/(2*x**2-x+3),x)
```

```
[Out] 5*x/2 + 11*log(x**2 - x/2 + 3/2)/8 - 33*sqrt(23)*atan(4*sqrt(23)*x/23 - sqr
t(23)/23)/92
```

3.38 $\int \frac{1}{(3-x+2x^2)(2+3x+5x^2)}\,dx$

Optimal. Leaf size=73

$$-\frac{1}{44}\log\left(2x^2-x+3\right)+\frac{1}{44}\log\left(5x^2+3x+2\right)+\frac{3\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{22\sqrt{23}}+\frac{13\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{22\sqrt{31}}$$

Rubi [A] time = 0.05, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {980, 634, 618, 204, 628}

$$-\frac{1}{44}\log\left(2x^2-x+3\right)+\frac{1}{44}\log\left(5x^2+3x+2\right)+\frac{3\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{22\sqrt{23}}+\frac{13\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{22\sqrt{31}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - x + 2*x^2)*(2 + 3*x + 5*x^2)),x]
```

```
[Out] (3*ArcTan[(1 - 4*x)/Sqrt[23]])/(22*Sqrt[23]) + (13*ArcTan[(3 + 10*x)/Sqrt[3
1]])/(22*Sqrt[31]) - Log[3 - x + 2*x^2]/44 + Log[2 + 3*x + 5*x^2]/44
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 980

```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2
)), x_Symbol] :> With[{q = c^2*d^2 - b*c*d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*
f - a*b*e*f + a^2*f^2}, Dist[1/q, Int[(c^2*d - b*c*e + b^2*f - a*c*f - (c^2
*e - b*c*f)*x)/(a + b*x + c*x^2), x], x] + Dist[1/q, Int[(c*e^2 - c*d*f - b
*e*f + a*f^2 + (c*e*f - b*f^2)*x)/(d + e*x + f*x^2), x], x] /; NeQ[q, 0]] /
; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0
]
```

Rubi steps

$$\int \frac{1}{\left(3-x+2x^2\right)\left(2+3x+5x^2\right)}\,dx = \frac{1}{242}\int \frac{-11-22x}{3-x+2x^2}\,dx + \frac{1}{242}\int \frac{88+55x}{2+3x+5x^2}\,dx$$

$$= -\left(\frac{1}{44}\int \frac{-1+4x}{3-x+2x^2}\,dx\right) + \frac{1}{44}\int \frac{3+10x}{2+3x+5x^2}\,dx - \frac{3}{44}\int \frac{1}{3-x+2x^2}\,d$$

$$= -\frac{1}{44}\log\left(3-x+2x^2\right) + \frac{1}{44}\log\left(2+3x+5x^2\right) + \frac{3}{22}\,\text{Subst}\left(\int \frac{1}{-23-x^2}\right.$$

$$= \frac{3\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{22\sqrt{23}} + \frac{13\tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)}{22\sqrt{31}} - \frac{1}{44}\log\left(3-x+2x^2\right) + \frac{1}{44}\log\left(2+\right.$$

Mathematica [A] time = 0.03, size = 73, normalized size = 1.00

$$-\frac{1}{44}\log\left(2x^2-x+3\right)+\frac{1}{44}\log\left(5x^2+3x+2\right)-\frac{3\tan^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{22\sqrt{23}}+\frac{13\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{22\sqrt{31}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - x + 2*x^2)*(2 + 3*x + 5*x^2)),x]
```

```
[Out] (-3*ArcTan[(-1 + 4*x)/Sqrt[23]])/(22*Sqrt[23]) + (13*ArcTan[(3 + 10*x)/Sqrt
[31]])/(22*Sqrt[31]) - Log[3 - x + 2*x^2]/44 + Log[2 + 3*x + 5*x^2]/44
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(3-x+2x^2\right)\left(2+3x+5x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((3 - x + 2*x^2)*(2 + 3*x + 5*x^2)),x]
```

```
[Out] IntegrateAlgebraic[1/((3 - x + 2*x^2)*(2 + 3*x + 5*x^2)), x]
```

fricas [A] time = 0.41, size = 59, normalized size = 0.81

$$\frac{13}{682}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right)-\frac{3}{506}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right)+\frac{1}{44}\log\left(5x^2+3x+2\right)-\frac{1}{44}\log\left(2x^2-x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)/(5*x^2+3*x+2),x, algorithm="fricas")
```

```
[Out] 13/682*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) - 3/506*sqrt(23)*arctan(1/
23*sqrt(23)*(4*x - 1)) + 1/44*log(5*x^2 + 3*x + 2) - 1/44*log(2*x^2 - x + 3
)
```

giac [A] time = 0.19, size = 59, normalized size = 0.81

$$\frac{13}{682}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right)-\frac{3}{506}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right)+\frac{1}{44}\log\left(5x^2+3x+2\right)-\frac{1}{44}\log\left(2x^2-x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)/(5*x^2+3*x+2),x, algorithm="giac")
```

```
[Out] 13/682*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) - 3/506*sqrt(23)*arctan(1/
23*sqrt(23)*(4*x - 1)) + 1/44*log(5*x^2 + 3*x + 2) - 1/44*log(2*x^2 - x + 3
)
```

maple [A] time = 0.01, size = 60, normalized size = 0.82

$$\frac{13\sqrt{31}\ \arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)}{682}-\frac{3\sqrt{23}\ \arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{506}-\frac{\ln\left(2x^2-x+3\right)}{44}+\frac{\ln\left(5x^2+3x+2\right)}{44}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^2-x+3)/(5*x^2+3*x+2),x)
```

```
[Out] 1/44*ln(5*x^2+3*x+2)+13/682*31^(1/2)*arctan(1/31*(10*x+3)*31^(1/2))-1/44*ln
(2*x^2-x+3)-3/506*23^(1/2)*arctan(1/23*(4*x-1)*23^(1/2))
```

maxima [A] time = 0.96, size = 59, normalized size = 0.81

$$\frac{13}{682}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right)-\frac{3}{506}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right)+\frac{1}{44}\log\left(5x^2+3x+2\right)-\frac{1}{44}\log\left(2x^2-x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)/(5*x^2+3*x+2),x, algorithm="maxima")
```

```
[Out] 13/682*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) - 3/506*sqrt(23)*arctan(1/
23*sqrt(23)*(4*x - 1)) + 1/44*log(5*x^2 + 3*x + 2) - 1/44*log(2*x^2 - x + 3
)
```

mupad [B] time = 0.19, size = 79, normalized size = 1.08

$$\ln\left(x-\frac{1}{4}-\frac{\sqrt{23}\,1\mathrm{i}}{4}\right)\left(-\frac{1}{44}+\frac{\sqrt{23}\,3\mathrm{i}}{1012}\right)-\ln\left(x-\frac{1}{4}+\frac{\sqrt{23}\,1\mathrm{i}}{4}\right)\left(\frac{1}{44}+\frac{\sqrt{23}\,3\mathrm{i}}{1012}\right)-\ln\left(x+\frac{3}{10}-\frac{\sqrt{31}\,1\mathrm{i}}{10}\right)\left(-\frac{1}{44}+\frac{\sqrt{31}\,13\mathrm{i}}{1364}\right)+\ln\left(x+\frac{3}{10}+\frac{\sqrt{31}\,1\mathrm{i}}{10}\right)\left(\frac{1}{44}+\frac{\sqrt{31}\,13\mathrm{i}}{1364}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*x^2 - x + 3)*(3*x + 5*x^2 + 2)),x)
```

```
[Out] log(x - (23^(1/2)*1i)/4 - 1/4)*((23^(1/2)*3i)/1012 - 1/44) - log(x + (23^(1
/2)*1i)/4 - 1/4)*((23^(1/2)*3i)/1012 + 1/44) - log(x - (31^(1/2)*1i)/10 + 3
/10)*((31^(1/2)*13i)/1364 - 1/44) + log(x + (31^(1/2)*1i)/10 + 3/10)*((31^(
1/2)*13i)/1364 + 1/44)
```

sympy [A] time = 0.24, size = 83, normalized size = 1.14

$$-\frac{\log\left(x^2-\frac{x}{2}+\frac{3}{2}\right)}{44}+\frac{\log\left(x^2+\frac{3x}{5}+\frac{2}{5}\right)}{44}-\frac{3\sqrt{23}\operatorname{atan}\left(\frac{4\sqrt{23}\,x}{23}-\frac{\sqrt{23}}{23}\right)}{506}+\frac{13\sqrt{31}\operatorname{atan}\left(\frac{10\sqrt{31}\,x}{31}+\frac{3\sqrt{31}}{31}\right)}{682}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**2-x+3)/(5*x**2+3*x+2),x)
```

```
[Out] -log(x**2 - x/2 + 3/2)/44 + log(x**2 + 3*x/5 + 2/5)/44 - 3*sqrt(23)*atan(4*
sqrt(23)*x/23 - sqrt(23)/23)/506 + 13*sqrt(31)*atan(10*sqrt(31)*x/31 + 3*sq
rt(31)/31)/682
```

3.39 $\int \frac{1}{(3-x+2x^2)(2+3x+5x^2)^2} \, dx$

Optimal. Leaf size=94

$$\frac{65x+4}{682(5x^2+3x+2)}+\frac{3}{968}\log(2x^2-x+3)-\frac{3}{968}\log(5x^2+3x+2)+\frac{7\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{484\sqrt{23}}+\frac{2891\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{15004\sqrt{31}}$$

Rubi [A] time = 0.09, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {974, 1072, 634, 618, 204, 628}

$$\frac{65x+4}{682(5x^2+3x+2)}+\frac{3}{968}\log(2x^2-x+3)-\frac{3}{968}\log(5x^2+3x+2)+\frac{7\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{484\sqrt{23}}+\frac{2891\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{15004\sqrt{31}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - x + 2*x^2)*(2 + 3*x + 5*x^2)^2),x]
```

```
[Out] (4 + 65*x)/(682*(2 + 3*x + 5*x^2)) + (7*ArcTan[(1 - 4*x)/Sqrt[23]])/(484*Sq
rt[23]) + (2891*ArcTan[(3 + 10*x)/Sqrt[31]])/(15004*Sqrt[31]) + (3*Log[3 -
x + 2*x^2])/968 - (3*Log[2 + 3*x + 5*x^2])/968
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 974

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*
a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*(
d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q,
 0]
```

Rule 1072

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)), x_Symbol] :> With[{q = c^2*d^2 - b*c*
d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*e*f + a^2*f^2}, Dist[1/q, Int[(A*
c^2*d - a*c*C*d - A*b*c*e + a*B*c*e + A*b^2*f - a*b*B*f - a*A*c*f + a^2*C*f
 + c*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B*f)*x)/(a + b*x + c*x^2),
x], x] + Dist[1/q, Int[(c*C*d^2 - B*c*d*e + A*c*e^2 + b*B*d*f - A*c*d*f - a
*C*d*f - A*b*e*f + a*A*f^2 - f*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B
*f)*x)/(d + e*x + f*x^2), x], x] /; NeQ[q, 0]] /; FreeQ[{a, b, c, d, e, f,
A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$\int \frac{1}{\left(3-x+2x^2\right)\left(2+3x+5x^2\right)^2}\,dx = \frac{4+65x}{682\left(2+3x+5x^2\right)} - \frac{\int \frac{-1804+1397x-1430x^2}{\left(3-x+2x^2\right)\left(2+3x+5x^2\right)}\,dx}{7502}$$

$$= \frac{4+65x}{682\left(2+3x+5x^2\right)} - \frac{\int \frac{18755-22506x}{3-x+2x^2}\,dx}{1815484} - \frac{\int \frac{-158026+56265x}{2+3x+5x^2}\,dx}{1815484}$$

$$= \frac{4+65x}{682\left(2+3x+5x^2\right)} + \frac{3}{968}\int \frac{-1+4x}{3-x+2x^2}\,dx - \frac{3}{968}\int \frac{3+10x}{2+3x+5x^2}\,dx -$$

$$= \frac{4+65x}{682\left(2+3x+5x^2\right)} + \frac{3}{968}\log\left(3-x+2x^2\right) - \frac{3}{968}\log\left(2+3x+5x^2\right) +$$

$$= \frac{4+65x}{682\left(2+3x+5x^2\right)} + \frac{7\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{484\sqrt{23}} + \frac{2891\tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)}{15004\sqrt{31}} + \frac{3}{968}\log\left(3-\right.$$

Mathematica [A] time = 0.08, size = 94, normalized size = 1.00

$$\frac{65x+4}{682\left(5x^2+3x+2\right)} + \frac{3}{968}\log\left(2x^2-x+3\right) - \frac{3}{968}\log\left(5x^2+3x+2\right) - \frac{7\tan^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{484\sqrt{23}} + \frac{2891\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{15004\sqrt{31}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - x + 2*x^2)*(2 + 3*x + 5*x^2)^2),x]
```

```
[Out] (4 + 65*x)/(682*(2 + 3*x + 5*x^2)) - (7*ArcTan[(-1 + 4*x)/Sqrt[23]])/(484*S
qrt[23]) + (2891*ArcTan[(3 + 10*x)/Sqrt[31]])/(15004*Sqrt[31]) + (3*Log[3 -
 x + 2*x^2])/968 - (3*Log[2 + 3*x + 5*x^2])/968
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(3-x+2x^2\right)\left(2+3x+5x^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((3 - x + 2*x^2)*(2 + 3*x + 5*x^2)^2),x]
```

```
[Out] IntegrateAlgebraic[1/((3 - x + 2*x^2)*(2 + 3*x + 5*x^2)^2), x]
```

fricas [A] time = 0.41, size = 117, normalized size = 1.24

$$\frac{132986 \sqrt{31} \left(5 x^2 + 3 x + 2\right) \arctan\left(\frac{1}{31} \sqrt{31} (10 x + 3)\right) - 13454 \sqrt{23} \left(5 x^2 + 3 x + 2\right) \arctan\left(\frac{1}{23} \sqrt{23} (4 x - 1)\right) - 66309 \left(5 x^2 + 3 x + 2\right) \log\left(5 x^2 + 3 x + 2\right) + 66309 \left(5 x^2 + 3 x + 2\right) \log\left(2 x^2 - x + 3\right) + 2039180 x + 125488}{21395704 \left(5 x^2 + 3 x + 2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)/(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

```
[Out] 1/21395704*(132986*sqrt(31)*(5*x^2 + 3*x + 2)*arctan(1/31*sqrt(31)*(10*x +
3)) - 13454*sqrt(23)*(5*x^2 + 3*x + 2)*arctan(1/23*sqrt(23)*(4*x - 1)) - 66
309*(5*x^2 + 3*x + 2)*log(5*x^2 + 3*x + 2) + 66309*(5*x^2 + 3*x + 2)*log(2*
x^2 - x + 3) + 2039180*x + 125488)/(5*x^2 + 3*x + 2)
```

giac [A] time = 0.20, size = 78, normalized size = 0.83

$$\frac{2891}{465124} \sqrt{31} \arctan\left(\frac{1}{31} \sqrt{31} (10 x + 3)\right) - \frac{7}{11132} \sqrt{23} \arctan\left(\frac{1}{23} \sqrt{23} (4 x - 1)\right) + \frac{65 x + 4}{682 \left(5 x^2 + 3 x + 2\right)} - \frac{3}{968} \log\left(5 x^2 + 3 x + 2\right) + \frac{3}{968} \log\left(2 x^2 - x + 3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)/(5*x^2+3*x+2)^2,x, algorithm="giac")
```

```
[Out] 2891/465124*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) - 7/11132*sqrt(23)*ar
ctan(1/23*sqrt(23)*(4*x - 1)) + 1/682*(65*x + 4)/(5*x^2 + 3*x + 2) - 3/968*
log(5*x^2 + 3*x + 2) + 3/968*log(2*x^2 - x + 3)
```

maple [A] time = 0.01, size = 77, normalized size = 0.82

$$\frac{2891 \sqrt{31}\, \arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)}{465124} - \frac{7\sqrt{23}\, \arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{11132} + \frac{3 \ln\left(2x^2 - x + 3\right)}{968} - \frac{3 \ln\left(5x^2 + 3x + 2\right)}{968} - \frac{-\frac{286x}{31} - \frac{88}{155}}{484 \left(x^2 + \frac{3}{5}x + \frac{2}{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^2-x+3)/(5*x^2+3*x+2)^2,x)
```

```
[Out] -1/484*(-286/31*x-88/155)/(x^2+3/5*x+2/5)-3/968*ln(5*x^2+3*x+2)+2891/465124
*31^(1/2)*arctan(1/31*(10*x+3)*31^(1/2))+3/968*ln(2*x^2-x+3)-7/11132*23^(1/
2)*arctan(1/23*(4*x-1)*23^(1/2))
```

maxima [A] time = 0.96, size = 78, normalized size = 0.83

$$\frac{2891}{465124} \sqrt{31} \arctan\left(\frac{1}{31} \sqrt{31} (10 x + 3)\right) - \frac{7}{11132} \sqrt{23} \arctan\left(\frac{1}{23} \sqrt{23} (4 x - 1)\right) + \frac{65 x + 4}{682 \left(5 x^2 + 3 x + 2\right)} - \frac{3}{968} \log\left(5 x^2 + 3 x + 2\right) + \frac{3}{968} \log\left(2 x^2 - x + 3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)/(5*x^2+3*x+2)^2,x, algorithm="maxima")
```

```
[Out] 2891/465124*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) - 7/11132*sqrt(23)*ar
ctan(1/23*sqrt(23)*(4*x - 1)) + 1/682*(65*x + 4)/(5*x^2 + 3*x + 2) - 3/968*
log(5*x^2 + 3*x + 2) + 3/968*log(2*x^2 - x + 3)
```

mupad [B] time = 3.57, size = 95, normalized size = 1.01

$$\frac{\frac{13\,x}{682}+\frac{2}{1705}}{x^2+\frac{3\,x}{5}+\frac{2}{5}}+\ln\left(x-\frac{1}{4}-\frac{\sqrt{23}\,1\mathrm{i}}{4}\right)\left(\frac{3}{968}+\frac{\sqrt{23}\,7\mathrm{i}}{22264}\right)-\ln\left(x-\frac{1}{4}+\frac{\sqrt{23}\,1\mathrm{i}}{4}\right)\left(-\frac{3}{968}+\frac{\sqrt{23}\,7\mathrm{i}}{22264}\right)-\ln\left(x+\frac{3}{10}-\frac{\sqrt{31}\,1\mathrm{i}}{10}\right)\left(\frac{3}{968}+\frac{\sqrt{31}\,2891\mathrm{i}}{930248}\right)+\ln\left(x+\frac{3}{10}+\frac{\sqrt{31}\,1\mathrm{i}}{10}\right)\left(-\frac{3}{968}+\frac{\sqrt{31}\,2891\mathrm{i}}{930248}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*x^2 - x + 3)*(3*x + 5*x^2 + 2)^2),x)
```

```
[Out] ((13*x)/682 + 2/1705)/((3*x)/5 + x^2 + 2/5) + log(x - (23^(1/2)*1i)/4 - 1/4
)*((23^(1/2)*7i)/22264 + 3/968) - log(x + (23^(1/2)*1i)/4 - 1/4)*((23^(1/2)
*7i)/22264 - 3/968) - log(x - (31^(1/2)*1i)/10 + 3/10)*((31^(1/2)*2891i)/93
0248 + 3/968) + log(x + (31^(1/2)*1i)/10 + 3/10)*((31^(1/2)*2891i)/930248 -
 3/968)
```

sympy [A] time = 0.32, size = 102, normalized size = 1.09

$$\frac{65x+4}{3410x^2+2046x+1364}+\frac{3\log\left(x^2-\frac{x}{2}+\frac{3}{2}\right)}{968}-\frac{3\log\left(x^2+\frac{3x}{5}+\frac{2}{5}\right)}{968}-\frac{7\sqrt{23}\,\operatorname{atan}\left(\frac{4\sqrt{23}\,x}{23}-\frac{\sqrt{23}}{23}\right)}{11132}+\frac{2891\sqrt{31}\,\operatorname{atan}\left(\frac{10\sqrt{31}\,x}{31}+\frac{3\sqrt{31}}{31}\right)}{465124}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**2-x+3)/(5*x**2+3*x+2)**2,x)
```

```
[Out] (65*x + 4)/(3410*x**2 + 2046*x + 1364) + 3*log(x**2 - x/2 + 3/2)/968 - 3*lo
g(x**2 + 3*x/5 + 2/5)/968 - 7*sqrt(23)*atan(4*sqrt(23)*x/23 - sqrt(23)/23)/
11132 + 2891*sqrt(31)*atan(10*sqrt(31)*x/31 + 3*sqrt(31)/31)/465124
```

3.40 $\int \frac{1}{(3-x+2x^2)(2+3x+5x^2)^3} \, dx$

Optimal. Leaf size=115

$$\frac{65x+4}{1364\left(5x^2+3x+2\right)^2}+\frac{21605x+7923}{465124\left(5x^2+3x+2\right)}-\frac{\log\left(2x^2-x+3\right)}{21296}+\frac{\log\left(5x^2+3x+2\right)}{21296}-\frac{45\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{10648\sqrt{23}}+\frac{84779}{10}$$

Rubi [A] time = 0.12, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {974, 1060, 1072, 634, 618, 204, 628}

$$\frac{65x+4}{1364\left(5x^2+3x+2\right)^2}+\frac{21605x+7923}{465124\left(5x^2+3x+2\right)}-\frac{\log\left(2x^2-x+3\right)}{21296}+\frac{\log\left(5x^2+3x+2\right)}{21296}-\frac{45\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{10648\sqrt{23}}+\frac{847793\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{10232728\sqrt{31}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - x + 2*x^2)*(2 + 3*x + 5*x^2)^3),x]
```

```
[Out] (4 + 65*x)/(1364*(2 + 3*x + 5*x^2)^2) + (7923 + 21605*x)/(465124*(2 + 3*x +
 5*x^2)) - (45*ArcTan[(1 - 4*x)/Sqrt[23]])/(10648*Sqrt[23]) + (847793*ArcTa
n[(3 + 10*x)/Sqrt[31]])/(10232728*Sqrt[31]) - Log[3 - x + 2*x^2]/21296 + Lo
g[2 + 3*x + 5*x^2]/21296
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 974

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*
a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*(
d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q,
 0]
```

Rule 1060

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x +
 c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
2*a*(c*d - a*f)))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !
```

```
IGtQ[q, 0]
```

Rule 1072

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)), x_Symbol] :> With[{q = c^2*d^2 - b*c*
d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*e*f + a^2*f^2}, Dist[1/q, Int[(A*
c^2*d - a*c*C*d - A*b*c*e + a*B*c*e + A*b^2*f - a*b*B*f - a*A*c*f + a^2*C*f
 + c*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B*f)*x)/(a + b*x + c*x^2),
x], x] + Dist[1/q, Int[(c*C*d^2 - B*c*d*e + A*c*e^2 + b*B*d*f - A*c*d*f - a
*C*d*f - A*b*e*f + a*A*f^2 - f*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B
*f)*x)/(d + e*x + f*x^2), x], x] /; NeQ[q, 0]] /; FreeQ[{a, b, c, d, e, f,
A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$\int \frac{1}{(3-x+2x^2)(2+3x+5x^2)^3}\,dx = \frac{4+65x}{1364(2+3x+5x^2)^2} - \frac{\int \frac{-5753+3509x-4290x^2}{(3-x+2x^2)(2+3x+5x^2)^2}\,dx}{15004}$$

$$= \frac{4+65x}{1364(2+3x+5x^2)^2} + \frac{7923+21605x}{465124(2+3x+5x^2)} - \frac{\int \frac{-14522420+3833038x-10\cdots}{(3-x+2x^2)(2+3x+5\cdots}}{112560008\cdots}$$

$$= \frac{4+65x}{1364(2+3x+5x^2)^2} + \frac{7923+21605x}{465124(2+3x+5x^2)} - \frac{\int \frac{-58838186+5116364x}{3-x+2x^2}\,dx\cdots}{27239521936}\cdots$$

$$= \frac{4+65x}{1364(2+3x+5x^2)^2} + \frac{7923+21605x}{465124(2+3x+5x^2)} - \frac{\int \frac{-1+4x}{3-x+2x^2}\,dx}{21296} + \frac{\int \frac{3+1\cdots}{2+3x\cdots}}{212\cdots}$$

$$= \frac{4+65x}{1364(2+3x+5x^2)^2} + \frac{7923+21605x}{465124(2+3x+5x^2)} - \frac{\log(3-x+2x^2)}{21296} + \frac{\mathrm{lo}\cdots}{\cdots}$$

$$= \frac{4+65x}{1364(2+3x+5x^2)^2} + \frac{7923+21605x}{465124(2+3x+5x^2)} - \frac{45\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{10648\sqrt{23}} + \frac{847\cdots}{\cdots}$$

Mathematica [A] time = 0.16, size = 104, normalized size = 0.90

$$\frac{31\left(-961\log(2x^2-x+3)+961\log(5x^2+3x+2)+\frac{44(108025x^3+104430x^2+89144x+17210)}{(5x^2+3x+2)^2}\right)+1695586\sqrt{31}\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{634429136}+\frac{45\tan^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{10648\sqrt{23}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - x + 2*x^2)*(2 + 3*x + 5*x^2)^3),x]
```

```
[Out] (45*ArcTan[(-1 + 4*x)/Sqrt[23]])/(10648*Sqrt[23]) + (1695586*Sqrt[31]*ArcTa
n[(3 + 10*x)/Sqrt[31]] + 31*((44*(17210 + 89144*x + 104430*x^2 + 108025*x^3
))/(2 + 3*x + 5*x^2)^2 - 961*Log[3 - x + 2*x^2] + 961*Log[2 + 3*x + 5*x^2])
)/634429136
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(3-x+2x^2)(2+3x+5x^2)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((3 - x + 2*x^2)*(2 + 3*x + 5*x^2)^3),x]
```

```
[Out] IntegrateAlgebraic[1/((3 - x + 2*x^2)*(2 + 3*x + 5*x^2)^3), x]
```

fricas [A] time = 0.40, size = 177, normalized size = 1.54

$$\frac{3388960300\,x^3 + 38998478\,\sqrt{31}\,(25\,x^4 + 30\,x^3 + 29\,x^2 + 12\,x + 4)\arctan\left(\frac{1}{31}\,\sqrt{31}\,(10\,x + 3)\right) + 2681190\,\sqrt{23}\,(25\,x^4 + 30\,x^3 + 29\,x^2 + 12\,x + 4)\arctan\left(\frac{1}{23}\,\sqrt{23}\,(4\,x - 1)\right) + 3276177960\,x^2 + 685193\,(25\,x^4 + 30\,x^3 + 29\,x^2 + 12\,x + 4)\log\left(5\,x^2 + 3\,x + 2\right) - 685193\,(25\,x^4 + 30\,x^3 + 29\,x^2 + 12\,x + 4)\log\left(2\,x^2 - x + 3\right) + 2796625568\,x + 539912120}{14591870128\,(25\,x^4 + 30\,x^3 + 29\,x^2 + 12\,x + 4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)/(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

```
[Out] 1/14591870128*(3388960300*x^3 + 38998478*sqrt(31)*(25*x^4 + 30*x^3 + 29*x^2
 + 12*x + 4)*arctan(1/31*sqrt(31)*(10*x + 3)) + 2681190*sqrt(23)*(25*x^4 +
30*x^3 + 29*x^2 + 12*x + 4)*arctan(1/23*sqrt(23)*(4*x - 1)) + 3276177960*x^
2 + 685193*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)*log(5*x^2 + 3*x + 2) - 685
193*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)*log(2*x^2 - x + 3) + 2796625568*x
 + 539912120)/(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)
```

giac [A] time = 0.18, size = 88, normalized size = 0.77

$$\frac{847793}{317214568}\,\sqrt{31}\,\arctan\left(\frac{1}{31}\,\sqrt{31}\,(10\,x + 3)\right) + \frac{45}{244904}\,\sqrt{23}\,\arctan\left(\frac{1}{23}\,\sqrt{23}\,(4\,x - 1)\right) + \frac{108025\,x^3 + 104430\,x^2 + 89144\,x + 17210}{465124\,(5\,x^2 + 3\,x + 2)^2} + \frac{1}{21296}\,\log\left(5\,x^2 + 3\,x + 2\right) - \frac{1}{21296}\,\log\left(2\,x^2 - x + 3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)/(5*x^2+3*x+2)^3,x, algorithm="giac")
```

```
[Out] 847793/317214568*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 45/244904*sqrt
(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 1/465124*(108025*x^3 + 104430*x^2 +
89144*x + 17210)/(5*x^2 + 3*x + 2)^2 + 1/21296*log(5*x^2 + 3*x + 2) - 1/212
96*log(2*x^2 - x + 3)
```

maple [A] time = 0.01, size = 89, normalized size = 0.77

$$\frac{847793\sqrt{31}\,\arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)}{317214568}+\frac{45\sqrt{23}\,\arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{244904}-\frac{\ln\left(2x^2-x+3\right)}{21296}+\frac{\ln\left(5x^2+3x+2\right)}{21296}+\frac{\frac{108025}{465124}x^3+\frac{52215}{232562}x^2+\frac{2026}{10571}x+\frac{8605}{232562}}{\left(5x^2+3x+2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^2-x+3)/(5*x^2+3*x+2)^3,x)
```

```
[Out] 25/10648*(95062/961*x^3+459492/4805*x^2+1961168/24025*x+75724/4805)/(5*x^2+
3*x+2)^2+1/21296*ln(5*x^2+3*x+2)+847793/317214568*31^(1/2)*arctan(1/31*(10*
x+3)*31^(1/2))-1/21296*ln(2*x^2-x+3)+45/244904*23^(1/2)*arctan(1/23*(4*x-1)
*23^(1/2))
```

maxima [A] time = 0.96, size = 98, normalized size = 0.85

$$\frac{847793}{317214568}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}\,(10\,x+3)\right)+\frac{45}{244904}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}\,(4\,x-1)\right)+\frac{108025\,x^3+104430\,x^2+89144\,x+17210}{465124\left(25\,x^4+30\,x^3+29\,x^2+12\,x+4\right)}+\frac{1}{21296}\log\left(5\,x^2+3\,x+2\right)-\frac{1}{21296}\log\left(2\,x^2-x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)/(5*x^2+3*x+2)^3,x, algorithm="maxima")
```

```
[Out] 847793/317214568*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 45/244904*sqrt
(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 1/465124*(108025*x^3 + 104430*x^2 +
89144*x + 17210)/(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4) + 1/21296*log(5*x^2
+ 3*x + 2) - 1/21296*log(2*x^2 - x + 3)
```

mupad [B] time = 0.18, size = 115, normalized size = 1.00

$$\frac{\frac{4321\,x^3}{465124}+\frac{10443\,x^2}{1162810}+\frac{2026\,x}{264275}+\frac{1721}{1162810}}{x^4+\frac{6\,x^3}{5}+\frac{29\,x^2}{25}+\frac{12\,x}{25}+\frac{4}{25}}+\ln\left(x-\frac{1}{4}+\frac{\sqrt{23}\,1\mathrm{i}}{4}\right)\left(-\frac{1}{21296}+\frac{\sqrt{23}\,45\mathrm{i}}{489808}\right)-\ln\left(x+\frac{3}{10}-\frac{\sqrt{31}\,1\mathrm{i}}{10}\right)\left(-\frac{1}{21296}+\frac{\sqrt{31}\,847793\mathrm{i}}{634429136}\right)+\ln\left(x+\frac{3}{10}+\frac{\sqrt{31}\,1\mathrm{i}}{10}\right)\left(\frac{1}{21296}+\frac{\sqrt{31}\,847793\mathrm{i}}{634429136}\right)-\ln\left(x-\frac{1}{4}-\frac{\sqrt{23}\,1\mathrm{i}}{4}\right)\left(\frac{1}{21296}+\frac{\sqrt{23}\,45\mathrm{i}}{489808}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*x^2 - x + 3)*(3*x + 5*x^2 + 2)^3),x)
```

```
[Out] log(x + (23^(1/2)*1i)/4 - 1/4)*((23^(1/2)*45i)/489808 - 1/21296) - log(x -
(23^(1/2)*1i)/4 - 1/4)*((23^(1/2)*45i)/489808 + 1/21296) - log(x - (31^(1/2
)*1i)/10 + 3/10)*((31^(1/2)*847793i)/634429136 - 1/21296) + log(x + (31^(1/
2)*1i)/10 + 3/10)*((31^(1/2)*847793i)/634429136 + 1/21296) + ((2026*x)/2642
75 + (10443*x^2)/1162810 + (4321*x^3)/465124 + 1721/1162810)/((12*x)/25 + (
29*x^2)/25 + (6*x^3)/5 + x^4 + 4/25)
```

sympy [A] time = 0.36, size = 119, normalized size = 1.03

$$\frac{108025x^3+104430x^2+89144x+17210}{11628100x^4+13953720x^3+13488596x^2+5581488x+1860496}-\frac{\log\left(x^2-\frac{x}{2}+\frac{3}{2}\right)}{21296}+\frac{\log\left(x^2+\frac{3x}{5}+\frac{2}{5}\right)}{21296}+\frac{45\sqrt{23}\,\mathrm{atan}\left(\frac{4\sqrt{23}\,x}{23}-\frac{\sqrt{23}}{23}\right)}{244904}+\frac{847793\sqrt{31}\,\mathrm{atan}\left(\frac{10\sqrt{31}\,x}{31}+\frac{3\sqrt{31}}{31}\right)}{317214568}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**2-x+3)/(5*x**2+3*x+2)**3,x)
```

```
[Out] (108025*x**3 + 104430*x**2 + 89144*x + 17210)/(11628100*x**4 + 13953720*x**
3 + 13488596*x**2 + 5581488*x + 1860496) - log(x**2 - x/2 + 3/2)/21296 + lo
g(x**2 + 3*x/5 + 2/5)/21296 + 45*sqrt(23)*atan(4*sqrt(23)*x/23 - sqrt(23)/2
3)/244904 + 847793*sqrt(31)*atan(10*sqrt(31)*x/31 + 3*sqrt(31)/31)/31721456
8
```

3.41 $\int \frac{\left(2+3x+5x^2\right)^4}{\left(3-x+2x^2\right)^2} dx$

Optimal. Leaf size=91

$$\frac{125x^5}{4}+\frac{2125x^4}{16}+\frac{9775x^3}{48}-\frac{1185x^2}{8}-\frac{14641(79x+101)}{2944\left(2x^2-x+3\right)}-\frac{30613}{128}\log\left(2x^2-x+3\right)-\frac{89359x}{64}-\frac{13292697\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{1472\sqrt{23}}$$

Rubi [A] time = 0.09, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1660, 1657, 634, 618, 204, 628}

$$\frac{125x^5}{4}+\frac{2125x^4}{16}+\frac{9775x^3}{48}-\frac{1185x^2}{8}-\frac{14641(79x+101)}{2944\left(2x^2-x+3\right)}-\frac{30613}{128}\log\left(2x^2-x+3\right)-\frac{89359x}{64}-\frac{13292697\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{1472\sqrt{23}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + 5*x^2)^4/(3 - x + 2*x^2)^2,x]
```

```
[Out] (-89359*x)/64 - (1185*x^2)/8 + (9775*x^3)/48 + (2125*x^4)/16 + (125*x^5)/4
- (14641*(101 + 79*x))/(2944*(3 - x + 2*x^2)) - (13292697*ArcTan[(1 - 4*x)/
Sqrt[23]])/(1472*Sqrt[23]) - (30613*Log[3 - x + 2*x^2])/128
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{\left(2+3x+5x^2\right)^4}{\left(3-x+2x^2\right)^2}\,dx = -\frac{14641(101+79x)}{2944\left(3-x+2x^2\right)} + \frac{1}{23}\int \frac{\frac{832627}{64} - \frac{661181x}{64} - \frac{488267x^2}{32} + \frac{143635x^3}{16} + \frac{213325x^4}{8} + \frac{83375x^5}{4}}{3-x+2x^2}$$

$$= -\frac{14641(101+79x)}{2944\left(3-x+2x^2\right)} + \frac{1}{23}\int\left(-\frac{2055257}{64} - \frac{27255x}{4} + \frac{224825x^2}{16} + \frac{48875x^3}{4} + \frac{14375}{4}\right.$$

$$= -\frac{89359x}{64} - \frac{1185x^2}{8} + \frac{9775x^3}{48} + \frac{2125x^4}{16} + \frac{125x^5}{4} - \frac{14641(101+79x)}{2944\left(3-x+2x^2\right)} + \frac{1331}{736}\int \frac{262}{3-}$$

$$= -\frac{89359x}{64} - \frac{1185x^2}{8} + \frac{9775x^3}{48} + \frac{2125x^4}{16} + \frac{125x^5}{4} - \frac{14641(101+79x)}{2944\left(3-x+2x^2\right)} - \frac{30613}{128}\int \frac{}{3}$$

$$= -\frac{89359x}{64} - \frac{1185x^2}{8} + \frac{9775x^3}{48} + \frac{2125x^4}{16} + \frac{125x^5}{4} - \frac{14641(101+79x)}{2944\left(3-x+2x^2\right)} - \frac{30613}{128}\log\left(\right.$$

$$= -\frac{89359x}{64} - \frac{1185x^2}{8} + \frac{9775x^3}{48} + \frac{2125x^4}{16} + \frac{125x^5}{4} - \frac{14641(101+79x)}{2944\left(3-x+2x^2\right)} - \frac{13292697\ \mathrm{ta}}{1472}$$

Mathematica [A] time = 0.05, size = 91, normalized size = 1.00

$$\frac{125x^5}{4} + \frac{2125x^4}{16} + \frac{9775x^3}{48} - \frac{1185x^2}{8} - \frac{14641(79x+101)}{2944\left(2x^2-x+3\right)} - \frac{30613}{128}\log\left(2x^2-x+3\right) - \frac{89359x}{64} + \frac{13292697\tan^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{1472\sqrt{23}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)^4/(3 - x + 2*x^2)^2,x]
```

```
[Out] (-89359*x)/64 - (1185*x^2)/8 + (9775*x^3)/48 + (2125*x^4)/16 + (125*x^5)/4
- (14641*(101 + 79*x))/(2944*(3 - x + 2*x^2)) + (13292697*ArcTan[(-1 + 4*x)
/Sqrt[23]])/(1472*Sqrt[23]) - (30613*Log[3 - x + 2*x^2])/128
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2+3x+5x^2\right)^4}{\left(3-x+2x^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(2 + 3*x + 5*x^2)^4/(3 - x + 2*x^2)^2,x]
```

```
[Out] IntegrateAlgebraic[(2 + 3*x + 5*x^2)^4/(3 - x + 2*x^2)^2, x]
```

fricas [A] time = 0.39, size = 98, normalized size = 1.08

$$\frac{12696000\,x^{7}+47610000\,x^{6}+74800600\,x^{5}-20609840\,x^{4}-413058012\,x^{3}+79756182\,\sqrt{23}\left(2\,x^{2}-x+3\right)\arctan\left(\frac{1}{23}\,\sqrt{23}\left(4\,x-1\right)\right)+193356906\,x^{2}-48582831\left(2\,x^{2}-x+3\right)\log\left(2\,x^{2}-x+3\right)-930684489\,x-102033129}{203136\left(2\,x^{2}-x+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^2,x, algorithm="fricas")
```

```
[Out] 1/203136*(12696000*x^7 + 47610000*x^6 + 74800600*x^5 - 20609840*x^4 - 41305
8012*x^3 + 79756182*sqrt(23)*(2*x^2 - x + 3)*arctan(1/23*sqrt(23)*(4*x - 1)
) + 193356906*x^2 - 48582831*(2*x^2 - x + 3)*log(2*x^2 - x + 3) - 930684489
*x - 102033129)/(2*x^2 - x + 3)
```

giac [A] time = 0.18, size = 72, normalized size = 0.79

$$\frac{125}{4}\,x^{5}+\frac{2125}{16}\,x^{4}+\frac{9775}{48}\,x^{3}-\frac{1185}{8}\,x^{2}+\frac{13292697}{33856}\,\sqrt{23}\arctan\left(\frac{1}{23}\,\sqrt{23}\left(4\,x-1\right)\right)-\frac{89359}{64}\,x-\frac{14641\left(79\,x+101\right)}{2944\left(2\,x^{2}-x+3\right)}-\frac{30613}{128}\,\log\left(2\,x^{2}-x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^2,x, algorithm="giac")
```

```
[Out] 125/4*x^5 + 2125/16*x^4 + 9775/48*x^3 - 1185/8*x^2 + 13292697/33856*sqrt(23
)*arctan(1/23*sqrt(23)*(4*x - 1)) - 89359/64*x - 14641/2944*(79*x + 101)/(2
*x^2 - x + 3) - 30613/128*log(2*x^2 - x + 3)
```

maple [A] time = 0.01, size = 71, normalized size = 0.78

$$\frac{125x^{5}}{4}+\frac{2125x^{4}}{16}+\frac{9775x^{3}}{48}-\frac{1185x^{2}}{8}-\frac{89359x}{64}+\frac{13292697\sqrt{23}\,\arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{33856}-\frac{30613\ln\left(2x^{2}-x+3\right)}{128}-\frac{1331\left(\frac{869x}{92}+\frac{1111}{92}\right)}{64\left(x^{2}-\frac{1}{2}x+\frac{3}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)^4/(2*x^2-x+3)^2,x)
```

```
[Out] 125/4*x^5+2125/16*x^4+9775/48*x^3-1185/8*x^2-89359/64*x-1331/64*(869/92*x+1
111/92)/(x^2-1/2*x+3/2)-30613/128*ln(2*x^2-x+3)+13292697/33856*23^(1/2)*arc
tan(1/23*(4*x-1)*23^(1/2))
```

maxima [A] time = 0.96, size = 72, normalized size = 0.79

$$\frac{125}{4}\,x^{5}+\frac{2125}{16}\,x^{4}+\frac{9775}{48}\,x^{3}-\frac{1185}{8}\,x^{2}+\frac{13292697}{33856}\,\sqrt{23}\arctan\left(\frac{1}{23}\,\sqrt{23}\left(4\,x-1\right)\right)-\frac{89359}{64}\,x-\frac{14641\left(79\,x+101\right)}{2944\left(2\,x^{2}-x+3\right)}-\frac{30613}{128}\,\log\left(2\,x^{2}-x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^2,x, algorithm="maxima")
```

```
[Out] 125/4*x^5 + 2125/16*x^4 + 9775/48*x^3 - 1185/8*x^2 + 13292697/33856*sqrt(23
)*arctan(1/23*sqrt(23)*(4*x - 1)) - 89359/64*x - 14641/2944*(79*x + 101)/(2
*x^2 - x + 3) - 30613/128*log(2*x^2 - x + 3)
```

mupad [B] time = 3.46, size = 72, normalized size = 0.79

$$\frac{13292697\sqrt{23}\,\operatorname{atan}\left(\frac{4\sqrt{23}\,x}{23}-\frac{\sqrt{23}}{23}\right)}{33856}-\frac{30613\,\ln\left(2\,x^2-x+3\right)}{128}-\frac{\frac{1156639\,x}{5888}+\frac{1478741}{5888}}{x^2-\frac{x}{2}+\frac{3}{2}}-\frac{89359\,x}{64}-\frac{1185\,x^2}{8}+\frac{9775\,x^3}{48}+\frac{2125\,x^4}{16}+\frac{125\,x^5}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 5*x^2 + 2)^4/(2*x^2 - x + 3)^2,x)
```

```
[Out] (13292697*23^(1/2)*atan((4*23^(1/2)*x)/23 - 23^(1/2)/23))/33856 - (30613*lo
g(2*x^2 - x + 3))/128 - ((1156639*x)/5888 + 1478741/5888)/(x^2 - x/2 + 3/2)
 - (89359*x)/64 - (1185*x^2)/8 + (9775*x^3)/48 + (2125*x^4)/16 + (125*x^5)/
4
```

sympy [A] time = 0.20, size = 90, normalized size = 0.99

$$\frac{125x^5}{4}+\frac{2125x^4}{16}+\frac{9775x^3}{48}-\frac{1185x^2}{8}-\frac{89359x}{64}+\frac{-1156639x-1478741}{5888x^2-2944x+8832}-\frac{30613\log\left(x^2-\frac{x}{2}+\frac{3}{2}\right)}{128}+\frac{13292697\sqrt{23}\,\operatorname{atan}\left(\frac{4\sqrt{23}\,x}{23}-\frac{\sqrt{23}}{23}\right)}{33856}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)**4/(2*x**2-x+3)**2,x)
```

```
[Out] 125*x**5/4 + 2125*x**4/16 + 9775*x**3/48 - 1185*x**2/8 - 89359*x/64 + (-115
6639*x - 1478741)/(5888*x**2 - 2944*x + 8832) - 30613*log(x**2 - x/2 + 3/2)
/128 + 13292697*sqrt(23)*atan(4*sqrt(23)*x/23 - sqrt(23)/23)/33856
```

3.42 $\int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^2}\,dx$

Optimal. Leaf size=77

$$\frac{125x^3}{12}+\frac{175x^2}{4}-\frac{1331(17-45x)}{736\left(2x^2-x+3\right)}-\frac{2057}{32}\log\left(2x^2-x+3\right)+\frac{915x}{16}+\frac{223971\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{368\sqrt{23}}$$

Rubi [A] time = 0.07, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1660, 1657, 634, 618, 204, 628}

$$\frac{125x^3}{12}+\frac{175x^2}{4}-\frac{1331(17-45x)}{736\left(2x^2-x+3\right)}-\frac{2057}{32}\log\left(2x^2-x+3\right)+\frac{915x}{16}+\frac{223971\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{368\sqrt{23}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + 5*x^2)^3/(3 - x + 2*x^2)^2,x]
```

```
[Out] (915*x)/16 + (175*x^2)/4 + (125*x^3)/12 - (1331*(17 - 45*x))/(736*(3 - x +
2*x^2)) + (223971*ArcTan[(1 - 4*x)/Sqrt[23]])/(368*Sqrt[23]) - (2057*Log[3
- x + 2*x^2])/32
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{\left(2+3x+5x^2\right)^3}{\left(3-x+2x^2\right)^2}\,dx = -\frac{1331(17-45x)}{736\left(3-x+2x^2\right)}+\frac{1}{23}\int \frac{-\frac{25195}{16}-\frac{19067x}{16}+\frac{22195x^2}{8}+\frac{13225x^3}{4}+\frac{2875x^4}{2}}{3-x+2x^2}\,dx$$

$$= -\frac{1331(17-45x)}{736\left(3-x+2x^2\right)}+\frac{1}{23}\int \left(\frac{21045}{16}+\frac{4025x}{2}+\frac{2875x^2}{4}-\frac{121(365+391x)}{8\left(3-x+2x^2\right)}\right)dx$$

$$= \frac{915x}{16}+\frac{175x^2}{4}+\frac{125x^3}{12}-\frac{1331(17-45x)}{736\left(3-x+2x^2\right)}-\frac{121}{184}\int \frac{365+391x}{3-x+2x^2}\,dx$$

$$= \frac{915x}{16}+\frac{175x^2}{4}+\frac{125x^3}{12}-\frac{1331(17-45x)}{736\left(3-x+2x^2\right)}-\frac{2057}{32}\int \frac{-1+4x}{3-x+2x^2}\,dx-\frac{223971}{736}\int \frac{}{3}$$

$$= \frac{915x}{16}+\frac{175x^2}{4}+\frac{125x^3}{12}-\frac{1331(17-45x)}{736\left(3-x+2x^2\right)}-\frac{2057}{32}\log\left(3-x+2x^2\right)+\frac{223971}{368}\,\mathrm{Subs}$$

$$= \frac{915x}{16}+\frac{175x^2}{4}+\frac{125x^3}{12}-\frac{1331(17-45x)}{736\left(3-x+2x^2\right)}+\frac{223971\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{368\sqrt{23}}-\frac{2057}{32}\log\left(3-x\right.$$

Mathematica [A] time = 0.03, size = 77, normalized size = 1.00

$$\frac{125x^3}{12}+\frac{175x^2}{4}+\frac{1331(45x-17)}{736\left(2x^2-x+3\right)}-\frac{2057}{32}\log\left(2x^2-x+3\right)+\frac{915x}{16}-\frac{223971\tan^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{368\sqrt{23}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)^3/(3 - x + 2*x^2)^2,x]
```

```
[Out] (915*x)/16 + (175*x^2)/4 + (125*x^3)/12 + (1331*(-17 + 45*x))/(736*(3 - x +
 2*x^2)) - (223971*ArcTan[(-1 + 4*x)/Sqrt[23]])/(368*Sqrt[23]) - (2057*Log[
3 - x + 2*x^2])/32
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(2+3x+5x^2\right)^3}{\left(3-x+2x^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(2 + 3*x + 5*x^2)^3/(3 - x + 2*x^2)^2,x]
```

```
[Out] IntegrateAlgebraic[(2 + 3*x + 5*x^2)^3/(3 - x + 2*x^2)^2, x]
```

fricas [A] time = 0.43, size = 88, normalized size = 1.14

$$\frac{1058000\,x^5+3914600\,x^4+5173620\,x^3-1343826\,\sqrt{23}\left(2\,x^2-x+3\right)\arctan\left(\frac{1}{23}\,\sqrt{23}\,(4\,x-1)\right)+3761190\,x^2-3264459\left(2\,x^2-x+3\right)\log\left(2\,x^2-x+3\right)+12845385\,x-1561263}{50784\left(2\,x^2-x+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^2,x, algorithm="fricas")
```

```
[Out] 1/50784*(1058000*x^5 + 3914600*x^4 + 5173620*x^3 - 1343826*sqrt(23)*(2*x^2
- x + 3)*arctan(1/23*sqrt(23)*(4*x - 1)) + 3761190*x^2 - 3264459*(2*x^2 - x
 + 3)*log(2*x^2 - x + 3) + 12845385*x - 1561263)/(2*x^2 - x + 3)
```

giac [A] time = 0.19, size = 62, normalized size = 0.81

$$\frac{125}{12}\,x^3+\frac{175}{4}\,x^2-\frac{223971}{8464}\,\sqrt{23}\,\arctan\left(\frac{1}{23}\,\sqrt{23}\,(4\,x-1)\right)+\frac{915}{16}\,x+\frac{1331\,(45\,x-17)}{736\left(2\,x^2-x+3\right)}-\frac{2057}{32}\,\log\left(2\,x^2-x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^2,x, algorithm="giac")
```

```
[Out] 125/12*x^3 + 175/4*x^2 - 223971/8464*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1
)) + 915/16*x + 1331/736*(45*x - 17)/(2*x^2 - x + 3) - 2057/32*log(2*x^2 -
x + 3)
```

maple [A] time = 0.01, size = 61, normalized size = 0.79

$$\frac{125x^3}{12}+\frac{175x^2}{4}+\frac{915x}{16}-\frac{223971\sqrt{23}\ \arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{8464}-\frac{2057\ln\left(2x^2-x+3\right)}{32}-\frac{121\left(-\frac{495x}{92}+\frac{187}{92}\right)}{16\left(x^2-\frac{1}{2}x+\frac{3}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)^3/(2*x^2-x+3)^2,x)
```

```
[Out] 125/12*x^3+175/4*x^2+915/16*x-121/16*(-495/92*x+187/92)/(x^2-1/2*x+3/2)-205
7/32*ln(2*x^2-x+3)-223971/8464*23^(1/2)*arctan(1/23*(4*x-1)*23^(1/2))
```

maxima [A] time = 0.96, size = 62, normalized size = 0.81

$$\frac{125}{12}\,x^3+\frac{175}{4}\,x^2-\frac{223971}{8464}\,\sqrt{23}\,\arctan\left(\frac{1}{23}\,\sqrt{23}\,(4\,x-1)\right)+\frac{915}{16}\,x+\frac{1331\,(45\,x-17)}{736\,\left(2\,x^2-x+3\right)}-\frac{2057}{32}\,\log\left(2\,x^2-x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^2,x, algorithm="maxima")
```

```
[Out] 125/12*x^3 + 175/4*x^2 - 223971/8464*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1
)) + 915/16*x + 1331/736*(45*x - 17)/(2*x^2 - x + 3) - 2057/32*log(2*x^2 -
x + 3)
```

mupad [B] time = 3.42, size = 61, normalized size = 0.79

$$\frac{915\,x}{16}-\frac{2057\,\ln\left(2\,x^2-x+3\right)}{32}+\frac{\frac{59895\,x}{1472}-\frac{22627}{1472}}{x^2-\frac{x}{2}+\frac{3}{2}}-\frac{223971\,\sqrt{23}\,\mathrm{atan}\left(\frac{4\,\sqrt{23}\,x}{23}-\frac{\sqrt{23}}{23}\right)}{8464}+\frac{175\,x^2}{4}+\frac{125\,x^3}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 5*x^2 + 2)^3/(2*x^2 - x + 3)^2,x)
```

```
[Out] (915*x)/16 - (2057*log(2*x^2 - x + 3))/32 + ((59895*x)/1472 - 22627/1472)/(
x^2 - x/2 + 3/2) - (223971*23^(1/2)*atan((4*23^(1/2)*x)/23 - 23^(1/2)/23))/
8464 + (175*x^2)/4 + (125*x^3)/12
```

sympy [A] time = 0.19, size = 75, normalized size = 0.97

$$\frac{125x^3}{12}+\frac{175x^2}{4}+\frac{915x}{16}+\frac{59895x-22627}{1472x^2-736x+2208}-\frac{2057\log\left(x^2-\frac{x}{2}+\frac{3}{2}\right)}{32}-\frac{223971\sqrt{23}\ \mathrm{atan}\left(\frac{4\sqrt{23}\,x}{23}-\frac{\sqrt{23}}{23}\right)}{8464}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)**3/(2*x**2-x+3)**2,x)
```

```
[Out] 125*x**3/12 + 175*x**2/4 + 915*x/16 + (59895*x - 22627)/(1472*x**2 - 736*x
+ 2208) - 2057*log(x**2 - x/2 + 3/2)/32 - 223971*sqrt(23)*atan(4*sqrt(23)*x
/23 - sqrt(23)/23)/8464
```

3.43 $\int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^2} dx$

Optimal. Leaf size=63

$$\frac{121(19-7x)}{184(2x^2-x+3)}+\frac{55}{8}\log(2x^2-x+3)+\frac{25x}{4}+\frac{1859\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{92\sqrt{23}}$$

Rubi [A] time = 0.06, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1660, 1657, 634, 618, 204, 628}

$$\frac{121(19-7x)}{184(2x^2-x+3)}+\frac{55}{8}\log(2x^2-x+3)+\frac{25x}{4}+\frac{1859\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{92\sqrt{23}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + 5*x^2)^2/(3 - x + 2*x^2)^2,x]
```

```
[Out] (25*x)/4 + (121*(19 - 7*x))/(184*(3 - x + 2*x^2)) + (1859*ArcTan[(1 - 4*x)/
Sqrt[23]])/(92*Sqrt[23]) + (55*Log[3 - x + 2*x^2])/8
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(2+3x+5x^2\right)^2}{\left(3-x+2x^2\right)^2}\,dx &= \frac{121(19-7x)}{184\left(3-x+2x^2\right)}+\frac{1}{23}\int \frac{\frac{163}{4}+\frac{1955x}{4}+\frac{575x^2}{2}}{3-x+2x^2}\,dx \\
&= \frac{121(19-7x)}{184\left(3-x+2x^2\right)}+\frac{1}{23}\int \left(\frac{575}{4}-\frac{11(71-115x)}{2\left(3-x+2x^2\right)}\right)dx \\
&= \frac{25x}{4}+\frac{121(19-7x)}{184\left(3-x+2x^2\right)}-\frac{11}{46}\int \frac{71-115x}{3-x+2x^2}\,dx \\
&= \frac{25x}{4}+\frac{121(19-7x)}{184\left(3-x+2x^2\right)}+\frac{55}{8}\int \frac{-1+4x}{3-x+2x^2}\,dx-\frac{1859}{184}\int \frac{1}{3-x+2x^2}\,dx \\
&= \frac{25x}{4}+\frac{121(19-7x)}{184\left(3-x+2x^2\right)}+\frac{55}{8}\log\left(3-x+2x^2\right)+\frac{1859}{92}\text{Subst}\left(\int \frac{1}{-23-x^2}\,dx,x,-1+\right. \\
&= \frac{25x}{4}+\frac{121(19-7x)}{184\left(3-x+2x^2\right)}+\frac{1859\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{92\sqrt{23}}+\frac{55}{8}\log\left(3-x+2x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 63, normalized size = 1.00

$$-\frac{121(7x-19)}{184\left(2x^2-x+3\right)}+\frac{55}{8}\log\left(2x^2-x+3\right)+\frac{25x}{4}-\frac{1859\tan^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{92\sqrt{23}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)^2/(3 - x + 2*x^2)^2,x]
```

```
[Out] (25*x)/4 - (121*(-19 + 7*x))/(184*(3 - x + 2*x^2)) - (1859*ArcTan[(-1 + 4*x
)/Sqrt[23]])/(92*Sqrt[23]) + (55*Log[3 - x + 2*x^2])/8
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(2+3x+5x^2\right)^2}{\left(3-x+2x^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(2 + 3*x + 5*x^2)^2/(3 - x + 2*x^2)^2,x]
```

```
[Out] IntegrateAlgebraic[(2 + 3*x + 5*x^2)^2/(3 - x + 2*x^2)^2, x]
```

fricas [A] time = 0.41, size = 78, normalized size = 1.24

$$\frac{52900\,x^3-3718\,\sqrt{23}\left(2\,x^2-x+3\right)\arctan\left(\frac{1}{23}\,\sqrt{23}\,(4\,x-1)\right)-26450\,x^2+29095\left(2\,x^2-x+3\right)\log\left(2\,x^2-x+3\right)+59869\,x+52877}{4232\left(2\,x^2-x+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^2,x, algorithm="fricas")
```

```
[Out] 1/4232*(52900*x^3 - 3718*sqrt(23)*(2*x^2 - x + 3)*arctan(1/23*sqrt(23)*(4*x
 - 1)) - 26450*x^2 + 29095*(2*x^2 - x + 3)*log(2*x^2 - x + 3) + 59869*x + 5
2877)/(2*x^2 - x + 3)
```

giac [A] time = 0.17, size = 52, normalized size = 0.83

$$-\frac{1859}{2116}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}\,(4\,x-1)\right)+\frac{25}{4}\,x-\frac{121\,(7\,x-19)}{184\left(2\,x^2-x+3\right)}+\frac{55}{8}\log\left(2\,x^2-x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^2,x, algorithm="giac")
```

```
[Out] -1859/2116*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 25/4*x - 121/184*(7*x
 - 19)/(2*x^2 - x + 3) + 55/8*log(2*x^2 - x + 3)
```

maple [A] time = 0.01, size = 51, normalized size = 0.81

$$\frac{25x}{4}-\frac{1859\sqrt{23}\,\arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{2116}+\frac{55\ln\left(2x^2-x+3\right)}{8}+\frac{-\frac{847x}{368}+\frac{2299}{368}}{x^2-\frac{1}{2}x+\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)^2/(2*x^2-x+3)^2,x)
```

```
[Out] 25/4*x+11/4*(-77/92*x+209/92)/(x^2-1/2*x+3/2)+55/8*ln(2*x^2-x+3)-1859/2116*
23^(1/2)*arctan(1/23*(4*x-1)*23^(1/2))
```

maxima [A] time = 0.97, size = 52, normalized size = 0.83

$$-\frac{1859}{2116}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right)+\frac{25}{4}x-\frac{121(7x-19)}{184\left(2x^2-x+3\right)}+\frac{55}{8}\log\left(2x^2-x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^2,x, algorithm="maxima")
```

```
[Out] -1859/2116*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 25/4*x - 121/184*(7*x
 - 19)/(2*x^2 - x + 3) + 55/8*log(2*x^2 - x + 3)
```

mupad [B] time = 3.40, size = 52, normalized size = 0.83

$$\frac{25x}{4}+\frac{55\ln\left(2x^2-x+3\right)}{8}-\frac{\frac{847x}{368}-\frac{2299}{368}}{x^2-\frac{x}{2}+\frac{3}{2}}-\frac{1859\sqrt{23}\,\mathrm{atan}\left(\frac{4\sqrt{23}\,x}{23}-\frac{\sqrt{23}}{23}\right)}{2116}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 5*x^2 + 2)^2/(2*x^2 - x + 3)^2,x)
```

```
[Out] (25*x)/4 + (55*log(2*x^2 - x + 3))/8 - ((847*x)/368 - 2299/368)/(x^2 - x/2
+ 3/2) - (1859*23^(1/2)*atan((4*23^(1/2)*x)/23 - 23^(1/2)/23))/2116
```

sympy [A] time = 0.19, size = 61, normalized size = 0.97

$$\frac{25x}{4}+\frac{2299-847x}{368x^2-184x+552}+\frac{55\log\left(x^2-\frac{x}{2}+\frac{3}{2}\right)}{8}-\frac{1859\sqrt{23}\,\mathrm{atan}\left(\frac{4\sqrt{23}\,x}{23}-\frac{\sqrt{23}}{23}\right)}{2116}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)**2/(2*x**2-x+3)**2,x)
```

```
[Out] 25*x/4 + (2299 - 847*x)/(368*x**2 - 184*x + 552) + 55*log(x**2 - x/2 + 3/2)
/8 - 1859*sqrt(23)*atan(4*sqrt(23)*x/23 - sqrt(23)/23)/2116
```

3.44 $\int \frac{2+3x+5x^2}{\left(3-x+2x^2\right)^2} \, dx$

Optimal. Leaf size=43

$$-\frac{11(3x+5)}{46\left(2x^2-x+3\right)}-\frac{82\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{23\sqrt{23}}$$

Rubi [A] time = 0.03, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {1660, 12, 618, 204}

$$-\frac{11(3x+5)}{46\left(2x^2-x+3\right)}-\frac{82\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{23\sqrt{23}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + 5*x^2)/(3 - x + 2*x^2)^2,x]
```

```
[Out] (-11*(5 + 3*x))/(46*(3 - x + 2*x^2)) - (82*ArcTan[(1 - 4*x)/Sqrt[23]])/(23*
Sqrt[23])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
```

```
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{2+3x+5x^2}{\left(3-x+2x^2\right)^2}\,dx &= -\frac{11(5+3x)}{46\left(3-x+2x^2\right)}+\frac{1}{23}\int \frac{41}{3-x+2x^2}\,dx \\
&= -\frac{11(5+3x)}{46\left(3-x+2x^2\right)}+\frac{41}{23}\int \frac{1}{3-x+2x^2}\,dx \\
&= -\frac{11(5+3x)}{46\left(3-x+2x^2\right)}-\frac{82}{23}\text{Subst}\left(\int \frac{1}{-23-x^2}\,dx,x,-1+4x\right) \\
&= -\frac{11(5+3x)}{46\left(3-x+2x^2\right)}-\frac{82\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{23\sqrt{23}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 43, normalized size = 1.00

$$\frac{82\tan^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{23\sqrt{23}}-\frac{11(3x+5)}{46\left(2x^2-x+3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)/(3 - x + 2*x^2)^2,x]
```

```
[Out] (-11*(5 + 3*x))/(46*(3 - x + 2*x^2)) + (82*ArcTan[(-1 + 4*x)/Sqrt[23]])/(23
*Sqrt[23])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2+3x+5x^2}{\left(3-x+2x^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(2 + 3*x + 5*x^2)/(3 - x + 2*x^2)^2,x]
```

```
[Out] IntegrateAlgebraic[(2 + 3*x + 5*x^2)/(3 - x + 2*x^2)^2, x]
```

fricas [A] time = 0.40, size = 45, normalized size = 1.05

$$\frac{164\sqrt{23}\left(2x^2-x+3\right)\arctan\left(\frac{1}{23}\sqrt{23}\,(4x-1)\right)-759x-1265}{1058\left(2x^2-x+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)/(2*x^2-x+3)^2,x, algorithm="fricas")
```

```
[Out] 1/1058*(164*sqrt(23)*(2*x^2 - x + 3)*arctan(1/23*sqrt(23)*(4*x - 1)) - 759*
x - 1265)/(2*x^2 - x + 3)
```

giac [A] time = 0.19, size = 36, normalized size = 0.84

$$\frac{82}{529}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}\,(4x-1)\right)-\frac{11\,(3x+5)}{46\left(2x^2-x+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)/(2*x^2-x+3)^2,x, algorithm="giac")
```

```
[Out] 82/529*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) - 11/46*(3*x + 5)/(2*x^2 -
x + 3)
```

maple [A] time = 0.00, size = 34, normalized size = 0.79

$$\frac{82\sqrt{23}\arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{529}+\frac{-\frac{33x}{92}-\frac{55}{92}}{x^2-\frac{1}{2}x+\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)/(2*x^2-x+3)^2,x)
```

```
[Out] (-33/92*x-55/92)/(x^2-1/2*x+3/2)+82/529*23^(1/2)*arctan(1/23*(4*x-1)*23^(1/
2))
```

maxima [A] time = 0.96, size = 36, normalized size = 0.84

$$\frac{82}{529}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}\,(4x-1)\right)-\frac{11\,(3x+5)}{46\left(2x^2-x+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)/(2*x^2-x+3)^2,x, algorithm="maxima")
```

```
[Out] 82/529*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) - 11/46*(3*x + 5)/(2*x^2 -
x + 3)
```

mupad [B] time = 0.04, size = 36, normalized size = 0.84

$$\frac{82\sqrt{23}\,\mathrm{atan}\left(\frac{4\sqrt{23}\,x}{23}-\frac{\sqrt{23}}{23}\right)}{529}-\frac{\frac{33\,x}{92}+\frac{55}{92}}{x^2-\frac{x}{2}+\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 5*x^2 + 2)/(2*x^2 - x + 3)^2,x)
```

```
[Out] (82*23^(1/2)*atan((4*23^(1/2)*x)/23 - 23^(1/2)/23))/529 - ((33*x)/92 + 55/9
2)/(x^2 - x/2 + 3/2)
```

sympy [A] time = 0.15, size = 42, normalized size = 0.98

$$\frac{-33x-55}{92x^2-46x+138}+\frac{82\sqrt{23}\,\mathrm{atan}\left(\frac{4\sqrt{23}\,x}{23}-\frac{\sqrt{23}}{23}\right)}{529}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)/(2*x**2-x+3)**2,x)
```

```
[Out] (-33*x - 55)/(92*x**2 - 46*x + 138) + 82*sqrt(23)*atan(4*sqrt(23)*x/23 - sq
rt(23)/23)/529
```

3.45 $\int \frac{1}{(3-x+2x^2)^2 (2+3x+5x^2)}\,dx$

Optimal. Leaf size=94

$$\frac{13-6x}{506\left(2x^2-x+3\right)}-\frac{13}{968}\log\left(2x^2-x+3\right)+\frac{13}{968}\log\left(5x^2+3x+2\right)+\frac{241\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{11132\sqrt{23}}+\frac{69\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{484\sqrt{31}}$$

Rubi [A] time = 0.09, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {974, 1072, 634, 618, 204, 628}

$$\frac{13-6x}{506\left(2x^2-x+3\right)}-\frac{13}{968}\log\left(2x^2-x+3\right)+\frac{13}{968}\log\left(5x^2+3x+2\right)+\frac{241\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{11132\sqrt{23}}+\frac{69\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{484\sqrt{31}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)),x]
```

```
[Out] (13 - 6*x)/(506*(3 - x + 2*x^2)) + (241*ArcTan[(1 - 4*x)/Sqrt[23]])/(11132*
Sqrt[23]) + (69*ArcTan[(3 + 10*x)/Sqrt[31]])/(484*Sqrt[31]) - (13*Log[3 - x
 + 2*x^2])/968 + (13*Log[2 + 3*x + 5*x^2])/968
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 974

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*
a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*(
d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q,
 0]
```

Rule 1072

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)), x_Symbol] :> With[{q = c^2*d^2 - b*c*
d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*e*f + a^2*f^2}, Dist[1/q, Int[(A*
c^2*d - a*c*C*d - A*b*c*e + a*B*c*e + A*b^2*f - a*b*B*f - a*A*c*f + a^2*C*f
 + c*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B*f)*x)/(a + b*x + c*x^2),
x], x] + Dist[1/q, Int[(c*C*d^2 - B*c*d*e + A*c*e^2 + b*B*d*f - A*c*d*f - a
*C*d*f - A*b*e*f + a*A*f^2 - f*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B
*f)*x)/(d + e*x + f*x^2), x], x] /; NeQ[q, 0]] /; FreeQ[{a, b, c, d, e, f,
A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$\int \frac{1}{\left(3-x+2x^2\right)^2\left(2+3x+5x^2\right)}\,dx = \frac{13-6x}{506\left(3-x+2x^2\right)} - \frac{\int \frac{-1892-1067x+330x^2}{\left(3-x+2x^2\right)\left(2+3x+5x^2\right)}\,dx}{5566}$$

$$= \frac{13-6x}{506\left(3-x+2x^2\right)} - \frac{\int \frac{-3509+72358x}{3-x+2x^2}\,dx}{1346972} - \frac{\int \frac{-150282-180895x}{2+3x+5x^2}\,dx}{1346972}$$

$$= \frac{13-6x}{506\left(3-x+2x^2\right)} - \frac{241\int \frac{1}{3-x+2x^2}\,dx}{22264} - \frac{13}{968}\int \frac{-1+4x}{3-x+2x^2}\,dx + \frac{13}{968}\int$$

$$= \frac{13-6x}{506\left(3-x+2x^2\right)} - \frac{13}{968}\log\left(3-x+2x^2\right) + \frac{13}{968}\log\left(2+3x+5x^2\right) +$$

$$= \frac{13-6x}{506\left(3-x+2x^2\right)} + \frac{241\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{11132\sqrt{23}} + \frac{69\tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)}{484\sqrt{31}} - \frac{13}{968}\log\left(3-\right.$$

Mathematica [A] time = 0.06, size = 94, normalized size = 1.00

$$\frac{13-6x}{506\left(2x^2-x+3\right)} - \frac{13}{968}\log\left(2x^2-x+3\right) + \frac{13}{968}\log\left(5x^2+3x+2\right) - \frac{241\tan^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{11132\sqrt{23}} + \frac{69\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{484\sqrt{31}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)),x]
```

```
[Out] (13 - 6*x)/(506*(3 - x + 2*x^2)) - (241*ArcTan[(-1 + 4*x)/Sqrt[23]])/(11132
*Sqrt[23]) + (69*ArcTan[(3 + 10*x)/Sqrt[31]])/(484*Sqrt[31]) - (13*Log[3 -
x + 2*x^2])/968 + (13*Log[2 + 3*x + 5*x^2])/968
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(3-x+2x^2\right)^2\left(2+3x+5x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)),x]
```

```
[Out] IntegrateAlgebraic[1/((3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)), x]
```

fricas [A] time = 0.41, size = 117, normalized size = 1.24

$$\frac{73002\,\sqrt{31}\,\left(2\,x^2-x+3\right)\arctan\left(\frac{1}{31}\,\sqrt{31}\,(10\,x+3)\right)-14942\,\sqrt{23}\,\left(2\,x^2-x+3\right)\arctan\left(\frac{1}{23}\,\sqrt{23}\,(4\,x-1)\right)+213187\,\left(2\,x^2-x+3\right)\log\left(5\,x^2+3\,x+2\right)-213187\,\left(2\,x^2-x+3\right)\log\left(2\,x^2-x+3\right)-188232\,x+407836}{15874232\,\left(2\,x^2-x+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^2/(5*x^2+3*x+2),x, algorithm="fricas")
```

```
[Out] 1/15874232*(73002*sqrt(31)*(2*x^2 - x + 3)*arctan(1/31*sqrt(31)*(10*x + 3))
 - 14942*sqrt(23)*(2*x^2 - x + 3)*arctan(1/23*sqrt(23)*(4*x - 1)) + 213187*
(2*x^2 - x + 3)*log(5*x^2 + 3*x + 2) - 213187*(2*x^2 - x + 3)*log(2*x^2 - x
 + 3) - 188232*x + 407836)/(2*x^2 - x + 3)
```

giac [A] time = 0.19, size = 78, normalized size = 0.83

$$\frac{69}{15004}\,\sqrt{31}\,\arctan\left(\frac{1}{31}\,\sqrt{31}\,(10\,x+3)\right)-\frac{241}{256036}\,\sqrt{23}\,\arctan\left(\frac{1}{23}\,\sqrt{23}\,(4\,x-1)\right)-\frac{6\,x-13}{506\,\left(2\,x^2-x+3\right)}+\frac{13}{968}\,\log\left(5\,x^2+3\,x+2\right)-\frac{13}{968}\,\log\left(2\,x^2-x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^2/(5*x^2+3*x+2),x, algorithm="giac")
```

```
[Out] 69/15004*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) - 241/256036*sqrt(23)*ar
ctan(1/23*sqrt(23)*(4*x - 1)) - 1/506*(6*x - 13)/(2*x^2 - x + 3) + 13/968*l
og(5*x^2 + 3*x + 2) - 13/968*log(2*x^2 - x + 3)
```

maple [A] time = 0.01, size = 77, normalized size = 0.82

$$\frac{69\sqrt{31}\,\arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)}{15004}-\frac{241\sqrt{23}\,\arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{256036}-\frac{13\ln\left(2x^2-x+3\right)}{968}+\frac{13\ln\left(5x^2+3x+2\right)}{968}-\frac{\frac{66x}{23}-\frac{143}{23}}{484\left(x^2-\frac{1}{2}x+\frac{3}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^2-x+3)^2/(5*x^2+3*x+2),x)
```

```
[Out] 13/968*ln(5*x^2+3*x+2)+69/15004*31^(1/2)*arctan(1/31*(10*x+3)*31^(1/2))-1/4
84*(66/23*x-143/23)/(x^2-1/2*x+3/2)-13/968*ln(2*x^2-x+3)-241/256036*23^(1/2
)*arctan(1/23*(4*x-1)*23^(1/2))
```

maxima [A] time = 0.96, size = 78, normalized size = 0.83

$$\frac{69}{15004}\,\sqrt{31}\,\arctan\left(\frac{1}{31}\,\sqrt{31}\,(10\,x+3)\right)-\frac{241}{256036}\,\sqrt{23}\,\arctan\left(\frac{1}{23}\,\sqrt{23}\,(4\,x-1)\right)-\frac{6\,x-13}{506\,\left(2\,x^2-x+3\right)}+\frac{13}{968}\,\log\left(5\,x^2+3\,x+2\right)-\frac{13}{968}\,\log\left(2\,x^2-x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^2/(5*x^2+3*x+2),x, algorithm="maxima")
```

```
[Out] 69/15004*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) - 241/256036*sqrt(23)*ar
ctan(1/23*sqrt(23)*(4*x - 1)) - 1/506*(6*x - 13)/(2*x^2 - x + 3) + 13/968*l
og(5*x^2 + 3*x + 2) - 13/968*log(2*x^2 - x + 3)
```

mupad [B] time = 3.58, size = 96, normalized size = 1.02

$$-\frac{\frac{3x}{506}-\frac{13}{1012}}{x^2-\frac{x}{2}+\frac{3}{2}}-\ln\left(x+\frac{3}{10}-\frac{\sqrt{31}\,1i}{10}\right)\left(-\frac{13}{968}+\frac{\sqrt{31}\,69i}{30008}\right)+\ln\left(x+\frac{3}{10}+\frac{\sqrt{31}\,1i}{10}\right)\left(\frac{13}{968}+\frac{\sqrt{31}\,69i}{30008}\right)+\ln\left(x-\frac{1}{4}-\frac{\sqrt{23}\,1i}{4}\right)\left(-\frac{13}{968}+\frac{\sqrt{23}\,241i}{512072}\right)-\ln\left(x-\frac{1}{4}+\frac{\sqrt{23}\,1i}{4}\right)\left(\frac{13}{968}+\frac{\sqrt{23}\,241i}{512072}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*x^2 - x + 3)^2*(3*x + 5*x^2 + 2)),x)
```

```
[Out] log(x + (31^(1/2)*1i)/10 + 3/10)*((31^(1/2)*69i)/30008 + 13/968) - log(x -
(31^(1/2)*1i)/10 + 3/10)*((31^(1/2)*69i)/30008 - 13/968) - ((3*x)/506 - 13/
1012)/(x^2 - x/2 + 3/2) + log(x - (23^(1/2)*1i)/4 - 1/4)*((23^(1/2)*241i)/5
12072 - 13/968) - log(x + (23^(1/2)*1i)/4 - 1/4)*((23^(1/2)*241i)/512072 +
13/968)
```

sympy [A] time = 0.32, size = 102, normalized size = 1.09

$$\frac{13-6x}{1012x^2-506x+1518}-\frac{13\log\left(x^2-\frac{x}{2}+\frac{3}{2}\right)}{968}+\frac{13\log\left(x^2+\frac{3x}{5}+\frac{2}{5}\right)}{968}-\frac{241\sqrt{23}\,\operatorname{atan}\left(\frac{4\sqrt{23}\,x}{23}-\frac{\sqrt{23}}{23}\right)}{256036}+\frac{69\sqrt{31}\,\operatorname{atan}\left(\frac{10\sqrt{31}\,x}{31}+\frac{3\sqrt{31}}{31}\right)}{15004}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**2-x+3)**2/(5*x**2+3*x+2),x)
```

```
[Out] (13 - 6*x)/(1012*x**2 - 506*x + 1518) - 13*log(x**2 - x/2 + 3/2)/968 + 13*l
og(x**2 + 3*x/5 + 2/5)/968 - 241*sqrt(23)*atan(4*sqrt(23)*x/23 - sqrt(23)/2
3)/256036 + 69*sqrt(31)*atan(10*sqrt(31)*x/31 + 3*sqrt(31)/31)/15004
```

3.46 $\int \frac{1}{(3-x+2x^2)^2 (2+3x+5x^2)^2} \, dx$

Optimal. Leaf size=127

$$-\frac{25(117-137x)}{172546(5x^2+3x+2)}+\frac{13-6x}{506(2x^2-x+3)(5x^2+3x+2)}+\frac{19\log(2x^2-x+3)}{10648}-\frac{19\log(5x^2+3x+2)}{10648}+\frac{2769\ldots}{12\ldots}$$

Rubi [A] time = 0.12, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {974, 1060, 1072, 634, 618, 204, 628}

$$-\frac{25(117-137x)}{172546(5x^2+3x+2)}+\frac{13-6x}{506(2x^2-x+3)(5x^2+3x+2)}+\frac{19\log(2x^2-x+3)}{10648}-\frac{19\log(5x^2+3x+2)}{10648}+\frac{2769\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{122452\sqrt{23}}+\frac{12643\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{165044\sqrt{31}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)^2),x]
```

```
[Out] (-25*(117 - 137*x))/(172546*(2 + 3*x + 5*x^2)) + (13 - 6*x)/(506*(3 - x + 2
*x^2)*(2 + 3*x + 5*x^2)) + (2769*ArcTan[(1 - 4*x)/Sqrt[23]])/(122452*Sqrt[2
3]) + (12643*ArcTan[(3 + 10*x)/Sqrt[31]])/(165044*Sqrt[31]) + (19*Log[3 - x
 + 2*x^2])/10648 - (19*Log[2 + 3*x + 5*x^2])/10648
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 974

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*
a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*(
d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q,
 0]
```

Rule 1060

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x +
 c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
2*a*(c*d - a*f)))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !
```

```
IGtQ[q, 0]
```

Rule 1072

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)), x_Symbol] :> With[{q = c^2*d^2 - b*c*
d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*e*f + a^2*f^2}, Dist[1/q, Int[(A*
c^2*d - a*c*C*d - A*b*c*e + a*B*c*e + A*b^2*f - a*b*B*f - a*A*c*f + a^2*C*f
 + c*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B*f)*x)/(a + b*x + c*x^2),
x], x] + Dist[1/q, Int[(c*C*d^2 - B*c*d*e + A*c*e^2 + b*B*d*f - A*c*d*f - a
*C*d*f - A*b*e*f + a*A*f^2 - f*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B
*f)*x)/(d + e*x + f*x^2), x], x] /; NeQ[q, 0]] /; FreeQ[{a, b, c, d, e, f,
A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$\int \frac{1}{\left(3-x+2x^2\right)^2\left(2+3x+5x^2\right)^2}\,dx = \frac{13-6x}{506\left(3-x+2x^2\right)\left(2+3x+5x^2\right)} - \frac{\int \frac{-2321-2299x+990x^2}{\left(3-x+2x^2\right)\left(2+3x+5x^2\right)^2}\,dx}{5566}$$

$$= -\frac{25(117-137x)}{172546\left(2+3x+5x^2\right)} + \frac{13-6x}{506\left(3-x+2x^2\right)\left(2+3x+5x^2\right)} - \cdots$$

$$= -\frac{25(117-137x)}{172546\left(2+3x+5x^2\right)} + \frac{13-6x}{506\left(3-x+2x^2\right)\left(2+3x+5x^2\right)} - \cdots$$

$$= -\frac{25(117-137x)}{172546\left(2+3x+5x^2\right)} + \frac{13-6x}{506\left(3-x+2x^2\right)\left(2+3x+5x^2\right)} + \cdots$$

$$= -\frac{25(117-137x)}{172546\left(2+3x+5x^2\right)} + \frac{13-6x}{506\left(3-x+2x^2\right)\left(2+3x+5x^2\right)} + \cdots$$

$$= -\frac{25(117-137x)}{172546\left(2+3x+5x^2\right)} + \frac{13-6x}{506\left(3-x+2x^2\right)\left(2+3x+5x^2\right)} + \cdots$$

Mathematica [A] time = 0.06, size = 106, normalized size = 0.83

$$\frac{9659011\log\left(2x^2-x+3\right)-9659011\log\left(5x^2+3x+2\right)+\frac{31372\left(6850x^3-9275x^2+11154x-4342\right)}{10x^4+x^3+16x^2+7x+6}-5322018\sqrt{23}\tan^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)+13376294\sqrt{31}\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{5413113112}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)^2),x]
```

```
[Out] ((31372*(-4342 + 11154*x - 9275*x^2 + 6850*x^3))/(6 + 7*x + 16*x^2 + x^3 +
10*x^4) - 5322018*Sqrt[23]*ArcTan[(-1 + 4*x)/Sqrt[23]] + 13376294*Sqrt[31]*
ArcTan[(3 + 10*x)/Sqrt[31]] + 9659011*Log[3 - x + 2*x^2] - 9659011*Log[2 +
3*x + 5*x^2])/5413113112
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(3-x+2x^2\right)^2\left(2+3x+5x^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)^2),x]
```

```
[Out] IntegrateAlgebraic[1/((3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)^2), x]
```

fricas [A] time = 0.43, size = 167, normalized size = 1.31

$$\frac{214898200\,x^3+13376294\,\sqrt{31}\left(10\,x^4+x^3+16\,x^2+7\,x+6\right)\arctan\left(\frac{1}{31}\,\sqrt{31}\left(10\,x+3\right)\right)-5322018\,\sqrt{23}\left(10\,x^4+x^3+16\,x^2+7\,x+6\right)\arctan\left(\frac{1}{23}\,\sqrt{23}\left(4\,x-1\right)\right)-290975300\,x^2-9659011\left(10\,x^4+x^3+16\,x^2+7\,x+6\right)\log\left(5\,x^2+3\,x+2\right)+9659011\left(10\,x^4+x^3+16\,x^2+7\,x+6\right)\log\left(2\,x^2-x+3\right)+349923288\,x-136217224}{5413113112\left(10\,x^4+x^3+16\,x^2+7\,x+6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^2/(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

```
[Out] 1/5413113112*(214898200*x^3 + 13376294*sqrt(31)*(10*x^4 + x^3 + 16*x^2 + 7*
x + 6)*arctan(1/31*sqrt(31)*(10*x + 3)) - 5322018*sqrt(23)*(10*x^4 + x^3 +
16*x^2 + 7*x + 6)*arctan(1/23*sqrt(23)*(4*x - 1)) - 290975300*x^2 - 9659011
*(10*x^4 + x^3 + 16*x^2 + 7*x + 6)*log(5*x^2 + 3*x + 2) + 9659011*(10*x^4 +
 x^3 + 16*x^2 + 7*x + 6)*log(2*x^2 - x + 3) + 349923288*x - 136217224)/(10*
x^4 + x^3 + 16*x^2 + 7*x + 6)
```

giac [A] time = 0.19, size = 96, normalized size = 0.76

$$\frac{12643}{5116364}\,\sqrt{31}\,\arctan\left(\frac{1}{31}\,\sqrt{31}\left(10\,x+3\right)\right)-\frac{2769}{2816396}\,\sqrt{23}\,\arctan\left(\frac{1}{23}\,\sqrt{23}\left(4\,x-1\right)\right)+\frac{6850\,x^3-9275\,x^2+11154\,x-4342}{172546\left(10\,x^4+x^3+16\,x^2+7\,x+6\right)}-\frac{19}{10648}\,\log\left(5\,x^2+3\,x+2\right)+\frac{19}{10648}\,\log\left(2\,x^2-x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^2/(5*x^2+3*x+2)^2,x, algorithm="giac")
```

```
[Out] 12643/5116364*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) - 2769/2816396*sqrt
(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 1/172546*(6850*x^3 - 9275*x^2 + 1115
4*x - 4342)/(10*x^4 + x^3 + 16*x^2 + 7*x + 6) - 19/10648*log(5*x^2 + 3*x +
2) + 19/10648*log(2*x^2 - x + 3)
```

maple [A] time = 0.01, size = 94, normalized size = 0.74

$$\frac{12643\sqrt{31}\arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)}{5116364}-\frac{2769\sqrt{23}\arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{2816396}+\frac{19\ln\left(2x^{2}-x+3\right)}{10648}-\frac{19\ln\left(5x^{2}+3x+2\right)}{10648}-\frac{-\frac{759x}{31}+\frac{1078}{155}}{5324\left(x^{2}+\frac{3}{5}x+\frac{2}{5}\right)}+\frac{-\frac{77x}{23}-\frac{341}{46}}{5324x^{2}-2662x+7986}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^2-x+3)^2/(5*x^2+3*x+2)^2,x)
```

```
[Out] -1/5324*(-759/31*x+1078/155)/(x^2+3/5*x+2/5)-19/10648*ln(5*x^2+3*x+2)+12643
/5116364*31^(1/2)*arctan(1/31*(10*x+3)*31^(1/2))+1/5324*(-77/23*x-341/46)/(
x^2-1/2*x+3/2)+19/10648*ln(2*x^2-x+3)-2769/2816396*23^(1/2)*arctan(1/23*(4*
x-1)*23^(1/2))
```

maxima [A] time = 0.96, size = 96, normalized size = 0.76

$$\frac{12643}{5116364}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}\,(10\,x+3)\right)-\frac{2769}{2816396}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}\,(4\,x-1)\right)+\frac{6850\,x^{3}-9275\,x^{2}+11154\,x-4342}{172546\,\left(10\,x^{4}+x^{3}+16\,x^{2}+7\,x+6\right)}-\frac{19}{10648}\log\left(5\,x^{2}+3\,x+2\right)+\frac{19}{10648}\log\left(2\,x^{2}-x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^2/(5*x^2+3*x+2)^2,x, algorithm="maxima")
```

```
[Out] 12643/5116364*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) - 2769/2816396*sqrt
(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 1/172546*(6850*x^3 - 9275*x^2 + 1115
4*x - 4342)/(10*x^4 + x^3 + 16*x^2 + 7*x + 6) - 19/10648*log(5*x^2 + 3*x +
2) + 19/10648*log(2*x^2 - x + 3)
```

mupad [B] time = 0.18, size = 115, normalized size = 0.91

$$\ln\left(x-\frac{1}{4}-\frac{\sqrt{23}\,1\mathrm{i}}{4}\right)\left(\frac{19}{10648}+\frac{\sqrt{23}\,2769\mathrm{i}}{5632792}\right)-\ln\left(x-\frac{1}{4}+\frac{\sqrt{23}\,1\mathrm{i}}{4}\right)\left(-\frac{19}{10648}+\frac{\sqrt{23}\,2769\mathrm{i}}{5632792}\right)-\ln\left(x+\frac{3}{10}-\frac{\sqrt{31}\,1\mathrm{i}}{10}\right)\left(\frac{19}{10648}+\frac{\sqrt{31}\,12643\mathrm{i}}{10232728}\right)+\ln\left(x+\frac{3}{10}+\frac{\sqrt{31}\,1\mathrm{i}}{10}\right)\left(-\frac{19}{10648}+\frac{\sqrt{31}\,12643\mathrm{i}}{10232728}\right)+\frac{\frac{685\,x^{3}}{172546}-\frac{1855\,x^{2}}{345092}+\frac{507\,x}{78430}-\frac{2171}{862730}}{x^{4}+\frac{x^{3}}{10}+\frac{8\,x^{2}}{5}+\frac{7\,x}{10}+\frac{3}{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*x^2 - x + 3)^2*(3*x + 5*x^2 + 2)^2),x)
```

```
[Out] log(x - (23^(1/2)*1i)/4 - 1/4)*((23^(1/2)*2769i)/5632792 + 19/10648) - log(
x + (23^(1/2)*1i)/4 - 1/4)*((23^(1/2)*2769i)/5632792 - 19/10648) - log(x -
(31^(1/2)*1i)/10 + 3/10)*((31^(1/2)*12643i)/10232728 + 19/10648) + log(x +
(31^(1/2)*1i)/10 + 3/10)*((31^(1/2)*12643i)/10232728 - 19/10648) + ((507*x)
/78430 - (1855*x^2)/345092 + (685*x^3)/172546 - 2171/862730)/((7*x)/10 + (8
*x^2)/5 + x^3/10 + x^4 + 3/5)
```

sympy [A] time = 0.36, size = 122, normalized size = 0.96

$$\frac{6850x^{3}-9275x^{2}+11154x-4342}{1725460x^{4}+172546x^{3}+2760736x^{2}+1207822x+1035276}+\frac{19\log\left(x^{2}-\frac{x}{2}+\frac{3}{2}\right)}{10648}-\frac{19\log\left(x^{2}+\frac{3x}{5}+\frac{2}{5}\right)}{10648}-\frac{2769\sqrt{23}\operatorname{atan}\left(\frac{4\sqrt{23}x}{23}-\frac{\sqrt{23}}{23}\right)}{2816396}+\frac{12643\sqrt{31}\operatorname{atan}\left(\frac{10\sqrt{31}x}{31}+\frac{3\sqrt{31}}{31}\right)}{5116364}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**2-x+3)**2/(5*x**2+3*x+2)**2,x)
```

```
[Out] (6850*x**3 - 9275*x**2 + 11154*x - 4342)/(1725460*x**4 + 172546*x**3 + 2760
736*x**2 + 1207822*x + 1035276) + 19*log(x**2 - x/2 + 3/2)/10648 - 19*log(x
**2 + 3*x/5 + 2/5)/10648 - 2769*sqrt(23)*atan(4*sqrt(23)*x/23 - sqrt(23)/23
)/2816396 + 12643*sqrt(31)*atan(10*sqrt(31)*x/31 + 3*sqrt(31)/31)/5116364
```

3.47 $\int \frac{1}{\left(3-x+2x^2\right)^2\left(2+3x+5x^2\right)^3}\,dx$

Optimal. Leaf size=148

$$\frac{13-6x}{506\left(2x^2-x+3\right)\left(5x^2+3x+2\right)^2}+\frac{3996965x+1765599}{235352744\left(5x^2+3x+2\right)}+\frac{5765x-9446}{690184\left(5x^2+3x+2\right)^2}+\frac{97\log\left(2x^2-x+3\right)}{468512}$$

Rubi [A] time = 0.16, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {974, 1060, 1072, 634, 618, 204, 628}

$$-\frac{9446-5765x}{690184\left(5x^2+3x+2\right)^2}+\frac{3996965x+1765599}{235352744\left(5x^2+3x+2\right)}+\frac{13-6x}{506\left(2x^2-x+3\right)\left(5x^2+3x+2\right)^2}+\frac{97\log\left(2x^2-x+3\right)}{468512}-\frac{97\log\left(5x^2+3x+2\right)}{468512}-\frac{25557\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{5387888\sqrt{23}}+\frac{4464079\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{225120016\sqrt{31}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)^3),x]
```

```
[Out] -(9446 - 5765*x)/(690184*(2 + 3*x + 5*x^2)^2) + (13 - 6*x)/(506*(3 - x + 2*
x^2)*(2 + 3*x + 5*x^2)^2) + (1765599 + 3996965*x)/(235352744*(2 + 3*x + 5*x
^2)) - (25557*ArcTan[(1 - 4*x)/Sqrt[23]])/(5387888*Sqrt[23]) + (4464079*Arc
Tan[(3 + 10*x)/Sqrt[31]])/(225120016*Sqrt[31]) + (97*Log[3 - x + 2*x^2])/46
8512 - (97*Log[2 + 3*x + 5*x^2])/468512
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 974

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*
a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*(
d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q,
 0]
```

Rule 1060

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x +
 c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
2*a*(c*d - a*f)))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !
```

```
IGtQ[q, 0]
```

Rule 1072

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)), x_Symbol] :> With[{q = c^2*d^2 - b*c*
d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*e*f + a^2*f^2}, Dist[1/q, Int[(A*
c^2*d - a*c*C*d - A*b*c*e + a*B*c*e + A*b^2*f - a*b*B*f - a*A*c*f + a^2*C*f
 + c*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B*f)*x)/(a + b*x + c*x^2),
x], x] + Dist[1/q, Int[(c*C*d^2 - B*c*d*e + A*c*e^2 + b*B*d*f - A*c*d*f - a
*C*d*f - A*b*e*f + a*A*f^2 - f*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B
*f)*x)/(d + e*x + f*x^2), x], x] /; NeQ[q, 0]] /; FreeQ[{a, b, c, d, e, f,
A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$\int \frac{1}{\left(3-x+2x^2\right)^2\left(2+3x+5x^2\right)^3}\,dx = \frac{13-6x}{506\left(3-x+2x^2\right)\left(2+3x+5x^2\right)^2} - \frac{\int \frac{-2750-3531x+1650x^2}{\left(3-x+2x^2\right)\left(2+3x+5x^2\right)^3}\,dx}{5566}$$

$$= -\frac{9446-5765x}{690184\left(2+3x+5x^2\right)^2} + \frac{13-6x}{506\left(3-x+2x^2\right)\left(2+3x+5x^2\right)^2} - \frac{\int \frac{-825}{}}{}$$

$$= -\frac{9446-5765x}{690184\left(2+3x+5x^2\right)^2} + \frac{13-6x}{506\left(3-x+2x^2\right)\left(2+3x+5x^2\right)^2} + \frac{176}{23535}$$

$$= -\frac{9446-5765x}{690184\left(2+3x+5x^2\right)^2} + \frac{13-6x}{506\left(3-x+2x^2\right)\left(2+3x+5x^2\right)^2} + \frac{176}{23535}$$

$$= -\frac{9446-5765x}{690184\left(2+3x+5x^2\right)^2} + \frac{13-6x}{506\left(3-x+2x^2\right)\left(2+3x+5x^2\right)^2} + \frac{176}{23535}$$

$$= -\frac{9446-5765x}{690184\left(2+3x+5x^2\right)^2} + \frac{13-6x}{506\left(3-x+2x^2\right)\left(2+3x+5x^2\right)^2} + \frac{176}{23535}$$

$$= -\frac{9446-5765x}{690184\left(2+3x+5x^2\right)^2} + \frac{13-6x}{506\left(3-x+2x^2\right)\left(2+3x+5x^2\right)^2} + \frac{176}{23535}$$

Mathematica [A] time = 0.08, size = 136, normalized size = 0.92

$$\frac{90x-11}{244904\left(2x^2-x+3\right)}+\frac{164380x+67573}{10232728\left(5x^2+3x+2\right)}+\frac{345x-98}{30008\left(5x^2+3x+2\right)^2}+\frac{97\log\left(2x^2-x+3\right)}{468512}-\frac{97\log\left(5x^2+3x+2\right)}{468512}+\frac{25557\tan^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{5387888\sqrt{23}}+\frac{4464079\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{225120016\sqrt{31}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)^3),x]
```

```
[Out] (-11 + 90*x)/(244904*(3 - x + 2*x^2)) + (-98 + 345*x)/(30008*(2 + 3*x + 5*x
^2)^2) + (67573 + 164380*x)/(10232728*(2 + 3*x + 5*x^2)) + (25557*ArcTan[(-
1 + 4*x)/Sqrt[23]])/(5387888*Sqrt[23]) + (4464079*ArcTan[(3 + 10*x)/Sqrt[31
]])/(225120016*Sqrt[31]) + (97*Log[3 - x + 2*x^2])/468512 - (97*Log[2 + 3*x
 + 5*x^2])/468512
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(3-x+2x^2\right)^2\left(2+3x+5x^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)^3),x]
```

```
[Out] IntegrateAlgebraic[1/((3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)^3), x]
```

fricas [A] time = 0.41, size = 237, normalized size = 1.60

$$\frac{1253927859800\,x^5+679296504260\,x^4+2185021181068\,x^3+4722995582\,\sqrt{31}\left(50\,x^6+35\,x^5+103\,x^4+85\,x^3+83\,x^2+32\,x+12\right)\arctan\left(\frac{1}{31}\,\sqrt{31}\,(10\,x+3)\right)+1522737174\,\sqrt{23}\left(50\,x^6+35\,x^5+103\,x^4+85\,x^3+83\,x^2+32\,x+12\right)\arctan\left(\frac{1}{23}\,\sqrt{23}\,(4\,x-1)\right)+1500218514344\,x^2-1528665583\left(50\,x^6+35\,x^5+103\,x^4+85\,x^3+83\,x^2+32\,x+12\right)\log\left(5\,x^2+3\,x+2\right)+1528665583\left(50\,x^6+35\,x^5+103\,x^4+85\,x^3+83\,x^2+32\,x+12\right)\log\left(2\,x^2-x+3\right)+1338609358240\,x+218880812656}{7383486284768\left(50\,x^6+35\,x^5+103\,x^4+85\,x^3+83\,x^2+32\,x+12\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^2/(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

```
[Out] 1/7383486284768*(1253927859800*x^5 + 679296504260*x^4 + 2185021181068*x^3 +
 4722995582*sqrt(31)*(50*x^6 + 35*x^5 + 103*x^4 + 85*x^3 + 83*x^2 + 32*x +
12)*arctan(1/31*sqrt(31)*(10*x + 3)) + 1522737174*sqrt(23)*(50*x^6 + 35*x^5
 + 103*x^4 + 85*x^3 + 83*x^2 + 32*x + 12)*arctan(1/23*sqrt(23)*(4*x - 1)) +
 1500218514344*x^2 - 1528665583*(50*x^6 + 35*x^5 + 103*x^4 + 85*x^3 + 83*x^
2 + 32*x + 12)*log(5*x^2 + 3*x + 2) + 1528665583*(50*x^6 + 35*x^5 + 103*x^4
 + 85*x^3 + 83*x^2 + 32*x + 12)*log(2*x^2 - x + 3) + 1338609358240*x + 2188
80812656)/(50*x^6 + 35*x^5 + 103*x^4 + 85*x^3 + 83*x^2 + 32*x + 12)
```

giac [A] time = 0.20, size = 110, normalized size = 0.74

$$\frac{4464079}{6978720496}\,\sqrt{31}\,\arctan\left(\frac{1}{31}\,\sqrt{31}\,(10\,x+3)\right)+\frac{25557}{123921424}\,\sqrt{23}\,\arctan\left(\frac{1}{23}\,\sqrt{23}\,(4\,x-1)\right)+\frac{39969650\,x^5+21652955\,x^4+69648769\,x^3+47820302\,x^2+42668920\,x+6976948}{235352744\left(5\,x^2+3\,x+2\right)^2\left(2\,x^2-x+3\right)}-\frac{97}{468512}\,\log\left(5\,x^2+3\,x+2\right)+\frac{97}{468512}\,\log\left(2\,x^2-x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^2/(5*x^2+3*x+2)^3,x, algorithm="giac")
```

```
[Out] 4464079/6978720496*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 25557/123921
424*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 1/235352744*(39969650*x^5 +
21652955*x^4 + 69648769*x^3 + 47820302*x^2 + 42668920*x + 6976948)/((5*x^2
+ 3*x + 2)^2*(2*x^2 - x + 3)) - 97/468512*log(5*x^2 + 3*x + 2) + 97/468512*
log(2*x^2 - x + 3)
```

maple [A] time = 0.01, size = 106, normalized size = 0.72

$$\frac{4464079\sqrt{31}\ \arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)}{6978720496}+\frac{25557\sqrt{23}\ \arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{123921424}+\frac{97\ln\left(2x^2-x+3\right)}{468512}-\frac{97\ln\left(5x^2+3x+2\right)}{468512}-\frac{25\left(-\frac{723272}{961}x^3-\frac{3656422}{4805}x^2-\frac{14280728}{24025}x-\frac{2238016}{24025}\right)}{234256\left(5x^2+3x+2\right)^2}+\frac{\frac{990x}{23}-\frac{121}{23}}{234256x^2-117128x+351384}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^2-x+3)^2/(5*x^2+3*x+2)^3,x)
```

```
[Out] -25/234256*(-723272/961*x^3-3656422/4805*x^2-14280728/24025*x-2238016/24025
)/(5*x^2+3*x+2)^2-97/468512*ln(5*x^2+3*x+2)+4464079/6978720496*31^(1/2)*arc
tan(1/31*(10*x+3)*31^(1/2))+1/234256*(990/23*x-121/23)/(x^2-1/2*x+3/2)+97/4
68512*ln(2*x^2-x+3)+25557/123921424*23^(1/2)*arctan(1/23*(4*x-1)*23^(1/2))
```

maxima [A] time = 0.97, size = 118, normalized size = 0.80

$$\frac{4464079}{6978720496}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}\left(10x+3\right)\right)+\frac{25557}{123921424}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}\left(4x-1\right)\right)+\frac{39969650x^5+21652955x^4+69648769x^3+47820302x^2+42668920x+6976948}{235352744\left(50x^6+35x^5+103x^4+85x^3+83x^2+32x+12\right)}-\frac{97}{468512}\log\left(5x^2+3x+2\right)+\frac{97}{468512}\log\left(2x^2-x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^2/(5*x^2+3*x+2)^3,x, algorithm="maxima")
```

```
[Out] 4464079/6978720496*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 25557/123921
424*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 1/235352744*(39969650*x^5 +
21652955*x^4 + 69648769*x^3 + 47820302*x^2 + 42668920*x + 6976948)/(50*x^6
+ 35*x^5 + 103*x^4 + 85*x^3 + 83*x^2 + 32*x + 12) - 97/468512*log(5*x^2 + 3
*x + 2) + 97/468512*log(2*x^2 - x + 3)
```

mupad [B] time = 3.59, size = 135, normalized size = 0.91

$$\frac{\frac{799393x^5}{235352744}+\frac{4330591x^4}{2353527440}+\frac{69648769x^3}{11767637200}+\frac{23910151x^2}{5883818600}+\frac{1066723x}{294190930}+\frac{158567}{267446300}}{x^6+\frac{7x^5}{10}+\frac{103x^4}{50}+\frac{17x^3}{10}+\frac{83x^2}{50}+\frac{16x}{25}+\frac{6}{25}}+\ln\left(x-\frac{1}{4}+\frac{\sqrt{23}\,1\mathrm{i}}{4}\right)\left(\frac{97}{468512}+\frac{\sqrt{23}\,25557\mathrm{i}}{247842848}\right)-\ln\left(x-\frac{1}{4}-\frac{\sqrt{23}\,1\mathrm{i}}{4}\right)\left(-\frac{97}{468512}+\frac{\sqrt{23}\,25557\mathrm{i}}{247842848}\right)-\ln\left(x+\frac{3}{10}-\frac{\sqrt{31}\,1\mathrm{i}}{10}\right)\left(\frac{97}{468512}+\frac{\sqrt{31}\,4464079\mathrm{i}}{13957440992}\right)+\ln\left(x+\frac{3}{10}+\frac{\sqrt{31}\,1\mathrm{i}}{10}\right)\left(-\frac{97}{468512}+\frac{\sqrt{31}\,4464079\mathrm{i}}{13957440992}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*x^2 - x + 3)^2*(3*x + 5*x^2 + 2)^3),x)
```

```
[Out] log(x + (23^(1/2)*1i)/4 - 1/4)*((23^(1/2)*25557i)/247842848 + 97/468512) -
log(x - (23^(1/2)*1i)/4 - 1/4)*((23^(1/2)*25557i)/247842848 - 97/468512) +
```

```
((1066723*x)/294190930 + (23910151*x^2)/5883818600 + (69648769*x^3)/1176763
7200 + (4330591*x^4)/2353527440 + (799393*x^5)/235352744 + 158567/267446300
)/((16*x)/25 + (83*x^2)/50 + (17*x^3)/10 + (103*x^4)/50 + (7*x^5)/10 + x^6
+ 6/25) - log(x - (31^(1/2)*1i)/10 + 3/10)*((31^(1/2)*4464079i)/13957440992
 + 97/468512) + log(x + (31^(1/2)*1i)/10 + 3/10)*((31^(1/2)*4464079i)/13957
440992 - 97/468512)
```

sympy [A] time = 0.40, size = 143, normalized size = 0.97

$$\frac{39969650x^5 + 21652955x^4 + 69648769x^3 + 47820302x^2 + 42668920x + 6976948}{11767637200x^6 + 8237346040x^5 + 24241332632x^4 + 20004983240x^3 + 19534277752x^2 + 7531287808x + 2824232928} + \frac{97\log\left(x^2 - \frac{x}{2} + \frac{3}{2}\right)}{468512} - \frac{97\log\left(x^2 + \frac{3x}{5} + \frac{2}{5}\right)}{468512} + \frac{25557\sqrt{23}\operatorname{atan}\left(\frac{4\sqrt{23}x}{23} - \frac{\sqrt{23}}{23}\right)}{123921424} + \frac{4464079\sqrt{31}\operatorname{atan}\left(\frac{10\sqrt{31}x}{31} + \frac{3\sqrt{31}}{31}\right)}{6978720496}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**2-x+3)**2/(5*x**2+3*x+2)**3,x)
```

```
[Out] (39969650*x**5 + 21652955*x**4 + 69648769*x**3 + 47820302*x**2 + 42668920*x
 + 6976948)/(11767637200*x**6 + 8237346040*x**5 + 24241332632*x**4 + 200049
83240*x**3 + 19534277752*x**2 + 7531287808*x + 2824232928) + 97*log(x**2 -
x/2 + 3/2)/468512 - 97*log(x**2 + 3*x/5 + 2/5)/468512 + 25557*sqrt(23)*atan
(4*sqrt(23)*x/23 - sqrt(23)/23)/123921424 + 4464079*sqrt(31)*atan(10*sqrt(3
1)*x/31 + 3*sqrt(31)/31)/6978720496
```

3.48 $\int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^3}\,dx$

Optimal. Leaf size=98

$$\frac{625x^3}{24}+\frac{4875x^2}{32}+\frac{1331(76420x+5229)}{135424(2x^2-x+3)}-\frac{14641(79x+101)}{5888(2x^2-x+3)^2}-\frac{13915}{64}\log(2x^2-x+3)+\frac{2725x}{8}+\frac{63799791 \tan}{16928\sqrt{}}$$

Rubi [A] time = 0.11, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1660, 1657, 634, 618, 204, 628}

$$\frac{625x^3}{24}+\frac{4875x^2}{32}+\frac{1331(76420x+5229)}{135424(2x^2-x+3)}-\frac{14641(79x+101)}{5888(2x^2-x+3)^2}-\frac{13915}{64}\log(2x^2-x+3)+\frac{2725x}{8}+\frac{63799791\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{16928\sqrt{23}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + 5*x^2)^4/(3 - x + 2*x^2)^3,x]
```

```
[Out] (2725*x)/8 + (4875*x^2)/32 + (625*x^3)/24 - (14641*(101 + 79*x))/(5888*(3 -
 x + 2*x^2)^2) + (1331*(5229 + 76420*x))/(135424*(3 - x + 2*x^2)) + (637997
91*ArcTan[(1 - 4*x)/Sqrt[23]])/(16928*Sqrt[23]) - (13915*Log[3 - x + 2*x^2]
)/64
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{\left(2+3x+5x^2\right)^4}{\left(3-x+2x^2\right)^3}\,dx = -\frac{14641(101+79x)}{5888\left(3-x+2x^2\right)^2} + \frac{1}{46}\int \frac{\frac{2173869}{128} - \frac{661181x}{32} - \frac{488267x^2}{16} + \frac{143635x^3}{8} + \frac{213325x^4}{4} + \frac{83375}{2}\cdots}{\left(3-x+2x^2\right)^2}$$

$$= -\frac{14641(101+79x)}{5888\left(3-x+2x^2\right)^2} + \frac{1331(5229+76420x)}{135424\left(3-x+2x^2\right)} + \frac{\int \frac{-\frac{5460539}{8} - \frac{626865x}{2} + \frac{5170975x^2}{8} + \frac{1124125x^3}{2} + 330\cdots}{3-x+2x^2}}{1058}$$

$$= -\frac{14641(101+79x)}{5888\left(3-x+2x^2\right)^2} + \frac{1331(5229+76420x)}{135424\left(3-x+2x^2\right)} + \frac{\int \left(\frac{1441525}{4} + \frac{2578875x}{8} + \frac{330625x^2}{4} - 121\cdots\right.}{1058}$$

$$= \frac{2725x}{8} + \frac{4875x^2}{32} + \frac{625x^3}{24} - \frac{14641(101+79x)}{5888\left(3-x+2x^2\right)^2} + \frac{1331(5229+76420x)}{135424\left(3-x+2x^2\right)} - \frac{121\int \frac{116\cdots}{}}{8\cdots}$$

$$= \frac{2725x}{8} + \frac{4875x^2}{32} + \frac{625x^3}{24} - \frac{14641(101+79x)}{5888\left(3-x+2x^2\right)^2} + \frac{1331(5229+76420x)}{135424\left(3-x+2x^2\right)} - \frac{13915}{64}\int \cdots$$

$$= \frac{2725x}{8} + \frac{4875x^2}{32} + \frac{625x^3}{24} - \frac{14641(101+79x)}{5888\left(3-x+2x^2\right)^2} + \frac{1331(5229+76420x)}{135424\left(3-x+2x^2\right)} - \frac{13915}{64}\log\cdots$$

$$= \frac{2725x}{8} + \frac{4875x^2}{32} + \frac{625x^3}{24} - \frac{14641(101+79x)}{5888\left(3-x+2x^2\right)^2} + \frac{1331(5229+76420x)}{135424\left(3-x+2x^2\right)} + \frac{6379979\cdots}{169\cdots}$$

Mathematica [A] time = 0.04, size = 98, normalized size = 1.00

$$\frac{625x^3}{24} + \frac{4875x^2}{32} + \frac{1331(76420x+5229)}{135424\left(2x^2-x+3\right)} - \frac{14641(79x+101)}{5888\left(2x^2-x+3\right)^2} - \frac{13915}{64}\log\left(2x^2-x+3\right) + \frac{2725x}{8} - \frac{63799791\tan^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{16928\sqrt{23}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)^4/(3 - x + 2*x^2)^3,x]
```

```
[Out] (2725*x)/8 + (4875*x^2)/32 + (625*x^3)/24 - (14641*(101 + 79*x))/(5888*(3 -
 x + 2*x^2)^2) + (1331*(5229 + 76420*x))/(135424*(3 - x + 2*x^2)) - (637997
91*ArcTan[(-1 + 4*x)/Sqrt[23]])/(16928*Sqrt[23]) - (13915*Log[3 - x + 2*x^2
])/64
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2+3x+5x^2\right)^4}{\left(3-x+2x^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(2 + 3*x + 5*x^2)^4/(3 - x + 2*x^2)^3,x]
```

```
[Out] IntegrateAlgebraic[(2 + 3*x + 5*x^2)^4/(3 - x + 2*x^2)^3, x]
```

fricas [A] time = 0.41, size = 128, normalized size = 1.31

$$\frac{486680000\,x^7+2360398000\,x^6+5100406400\,x^5+2157209100\,x^4+24531516180\,x^3-765597492\,\sqrt{23}\left(4\,x^4-4\,x^3+13\,x^2-6\,x+9\right)\arctan\left(\frac{1}{23}\,\sqrt{23}\,(4\,x-1)\right)-6171678159\,x^2-1015822830\left(4\,x^4-4\,x^3+13\,x^2-6\,x+9\right)\log\left(2\,x^2-x+3\right)+23692590858\,x-453041787}{4672128\left(4\,x^4-4\,x^3+13\,x^2-6\,x+9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^3,x, algorithm="fricas")
```

```
[Out] 1/4672128*(486680000*x^7 + 2360398000*x^6 + 5100406400*x^5 + 2157209100*x^4
 + 24531516180*x^3 - 765597492*sqrt(23)*(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)*
arctan(1/23*sqrt(23)*(4*x - 1)) - 6171678159*x^2 - 1015822830*(4*x^4 - 4*x^
3 + 13*x^2 - 6*x + 9)*log(2*x^2 - x + 3) + 23692590858*x - 453041787)/(4*x^
4 - 4*x^3 + 13*x^2 - 6*x + 9)
```

giac [A] time = 0.21, size = 72, normalized size = 0.73

$$\frac{625}{24}\,x^3+\frac{4875}{32}\,x^2-\frac{63799791}{389344}\,\sqrt{23}\,\arctan\left(\frac{1}{23}\,\sqrt{23}\,(4\,x-1)\right)+\frac{2725}{8}\,x+\frac{1331\left(76420\,x^3-32981\,x^2+102022\,x-4933\right)}{67712\left(2\,x^2-x+3\right)^2}-\frac{13915}{64}\,\log\left(2\,x^2-x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^3,x, algorithm="giac")
```

```
[Out] 625/24*x^3 + 4875/32*x^2 - 63799791/389344*sqrt(23)*arctan(1/23*sqrt(23)*(4
*x - 1)) + 2725/8*x + 1331/67712*(76420*x^3 - 32981*x^2 + 102022*x - 4933)/
(2*x^2 - x + 3)^2 - 13915/64*log(2*x^2 - x + 3)
```

maple [A] time = 0.01, size = 73, normalized size = 0.74

$$\frac{625x^3}{24}+\frac{4875x^2}{32}+\frac{2725x}{8}-\frac{63799791\sqrt{23}\,\arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{389344}-\frac{13915\ln\left(2x^2-x+3\right)}{64}-\frac{121\left(-\frac{210155}{4232}x^3+\frac{362791}{16928}x^2-\frac{561121}{8464}x+\frac{54263}{16928}\right)}{4\left(2x^2-x+3\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)^4/(2*x^2-x+3)^3,x)
```

```
[Out] 625/24*x^3+4875/32*x^2+2725/8*x-121/4*(-210155/4232*x^3+362791/16928*x^2-56
1121/8464*x+54263/16928)/(2*x^2-x+3)^2-13915/64*ln(2*x^2-x+3)-63799791/3893
44*23^(1/2)*arctan(1/23*(4*x-1)*23^(1/2))
```

maxima [A] time = 0.95, size = 82, normalized size = 0.84

$$\frac{625}{24}x^3+\frac{4875}{32}x^2-\frac{63799791}{389344}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right)+\frac{2725}{8}x+\frac{1331\left(76420x^3-32981x^2+102022x-4933\right)}{67712\left(4x^4-4x^3+13x^2-6x+9\right)}-\frac{13915}{64}\log\left(2x^2-x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^3,x, algorithm="maxima")
```

```
[Out] 625/24*x^3 + 4875/32*x^2 - 63799791/389344*sqrt(23)*arctan(1/23*sqrt(23)*(4
*x - 1)) + 2725/8*x + 1331/67712*(76420*x^3 - 32981*x^2 + 102022*x - 4933)/
(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9) - 13915/64*log(2*x^2 - x + 3)
```

mupad [B] time = 0.05, size = 81, normalized size = 0.83

$$\frac{2725x}{8}-\frac{13915\ln\left(2x^2-x+3\right)}{64}-\frac{63799791\sqrt{23}\,\mathrm{atan}\left(\frac{4\sqrt{23}x}{23}-\frac{\sqrt{23}}{23}\right)}{389344}+\frac{4875x^2}{32}+\frac{625x^3}{24}+\frac{\frac{25428755x^3}{67712}-\frac{43897711x^2}{270848}+\frac{67895641x}{135424}-\frac{6565823}{270848}}{x^4-x^3+\frac{13x^2}{4}-\frac{3x}{2}+\frac{9}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 5*x^2 + 2)^4/(2*x^2 - x + 3)^3,x)
```

```
[Out] (2725*x)/8 - (13915*log(2*x^2 - x + 3))/64 - (63799791*23^(1/2)*atan((4*23^
(1/2)*x)/23 - 23^(1/2)/23))/389344 + (4875*x^2)/32 + (625*x^3)/24 + ((67895
641*x)/135424 - (43897711*x^2)/270848 + (25428755*x^3)/67712 - 6565823/2708
48)/((13*x^2)/4 - (3*x)/2 - x^3 + x^4 + 9/4)
```

sympy [A] time = 0.24, size = 95, normalized size = 0.97

$$\frac{625x^3}{24}+\frac{4875x^2}{32}+\frac{2725x}{8}+\frac{101715020x^3-43897711x^2+135791282x-6565823}{270848x^4-270848x^3+880256x^2-406272x+609408}-\frac{13915\log\left(x^2-\frac{x}{2}+\frac{3}{2}\right)}{64}-\frac{63799791\sqrt{23}\,\mathrm{atan}\left(\frac{4\sqrt{23}x}{23}-\frac{\sqrt{23}}{23}\right)}{389344}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)**4/(2*x**2-x+3)**3,x)
```

```
[Out] 625*x**3/24 + 4875*x**2/32 + 2725*x/8 + (101715020*x**3 - 43897711*x**2 + 1
35791282*x - 6565823)/(270848*x**4 - 270848*x**3 + 880256*x**2 - 406272*x +
 609408) - 13915*log(x**2 - x/2 + 3/2)/64 - 63799791*sqrt(23)*atan(4*sqrt(2
3)*x/23 - sqrt(23)/23)/389344
```

3.49 $\int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^3}\,dx$

Optimal. Leaf size=84

$$\frac{121(21193-12828x)}{33856\left(2x^2-x+3\right)}-\frac{1331(17-45x)}{1472\left(2x^2-x+3\right)^2}+\frac{825}{32}\log\left(2x^2-x+3\right)+\frac{125x}{8}+\frac{165099\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{8464\sqrt{23}}$$

Rubi [A] time = 0.09, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1660, 1657, 634, 618, 204, 628}

$$\frac{121(21193-12828x)}{33856\left(2x^2-x+3\right)}-\frac{1331(17-45x)}{1472\left(2x^2-x+3\right)^2}+\frac{825}{32}\log\left(2x^2-x+3\right)+\frac{125x}{8}+\frac{165099\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{8464\sqrt{23}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + 5*x^2)^3/(3 - x + 2*x^2)^3,x]
```

```
[Out] (125*x)/8 - (1331*(17 - 45*x))/(1472*(3 - x + 2*x^2)^2) + (121*(21193 - 128
28*x))/(33856*(3 - x + 2*x^2)) + (165099*ArcTan[(1 - 4*x)/Sqrt[23]])/(8464*
Sqrt[23]) + (825*Log[3 - x + 2*x^2])/32
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{\left(2+3x+5x^2\right)^3}{\left(3-x+2x^2\right)^3}\,dx = -\frac{1331(17-45x)}{1472\left(3-x+2x^2\right)^2} + \frac{1}{46}\int \frac{-\frac{40885}{32}-\frac{19067x}{8}+\frac{22195x^2}{4}+\frac{13225x^3}{2}+2875x^4}{\left(3-x+2x^2\right)^2}\,dx$$

$$= -\frac{1331(17-45x)}{1472\left(3-x+2x^2\right)^2} + \frac{121(21193-12828x)}{33856\left(3-x+2x^2\right)} + \frac{\int \frac{\frac{23997}{2}+92575x+\frac{66125x^2}{2}}{3-x+2x^2}\,dx}{1058}$$

$$= -\frac{1331(17-45x)}{1472\left(3-x+2x^2\right)^2} + \frac{121(21193-12828x)}{33856\left(3-x+2x^2\right)} + \frac{\int \left(\frac{66125}{4}-\frac{33(4557-13225x)}{4\left(3-x+2x^2\right)}\right)dx}{1058}$$

$$= \frac{125x}{8} - \frac{1331(17-45x)}{1472\left(3-x+2x^2\right)^2} + \frac{121(21193-12828x)}{33856\left(3-x+2x^2\right)} - \frac{33\int \frac{4557-13225x}{3-x+2x^2}\,dx}{4232}$$

$$= \frac{125x}{8} - \frac{1331(17-45x)}{1472\left(3-x+2x^2\right)^2} + \frac{121(21193-12828x)}{33856\left(3-x+2x^2\right)} - \frac{165099\int \frac{1}{3-x+2x^2}\,dx}{16928} + \frac{825}{32}\int \frac{}{3}$$

$$= \frac{125x}{8} - \frac{1331(17-45x)}{1472\left(3-x+2x^2\right)^2} + \frac{121(21193-12828x)}{33856\left(3-x+2x^2\right)} + \frac{825}{32}\log\left(3-x+2x^2\right) + \frac{165099}{}$$

$$= \frac{125x}{8} - \frac{1331(17-45x)}{1472\left(3-x+2x^2\right)^2} + \frac{121(21193-12828x)}{33856\left(3-x+2x^2\right)} + \frac{165099\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{8464\sqrt{23}} + \frac{825}{32}\log($$

Mathematica [A] time = 0.04, size = 84, normalized size = 1.00

$$-\frac{121(12828x-21193)}{33856\left(2x^2-x+3\right)} + \frac{1331(45x-17)}{1472\left(2x^2-x+3\right)^2} + \frac{825}{32}\log\left(2x^2-x+3\right) + \frac{125x}{8} - \frac{165099\tan^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{8464\sqrt{23}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)^3/(3 - x + 2*x^2)^3,x]
```

```
[Out] (125*x)/8 + (1331*(-17 + 45*x))/(1472*(3 - x + 2*x^2)^2) - (121*(-21193 + 1
2828*x))/(33856*(3 - x + 2*x^2)) - (165099*ArcTan[(-1 + 4*x)/Sqrt[23]])/(84
64*Sqrt[23]) + (825*Log[3 - x + 2*x^2])/32
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2+3x+5x^2\right)^3}{\left(3-x+2x^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(2 + 3*x + 5*x^2)^3/(3 - x + 2*x^2)^3,x]
```

```
[Out] IntegrateAlgebraic[(2 + 3*x + 5*x^2)^3/(3 - x + 2*x^2)^3, x]
```

fricas [A] time = 0.39, size = 118, normalized size = 1.40

$$\frac{24334000\,x^5-24334000\,x^4+43385176\,x^3-330198\,\sqrt{23}\left(4\,x^4-4\,x^3+13\,x^2-6\,x+9\right)\arctan\left(\frac{1}{23}\,\sqrt{23}\,(4\,x-1)\right)+40329281\,x^2+10037775\left(4\,x^4-4\,x^3+13\,x^2-6\,x+9\right)\log\left(2\,x^2-x+3\right)-12446818\,x+82485337}{389344\left(4\,x^4-4\,x^3+13\,x^2-6\,x+9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^3,x, algorithm="fricas")
```

```
[Out] 1/389344*(24334000*x^5 - 24334000*x^4 + 43385176*x^3 - 330198*sqrt(23)*(4*x
^4 - 4*x^3 + 13*x^2 - 6*x + 9)*arctan(1/23*sqrt(23)*(4*x - 1)) + 40329281*x
^2 + 10037775*(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)*log(2*x^2 - x + 3) - 12446
818*x + 82485337)/(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)
```

giac [A] time = 0.18, size = 62, normalized size = 0.74

$$-\frac{165099}{194672}\,\sqrt{23}\,\arctan\left(\frac{1}{23}\,\sqrt{23}\,(4\,x-1)\right)+\frac{125}{8}\,x-\frac{121\left(12828\,x^3-27607\,x^2+24146\,x-29639\right)}{16928\left(2\,x^2-x+3\right)^2}+\frac{825}{32}\,\log\left(2\,x^2-x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^3,x, algorithm="giac")
```

```
[Out] -165099/194672*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 125/8*x - 121/169
28*(12828*x^3 - 27607*x^2 + 24146*x - 29639)/(2*x^2 - x + 3)^2 + 825/32*log
(2*x^2 - x + 3)
```

maple [A] time = 0.01, size = 63, normalized size = 0.75

$$\frac{125x}{8}-\frac{165099\sqrt{23}\,\arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{194672}+\frac{825\ln\left(2x^2-x+3\right)}{32}+\frac{-\frac{388047}{4232}x^3+\frac{3340447}{16928}x^2-\frac{1460833}{8464}x+\frac{3586319}{16928}}{\left(2x^2-x+3\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)^3/(2*x^2-x+3)^3,x)
```

```
[Out] 125/8*x+11/2*(-35277/2116*x^3+303677/8464*x^2-132803/4232*x+326029/8464)/(2
*x^2-x+3)^2+825/32*ln(2*x^2-x+3)-165099/194672*23^(1/2)*arctan(1/23*(4*x-1)
*23^(1/2))
```

maxima [A] time = 0.96, size = 72, normalized size = 0.86

$$-\frac{165099}{194672}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right)+\frac{125}{8}x-\frac{121\left(12828x^3-27607x^2+24146x-29639\right)}{16928\left(4x^4-4x^3+13x^2-6x+9\right)}+\frac{825}{32}\log\left(2x^2-x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^3,x, algorithm="maxima")
```

```
[Out] -165099/194672*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 125/8*x - 121/169
28*(12828*x^3 - 27607*x^2 + 24146*x - 29639)/(4*x^4 - 4*x^3 + 13*x^2 - 6*x
+ 9) + 825/32*log(2*x^2 - x + 3)
```

mupad [B] time = 0.05, size = 72, normalized size = 0.86

$$\frac{125x}{8}+\frac{825\ln\left(2x^2-x+3\right)}{32}-\frac{165099\sqrt{23}\operatorname{atan}\left(\frac{4\sqrt{23}x}{23}-\frac{\sqrt{23}}{23}\right)}{194672}-\frac{\frac{388047x^3}{16928}-\frac{3340447x^2}{67712}+\frac{1460833x}{33856}-\frac{3586319}{67712}}{x^4-x^3+\frac{13x^2}{4}-\frac{3x}{2}+\frac{9}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 5*x^2 + 2)^3/(2*x^2 - x + 3)^3,x)
```

```
[Out] (125*x)/8 + (825*log(2*x^2 - x + 3))/32 - (165099*23^(1/2)*atan((4*23^(1/2)
*x)/23 - 23^(1/2)/23))/194672 - ((1460833*x)/33856 - (3340447*x^2)/67712 +
(388047*x^3)/16928 - 3586319/67712)/((13*x^2)/4 - (3*x)/2 - x^3 + x^4 + 9/4
)
```

sympy [A] time = 0.23, size = 82, normalized size = 0.98

$$\frac{125x}{8}+\frac{-1552188x^3+3340447x^2-2921666x+3586319}{67712x^4-67712x^3+220064x^2-101568x+152352}+\frac{825\log\left(x^2-\frac{x}{2}+\frac{3}{2}\right)}{32}-\frac{165099\sqrt{23}\operatorname{atan}\left(\frac{4\sqrt{23}x}{23}-\frac{\sqrt{23}}{23}\right)}{194672}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)**3/(2*x**2-x+3)**3,x)
```

```
[Out] 125*x/8 + (-1552188*x**3 + 3340447*x**2 - 2921666*x + 3586319)/(67712*x**4
- 67712*x**3 + 220064*x**2 - 101568*x + 152352) + 825*log(x**2 - x/2 + 3/2)
/32 - 165099*sqrt(23)*atan(4*sqrt(23)*x/23 - sqrt(23)/23)/194672
```

3.50 $\int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^3} \, dx$

Optimal. Leaf size=64

$$\frac{121(19-7x)}{368\left(2x^2-x+3\right)^2}-\frac{55(332x+975)}{8464\left(2x^2-x+3\right)}-\frac{4330\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{529\sqrt{23}}$$

Rubi [A] time = 0.05, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1660, 12, 618, 204}

$$\frac{121(19-7x)}{368\left(2x^2-x+3\right)^2}-\frac{55(332x+975)}{8464\left(2x^2-x+3\right)}-\frac{4330\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{529\sqrt{23}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + 5*x^2)^2/(3 - x + 2*x^2)^3,x]
```

```
[Out] (121*(19 - 7*x))/(368*(3 - x + 2*x^2)^2) - (55*(975 + 332*x))/(8464*(3 - x
+ 2*x^2)) - (4330*ArcTan[(1 - 4*x)/Sqrt[23]])/(529*Sqrt[23])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
```

```
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(2+3x+5x^2\right)^2}{\left(3-x+2x^2\right)^3}\,dx &= \frac{121(19-7x)}{368\left(3-x+2x^2\right)^2}+\frac{1}{46}\int \frac{-\frac{195}{8}+\frac{1955x}{2}+575x^2}{\left(3-x+2x^2\right)^2}\,dx\\
&= \frac{121(19-7x)}{368\left(3-x+2x^2\right)^2}-\frac{55(975+332x)}{8464\left(3-x+2x^2\right)}+\frac{\int \frac{4330}{3-x+2x^2}\,dx}{1058}\\
&= \frac{121(19-7x)}{368\left(3-x+2x^2\right)^2}-\frac{55(975+332x)}{8464\left(3-x+2x^2\right)}+\frac{2165}{529}\int \frac{1}{3-x+2x^2}\,dx\\
&= \frac{121(19-7x)}{368\left(3-x+2x^2\right)^2}-\frac{55(975+332x)}{8464\left(3-x+2x^2\right)}-\frac{4330}{529}\,\mathrm{Subst}\left(\int \frac{1}{-23-x^2}\,dx,x,-1+4x\right)\\
&= \frac{121(19-7x)}{368\left(3-x+2x^2\right)^2}-\frac{55(975+332x)}{8464\left(3-x+2x^2\right)}-\frac{4330\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{529\sqrt{23}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 51, normalized size = 0.80

$$\frac{4330\tan^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{529\sqrt{23}}-\frac{11\left(1660x^3+4045x^2+938x+4909\right)}{4232\left(-2x^2+x-3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)^2/(3 - x + 2*x^2)^3,x]
```

```
[Out] (-11*(4909 + 938*x + 4045*x^2 + 1660*x^3))/(4232*(-3 + x - 2*x^2)^2) + (433
0*ArcTan[(-1 + 4*x)/Sqrt[23]])/(529*Sqrt[23])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2+3x+5x^2\right)^2}{\left(3-x+2x^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(2 + 3*x + 5*x^2)^2/(3 - x + 2*x^2)^3,x]
```

```
[Out] IntegrateAlgebraic[(2 + 3*x + 5*x^2)^2/(3 - x + 2*x^2)^3, x]
```

fricas [A] time = 0.39, size = 75, normalized size = 1.17

$$-\frac{419980\,x^3 - 34640\,\sqrt{23}\left(4\,x^4 - 4\,x^3 + 13\,x^2 - 6\,x + 9\right)\arctan\left(\frac{1}{23}\,\sqrt{23}\,(4\,x - 1)\right) + 1023385\,x^2 + 237314\,x + 1241977}{97336\left(4\,x^4 - 4\,x^3 + 13\,x^2 - 6\,x + 9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^3,x, algorithm="fricas")
```

```
[Out] -1/97336*(419980*x^3 - 34640*sqrt(23)*(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)*ar
ctan(1/23*sqrt(23)*(4*x - 1)) + 1023385*x^2 + 237314*x + 1241977)/(4*x^4 -
4*x^3 + 13*x^2 - 6*x + 9)
```

giac [A] time = 0.19, size = 46, normalized size = 0.72

$$\frac{4330}{12167}\,\sqrt{23}\,\arctan\left(\frac{1}{23}\,\sqrt{23}\,(4\,x - 1)\right) - \frac{11\left(1660\,x^3 + 4045\,x^2 + 938\,x + 4909\right)}{4232\left(2\,x^2 - x + 3\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^3,x, algorithm="giac")
```

```
[Out] 4330/12167*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) - 11/4232*(1660*x^3 + 4
045*x^2 + 938*x + 4909)/(2*x^2 - x + 3)^2
```

maple [A] time = 0.01, size = 47, normalized size = 0.73

$$\frac{4330\sqrt{23}\,\arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{12167} + \frac{-\frac{4565}{1058}x^3 - \frac{44495}{4232}x^2 - \frac{5159}{2116}x - \frac{53999}{4232}}{\left(2x^2 - x + 3\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)^2/(2*x^2-x+3)^3,x)
```

```
[Out] 4*(-4565/4232*x^3-44495/16928*x^2-5159/8464*x-53999/16928)/(2*x^2-x+3)^2+43
30/12167*23^(1/2)*arctan(1/23*(4*x-1)*23^(1/2))
```

maxima [A] time = 0.96, size = 56, normalized size = 0.88

$$\frac{4330}{12167}\,\sqrt{23}\,\arctan\left(\frac{1}{23}\,\sqrt{23}\,(4\,x - 1)\right) - \frac{11\left(1660\,x^3 + 4045\,x^2 + 938\,x + 4909\right)}{4232\left(4\,x^4 - 4\,x^3 + 13\,x^2 - 6\,x + 9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^3,x, algorithm="maxima")
```

```
[Out] 4330/12167*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) - 11/4232*(1660*x^3 + 4
045*x^2 + 938*x + 4909)/(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)
```

mupad [B] time = 3.47, size = 56, normalized size = 0.88

$$\frac{4330\,\sqrt{23}\,\mathrm{atan}\left(\frac{4\,\sqrt{23}\,x}{23}-\frac{\sqrt{23}}{23}\right)}{12167}-\frac{\frac{4565\,x^3}{4232}+\frac{44495\,x^2}{16928}+\frac{5159\,x}{8464}+\frac{53999}{16928}}{x^4-x^3+\frac{13\,x^2}{4}-\frac{3\,x}{2}+\frac{9}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 5*x^2 + 2)^2/(2*x^2 - x + 3)^3,x)
```

```
[Out] (4330*23^(1/2)*atan((4*23^(1/2)*x)/23 - 23^(1/2)/23))/12167 - ((5159*x)/846
4 + (44495*x^2)/16928 + (4565*x^3)/4232 + 53999/16928)/((13*x^2)/4 - (3*x)/
2 - x^3 + x^4 + 9/4)
```

sympy [A] time = 0.20, size = 63, normalized size = 0.98

$$\frac{-18260x^3-44495x^2-10318x-53999}{16928x^4-16928x^3+55016x^2-25392x+38088}+\frac{4330\sqrt{23}\,\mathrm{atan}\left(\frac{4\sqrt{23}\,x}{23}-\frac{\sqrt{23}}{23}\right)}{12167}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)**2/(2*x**2-x+3)**3,x)
```

```
[Out] (-18260*x**3 - 44495*x**2 - 10318*x - 53999)/(16928*x**4 - 16928*x**3 + 550
16*x**2 - 25392*x + 38088) + 4330*sqrt(23)*atan(4*sqrt(23)*x/23 - sqrt(23)/
23)/12167
```

3.51 $\int \frac{2+3x+5x^2}{\left(3-x+2x^2\right)^3}\,dx$

Optimal. Leaf size=64

$$-\frac{131(1-4x)}{2116\left(2x^2-x+3\right)}-\frac{11(3x+5)}{92\left(2x^2-x+3\right)^2}-\frac{262\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{529\sqrt{23}}$$

Rubi [A] time = 0.03, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {1660, 12, 614, 618, 204}

$$-\frac{131(1-4x)}{2116\left(2x^2-x+3\right)}-\frac{11(3x+5)}{92\left(2x^2-x+3\right)^2}-\frac{262\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{529\sqrt{23}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + 5*x^2)/(3 - x + 2*x^2)^3,x]
```

```
[Out] (-11*(5 + 3*x))/(92*(3 - x + 2*x^2)^2) - (131*(1 - 4*x))/(2116*(3 - x + 2*x
^2)) - (262*ArcTan[(1 - 4*x)/Sqrt[23]])/(529*Sqrt[23])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{2+3x+5x^2}{\left(3-x+2x^2\right)^3}\,dx &= -\frac{11(5+3x)}{92\left(3-x+2x^2\right)^2}+\frac{1}{46}\int \frac{131}{2\left(3-x+2x^2\right)^2}\,dx \\
&= -\frac{11(5+3x)}{92\left(3-x+2x^2\right)^2}+\frac{131}{92}\int \frac{1}{\left(3-x+2x^2\right)^2}\,dx \\
&= -\frac{11(5+3x)}{92\left(3-x+2x^2\right)^2}-\frac{131(1-4x)}{2116\left(3-x+2x^2\right)}+\frac{131}{529}\int \frac{1}{3-x+2x^2}\,dx \\
&= -\frac{11(5+3x)}{92\left(3-x+2x^2\right)^2}-\frac{131(1-4x)}{2116\left(3-x+2x^2\right)}-\frac{262}{529}\,\text{Subst}\left(\int \frac{1}{-23-x^2}\,dx,x,-1+4x\right) \\
&= -\frac{11(5+3x)}{92\left(3-x+2x^2\right)^2}-\frac{131(1-4x)}{2116\left(3-x+2x^2\right)}-\frac{262\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{529\sqrt{23}}
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 51, normalized size = 0.80

$$
\frac{\frac{46\left(524x^3-393x^2+472x-829\right)}{\left(-2x^2+x-3\right)^2}+1048\sqrt{23}\,\tan^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{48668}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)/(3 - x + 2*x^2)^3,x]
```

```
[Out] ((46*(-829 + 472*x - 393*x^2 + 524*x^3))/(-3 + x - 2*x^2)^2 + 1048*Sqrt[23]
*ArcTan[(-1 + 4*x)/Sqrt[23]])/48668
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2+3x+5x^2}{\left(3-x+2x^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(2 + 3*x + 5*x^2)/(3 - x + 2*x^2)^3,x]
```

```
[Out] IntegrateAlgebraic[(2 + 3*x + 5*x^2)/(3 - x + 2*x^2)^3, x]
```

fricas [A] time = 0.42, size = 75, normalized size = 1.17

$$\frac{12052\,x^3+524\,\sqrt{23}\left(4\,x^4-4\,x^3+13\,x^2-6\,x+9\right)\arctan\left(\frac{1}{23}\,\sqrt{23}\,(4\,x-1)\right)-9039\,x^2+10856\,x-19067}{24334\left(4\,x^4-4\,x^3+13\,x^2-6\,x+9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)/(2*x^2-x+3)^3,x, algorithm="fricas")
```

```
[Out] 1/24334*(12052*x^3 + 524*sqrt(23)*(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)*arctan
(1/23*sqrt(23)*(4*x - 1)) - 9039*x^2 + 10856*x - 19067)/(4*x^4 - 4*x^3 + 13
*x^2 - 6*x + 9)
```

giac [A] time = 0.21, size = 46, normalized size = 0.72

$$\frac{262}{12167}\,\sqrt{23}\,\arctan\left(\frac{1}{23}\,\sqrt{23}\,(4\,x-1)\right)+\frac{524\,x^3-393\,x^2+472\,x-829}{1058\left(2\,x^2-x+3\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)/(2*x^2-x+3)^3,x, algorithm="giac")
```

```
[Out] 262/12167*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 1/1058*(524*x^3 - 393*
x^2 + 472*x - 829)/(2*x^2 - x + 3)^2
```

maple [A] time = 0.01, size = 47, normalized size = 0.73

$$\frac{262\sqrt{23}\,\arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{12167}+\frac{\frac{262}{529}x^3-\frac{393}{1058}x^2+\frac{236}{529}x-\frac{829}{1058}}{\left(2x^2-x+3\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)/(2*x^2-x+3)^3,x)
```

```
[Out] 4*(131/1058*x^3-393/4232*x^2+59/529*x-829/4232)/(2*x^2-x+3)^2+262/12167*23^
(1/2)*arctan(1/23*(4*x-1)*23^(1/2))
```

maxima [A] time = 0.96, size = 56, normalized size = 0.88

$$\frac{262}{12167}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right)+\frac{524x^3-393x^2+472x-829}{1058\left(4x^4-4x^3+13x^2-6x+9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)/(2*x^2-x+3)^3,x, algorithm="maxima")
```

```
[Out] 262/12167*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 1/1058*(524*x^3 - 393*
x^2 + 472*x - 829)/(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)
```

mupad [B] time = 0.04, size = 55, normalized size = 0.86

$$\frac{262\sqrt{23}\,\mathrm{atan}\left(\frac{4\sqrt{23}\,x}{23}-\frac{\sqrt{23}}{23}\right)}{12167}+\frac{\frac{131x^3}{1058}-\frac{393x^2}{4232}+\frac{59x}{529}-\frac{829}{4232}}{x^4-x^3+\frac{13x^2}{4}-\frac{3x}{2}+\frac{9}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 5*x^2 + 2)/(2*x^2 - x + 3)^3,x)
```

```
[Out] (262*23^(1/2)*atan((4*23^(1/2)*x)/23 - 23^(1/2)/23))/12167 + ((59*x)/529 -
(393*x^2)/4232 + (131*x^3)/1058 - 829/4232)/((13*x^2)/4 - (3*x)/2 - x^3 + x
^4 + 9/4)
```

sympy [A] time = 0.18, size = 61, normalized size = 0.95

$$\frac{524x^3-393x^2+472x-829}{4232x^4-4232x^3+13754x^2-6348x+9522}+\frac{262\sqrt{23}\,\mathrm{atan}\left(\frac{4\sqrt{23}\,x}{23}-\frac{\sqrt{23}}{23}\right)}{12167}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)/(2*x**2-x+3)**3,x)
```

```
[Out] (524*x**3 - 393*x**2 + 472*x - 829)/(4232*x**4 - 4232*x**3 + 13754*x**2 - 6
348*x + 9522) + 262*sqrt(23)*atan(4*sqrt(23)*x/23 - sqrt(23)/23)/12167
```

3.52 $\int \frac{1}{(3-x+2x^2)^3 (2+3x+5x^2)} \, dx$

Optimal. Leaf size=115

$$\frac{3625-746x}{256036\,(2x^2-x+3)}+\frac{13-6x}{1012\,(2x^2-x+3)^2}-\frac{119\log(2x^2-x+3)}{21296}+\frac{119\log(5x^2+3x+2)}{21296}-\frac{53403\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{5632792\sqrt{23}}$$

Rubi [A] time = 0.12, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {974, 1060, 1072, 634, 618, 204, 628}

$$\frac{3625-746x}{256036\,(2x^2-x+3)}+\frac{13-6x}{1012\,(2x^2-x+3)^2}-\frac{119\log(2x^2-x+3)}{21296}+\frac{119\log(5x^2+3x+2)}{21296}-\frac{53403\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{5632792\sqrt{23}}+\frac{247\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{10648\sqrt{31}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2)),x]
```

```
[Out] (13 - 6*x)/(1012*(3 - x + 2*x^2)^2) + (3625 - 746*x)/(256036*(3 - x + 2*x^2
)) - (53403*ArcTan[(1 - 4*x)/Sqrt[23]])/(5632792*Sqrt[23]) + (247*ArcTan[(3
 + 10*x)/Sqrt[31]])/(10648*Sqrt[31]) - (119*Log[3 - x + 2*x^2])/21296 + (11
9*Log[2 + 3*x + 5*x^2])/21296
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 974

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*
a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*(
d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q,
 0]
```

Rule 1060

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x +
 c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
2*a*(c*d - a*f)))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !
```

```
IGtQ[q, 0]
```

Rule 1072

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)), x_Symbol] :> With[{q = c^2*d^2 - b*c*
d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*e*f + a^2*f^2}, Dist[1/q, Int[(A*
c^2*d - a*c*C*d - A*b*c*e + a*B*c*e + A*b^2*f - a*b*B*f - a*A*c*f + a^2*C*f
 + c*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B*f)*x)/(a + b*x + c*x^2),
x], x] + Dist[1/q, Int[(c*C*d^2 - B*c*d*e + A*c*e^2 + b*B*d*f - A*c*d*f - a
*C*d*f - A*b*e*f + a*A*f^2 - f*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B
*f)*x)/(d + e*x + f*x^2), x], x] /; NeQ[q, 0]] /; FreeQ[{a, b, c, d, e, f,
A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$\int \frac{1}{\left(3-x+2x^2\right)^3\left(2+3x+5x^2\right)}\,dx = \frac{13-6x}{1012\left(3-x+2x^2\right)^2} - \frac{\int \frac{-3652-1936x+990x^2}{\left(3-x+2x^2\right)^2\left(2+3x+5x^2\right)}\,dx}{11132}$$

$$= \frac{13-6x}{1012\left(3-x+2x^2\right)^2} + \frac{3625-746x}{256036\left(3-x+2x^2\right)} - \frac{\int \frac{-6551908-7779574x+902660}{\left(3-x+2x^2\right)\left(2+3x+5x^2\right)}}{61960712}$$

$$= \frac{13-6x}{1012\left(3-x+2x^2\right)^2} + \frac{3625-746x}{256036\left(3-x+2x^2\right)} - \frac{\int \frac{-154867174+335151124x}{3-x+2x^2}\,dx}{14994492304}$$

$$= \frac{13-6x}{1012\left(3-x+2x^2\right)^2} + \frac{3625-746x}{256036\left(3-x+2x^2\right)} + \frac{53403\int \frac{1}{3-x+2x^2}\,dx}{11265584} - 119$$

$$= \frac{13-6x}{1012\left(3-x+2x^2\right)^2} + \frac{3625-746x}{256036\left(3-x+2x^2\right)} - \frac{119\log\left(3-x+2x^2\right)}{21296} + 1$$

$$= \frac{13-6x}{1012\left(3-x+2x^2\right)^2} + \frac{3625-746x}{256036\left(3-x+2x^2\right)} - \frac{53403\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{5632792\sqrt{23}} + 24$$

Mathematica [A] time = 0.16, size = 99, normalized size = 0.86

$$\frac{713\left(-62951\log\left(2x^2-x+3\right)+62951\log\left(5x^2+3x+2\right)-\frac{44\left(1492x^3-7996x^2+7381x-14164\right)}{\left(-2x^2+x-3\right)^2}\right)+3310986\sqrt{23}\tan^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)+6010498\sqrt{31}\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{8032361392}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2)),x]
```

```
[Out] (3310986*Sqrt[23]*ArcTan[(-1 + 4*x)/Sqrt[23]] + 6010498*Sqrt[31]*ArcTan[(3
+ 10*x)/Sqrt[31]] + 713*((-44*(-14164 + 7381*x - 7996*x^2 + 1492*x^3))/(-3
+ x - 2*x^2)^2 - 62951*Log[3 - x + 2*x^2] + 62951*Log[2 + 3*x + 5*x^2]))/80
32361392
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(3-x+2x^2\right)^3\left(2+3x+5x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2)),x]
```

```
[Out] IntegrateAlgebraic[1/((3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2)), x]
```

fricas [A] time = 0.43, size = 177, normalized size = 1.54

$$-\frac{46807024\,x^{3}-6010498\,\sqrt{31}\left(4\,x^{4}-4\,x^{3}+13\,x^{2}-6\,x+9\right)\arctan\left(\frac{1}{31}\,\sqrt{31}\,(10\,x+3)\right)-3310986\,\sqrt{23}\left(4\,x^{4}-4\,x^{3}+13\,x^{2}-6\,x+9\right)\arctan\left(\frac{1}{23}\,\sqrt{23}\,(4\,x-1)\right)-250850512\,x^{2}-44884063\left(4\,x^{4}-4\,x^{3}+13\,x^{2}-6\,x+9\right)\log\left(5\,x^{2}+3\,x+2\right)+44884063\left(4\,x^{4}-4\,x^{3}+13\,x^{2}-6\,x+9\right)\log\left(2\,x^{2}-x+3\right)+231556732\,x-444353008}{8032361392\left(4\,x^{4}-4\,x^{3}+13\,x^{2}-6\,x+9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^3/(5*x^2+3*x+2),x, algorithm="fricas")
```

```
[Out] -1/8032361392*(46807024*x^3 - 6010498*sqrt(31)*(4*x^4 - 4*x^3 + 13*x^2 - 6*
x + 9)*arctan(1/31*sqrt(31)*(10*x + 3)) - 3310986*sqrt(23)*(4*x^4 - 4*x^3 +
 13*x^2 - 6*x + 9)*arctan(1/23*sqrt(23)*(4*x - 1)) - 250850512*x^2 - 448840
63*(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)*log(5*x^2 + 3*x + 2) + 44884063*(4*x^
4 - 4*x^3 + 13*x^2 - 6*x + 9)*log(2*x^2 - x + 3) + 231556732*x - 444353008)
/(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)
```

giac [A] time = 0.21, size = 88, normalized size = 0.77

$$\frac{247}{330088}\,\sqrt{31}\arctan\left(\frac{1}{31}\,\sqrt{31}\,(10\,x+3)\right)+\frac{53403}{129554216}\,\sqrt{23}\arctan\left(\frac{1}{23}\,\sqrt{23}\,(4\,x-1)\right)-\frac{1492\,x^{3}-7996\,x^{2}+7381\,x-14164}{256036\left(2\,x^{2}-x+3\right)^{2}}+\frac{119}{21296}\log\left(5\,x^{2}+3\,x+2\right)-\frac{119}{21296}\log\left(2\,x^{2}-x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^3/(5*x^2+3*x+2),x, algorithm="giac")
```

```
[Out] 247/330088*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 53403/129554216*sqrt
(23)*arctan(1/23*sqrt(23)*(4*x - 1)) - 1/256036*(1492*x^3 - 7996*x^2 + 7381
*x - 14164)/(2*x^2 - x + 3)^2 + 119/21296*log(5*x^2 + 3*x + 2) - 119/21296*
log(2*x^2 - x + 3)
```

maple [A] time = 0.01, size = 89, normalized size = 0.77

$$\frac{247\sqrt{31}\ \arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)}{330088}+\frac{53403\sqrt{23}\ \arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{129554216}-\frac{119\ln\left(2x^2-x+3\right)}{21296}+\frac{119\ln\left(5x^2+3x+2\right)}{21296}-\frac{\frac{8206}{529}x^3-\frac{43978}{529}x^2+\frac{81191}{1058}x-\frac{77902}{529}}{2662\left(2x^2-x+3\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^2-x+3)^3/(5*x^2+3*x+2),x)
```

```
[Out] 119/21296*ln(5*x^2+3*x+2)+247/330088*31^(1/2)*arctan(1/31*(10*x+3)*31^(1/2)
)-1/2662*(8206/529*x^3-43978/529*x^2+81191/1058*x-77902/529)/(2*x^2-x+3)^2-
119/21296*ln(2*x^2-x+3)+53403/129554216*23^(1/2)*arctan(1/23*(4*x-1)*23^(1/
2))
```

maxima [A] time = 0.97, size = 98, normalized size = 0.85

$$\frac{247}{330088}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right)+\frac{53403}{129554216}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right)-\frac{1492x^3-7996x^2+7381x-14164}{256036\left(4x^4-4x^3+13x^2-6x+9\right)}+\frac{119}{21296}\log\left(5x^2+3x+2\right)-\frac{119}{21296}\log\left(2x^2-x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^3/(5*x^2+3*x+2),x, algorithm="maxima")
```

```
[Out] 247/330088*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 53403/129554216*sqrt
(23)*arctan(1/23*sqrt(23)*(4*x - 1)) - 1/256036*(1492*x^3 - 7996*x^2 + 7381
*x - 14164)/(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9) + 119/21296*log(5*x^2 + 3*x
+ 2) - 119/21296*log(2*x^2 - x + 3)
```

mupad [B] time = 3.58, size = 116, normalized size = 1.01

$$-\ln\left(x+\frac{3}{10}-\frac{\sqrt{31}\,1\mathrm{i}}{10}\right)\left(-\frac{119}{21296}+\frac{\sqrt{31}\,247\mathrm{i}}{660176}\right)+\ln\left(x+\frac{3}{10}+\frac{\sqrt{31}\,1\mathrm{i}}{10}\right)\left(\frac{119}{21296}+\frac{\sqrt{31}\,247\mathrm{i}}{660176}\right)-\ln\left(x-\frac{1}{4}-\frac{\sqrt{23}\,1\mathrm{i}}{4}\right)\left(\frac{119}{21296}+\frac{\sqrt{23}\,53403\mathrm{i}}{259108432}\right)+\ln\left(x-\frac{1}{4}+\frac{\sqrt{23}\,1\mathrm{i}}{4}\right)\left(-\frac{119}{21296}+\frac{\sqrt{23}\,53403\mathrm{i}}{259108432}\right)-\frac{\frac{373x^3}{256036}-\frac{1999x^2}{256036}+\frac{61x}{8464}-\frac{3541}{256036}}{x^4-x^3+\frac{13x^2}{4}-\frac{3x}{2}+\frac{9}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*x^2 - x + 3)^3*(3*x + 5*x^2 + 2)),x)
```

```
[Out] log(x + (31^(1/2)*1i)/10 + 3/10)*((31^(1/2)*247i)/660176 + 119/21296) - log
(x - (31^(1/2)*1i)/10 + 3/10)*((31^(1/2)*247i)/660176 - 119/21296) - log(x
- (23^(1/2)*1i)/4 - 1/4)*((23^(1/2)*53403i)/259108432 + 119/21296) + log(x
+ (23^(1/2)*1i)/4 - 1/4)*((23^(1/2)*53403i)/259108432 - 119/21296) - ((61*x
)/8464 - (1999*x^2)/256036 + (373*x^3)/256036 - 3541/256036)/((13*x^2)/4 -
(3*x)/2 - x^3 + x^4 + 9/4)
```

sympy [A] time = 0.36, size = 122, normalized size = 1.06

$$\frac{-1492x^3+7996x^2-7381x+14164}{1024144x^4-1024144x^3+3328468x^2-1536216x+2304324}-\frac{119\log\left(x^2-\frac{x}{2}+\frac{3}{2}\right)}{21296}+\frac{119\log\left(x^2+\frac{3x}{5}+\frac{2}{5}\right)}{21296}+\frac{53403\sqrt{23}\ \operatorname{atan}\left(\frac{4\sqrt{23}\,x}{23}-\frac{\sqrt{23}}{23}\right)}{129554216}+\frac{247\sqrt{31}\ \operatorname{atan}\left(\frac{10\sqrt{31}\,x}{31}+\frac{3\sqrt{31}}{31}\right)}{330088}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**2-x+3)**3/(5*x**2+3*x+2),x)
```

```
[Out] (-1492*x**3 + 7996*x**2 - 7381*x + 14164)/(1024144*x**4 - 1024144*x**3 + 33
28468*x**2 - 1536216*x + 2304324) - 119*log(x**2 - x/2 + 3/2)/21296 + 119*l
og(x**2 + 3*x/5 + 2/5)/21296 + 53403*sqrt(23)*atan(4*sqrt(23)*x/23 - sqrt(2
3)/23)/129554216 + 247*sqrt(31)*atan(10*sqrt(31)*x/31 + 3*sqrt(31)/31)/3300
88
```

3.53 $\int \frac{1}{\left(3-x+2x^2\right)^3\left(2+3x+5x^2\right)^2}\,dx$

Optimal. Leaf size=160

$$\frac{-252815x-2328909}{174616552\left(5x^2+3x+2\right)}+\frac{9665-1446x}{512072\left(2x^2-x+3\right)\left(5x^2+3x+2\right)}+\frac{13-6x}{1012\left(2x^2-x+3\right)^2\left(5x^2+3x+2\right)}+\frac{181\log}{}$$

Rubi [A] time = 0.16, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {974, 1060, 1072, 634, 618, 204, 628}

$$\frac{9665-1446x}{512072\left(2x^2-x+3\right)\left(5x^2+3x+2\right)}-\frac{252815x+2328909}{174616552\left(5x^2+3x+2\right)}+\frac{13-6x}{1012\left(2x^2-x+3\right)^2\left(5x^2+3x+2\right)}+\frac{181\log\left(2x^2-x+3\right)}{468512}-\frac{181\log\left(5x^2+3x+2\right)}{468512}+\frac{2038497\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{123921424\sqrt{23}}+\frac{246757\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{7261936\sqrt{31}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2)^2),x]
```

```
[Out] -(2328909 + 252815*x)/(174616552*(2 + 3*x + 5*x^2)) + (13 - 6*x)/(1012*(3 -
 x + 2*x^2)^2*(2 + 3*x + 5*x^2)) + (9665 - 1446*x)/(512072*(3 - x + 2*x^2)*
(2 + 3*x + 5*x^2)) + (2038497*ArcTan[(1 - 4*x)/Sqrt[23]])/(123921424*Sqrt[2
3]) + (246757*ArcTan[(3 + 10*x)/Sqrt[31]])/(7261936*Sqrt[31]) + (181*Log[3
- x + 2*x^2])/468512 - (181*Log[2 + 3*x + 5*x^2])/468512
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 974

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*
a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*(
d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q,
 0]
```

Rule 1060

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x +
 c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
2*a*(c*d - a*f)))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !
```

```
IGtQ[q, 0]
```

Rule 1072

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)), x_Symbol] :> With[{q = c^2*d^2 - b*c*
d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*e*f + a^2*f^2}, Dist[1/q, Int[(A*
c^2*d - a*c*C*d - A*b*c*e + a*B*c*e + A*b^2*f - a*b*B*f - a*A*c*f + a^2*C*f
 + c*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B*f)*x)/(a + b*x + c*x^2),
x], x] + Dist[1/q, Int[(c*C*d^2 - B*c*d*e + A*c*e^2 + b*B*d*f - A*c*d*f - a
*C*d*f - A*b*e*f + a*A*f^2 - f*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B
*f)*x)/(d + e*x + f*x^2), x], x] /; NeQ[q, 0]] /; FreeQ[{a, b, c, d, e, f,
A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$\int \frac{1}{\left(3-x+2x^2\right)^3\left(2+3x+5x^2\right)^2}\,dx = \frac{13-6x}{1012\left(3-x+2x^2\right)^2\left(2+3x+5x^2\right)} - \frac{\int \frac{-4081-3168x+1650x^2}{\left(3-x+2x^2\right)^2\left(2+3x+5x^2\right)^2}\,dx}{11132}$$

$$= \frac{13-6x}{1012\left(3-x+2x^2\right)^2\left(2+3x+5x^2\right)} + \frac{9665-1446x}{512072\left(3-x+2x^2\right)\left(2+3x+5\right.}$$

$$= -\frac{2328909+252815x}{174616552\left(2+3x+5x^2\right)} + \frac{13-6x}{1012\left(3-x+2x^2\right)^2\left(2+3x+5x^2\right)} + \frac{}{51}$$

$$= -\frac{2328909+252815x}{174616552\left(2+3x+5x^2\right)} + \frac{13-6x}{1012\left(3-x+2x^2\right)^2\left(2+3x+5x^2\right)} + \frac{}{51}$$

$$= -\frac{2328909+252815x}{174616552\left(2+3x+5x^2\right)} + \frac{13-6x}{1012\left(3-x+2x^2\right)^2\left(2+3x+5x^2\right)} + \frac{}{51}$$

$$= -\frac{2328909+252815x}{174616552\left(2+3x+5x^2\right)} + \frac{13-6x}{1012\left(3-x+2x^2\right)^2\left(2+3x+5x^2\right)} + \frac{}{51}$$

$$= -\frac{2328909+252815x}{174616552\left(2+3x+5x^2\right)} + \frac{13-6x}{1012\left(3-x+2x^2\right)^2\left(2+3x+5x^2\right)} + \frac{}{51}$$

Mathematica [A] time = 0.12, size = 136, normalized size = 0.85

$$\frac{-2923x-1782}{1408198\left(2x^2-x+3\right)}+\frac{1235x-1474}{330088\left(5x^2+3x+2\right)}+\frac{-14x-31}{22264\left(2x^2-x+3\right)^2}+\frac{181\log\left(2x^2-x+3\right)}{468512}-\frac{181\log\left(5x^2+3x+2\right)}{468512}-\frac{2038497\tan^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{123921424\sqrt{23}}+\frac{246757\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{7261936\sqrt{31}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2)^2),x]
```

```
[Out] (-31 - 14*x)/(22264*(3 - x + 2*x^2)^2) + (-1782 - 2923*x)/(1408198*(3 - x +
 2*x^2)) + (-1474 + 1235*x)/(330088*(2 + 3*x + 5*x^2)) - (2038497*ArcTan[(-
1 + 4*x)/Sqrt[23]])/(123921424*Sqrt[23]) + (246757*ArcTan[(3 + 10*x)/Sqrt[3
1]])/(7261936*Sqrt[31]) + (181*Log[3 - x + 2*x^2])/468512 - (181*Log[2 + 3*
x + 5*x^2])/468512
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(3-x+2x^2\right)^3\left(2+3x+5x^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2)^2),x]
```

```
[Out] IntegrateAlgebraic[1/((3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2)^2), x]
```

fricas [A] time = 0.43, size = 227, normalized size = 1.42

$$-\frac{31725248720\,x^5+260524883872\,x^4-158204886268\,x^3-6004584838\,\sqrt{31}\left(20\,x^6-8\,x^5+61\,x^4+x^3+53\,x^2+15\,x+18\right)\arctan\left(\frac{1}{31}\,\sqrt{31}\,(10\,x+3)\right)+3917991234\,\sqrt{23}\left(20\,x^6-8\,x^5+61\,x^4+x^3+53\,x^2+15\,x+18\right)\arctan\left(\frac{1}{23}\,\sqrt{23}\,(4\,x-1)\right)+679966484692\,x^2+2116340147\left(20\,x^6-8\,x^5+61\,x^4+x^3+53\,x^2+15\,x+18\right)\log\left(5\,x^2+3\,x+2\right)-2116340147\left(20\,x^6-8\,x^5+61\,x^4+x^3+53\,x^2+15\,x+18\right)\log\left(2\,x^2-x+3\right)-184712689040\,x+277008109136}{5478070469344\left(20\,x^6-8\,x^5+61\,x^4+x^3+53\,x^2+15\,x+18\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^3/(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

```
[Out] -1/5478070469344*(31725248720*x^5 + 260524883872*x^4 - 158204886268*x^3 - 6
004584838*sqrt(31)*(20*x^6 - 8*x^5 + 61*x^4 + x^3 + 53*x^2 + 15*x + 18)*arc
tan(1/31*sqrt(31)*(10*x + 3)) + 3917991234*sqrt(23)*(20*x^6 - 8*x^5 + 61*x^
4 + x^3 + 53*x^2 + 15*x + 18)*arctan(1/23*sqrt(23)*(4*x - 1)) + 67996648469
2*x^2 + 2116340147*(20*x^6 - 8*x^5 + 61*x^4 + x^3 + 53*x^2 + 15*x + 18)*log
(5*x^2 + 3*x + 2) - 2116340147*(20*x^6 - 8*x^5 + 61*x^4 + x^3 + 53*x^2 + 15
*x + 18)*log(2*x^2 - x + 3) - 184712689040*x + 277008109136)/(20*x^6 - 8*x^
5 + 61*x^4 + x^3 + 53*x^2 + 15*x + 18)
```

giac [A] time = 0.22, size = 110, normalized size = 0.69

$$\frac{246757}{225120016}\,\sqrt{31}\arctan\left(\frac{1}{31}\,\sqrt{31}\,(10\,x+3)\right)-\frac{2038497}{2850192752}\,\sqrt{23}\arctan\left(\frac{1}{23}\,\sqrt{23}\,(4\,x-1)\right)-\frac{1011260\,x^5+8304376\,x^4-5042869\,x^3+21674311\,x^2-5887820\,x+8829788}{174616552\left(5\,x^2+3\,x+2\right)\left(2\,x^2-x+3\right)^2}-\frac{181}{468512}\log\left(5\,x^2+3\,x+2\right)+\frac{181}{468512}\log\left(2\,x^2-x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^3/(5*x^2+3*x+2)^2,x, algorithm="giac")
```

```
[Out] 246757/225120016*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) - 2038497/285019
2752*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) - 1/174616552*(1011260*x^5 +
8304376*x^4 - 5042869*x^3 + 21674311*x^2 - 5887820*x + 8829788)/((5*x^2 + 3
*x + 2)*(2*x^2 - x + 3)^2) - 181/468512*log(5*x^2 + 3*x + 2) + 181/468512*l
og(2*x^2 - x + 3)
```

maple [A] time = 0.01, size = 106, normalized size = 0.66

$$\frac{246757\sqrt{31}\ \arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)}{225120016}-\frac{2038497\sqrt{23}\ \arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{2850192752}+\frac{181\ln\left(2x^2-x+3\right)}{468512}-\frac{181\ln\left(5x^2+3x+2\right)}{468512}-\frac{-\frac{5434x}{31}+\frac{32428}{155}}{234256\left(x^2+\frac{3}{5}x+\frac{2}{5}\right)}+\frac{-\frac{128612}{529}x^3-\frac{14102}{529}x^2-\frac{173195}{529}x-\frac{321497}{1058}}{58564\left(2x^2-x+3\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^2-x+3)^3/(5*x^2+3*x+2)^2,x)
```

```
[Out] -1/234256*(-5434/31*x+32428/155)/(x^2+3/5*x+2/5)-181/468512*ln(5*x^2+3*x+2)
+246757/225120016*31^(1/2)*arctan(1/31*(10*x+3)*31^(1/2))+1/58564*(-128612/
529*x^3-14102/529*x^2-173195/529*x-321497/1058)/(2*x^2-x+3)^2+181/468512*ln
(2*x^2-x+3)-2038497/2850192752*23^(1/2)*arctan(1/23*(4*x-1)*23^(1/2))
```

maxima [A] time = 0.97, size = 116, normalized size = 0.72

$$\frac{246757}{225120016}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}\,(10\,x+3)\right)-\frac{2038497}{2850192752}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}\,(4\,x-1)\right)-\frac{1011260\,x^5+8304376\,x^4-5042869\,x^3+21674311\,x^2-5887820\,x+8829788}{174616552\left(20\,x^6-8\,x^5+61\,x^4+x^3+53\,x^2+15\,x+18\right)}-\frac{181}{468512}\log\left(5\,x^2+3\,x+2\right)+\frac{181}{468512}\log\left(2\,x^2-x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^3/(5*x^2+3*x+2)^2,x, algorithm="maxima")
```

```
[Out] 246757/225120016*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) - 2038497/285019
2752*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) - 1/174616552*(1011260*x^5 +
8304376*x^4 - 5042869*x^3 + 21674311*x^2 - 5887820*x + 8829788)/(20*x^6 - 8
*x^5 + 61*x^4 + x^3 + 53*x^2 + 15*x + 18) - 181/468512*log(5*x^2 + 3*x + 2)
 + 181/468512*log(2*x^2 - x + 3)
```

mupad [B] time = 3.60, size = 136, normalized size = 0.85

$$-\frac{\frac{50563\,x^5}{174616552}+\frac{1038047\,x^4}{436541380}-\frac{5042869\,x^3}{3492331040}+\frac{21674311\,x^2}{3492331040}-\frac{294391\,x}{174616552}+\frac{200677}{79371160}}{x^6-\frac{2\,x^5}{5}+\frac{61\,x^4}{20}+\frac{x^3}{20}+\frac{53\,x^2}{20}+\frac{3\,x}{4}+\frac{9}{10}}-\ln\left(x+\frac{3}{10}-\frac{\sqrt{31}\,1\mathrm{i}}{10}\right)\left(\frac{181}{468512}+\frac{\sqrt{31}\,246757\mathrm{i}}{450240032}\right)+\ln\left(x+\frac{3}{10}+\frac{\sqrt{31}\,1\mathrm{i}}{10}\right)\left(-\frac{181}{468512}+\frac{\sqrt{31}\,246757\mathrm{i}}{450240032}\right)+\ln\left(x-\frac{1}{4}-\frac{\sqrt{23}\,1\mathrm{i}}{4}\right)\left(\frac{181}{468512}+\frac{\sqrt{23}\,2038497\mathrm{i}}{5700385504}\right)-\ln\left(x-\frac{1}{4}+\frac{\sqrt{23}\,1\mathrm{i}}{4}\right)\left(-\frac{181}{468512}+\frac{\sqrt{23}\,2038497\mathrm{i}}{5700385504}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*x^2 - x + 3)^3*(3*x + 5*x^2 + 2)^2),x)
```

```
[Out] log(x + (31^(1/2)*1i)/10 + 3/10)*((31^(1/2)*246757i)/450240032 - 181/468512
) - log(x - (31^(1/2)*1i)/10 + 3/10)*((31^(1/2)*246757i)/450240032 + 181/46
```

```
8512) - ((21674311*x^2)/3492331040 - (294391*x)/174616552 - (5042869*x^3)/3
492331040 + (1038047*x^4)/436541380 + (50563*x^5)/174616552 + 200677/793711
60)/((3*x)/4 + (53*x^2)/20 + x^3/20 + (61*x^4)/20 - (2*x^5)/5 + x^6 + 9/10)
 + log(x - (23^(1/2)*1i)/4 - 1/4)*((23^(1/2)*2038497i)/5700385504 + 181/468
512) - log(x + (23^(1/2)*1i)/4 - 1/4)*((23^(1/2)*2038497i)/5700385504 - 181
/468512)
```

sympy [A] time = 0.43, size = 143, normalized size = 0.89

$$\frac{-1011260x^5 - 8304376x^4 + 5042869x^3 - 21674311x^2 + 5887820x - 8829788}{3492331040x^6 - 1396932416x^5 + 10651609672x^4 + 174616552x^3 + 9254677256x^2 + 2619248280x + 3143097936} + \frac{181\log\left(x^2 - \frac{x}{2} + \frac{3}{2}\right)}{468512} - \frac{181\log\left(x^2 + \frac{3x}{5} + \frac{2}{5}\right)}{468512} - \frac{2038497\sqrt{23}\operatorname{atan}\left(\frac{4\sqrt{23}x}{23} - \frac{\sqrt{23}}{23}\right)}{2850192752} + \frac{246757\sqrt{31}\operatorname{atan}\left(\frac{10\sqrt{31}x}{31} + \frac{3\sqrt{31}}{31}\right)}{225120016}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**2-x+3)**3/(5*x**2+3*x+2)**2,x)
```

```
[Out] (-1011260*x**5 - 8304376*x**4 + 5042869*x**3 - 21674311*x**2 + 5887820*x -
8829788)/(3492331040*x**6 - 1396932416*x**5 + 10651609672*x**4 + 174616552*
x**3 + 9254677256*x**2 + 2619248280*x + 3143097936) + 181*log(x**2 - x/2 +
3/2)/468512 - 181*log(x**2 + 3*x/5 + 2/5)/468512 - 2038497*sqrt(23)*atan(4*
sqrt(23)*x/23 - sqrt(23)/23)/2850192752 + 246757*sqrt(31)*atan(10*sqrt(31)*
x/31 + 3*sqrt(31)/31)/225120016
```

3.54 $\int \frac{1}{\left(3-x+2x^2\right)^3\left(2+3x+5x^2\right)^3}\,dx$

Optimal. Leaf size=181

$$\frac{5(302-35x)}{64009\left(2x^2-x+3\right)\left(5x^2+3x+2\right)^2}+\frac{15(7140435x+2618306)}{14886061058\left(5x^2+3x+2\right)}-\frac{5(77020x+223707)}{87308276\left(5x^2+3x+2\right)^2}+\frac{}{1012\left(2x^2-x\right.}$$

Rubi [A] time = 0.20, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {974, 1060, 1072, 634, 618, 204, 628}

$$\frac{5(302-35x)}{64009\left(2x^2-x+3\right)\left(5x^2+3x+2\right)^2}+\frac{15(7140435x+2618306)}{14886061058\left(5x^2+3x+2\right)}-\frac{5(77020x+223707)}{87308276\left(5x^2+3x+2\right)^2}+\frac{13-6x}{1012\left(2x^2-x+3\right)^2\left(5x^2+3x+2\right)^2}+\frac{405\log\left(2x^2-x+3\right)}{1288408}-\frac{405\log\left(5x^2+3x+2\right)}{1288408}-\frac{880575\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{340783916\sqrt{23}}+\frac{2768835\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{619080044\sqrt{31}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2)^3),x]
```

```
[Out] (-5*(223707 + 77020*x))/(87308276*(2 + 3*x + 5*x^2)^2) + (13 - 6*x)/(1012*(
3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)^2) + (5*(302 - 35*x))/(64009*(3 - x + 2*
x^2)*(2 + 3*x + 5*x^2)^2) + (15*(2618306 + 7140435*x))/(14886061058*(2 + 3*
x + 5*x^2)) - (880575*ArcTan[(1 - 4*x)/Sqrt[23]])/(340783916*Sqrt[23]) + (2
768835*ArcTan[(3 + 10*x)/Sqrt[31]])/(619080044*Sqrt[31]) + (405*Log[3 - x +
 2*x^2])/1288408 - (405*Log[2 + 3*x + 5*x^2])/1288408
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 974

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*
a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*(
d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q,
 0]
```

Rule 1060

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x +
 c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
2*a*(c*d - a*f)))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !
```

```
IGtQ[q, 0]
```

Rule 1072

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)), x_Symbol] :> With[{q = c^2*d^2 - b*c*
d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*e*f + a^2*f^2}, Dist[1/q, Int[(A*
c^2*d - a*c*C*d - A*b*c*e + a*B*c*e + A*b^2*f - a*b*B*f - a*A*c*f + a^2*C*f
 + c*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B*f)*x)/(a + b*x + c*x^2),
x], x] + Dist[1/q, Int[(c*C*d^2 - B*c*d*e + A*c*e^2 + b*B*d*f - A*c*d*f - a
*C*d*f - A*b*e*f + a*A*f^2 - f*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B
*f)*x)/(d + e*x + f*x^2), x], x] /; NeQ[q, 0]] /; FreeQ[{a, b, c, d, e, f,
A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$\int \frac{1}{\left(3-x+2x^2\right)^3\left(2+3x+5x^2\right)^3}\,dx = \frac{13-6x}{1012\left(3-x+2x^2\right)^2\left(2+3x+5x^2\right)^2} - \frac{\int \frac{-4510-4400x+2310x^2}{\left(3-x+2x^2\right)^2\left(2+3x+5x^2\right)^3}\,dx}{11132}$$

$$= \frac{13-6x}{1012\left(3-x+2x^2\right)^2\left(2+3x+5x^2\right)^2} + \frac{5(302-35x)}{64009\left(3-x+2x^2\right)\left(2+3x+5x^{2}\right.}$$

$$= -\frac{5(223707+77020x)}{87308276\left(2+3x+5x^2\right)^2} + \frac{13-6x}{1012\left(3-x+2x^2\right)^2\left(2+3x+5x^2\right)^2} + \frac{}{64}$$

$$= -\frac{5(223707+77020x)}{87308276\left(2+3x+5x^2\right)^2} + \frac{13-6x}{1012\left(3-x+2x^2\right)^2\left(2+3x+5x^2\right)^2} + \frac{}{64}$$

$$= -\frac{5(223707+77020x)}{87308276\left(2+3x+5x^2\right)^2} + \frac{13-6x}{1012\left(3-x+2x^2\right)^2\left(2+3x+5x^2\right)^2} + \frac{}{64}$$

$$= -\frac{5(223707+77020x)}{87308276\left(2+3x+5x^2\right)^2} + \frac{13-6x}{1012\left(3-x+2x^2\right)^2\left(2+3x+5x^2\right)^2} + \frac{}{64}$$

$$= -\frac{5(223707+77020x)}{87308276\left(2+3x+5x^2\right)^2} + \frac{13-6x}{1012\left(3-x+2x^2\right)^2\left(2+3x+5x^2\right)^2} + \frac{}{64}$$

$$= -\frac{5(223707+77020x)}{87308276\left(2+3x+5x^2\right)^2} + \frac{13-6x}{1012\left(3-x+2x^2\right)^2\left(2+3x+5x^2\right)^2} + \frac{}{64}$$

Mathematica [A] time = 0.10, size = 151, normalized size = 0.83

$$\frac{405\log\left(2x^2-x+3\right)}{1288408} - \frac{405\log\left(5x^2+3x+2\right)}{1288408} + \frac{6850x^3-9275x^2+11154x-4342}{345092\left(10x^4+x^3+16x^2+7x+6\right)^2} + \frac{5\left(42842610x^3-5711469x^2+51156233x+14085977\right)}{14886061058\left(10x^4+x^3+16x^2+7x+6\right)} + \frac{880575\tan^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{340783916\sqrt{23}} + \frac{2768835\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{619080044\sqrt{31}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2)^3),x]
```

```
[Out] (-4342 + 11154*x - 9275*x^2 + 6850*x^3)/(345092*(6 + 7*x + 16*x^2 + x^3 + 1
0*x^4)^2) + (5*(14085977 + 51156233*x - 5711469*x^2 + 42842610*x^3))/(14886
061058*(6 + 7*x + 16*x^2 + x^3 + 10*x^4)) + (880575*ArcTan[(-1 + 4*x)/Sqrt[
```

```
23]])/(340783916*Sqrt[23]) + (2768835*ArcTan[(3 + 10*x)/Sqrt[31]])/(6190800
44*Sqrt[31]) + (405*Log[3 - x + 2*x^2])/1288408 - (405*Log[2 + 3*x + 5*x^2]
)/1288408
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(3-x+2x^2\right)^3\left(2+3x+5x^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2)^3),x]
```

```
[Out] IntegrateAlgebraic[1/((3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2)^3), x]
```

fricas [A] time = 0.43, size = 297, normalized size = 1.64

$$\frac{67202918046000\,x^{7}-2238718468800\,x^{6}+186872434930060\,x^{5}+62827256425340\,x^{4}+173919793526820\,x^{3}+67376830890\,\sqrt{31}\left(100\,x^{8}+20\,x^{7}+321\,x^{6}+172\,x^{5}+390\,x^{4}+236\,x^{3}+241\,x^{2}+84\,x+36\right)\arctan\left(\frac{1}{31}\,\sqrt{31}\,(10\,x+3)\right)+52466419650\,\sqrt{23}\left(100\,x^{8}+20\,x^{7}+321\,x^{6}+172\,x^{5}+390\,x^{4}+236\,x^{3}+241\,x^{2}+84\,x+36\right)\arctan\left(\frac{1}{23}\,\sqrt{23}\,(4\,x-1)\right)+73595926401690\,x^{2}-146799174285\left(100\,x^{8}+20\,x^{7}+321\,x^{6}+172\,x^{5}+390\,x^{4}+236\,x^{3}+241\,x^{2}+84\,x+36\right)\log\left(5\,x^{2}+3\,x+2\right)+146799174285\left(100\,x^{8}+20\,x^{7}+321\,x^{6}+172\,x^{5}+390\,x^{4}+236\,x^{3}+241\,x^{2}+84\,x+36\right)\log\left(2\,x^{2}-x+3\right)+78707350628632\,x+7381223830244}{467005507511576\left(100\,x^{8}+20\,x^{7}+321\,x^{6}+172\,x^{5}+390\,x^{4}+236\,x^{3}+241\,x^{2}+84\,x+36\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^3/(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

```
[Out] 1/467005507511576*(67202918046000*x^7 - 2238718468800*x^6 + 186872434930060
*x^5 + 62827256425340*x^4 + 173919793526820*x^3 + 67376830890*sqrt(31)*(100
*x^8 + 20*x^7 + 321*x^6 + 172*x^5 + 390*x^4 + 236*x^3 + 241*x^2 + 84*x + 36
)*arctan(1/31*sqrt(31)*(10*x + 3)) + 52466419650*sqrt(23)*(100*x^8 + 20*x^7
 + 321*x^6 + 172*x^5 + 390*x^4 + 236*x^3 + 241*x^2 + 84*x + 36)*arctan(1/23
*sqrt(23)*(4*x - 1)) + 73595926401690*x^2 - 146799174285*(100*x^8 + 20*x^7
+ 321*x^6 + 172*x^5 + 390*x^4 + 236*x^3 + 241*x^2 + 84*x + 36)*log(5*x^2 +
3*x + 2) + 146799174285*(100*x^8 + 20*x^7 + 321*x^6 + 172*x^5 + 390*x^4 + 2
36*x^3 + 241*x^2 + 84*x + 36)*log(2*x^2 - x + 3) + 78707350628632*x + 73812
23830244)/(100*x^8 + 20*x^7 + 321*x^6 + 172*x^5 + 390*x^4 + 236*x^3 + 241*x
^2 + 84*x + 36)
```

giac [A] time = 0.22, size = 116, normalized size = 0.64

$$\frac{2768835}{19191481364}\,\sqrt{31}\,\arctan\left(\frac{1}{31}\,\sqrt{31}\,(10\,x+3)\right)+\frac{880575}{7838030068}\,\sqrt{23}\,\arctan\left(\frac{1}{23}\,\sqrt{23}\,(4\,x-1)\right)+\frac{4284261000\,x^{7}-142720800\,x^{6}+11913326210\,x^{5}+4005307690\,x^{4}+11087580870\,x^{3}+4691822415\,x^{2}+5017681412\,x+470561254}{29772122116\left(10\,x^{4}+x^{3}+16\,x^{2}+7\,x+6\right)^{2}}-\frac{405}{1288408}\,\log\left(5\,x^{2}+3\,x+2\right)+\frac{405}{1288408}\,\log\left(2\,x^{2}-x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^3/(5*x^2+3*x+2)^3,x, algorithm="giac")
```

```
[Out] 2768835/19191481364*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 880575/7838
030068*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 1/29772122116*(4284261000
*x^7 - 142720800*x^6 + 11913326210*x^5 + 4005307690*x^4 + 11087580870*x^3 +
```

```
 4691822415*x^2 + 5017681412*x + 470561254)/(10*x^4 + x^3 + 16*x^2 + 7*x +
6)^2 - 405/1288408*log(5*x^2 + 3*x + 2) + 405/1288408*log(2*x^2 - x + 3)
```

maple [A]   time = 0.01, size = 118, normalized size = 0.65

$$\frac{2768835\sqrt{31}\ \arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)}{19191481364}+\frac{880575\sqrt{23}\ \arctan\left(\frac{(4x-1)\sqrt{23}}{23}\right)}{7838030068}+\frac{405\ln\left(2x^2-x+3\right)}{1288408}-\frac{405\ln\left(5x^2+3x+2\right)}{1288408}-\frac{25\left(-\frac{3013197}{961}x^3-\frac{14516062}{4805}x^2-\frac{51193868}{24025}x-\frac{5423968}{24025}\right)}{2576816\left(5x^2+3x+2\right)^2}+\frac{\frac{302907}{529}x^3-\frac{368291}{529}x^2+\frac{2501587}{2116}x-\frac{665819}{1058}}{644204\left(2x^2-x+3\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^2-x+3)^3/(5*x^2+3*x+2)^3,x)
```

```
[Out] -25/2576816*(-3013197/961*x^3-14516062/4805*x^2-51193868/24025*x-5423968/24
025)/(5*x^2+3*x+2)^2-405/1288408*ln(5*x^2+3*x+2)+2768835/19191481364*31^(1/
2)*arctan(1/31*(10*x+3)*31^(1/2))+1/644204*(302907/529*x^3-368291/529*x^2+2
501587/2116*x-665819/1058)/(2*x^2-x+3)^2+405/1288408*ln(2*x^2-x+3)+880575/7
838030068*23^(1/2)*arctan(1/23*(4*x-1)*23^(1/2))
```

maxima [A]   time = 0.97, size = 138, normalized size = 0.76

$$\frac{2768835}{19191481364}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right)+\frac{880575}{7838030068}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right)+\frac{4284261000x^7-142720800x^6+11913326210x^5+4005307690x^4+11087580870x^3+4691822415x^2+5017681412x+470561254}{29772122116\left(100x^8+20x^7+321x^6+172x^5+390x^4+236x^3+241x^2+84x+36\right)}-\frac{405}{1288408}\log\left(5x^2+3x+2\right)+\frac{405}{1288408}\log\left(2x^2-x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^3/(5*x^2+3*x+2)^3,x, algorithm="maxima")
```

```
[Out] 2768835/19191481364*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 880575/7838
030068*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 1/29772122116*(4284261000
*x^7 - 142720800*x^6 + 11913326210*x^5 + 4005307690*x^4 + 11087580870*x^3 +
 4691822415*x^2 + 5017681412*x + 470561254)/(100*x^8 + 20*x^7 + 321*x^6 + 1
72*x^5 + 390*x^4 + 236*x^3 + 241*x^2 + 84*x + 36) - 405/1288408*log(5*x^2 +
 3*x + 2) + 405/1288408*log(2*x^2 - x + 3)
```

mupad [B]   time = 3.59, size = 155, normalized size = 0.86

$$\frac{\frac{21421305x^7}{14886061058}-\frac{356802x^6}{7443030529}+\frac{1191332621x^5}{297721221160}+\frac{400530769x^4}{297721221160}+\frac{1108758087x^3}{297721221160}+\frac{938364483x^2}{595442442320}+\frac{1254420353x}{744303052900}+\frac{235280627}{1488606105800}}{x^8+\frac{x^7}{5}+\frac{321x^6}{100}+\frac{43x^5}{25}+\frac{39x^4}{10}+\frac{59x^3}{25}+\frac{241x^2}{100}+\frac{21x}{25}+\frac{9}{25}}+\ln\left(x-\frac{1}{4}+\frac{\sqrt{23}\,1\mathrm{i}}{4}\right)\left(\frac{405}{1288408}+\frac{\sqrt{23}\,880575\mathrm{i}}{15676060136}\right)-\ln\left(x+\frac{3}{10}-\frac{\sqrt{31}\,1\mathrm{i}}{10}\right)\left(\frac{405}{1288408}+\frac{\sqrt{31}\,2768835\mathrm{i}}{38382962728}\right)+\ln\left(x+\frac{3}{10}+\frac{\sqrt{31}\,1\mathrm{i}}{10}\right)\left(-\frac{405}{1288408}+\frac{\sqrt{31}\,2768835\mathrm{i}}{38382962728}\right)-\ln\left(x-\frac{1}{4}-\frac{\sqrt{23}\,1\mathrm{i}}{4}\right)\left(-\frac{405}{1288408}+\frac{\sqrt{23}\,880575\mathrm{i}}{15676060136}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*x^2 - x + 3)^3*(3*x + 5*x^2 + 2)^3),x)
```

```
[Out] log(x + (23^(1/2)*1i)/4 - 1/4)*((23^(1/2)*880575i)/15676060136 + 405/128840
8) - log(x - (23^(1/2)*1i)/4 - 1/4)*((23^(1/2)*880575i)/15676060136 - 405/1
288408) - log(x - (31^(1/2)*1i)/10 + 3/10)*((31^(1/2)*2768835i)/38382962728
 + 405/1288408) + log(x + (31^(1/2)*1i)/10 + 3/10)*((31^(1/2)*2768835i)/383
82962728 - 405/1288408) + ((1254420353*x)/744303052900 + (938364483*x^2)/59
5442442320 + (1108758087*x^3)/297721221160 + (400530769*x^4)/297721221160 +
 (1191332621*x^5)/297721221160 - (356802*x^6)/7443030529 + (21421305*x^7)/1
```

```
4886061058 + 235280627/1488606105800)/((21*x)/25 + (241*x^2)/100 + (59*x^3)
/25 + (39*x^4)/10 + (43*x^5)/25 + (321*x^6)/100 + x^7/5 + x^8 + 9/25)
```

<u>**sympy**</u> **[A]** time = 0.47, size = 163, normalized size = 0.90

$$\frac{4284261000x^7 - 142720800x^6 + 11913326210x^5 + 4005307690x^4 + 11087580870x^3 + 4691822415x^2 + 5017681412x + 470561254}{2977212211600x^8 + 595442442320x^7 + 9556851199236x^6 + 5120805003952x^5 + 11611127625240x^4 + 7026220819376x^3 + 7175081429956x^2 + 2500858257744x + 1071796396176} + \frac{405\log\left(x^2 - \frac{x}{2} + \frac{3}{2}\right)}{1288408} - \frac{405\log\left(x^2 + \frac{3x}{5} + \frac{2}{5}\right)}{1288408} + \frac{880575\sqrt{23}\operatorname{atan}\left(\frac{4\sqrt{23}x}{23} - \frac{\sqrt{23}}{23}\right)}{7838030068} + \frac{2768835\sqrt{31}\operatorname{atan}\left(\frac{10\sqrt{31}x}{31} + \frac{3\sqrt{31}}{31}\right)}{19191481364}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**2-x+3)**3/(5*x**2+3*x+2)**3,x)
```

```
[Out] (4284261000*x**7 - 142720800*x**6 + 11913326210*x**5 + 4005307690*x**4 + 11
087580870*x**3 + 4691822415*x**2 + 5017681412*x + 470561254)/(2977212211600
*x**8 + 595442442320*x**7 + 9556851199236*x**6 + 5120805003952*x**5 + 11611
127625240*x**4 + 7026220819376*x**3 + 7175081429956*x**2 + 2500858257744*x
+ 1071796396176) + 405*log(x**2 - x/2 + 3/2)/1288408 - 405*log(x**2 + 3*x/5
 + 2/5)/1288408 + 880575*sqrt(23)*atan(4*sqrt(23)*x/23 - sqrt(23)/23)/78380
30068 + 2768835*sqrt(31)*atan(10*sqrt(31)*x/31 + 3*sqrt(31)/31)/19191481364
```

3.55 $\int \sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^4\,dx$

Optimal. Leaf size=208

$$-\frac{83948353\left(2x^2-x+3\right)^{3/2}x^2}{2293760}+\frac{804243809\left(2x^2-x+3\right)^{3/2}x}{36700160}+\frac{27185733541\left(2x^2-x+3\right)^{3/2}}{440401920}-\frac{359471503(1-4\ldots}{67108\ldots}$$

Rubi [A] time = 0.31, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1661, 640, 612, 619, 215}

$$\frac{125}{4}\left(2x^2-x+3\right)^{3/2}x^7+\frac{14125}{144}\left(2x^2-x+3\right)^{3/2}x^6+\frac{233225\left(2x^2-x+3\right)^{3/2}x^5}{1536}+\frac{4796405\left(2x^2-x+3\right)^{3/2}x^4}{43008}+\frac{8325631\left(2x^2-x+3\right)^{3/2}x^3}{1032192}-\frac{83948353\left(2x^2-x+3\right)^{3/2}x^2}{2293760}+\frac{804243809\left(2x^2-x+3\right)^{3/2}x}{36700160}+\frac{27185733541\left(2x^2-x+3\right)^{3/2}}{440401920}-\frac{359471503(1-4x)\sqrt{2x^2-x+3}}{67108864}-\frac{8267844569\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{134217728\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)^4,x]
```

```
[Out] (-359471503*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/67108864 + (27185733541*(3 - x +
 2*x^2)^(3/2))/440401920 + (804243809*x*(3 - x + 2*x^2)^(3/2))/36700160 - (
83948353*x^2*(3 - x + 2*x^2)^(3/2))/2293760 + (8325631*x^3*(3 - x + 2*x^2)^
(3/2))/1032192 + (4796405*x^4*(3 - x + 2*x^2)^(3/2))/43008 + (233225*x^5*(3
 - x + 2*x^2)^(3/2))/1536 + (14125*x^6*(3 - x + 2*x^2)^(3/2))/144 + (125*x^
7*(3 - x + 2*x^2)^(3/2))/4 - (8267844569*ArcSinh[(1 - 4*x)/Sqrt[23]])/(1342
17728*Sqrt[2])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \sqrt{3-x+2x^2}\,\left(2+3x+5x^2\right)^4\,dx = \frac{125}{4}x^7\left(3-x+2x^2\right)^{3/2}+\frac{1}{20}\int\sqrt{3-x+2x^2}\,\Big(320+1920x+7520x^2+1$$

$$= \frac{14125}{144}x^6\left(3-x+2x^2\right)^{3/2}+\frac{125}{4}x^7\left(3-x+2x^2\right)^{3/2}+\frac{1}{360}\int\sqrt{3-x+2x^2}$$

$$= \frac{233225x^5\left(3-x+2x^2\right)^{3/2}}{1536}+\frac{14125}{144}x^6\left(3-x+2x^2\right)^{3/2}+\frac{125}{4}x^7\left(3-x+2\right.$$

$$= \frac{4796405x^4\left(3-x+2x^2\right)^{3/2}}{43008}+\frac{233225x^5\left(3-x+2x^2\right)^{3/2}}{1536}+\frac{14125}{144}x^6\left(3-\right.$$

$$= \frac{8325631x^3\left(3-x+2x^2\right)^{3/2}}{1032192}+\frac{4796405x^4\left(3-x+2x^2\right)^{3/2}}{43008}+\frac{233225x^5\left(3\right.}{15}$$

$$= -\frac{83948353x^2\left(3-x+2x^2\right)^{3/2}}{2293760}+\frac{8325631x^3\left(3-x+2x^2\right)^{3/2}}{1032192}+\frac{4796405x}{}$$

$$= \frac{804243809x\left(3-x+2x^2\right)^{3/2}}{36700160}-\frac{83948353x^2\left(3-x+2x^2\right)^{3/2}}{2293760}+\frac{8325631x}{1}$$

$$= \frac{27185733541\left(3-x+2x^2\right)^{3/2}}{440401920}+\frac{804243809x\left(3-x+2x^2\right)^{3/2}}{36700160}-\frac{8394835}{}$$

$$= -\frac{359471503(1-4x)\sqrt{3-x+2x^2}}{67108864}+\frac{27185733541\left(3-x+2x^2\right)^{3/2}}{440401920}+\frac{804}{}$$

$$= -\frac{359471503(1-4x)\sqrt{3-x+2x^2}}{67108864}+\frac{27185733541\left(3-x+2x^2\right)^{3/2}}{440401920}+\frac{804}{}$$

$$= -\frac{359471503(1-4x)\sqrt{3-x+2x^2}}{67108864}+\frac{27185733541\left(3-x+2x^2\right)^{3/2}}{440401920}+\frac{804}{}$$

Mathematica [A] time = 0.30, size = 85, normalized size = 0.41

$$\frac{4\sqrt{2x^2-x+3}\left(1321205760000x^9+3486515200000x^8+6327795712000x^7+7725962035200x^6+7612808028160x^5+5354741991424x^4+2211683657856x^3-174418077792x^2+537752185764x+3801512106459\right)-2604371039235\sqrt{2}\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{84557168640}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)^4,x]
```

```
[Out] (4*Sqrt[3 - x + 2*x^2]*(3801512106459 + 537752185764*x - 174418077792*x^2 +
 2211683657856*x^3 + 5354741991424*x^4 + 7612808028160*x^5 + 7725962035200*
```

```
x^6 + 6327795712000*x^7 + 3486515200000*x^8 + 1321205760000*x^9) - 26043710
39235*Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]])/84557168640
```

IntegrateAlgebraic [A] time = 1.08, size = 100, normalized size = 0.48

$$\frac{\sqrt{2x^2-x+3}\left(1321205760000x^9+3486515200000x^8+6327795712000x^7+7725962035200x^6+7612808028160x^5+5354741991424x^4+2211683657856x^3-174418077792x^2+537752185764x+3801512106459\right)}{21139292160}-\frac{8267844569\log\left(2\sqrt{2}\sqrt{2x^2-x+3}-4x+1\right)}{134217728\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)^4,x]
```

```
[Out] (Sqrt[3 - x + 2*x^2]*(3801512106459 + 537752185764*x - 174418077792*x^2 + 2
211683657856*x^3 + 5354741991424*x^4 + 7612808028160*x^5 + 7725962035200*x^
6 + 6327795712000*x^7 + 3486515200000*x^8 + 1321205760000*x^9))/21139292160
 - (8267844569*Log[1 - 4*x + 2*Sqrt[2]*Sqrt[3 - x + 2*x^2]])/(134217728*Sqr
t[2])
```

fricas [A] time = 0.44, size = 98, normalized size = 0.47

$$\frac{1}{21139292160}\left(1321205760000\,x^9+3486515200000\,x^8+6327795712000\,x^7+7725962035200\,x^6+7612808028160\,x^5+5354741991424\,x^4+2211683657856\,x^3-174418077792\,x^2+537752185764\,x+3801512106459\right)\sqrt{2\,x^2-x+3}+\frac{8267844569}{536870912}\sqrt{2}\log\left(-4\sqrt{2}\sqrt{2\,x^2-x+3}\,(4\,x-1)-32\,x^2+16\,x-25\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^4*(2*x^2-x+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/21139292160*(1321205760000*x^9 + 3486515200000*x^8 + 6327795712000*x^7 +
7725962035200*x^6 + 7612808028160*x^5 + 5354741991424*x^4 + 2211683657856*x
^3 - 174418077792*x^2 + 537752185764*x + 3801512106459)*sqrt(2*x^2 - x + 3)
 + 8267844569/536870912*sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1
) - 32*x^2 + 16*x - 25)
```

giac [A] time = 0.25, size = 93, normalized size = 0.45

$$\frac{1}{21139292160}\left(4\left(8\left(4\left(16\left(20\left(40\left(140\left(160\left(36\,x+95\right)x+27587\right)x+4715553\right)x+185859571\right)x+2614620113\right)x+17278778577\right)x-5450564931\right)x+134438046441\right)x+3801512106459\right)\sqrt{2\,x^2-x+3}-\frac{8267844569}{268435456}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}\,x-\sqrt{2\,x^2-x+3}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^4*(2*x^2-x+3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/21139292160*(4*(8*(4*(16*(20*(40*(140*(160*(36*x + 95)*x + 27587)*x + 471
5553)*x + 185859571)*x + 2614620113)*x + 17278778577)*x - 5450564931)*x + 1
34438046441)*x + 3801512106459)*sqrt(2*x^2 - x + 3) - 8267844569/268435456*
sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1)
```

maple [A] time = 0.03, size = 166, normalized size = 0.80

$$\frac{125\left(2x^2-x+3\right)^{\frac{3}{2}}x^7}{4}+\frac{14125\left(2x^2-x+3\right)^{\frac{3}{2}}x^6}{144}+\frac{233225\left(2x^2-x+3\right)^{\frac{3}{2}}x^5}{1536}+\frac{4796405\left(2x^2-x+3\right)^{\frac{3}{2}}x^4}{43008}+\frac{8325631\left(2x^2-x+3\right)^{\frac{3}{2}}x^3}{1032192}-\frac{83948353\left(2x^2-x+3\right)^{\frac{3}{2}}x^2}{2293760}+\frac{804243809\left(2x^2-x+3\right)^{\frac{3}{2}}x}{36700160}+\frac{8267844569\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{268435456}+\frac{27185733541\left(2x^2-x+3\right)^{\frac{3}{2}}}{440401920}+\frac{359471503\,(4x-1)\sqrt{2x^2-x+3}}{67108864}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)^4*(2*x^2-x+3)^(1/2),x)
```

```
[Out] 125/4*x^7*(2*x^2-x+3)^(3/2)+27185733541/440401920*(2*x^2-x+3)^(3/2)+14125/1
44*x^6*(2*x^2-x+3)^(3/2)+233225/1536*x^5*(2*x^2-x+3)^(3/2)+4796405/43008*x^
4*(2*x^2-x+3)^(3/2)+8325631/1032192*x^3*(2*x^2-x+3)^(3/2)-83948353/2293760*
x^2*(2*x^2-x+3)^(3/2)+804243809/36700160*x*(2*x^2-x+3)^(3/2)+8267844569/268
435456*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))+359471503/67108864*(4*x-1)*(2
*x^2-x+3)^(1/2)
```

maxima [A] time = 1.01, size = 177, normalized size = 0.85

$$\frac{125}{4}\left(2x^2-x+3\right)^{\frac{3}{2}}x^7+\frac{14125}{144}\left(2x^2-x+3\right)^{\frac{3}{2}}x^6+\frac{233225}{1536}\left(2x^2-x+3\right)^{\frac{3}{2}}x^5+\frac{4796405}{43008}\left(2x^2-x+3\right)^{\frac{3}{2}}x^4+\frac{8325631}{1032192}\left(2x^2-x+3\right)^{\frac{3}{2}}x^3-\frac{83948353}{2293760}\left(2x^2-x+3\right)^{\frac{3}{2}}x^2+\frac{804243809}{36700160}\left(2x^2-x+3\right)^{\frac{3}{2}}x+\frac{27185733541}{440401920}\left(2x^2-x+3\right)^{\frac{3}{2}}+\frac{359471503}{16777216}\sqrt{2x^2-x+3}\,x+\frac{8267844569}{268435456}\sqrt{2}\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}\left(4x-1\right)\right)-\frac{359471503}{67108864}\sqrt{2x^2-x+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^4*(2*x^2-x+3)^(1/2),x, algorithm="maxima")
```

```
[Out] 125/4*(2*x^2 - x + 3)^(3/2)*x^7 + 14125/144*(2*x^2 - x + 3)^(3/2)*x^6 + 233
225/1536*(2*x^2 - x + 3)^(3/2)*x^5 + 4796405/43008*(2*x^2 - x + 3)^(3/2)*x^
4 + 8325631/1032192*(2*x^2 - x + 3)^(3/2)*x^3 - 83948353/2293760*(2*x^2 - x
 + 3)^(3/2)*x^2 + 804243809/36700160*(2*x^2 - x + 3)^(3/2)*x + 27185733541/
440401920*(2*x^2 - x + 3)^(3/2) + 359471503/16777216*sqrt(2*x^2 - x + 3)*x
+ 8267844569/268435456*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 359471503
/67108864*sqrt(2*x^2 - x + 3)
```

mupad [B] time = 5.03, size = 221, normalized size = 1.06

$$\frac{8325631\,x^3\left(2x^2-x+3\right)^{3/2}}{1032192}-\frac{83948353\,x^2\left(2x^2-x+3\right)^{3/2}}{2293760}+\frac{4796405\,x^4\left(2x^2-x+3\right)^{3/2}}{43008}+\frac{233225\,x^5\left(2x^2-x+3\right)^{3/2}}{1536}+\frac{14125\,x^6\left(2x^2-x+3\right)^{3/2}}{144}+\frac{125\,x^7\left(2x^2-x+3\right)^{3/2}}{4}-\frac{41987163941\sqrt{2}\,\ln\left(\sqrt{2x^2-x+3}+\frac{\sqrt{2}\left(2x-\frac{1}{2}\right)}{2}\right)}{1174405120}-\frac{1825528867\left(\frac{x}{2}-\frac{1}{8}\right)\sqrt{2x^2-x+3}}{36700160}+\frac{27185733541\sqrt{2x^2-x+3}\,\left(32x^2-4x+45\right)}{7046430720}+\frac{804243809\,x\left(2x^2-x+3\right)^{3/2}}{36700160}+\frac{625271871443\sqrt{2}\,\ln\left(2\sqrt{2x^2-x+3}+\frac{\sqrt{2}\,(4x-1)}{2}\right)}{9395240960}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^(1/2)*(3*x + 5*x^2 + 2)^4,x)
```

```
[Out] (8325631*x^3*(2*x^2 - x + 3)^(3/2))/1032192 - (83948353*x^2*(2*x^2 - x + 3)
^(3/2))/2293760 + (4796405*x^4*(2*x^2 - x + 3)^(3/2))/43008 + (233225*x^5*(
2*x^2 - x + 3)^(3/2))/1536 + (14125*x^6*(2*x^2 - x + 3)^(3/2))/144 + (125*x
^7*(2*x^2 - x + 3)^(3/2))/4 - (41987163941*2^(1/2)*log((2*x^2 - x + 3)^(1/2
) + (2^(1/2)*(2*x - 1/2))/2))/1174405120 - (1825528867*(x/2 - 1/8)*(2*x^2 -
 x + 3)^(1/2))/36700160 + (27185733541*(2*x^2 - x + 3)^(1/2)*(32*x^2 - 4*x
+ 45))/7046430720 + (804243809*x*(2*x^2 - x + 3)^(3/2))/36700160 + (6252718
71443*2^(1/2)*log(2*(2*x^2 - x + 3)^(1/2) + (2^(1/2)*(4*x - 1))/2))/9395240
960
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{2x^2-x+3}\,\left(5x^2+3x+2\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)**4*(2*x**2-x+3)**(1/2),x)
[Out] Integral(sqrt(2*x**2 - x + 3)*(5*x**2 + 3*x + 2)**4, x)
```

3.56 $\int \sqrt{3-x+2x^2}\,\left(2+3x+5x^2\right)^3\,dx$

Optimal. Leaf size=166

$$\frac{531681\left(2x^2-x+3\right)^{3/2}x^2}{71680}-\frac{9627393\left(2x^2-x+3\right)^{3/2}x}{1146880}-\frac{22548119\left(2x^2-x+3\right)^{3/2}}{4587520}-\frac{6766097(1-4x)\sqrt{2x^2-x+}}{2097152}$$

Rubi [A] time = 0.18, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1661, 640, 612, 619, 215}

$$\frac{125}{16}\left(2x^2-x+3\right)^{3/2}x^5+\frac{8825}{448}\left(2x^2-x+3\right)^{3/2}x^4+\frac{247435\left(2x^2-x+3\right)^{3/2}x^3}{10752}+\frac{531681\left(2x^2-x+3\right)^{3/2}x^2}{71680}-\frac{9627393\left(2x^2-x+3\right)^{3/2}x}{1146880}-\frac{22548119\left(2x^2-x+3\right)^{3/2}}{4587520}-\frac{6766097(1-4x)\sqrt{2x^2-x+3}}{2097152}-\frac{155620231\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{4194304\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] (-6766097*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/2097152 - (22548119*(3 - x + 2*x^2
)^(3/2))/4587520 - (9627393*x*(3 - x + 2*x^2)^(3/2))/1146880 + (531681*x^2*
(3 - x + 2*x^2)^(3/2))/71680 + (247435*x^3*(3 - x + 2*x^2)^(3/2))/10752 + (
8825*x^4*(3 - x + 2*x^2)^(3/2))/448 + (125*x^5*(3 - x + 2*x^2)^(3/2))/16 -
(155620231*ArcSinh[(1 - 4*x)/Sqrt[23]])/(4194304*Sqrt[2])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
```

```
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^3 dx = \frac{125}{16}x^5\left(3-x+2x^2\right)^{3/2} + \frac{1}{16}\int \sqrt{3-x+2x^2}\left(128+576x+1824x^2+3\right.$$

$$= \frac{8825}{448}x^4\left(3-x+2x^2\right)^{3/2} + \frac{125}{16}x^5\left(3-x+2x^2\right)^{3/2} + \frac{1}{224}\int \sqrt{3-x+2x}$$

$$= \frac{247435x^3\left(3-x+2x^2\right)^{3/2}}{10752} + \frac{8825}{448}x^4\left(3-x+2x^2\right)^{3/2} + \frac{125}{16}x^5\left(3-x+2\right.$$

$$= \frac{531681x^2\left(3-x+2x^2\right)^{3/2}}{71680} + \frac{247435x^3\left(3-x+2x^2\right)^{3/2}}{10752} + \frac{8825}{448}x^4\left(3-x\right.$$

$$= -\frac{9627393x\left(3-x+2x^2\right)^{3/2}}{1146880} + \frac{531681x^2\left(3-x+2x^2\right)^{3/2}}{71680} + \frac{247435x^3\left(3\right.}{10}$$

$$= -\frac{22548119\left(3-x+2x^2\right)^{3/2}}{4587520} - \frac{9627393x\left(3-x+2x^2\right)^{3/2}}{1146880} + \frac{531681x^2\left(3\right.}{71}$$

$$= -\frac{6766097(1-4x)\sqrt{3-x+2x^2}}{2097152} - \frac{22548119\left(3-x+2x^2\right)^{3/2}}{4587520} - \frac{962739}{}$$

$$= -\frac{6766097(1-4x)\sqrt{3-x+2x^2}}{2097152} - \frac{22548119\left(3-x+2x^2\right)^{3/2}}{4587520} - \frac{962739}{}$$

$$= -\frac{6766097(1-4x)\sqrt{3-x+2x^2}}{2097152} - \frac{22548119\left(3-x+2x^2\right)^{3/2}}{4587520} - \frac{962739}{}$$

Mathematica [A] time = 0.17, size = 75, normalized size = 0.45

$$\frac{4\sqrt{2x^2-x+3}\left(3440640000x^7+6955008000x^6+10958233600x^5+11212171264x^4+9872163456x^3+4583812128x^2-1621307916x-3957369321\right)-16340124255\sqrt{2}\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{880803840}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] (4*Sqrt[3 - x + 2*x^2]*(-3957369321 - 1621307916*x + 4583812128*x^2 + 98721
63456*x^3 + 11212171264*x^4 + 10958233600*x^5 + 6955008000*x^6 + 3440640000
*x^7) - 16340124255*Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]])/880803840
```

IntegrateAlgebraic [A] time = 0.87, size = 90, normalized size = 0.54

$$\frac{\sqrt{2x^2-x+3}\left(3440640000x^7+6955008000x^6+10958233600x^5+11212171264x^4+9872163456x^3+4583812128x^2-1621307916x-3957369321\right)}{220200960}-\frac{155620231\log\left(2\sqrt{2}\sqrt{2x^2-x+3}-4x+1\right)}{4194304\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] (Sqrt[3 - x + 2*x^2]*(-3957369321 - 1621307916*x + 4583812128*x^2 + 9872163
456*x^3 + 11212171264*x^4 + 10958233600*x^5 + 6955008000*x^6 + 3440640000*x
^7))/220200960 - (155620231*Log[1 - 4*x + 2*Sqrt[2]*Sqrt[3 - x + 2*x^2]])/(
4194304*Sqrt[2])
```

fricas [A] time = 0.43, size = 88, normalized size = 0.53

$$\frac{1}{220200960}\left(3440640000x^7+6955008000x^6+10958233600x^5+11212171264x^4+9872163456x^3+4583812128x^2-1621307916x-3957369321\right)\sqrt{2x^2-x+3}+\frac{155620231}{16777216}\sqrt{2}\log\left(-4\sqrt{2}\sqrt{2x^2-x+3}(4x-1)-32x^2+16x-25\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^3*(2*x^2-x+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/220200960*(3440640000*x^7 + 6955008000*x^6 + 10958233600*x^5 + 1121217126
4*x^4 + 9872163456*x^3 + 4583812128*x^2 - 1621307916*x - 3957369321)*sqrt(2
*x^2 - x + 3) + 155620231/16777216*sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2 - x +
3)*(4*x - 1) - 32*x^2 + 16*x - 25)
```

giac [A] time = 0.25, size = 83, normalized size = 0.50

$$\frac{1}{220200960}(4(8(4(16(100(120(140x+283)x+53507)x+5474693)x+77126277)x+143244129)x-405326979)x-3957369321)\sqrt{2x^2-x+3}-\frac{155620231}{8388608}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^3*(2*x^2-x+3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/220200960*(4*(8*(4*(16*(100*(120*(140*x + 283)*x + 53507)*x + 5474693)*x
+ 77126277)*x + 143244129)*x - 405326979)*x - 3957369321)*sqrt(2*x^2 - x +
```

```
3) - 155620231/8388608*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x +
 3)) + 1)
```

maple [A] time = 0.01, size = 132, normalized size = 0.80

$$\frac{125\left(2x^{2}-x+3\right)^{\frac{3}{2}}x^{5}}{16}+\frac{8825\left(2x^{2}-x+3\right)^{\frac{3}{2}}x^{4}}{448}+\frac{247435\left(2x^{2}-x+3\right)^{\frac{3}{2}}x^{3}}{10752}+\frac{531681\left(2x^{2}-x+3\right)^{\frac{3}{2}}x^{2}}{71680}-\frac{9627393\left(2x^{2}-x+3\right)^{\frac{3}{2}}x}{1146880}+\frac{155620231\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{8388608}-\frac{22548119\left(2x^{2}-x+3\right)^{\frac{3}{2}}}{4587520}+\frac{6766097\left(4x-1\right)\sqrt{2x^{2}-x+3}}{2097152}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)^3*(2*x^2-x+3)^(1/2),x)
```

```
[Out] -22548119/4587520*(2*x^2-x+3)^(3/2)+125/16*(2*x^2-x+3)^(3/2)*x^5+8825/448*(
2*x^2-x+3)^(3/2)*x^4+247435/10752*(2*x^2-x+3)^(3/2)*x^3+531681/71680*(2*x^2
-x+3)^(3/2)*x^2-9627393/1146880*(2*x^2-x+3)^(3/2)*x+155620231/8388608*2^(1/
2)*arcsinh(4/23*23^(1/2)*(x-1/4))+6766097/2097152*(4*x-1)*(2*x^2-x+3)^(1/2)
```

maxima [A] time = 0.98, size = 143, normalized size = 0.86

$$\frac{125}{16}\left(2x^{2}-x+3\right)^{\frac{3}{2}}x^{5}+\frac{8825}{448}\left(2x^{2}-x+3\right)^{\frac{3}{2}}x^{4}+\frac{247435}{10752}\left(2x^{2}-x+3\right)^{\frac{3}{2}}x^{3}+\frac{531681}{71680}\left(2x^{2}-x+3\right)^{\frac{3}{2}}x^{2}-\frac{9627393}{1146880}\left(2x^{2}-x+3\right)^{\frac{3}{2}}x-\frac{22548119}{4587520}\left(2x^{2}-x+3\right)^{\frac{3}{2}}+\frac{6766097}{524288}\sqrt{2x^{2}-x+3}\,x+\frac{155620231}{8388608}\sqrt{2}\,\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}\left(4x-1\right)\right)-\frac{6766097}{2097152}\sqrt{2x^{2}-x+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^3*(2*x^2-x+3)^(1/2),x, algorithm="maxima")
```

```
[Out] 125/16*(2*x^2 - x + 3)^(3/2)*x^5 + 8825/448*(2*x^2 - x + 3)^(3/2)*x^4 + 247
435/10752*(2*x^2 - x + 3)^(3/2)*x^3 + 531681/71680*(2*x^2 - x + 3)^(3/2)*x^
2 - 9627393/1146880*(2*x^2 - x + 3)^(3/2)*x - 22548119/4587520*(2*x^2 - x +
 3)^(3/2) + 6766097/524288*sqrt(2*x^2 - x + 3)*x + 155620231/8388608*sqrt(2
)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 6766097/2097152*sqrt(2*x^2 - x + 3)
```

mupad [B] time = 4.69, size = 187, normalized size = 1.13

$$\frac{531681\,x^{2}\left(2x^{2}-x+3\right)^{3/2}}{71680}+\frac{247435\,x^{3}\left(2x^{2}-x+3\right)^{3/2}}{10752}+\frac{8825\,x^{4}\left(2x^{2}-x+3\right)^{3/2}}{448}+\frac{125\,x^{5}\left(2x^{2}-x+3\right)^{3/2}}{16}+\frac{875316037\sqrt{2}\,\ln\left(\sqrt{2x^{2}-x+3}+\frac{\sqrt{2}\left(2x-\frac{1}{2}\right)}{2}\right)}{36700160}+\frac{38057219\left(\frac{x}{2}-\frac{1}{8}\right)\sqrt{2x^{2}-x+3}}{1146880}-\frac{22548119\sqrt{2x^{2}-x+3}\left(32x^{2}-4x+45\right)}{73400320}-\frac{9627393\,x\left(2x^{2}-x+3\right)^{3/2}}{1146880}-\frac{1555820211\sqrt{2}\,\ln\left(2\sqrt{2x^{2}-x+3}+\frac{\sqrt{2}\left(4x-1\right)}{2}\right)}{293601280}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^(1/2)*(3*x + 5*x^2 + 2)^3,x)
```

```
[Out] (531681*x^2*(2*x^2 - x + 3)^(3/2))/71680 + (247435*x^3*(2*x^2 - x + 3)^(3/2
))/10752 + (8825*x^4*(2*x^2 - x + 3)^(3/2))/448 + (125*x^5*(2*x^2 - x + 3)^
(3/2))/16 + (875316037*2^(1/2)*log((2*x^2 - x + 3)^(1/2) + (2^(1/2)*(2*x -
1/2))/2))/36700160 + (38057219*(x/2 - 1/8)*(2*x^2 - x + 3)^(1/2))/1146880 -
 (22548119*(2*x^2 - x + 3)^(1/2)*(32*x^2 - 4*x + 45))/73400320 - (9627393*x
*(2*x^2 - x + 3)^(3/2))/1146880 - (1555820211*2^(1/2)*log(2*(2*x^2 - x + 3)
^(1/2) + (2^(1/2)*(4*x - 1))/2))/293601280
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{2x^2 - x + 3}\,\left(5x^2 + 3x + 2\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)**3*(2*x**2-x+3)**(1/2),x)
```

```
[Out] Integral(sqrt(2*x**2 - x + 3)*(5*x**2 + 3*x + 2)**3, x)
```

3.57 $\int \sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^2 dx$

Optimal. Leaf size=124

$$\frac{63}{16}\left(2x^2-x+3\right)^{3/2}x^2+\frac{769}{256}\left(2x^2-x+3\right)^{3/2}x-\frac{2107\left(2x^2-x+3\right)^{3/2}}{3072}+\frac{12371(1-4x)\sqrt{2x^2-x+3}}{16384}+\frac{25}{12}\left(2x^2-x\right.$$

Rubi [A] time = 0.10, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1661, 640, 612, 619, 215}

$$\frac{25}{12}\left(2x^2-x+3\right)^{3/2}x^3+\frac{63}{16}\left(2x^2-x+3\right)^{3/2}x^2+\frac{769}{256}\left(2x^2-x+3\right)^{3/2}x-\frac{2107\left(2x^2-x+3\right)^{3/2}}{3072}+\frac{12371(1-4x)\sqrt{2x^2-x+3}}{16384}+\frac{284533\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{32768\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] (12371*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/16384 - (2107*(3 - x + 2*x^2)^(3/2))/
3072 + (769*x*(3 - x + 2*x^2)^(3/2))/256 + (63*x^2*(3 - x + 2*x^2)^(3/2))/1
6 + (25*x^3*(3 - x + 2*x^2)^(3/2))/12 + (284533*ArcSinh[(1 - 4*x)/Sqrt[23]]
)/(32768*Sqrt[2])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
```

```
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^2 dx = \frac{25}{12}x^3\left(3-x+2x^2\right)^{3/2} + \frac{1}{12}\int \sqrt{3-x+2x^2}\left(48+144x+123x^2+\frac{945x^3}{2}\right.$$

$$= \frac{63}{16}x^2\left(3-x+2x^2\right)^{3/2} + \frac{25}{12}x^3\left(3-x+2x^2\right)^{3/2} + \frac{1}{120}\int \sqrt{3-x+2x^2}\left(48\right.$$

$$= \frac{769}{256}x\left(3-x+2x^2\right)^{3/2} + \frac{63}{16}x^2\left(3-x+2x^2\right)^{3/2} + \frac{25}{12}x^3\left(3-x+2x^2\right)^{3/2} +$$

$$= -\frac{2107\left(3-x+2x^2\right)^{3/2}}{3072} + \frac{769}{256}x\left(3-x+2x^2\right)^{3/2} + \frac{63}{16}x^2\left(3-x+2x^2\right)^{3/2} +$$

$$= \frac{12371(1-4x)\sqrt{3-x+2x^2}}{16384} - \frac{2107\left(3-x+2x^2\right)^{3/2}}{3072} + \frac{769}{256}x\left(3-x+2x^2\right.$$

$$= \frac{12371(1-4x)\sqrt{3-x+2x^2}}{16384} - \frac{2107\left(3-x+2x^2\right)^{3/2}}{3072} + \frac{769}{256}x\left(3-x+2x^2\right.$$

$$= \frac{12371(1-4x)\sqrt{3-x+2x^2}}{16384} - \frac{2107\left(3-x+2x^2\right)^{3/2}}{3072} + \frac{769}{256}x\left(3-x+2x^2\right.$$

Mathematica [A] time = 0.10, size = 65, normalized size = 0.52

$$\frac{4\sqrt{2x^2-x+3}\left(204800x^5+284672x^4+408960x^3+365536x^2+328204x-64023\right)+853599\sqrt{2}\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{196608}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] (4*Sqrt[3 - x + 2*x^2]*(-64023 + 328204*x + 365536*x^2 + 408960*x^3 + 28467
2*x^4 + 204800*x^5) + 853599*Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]])/196608
```

IntegrateAlgebraic [A] time = 0.56, size = 80, normalized size = 0.65

$$\frac{284533\log\left(2\sqrt{2}\sqrt{2x^2-x+3}-4x+1\right)}{32768\sqrt{2}}+\frac{\sqrt{2x^2-x+3}\left(204800x^5+284672x^4+408960x^3+365536x^2+328204x-64023\right)}{49152}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] (Sqrt[3 - x + 2*x^2]*(-64023 + 328204*x + 365536*x^2 + 408960*x^3 + 284672*
x^4 + 204800*x^5))/49152 + (284533*Log[1 - 4*x + 2*Sqrt[2]*Sqrt[3 - x + 2*x
^2]])/(32768*Sqrt[2])
```

fricas [A] time = 0.42, size = 78, normalized size = 0.63

$$\frac{1}{49152}\left(204800\,x^5+284672\,x^4+408960\,x^3+365536\,x^2+328204\,x-64023\right)\sqrt{2\,x^2-x+3}+\frac{284533}{131072}\sqrt{2}\log\left(4\sqrt{2}\sqrt{2\,x^2-x+3}(4\,x-1)-32\,x^2+16\,x-25\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^2*(2*x^2-x+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/49152*(204800*x^5 + 284672*x^4 + 408960*x^3 + 365536*x^2 + 328204*x - 640
23)*sqrt(2*x^2 - x + 3) + 284533/131072*sqrt(2)*log(4*sqrt(2)*sqrt(2*x^2 -
x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25)
```

giac [A] time = 0.22, size = 73, normalized size = 0.59

$$\frac{1}{49152}(4(8(4(16(100\,x+139)x+3195)x+11423)x+82051)x-64023)\sqrt{2\,x^2-x+3}+\frac{284533}{65536}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}\,x-\sqrt{2\,x^2-x+3}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^2*(2*x^2-x+3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/49152*(4*(8*(4*(16*(100*x + 139)*x + 3195)*x + 11423)*x + 82051)*x - 6402
3)*sqrt(2*x^2 - x + 3) + 284533/65536*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - s
qrt(2*x^2 - x + 3)) + 1)
```

maple [A] time = 0.01, size = 98, normalized size = 0.79

$$\frac{25\left(2x^2-x+3\right)^{\frac{3}{2}}x^3}{12}+\frac{63\left(2x^2-x+3\right)^{\frac{3}{2}}x^2}{16}+\frac{769\left(2x^2-x+3\right)^{\frac{3}{2}}x}{256}-\frac{284533\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{65536}-\frac{2107\left(2x^2-x+3\right)^{\frac{3}{2}}}{3072}-\frac{12371\,(4x-1)\sqrt{2x^2-x+3}}{16384}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)^2*(2*x^2-x+3)^(1/2),x)
```

```
[Out] 25/12*(2*x^2-x+3)^(3/2)*x^3+63/16*(2*x^2-x+3)^(3/2)*x^2+769/256*(2*x^2-x+3)
^(3/2)*x-2107/3072*(2*x^2-x+3)^(3/2)-12371/16384*(4*x-1)*(2*x^2-x+3)^(1/2)-
284533/65536*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))
```

maxima [A] time = 0.98, size = 109, normalized size = 0.88

$$\frac{25}{12}\left(2x^2-x+3\right)^{\frac{3}{2}}x^3+\frac{63}{16}\left(2x^2-x+3\right)^{\frac{3}{2}}x^2+\frac{769}{256}\left(2x^2-x+3\right)^{\frac{3}{2}}x-\frac{2107}{3072}\left(2x^2-x+3\right)^{\frac{3}{2}}-\frac{12371}{4096}\sqrt{2x^2-x+3}\,x-\frac{284533}{65536}\sqrt{2}\,\mathrm{arsinh}\left(\frac{1}{23}\sqrt{23}\,(4x-1)\right)+\frac{12371}{16384}\sqrt{2x^2-x+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^2*(2*x^2-x+3)^(1/2),x, algorithm="maxima")
```

```
[Out] 25/12*(2*x^2 - x + 3)^(3/2)*x^3 + 63/16*(2*x^2 - x + 3)^(3/2)*x^2 + 769/256
*(2*x^2 - x + 3)^(3/2)*x - 2107/3072*(2*x^2 - x + 3)^(3/2) - 12371/4096*sqr
t(2*x^2 - x + 3)*x - 284533/65536*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1))
+ 12371/16384*sqrt(2*x^2 - x + 3)
```

mupad [B] time = 4.19, size = 153, normalized size = 1.23

$$\frac{63x^2\left(2x^2-x+3\right)^{3/2}}{16}+\frac{25x^3\left(2x^2-x+3\right)^{3/2}}{12}-\frac{29509\sqrt{2}\,\ln\left(\sqrt{2x^2-x+3}+\frac{\sqrt{2}\left(2x-\frac{1}{2}\right)}{2}\right)}{8192}-\frac{1283\left(\frac{x}{2}-\frac{1}{8}\right)\sqrt{2x^2-x+3}}{256}-\frac{2107\sqrt{2x^2-x+3}\,\left(32x^2-4x+45\right)}{49152}+\frac{769x\left(2x^2-x+3\right)^{3/2}}{256}-\frac{48461\sqrt{2}\,\ln\left(2\sqrt{2x^2-x+3}+\frac{\sqrt{2}\,(4x-1)}{2}\right)}{65536}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^(1/2)*(3*x + 5*x^2 + 2)^2,x)
```

```
[Out] (63*x^2*(2*x^2 - x + 3)^(3/2))/16 + (25*x^3*(2*x^2 - x + 3)^(3/2))/12 - (29
509*2^(1/2)*log((2*x^2 - x + 3)^(1/2) + (2^(1/2)*(2*x - 1/2))/2))/8192 - (1
283*(x/2 - 1/8)*(2*x^2 - x + 3)^(1/2))/256 - (2107*(2*x^2 - x + 3)^(1/2)*(3
2*x^2 - 4*x + 45))/49152 + (769*x*(2*x^2 - x + 3)^(3/2))/256 - (48461*2^(1/
2)*log(2*(2*x^2 - x + 3)^(1/2) + (2^(1/2)*(4*x - 1))/2))/65536
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{2x^2-x+3}\,\left(5x^2+3x+2\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)**2*(2*x**2-x+3)**(1/2),x)
```

```
[Out] Integral(sqrt(2*x**2 - x + 3)*(5*x**2 + 3*x + 2)**2, x)
```

3.58 $\int \sqrt{3-x+2x^2}\,\left(2+3x+5x^2\right)\,dx$

Optimal. Leaf size=82

$$\frac{5}{8}x\left(2x^2-x+3\right)^{3/2}+\frac{73}{96}\left(2x^2-x+3\right)^{3/2}-\frac{81}{512}(1-4x)\sqrt{2x^2-x+3}-\frac{1863\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{1024\sqrt{2}}$$

Rubi [A] time = 0.04, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1661, 640, 612, 619, 215}

$$\frac{5}{8}x\left(2x^2-x+3\right)^{3/2}+\frac{73}{96}\left(2x^2-x+3\right)^{3/2}-\frac{81}{512}(1-4x)\sqrt{2x^2-x+3}-\frac{1863\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{1024\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2),x]
```

```
[Out] (-81*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/512 + (73*(3 - x + 2*x^2)^(3/2))/96 + (
5*x*(3 - x + 2*x^2)^(3/2))/8 - (1863*ArcSinh[(1 - 4*x)/Sqrt[23]])/(1024*Sqr
t[2])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
```

```
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \sqrt{3-x+2x^2}\left(2+3x+5x^2\right)dx = \frac{5}{8}x\left(3-x+2x^2\right)^{3/2}+\frac{1}{8}\int\left(1+\frac{73x}{2}\right)\sqrt{3-x+2x^2}\,dx$$

$$= \frac{73}{96}\left(3-x+2x^2\right)^{3/2}+\frac{5}{8}x\left(3-x+2x^2\right)^{3/2}+\frac{81}{64}\int\sqrt{3-x+2x^2}\,dx$$

$$= -\frac{81}{512}(1-4x)\sqrt{3-x+2x^2}+\frac{73}{96}\left(3-x+2x^2\right)^{3/2}+\frac{5}{8}x\left(3-x+2x^2\right)^{3/2}+ \cdots$$

$$= -\frac{81}{512}(1-4x)\sqrt{3-x+2x^2}+\frac{73}{96}\left(3-x+2x^2\right)^{3/2}+\frac{5}{8}x\left(3-x+2x^2\right)^{3/2}+ \cdots$$

$$= -\frac{81}{512}(1-4x)\sqrt{3-x+2x^2}+\frac{73}{96}\left(3-x+2x^2\right)^{3/2}+\frac{5}{8}x\left(3-x+2x^2\right)^{3/2}- \cdots$$

Mathematica [A] time = 0.05, size = 55, normalized size = 0.67

$$\frac{4\sqrt{2x^2-x+3}\left(1920x^3+1376x^2+2684x+3261\right)-5589\sqrt{2}\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{6144}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2),x]
```

```
[Out] (4*Sqrt[3 - x + 2*x^2]*(3261 + 2684*x + 1376*x^2 + 1920*x^3) - 5589*Sqrt[2]
*ArcSinh[(1 - 4*x)/Sqrt[23]])/6144
```

IntegrateAlgebraic [A] time = 0.37, size = 70, normalized size = 0.85

$$\frac{\sqrt{2x^2-x+3}\left(1920x^3+1376x^2+2684x+3261\right)}{1536}-\frac{1863\log\left(2\sqrt{2}\sqrt{2x^2-x+3}-4x+1\right)}{1024\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2),x]
```

```
[Out] (Sqrt[3 - x + 2*x^2]*(3261 + 2684*x + 1376*x^2 + 1920*x^3))/1536 - (1863*Lo
g[1 - 4*x + 2*Sqrt[2]*Sqrt[3 - x + 2*x^2]])/(1024*Sqrt[2])
```

fricas [A] time = 0.41, size = 68, normalized size = 0.83

$$\frac{1}{1536}\left(1920\,x^3+1376\,x^2+2684\,x+3261\right)\sqrt{2\,x^2-x+3}+\frac{1863}{4096}\sqrt{2}\log\left(-4\sqrt{2}\sqrt{2\,x^2-x+3}\,(4\,x-1)-32\,x^2+16\,x-25\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)*(2*x^2-x+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/1536*(1920*x^3 + 1376*x^2 + 2684*x + 3261)*sqrt(2*x^2 - x + 3) + 1863/409
6*sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25
)
```

giac [A] time = 0.24, size = 63, normalized size = 0.77

$$\frac{1}{1536}\left(4\left(8\left(60\,x+43\right)x+671\right)x+3261\right)\sqrt{2\,x^2-x+3}-\frac{1863}{2048}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}\,x-\sqrt{2\,x^2-x+3}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)*(2*x^2-x+3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/1536*(4*(8*(60*x + 43)*x + 671)*x + 3261)*sqrt(2*x^2 - x + 3) - 1863/2048
*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1)
```

maple [A] time = 0.01, size = 64, normalized size = 0.78

$$\frac{5\left(2x^2-x+3\right)^{\frac{3}{2}}x}{8}+\frac{1863\sqrt{2}\ \operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{2048}+\frac{73\left(2x^2-x+3\right)^{\frac{3}{2}}}{96}+\frac{81\,(4x-1)\sqrt{2x^2-x+3}}{512}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)*(2*x^2-x+3)^(1/2),x)
```

```
[Out] 5/8*(2*x^2-x+3)^(3/2)*x+73/96*(2*x^2-x+3)^(3/2)+81/512*(4*x-1)*(2*x^2-x+3)^
(1/2)+1863/2048*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))
```

maxima [A] time = 0.96, size = 75, normalized size = 0.91

$$\frac{5}{8}\left(2x^2-x+3\right)^{\frac{3}{2}}x+\frac{73}{96}\left(2x^2-x+3\right)^{\frac{3}{2}}+\frac{81}{128}\sqrt{2x^2-x+3}\,x+\frac{1863}{2048}\sqrt{2}\,\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}\,(4x-1)\right)-\frac{81}{512}\sqrt{2x^2-x+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)*(2*x^2-x+3)^(1/2),x, algorithm="maxima")
```

```
[Out] 5/8*(2*x^2 - x + 3)^(3/2)*x + 73/96*(2*x^2 - x + 3)^(3/2) + 81/128*sqrt(2*x
^2 - x + 3)*x + 1863/2048*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 81/512
*sqrt(2*x^2 - x + 3)
```

mupad [B] time = 3.84, size = 119, normalized size = 1.45

$$\frac{23\sqrt{2}\,\ln\left(\sqrt{2x^2-x+3}+\frac{\sqrt{2}\left(2x-\frac{1}{2}\right)}{2}\right)}{256}+\frac{\left(\frac{x}{2}-\frac{1}{8}\right)\sqrt{2x^2-x+3}}{8}+\frac{73\sqrt{2x^2-x+3}\,\left(32x^2-4x+45\right)}{1536}+\frac{5x\left(2x^2-x+3\right)^{3/2}}{8}+\frac{1679\sqrt{2}\,\ln\left(2\sqrt{2x^2-x+3}+\frac{\sqrt{2}\,(4x-1)}{2}\right)}{2048}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^(1/2)*(3*x + 5*x^2 + 2),x)
```

```
[Out] (23*2^(1/2)*log((2*x^2 - x + 3)^(1/2) + (2^(1/2)*(2*x - 1/2))/2))/256 + ((x
/2 - 1/8)*(2*x^2 - x + 3)^(1/2))/8 + (73*(2*x^2 - x + 3)^(1/2)*(32*x^2 - 4*
x + 45))/1536 + (5*x*(2*x^2 - x + 3)^(3/2))/8 + (1679*2^(1/2)*log(2*(2*x^2
- x + 3)^(1/2) + (2^(1/2)*(4*x - 1))/2))/2048
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{2x^2-x+3}\,\left(5x^2+3x+2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)*(2*x**2-x+3)**(1/2),x)
```

```
[Out] Integral(sqrt(2*x**2 - x + 3)*(5*x**2 + 3*x + 2), x)
```

3.59 $\int \frac{\sqrt{3-x+2x^2}}{2+3x+5x^2}\,dx$

Optimal. Leaf size=174

$$\frac{1}{5}\sqrt{\frac{11}{31}\left(13+10\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{11}{62\left(13+10\sqrt{2}\right)}}\left(\left(20+13\sqrt{2}\right)x+7\sqrt{2}+6\right)}{\sqrt{2x^2-x+3}}\right)-\frac{1}{5}\sqrt{\frac{11}{31}\left(10\sqrt{2}-13\right)}\tanh^{-1}\left(\sqrt{\frac{11}{62(1}}\right.$$

Rubi [A] time = 0.44, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {989, 619, 215, 1035, 1029, 206, 204}

$$\frac{1}{5}\sqrt{\frac{11}{31}\left(13+10\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{11}{62\left(13+10\sqrt{2}\right)}}\left(\left(20+13\sqrt{2}\right)x+7\sqrt{2}+6\right)}{\sqrt{2x^2-x+3}}\right)-\frac{1}{5}\sqrt{\frac{11}{31}\left(10\sqrt{2}-13\right)}\tanh^{-1}\left(\frac{\sqrt{\frac{11}{62\left(10\sqrt{2}-13\right)}}\left(\left(20-13\sqrt{2}\right)x-7\sqrt{2}+6\right)}{\sqrt{2x^2-x+3}}\right)-\frac{1}{5}\sqrt{2}\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3 - x + 2*x^2]/(2 + 3*x + 5*x^2),x]
```

```
[Out] -(Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]])/5 + (Sqrt[(11*(13 + 10*Sqrt[2]))/31]
*ArcTan[(Sqrt[11/(62*(13 + 10*Sqrt[2]))]*(6 + 7*Sqrt[2] + (20 + 13*Sqrt[2])
*x))/Sqrt[3 - x + 2*x^2]])/5 - (Sqrt[(11*(-13 + 10*Sqrt[2]))/31]*ArcTanh[(S
qrt[11/(62*(-13 + 10*Sqrt[2]))]*(6 - 7*Sqrt[2] + (20 - 13*Sqrt[2])*x))/Sqrt
[3 - x + 2*x^2]])/5
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 989

```
Int[Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]/((d_) + (e_.)*(x_) + (f_.)*(x_)^
2), x_Symbol] :> Dist[c/f, Int[1/Sqrt[a + b*x + c*x^2], x], x] - Dist[1/f,
Int[(c*d - a*f + (c*e - b*f)*x)/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 -
4*d*f, 0]
```

Rule 1029

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
 c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 1035

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

Rubi steps

$$\int \frac{\sqrt{3-x+2x^2}}{2+3x+5x^2}\,dx = -\left(\frac{1}{5}\int \frac{-11+11x}{\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)}\,dx\right)+\frac{2}{5}\int \frac{1}{\sqrt{3-x+2x^2}}\,dx$$

$$= \frac{1}{5}\sqrt{\frac{2}{23}}\,\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{23}}}\,dx, x, -1+4x\right)+\frac{\int \frac{121\left(2+\sqrt{2}\right)-121\sqrt{2}\,x}{\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)}\,dx}{110\sqrt{2}}-\frac{\int \frac{121\left(2-\sqrt{2}\right)+121}{\sqrt{3-x+2x^2}\left(2+3x\right.}}{110\sqrt{2}}$$

$$= -\frac{1}{5}\sqrt{2}\,\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)-\frac{1}{5}\left(1331\left(20-13\sqrt{2}\right)\right)\text{Subst}\left(\int \frac{1}{-907742\left(13-10\sqrt{2}\right)-11x^2}\right.$$

$$= -\frac{1}{5}\sqrt{2}\,\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)+\frac{1}{5}\sqrt{\frac{11}{31}\left(13+10\sqrt{2}\right)}\,\tan^{-1}\left(\frac{\sqrt{\frac{11}{62\left(13+10\sqrt{2}\right)}}\left(6+7\sqrt{2}+\left(20+\right.\right.}{\sqrt{3-x+2x^2}}\right.$$

Mathematica [C] time = 0.46, size = 185, normalized size = 1.06

$$-\frac{1}{5}\sqrt{2}\,\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)-\frac{1}{5}i\sqrt{\frac{11}{62}}\left(\sqrt{13+i\sqrt{31}}\,\tanh^{-1}\left(\frac{-4i\sqrt{31}\,x-22x+i\sqrt{31}+63}{2\sqrt{286+22i\sqrt{31}}\,\sqrt{2x^2-x+3}}\right)-\sqrt{13-i\sqrt{31}}\,\tanh^{-1}\left(\frac{4i\sqrt{31}\,x-22x-i\sqrt{31}+63}{2\sqrt{286-22i\sqrt{31}}\,\sqrt{2x^2-x+3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3 - x + 2*x^2]/(2 + 3*x + 5*x^2),x]
```

```
[Out] -1/5*(Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]]) - (I/5)*Sqrt[11/62]*(Sqrt[13 + I
*Sqrt[31]]*ArcTanh[(63 + I*Sqrt[31] - 22*x - (4*I)*Sqrt[31]*x)/(2*Sqrt[286
+ (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])] - Sqrt[13 - I*Sqrt[31]]*ArcTanh[(6
3 - I*Sqrt[31] - 22*x + (4*I)*Sqrt[31]*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sq
rt[3 - x + 2*x^2])])
```

IntegrateAlgebraic [C] time = 0.35, size = 211, normalized size = 1.21

$$\frac{11}{5}\text{RootSum}\left[-5\#1^4+6\sqrt{2}\#1^3+17\#1^2-26\sqrt{2}\#1-56\&,\frac{\#1^2\log\left(-\#1+\sqrt{2x^2-x+3}-\sqrt{2}x\right)+2\sqrt{2}\#1\log\left(-\#1+\sqrt{2x^2-x+3}-\sqrt{2}x\right)-2\log\left(-\#1+\sqrt{2x^2-x+3}-\sqrt{2}x\right)}{-10\#1^3+9\sqrt{2}\#1^2+17\#1-13\sqrt{2}}\&\right]-\frac{1}{5}\sqrt{2}\log\left(2\sqrt{2}\sqrt{2x^2-x+3}-4x+1\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[3 - x + 2*x^2]/(2 + 3*x + 5*x^2),x]
```

```
[Out] -1/5*(Sqrt[2]*Log[1 - 4*x + 2*Sqrt[2]*Sqrt[3 - x + 2*x^2]]) + (11*RootSum[-
56 - 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3 - 5*#1^4 & , (-2*Log[-(Sqrt[2
]*x) + Sqrt[3 - x + 2*x^2] - #1] + 2*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x
```

```
+ 2*x^2] - #1]*#1 + Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1^2)/(-13
*Sqrt[2] + 17*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3) & ])/5
```

fricas [B] time = 0.99, size = 2016, normalized size = 11.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(1/2)/(5*x^2+3*x+2),x, algorithm="fricas")
```

```
[Out] 1/1550*6050^(1/4)*sqrt(31)*sqrt(5)*sqrt(2)*sqrt(13*sqrt(2) + 20)*arctan(1/8
9125*(460*sqrt(5)*(4*6050^(3/4)*sqrt(31)*(4702*x^7 - 19541*x^6 + 40352*x^5
- 68777*x^4 + 35480*x^3 - 19080*x^2 - sqrt(2)*(4028*x^7 - 14488*x^6 + 30919
*x^5 - 46671*x^4 + 22688*x^3 - 9144*x^2 - 27648*x + 17280) - 34560*x + 2764
8) + 5*6050^(1/4)*sqrt(31)*(22836*x^7 - 355266*x^6 + 1914360*x^5 - 4475096*
x^4 + 5840640*x^3 - 4011840*x^2 - sqrt(2)*(18463*x^7 - 280047*x^6 + 1453472
*x^5 - 3238500*x^4 + 4140576*x^3 - 2378592*x^2 - 3068928*x + 1990656) - 398
1312*x + 3068928))*sqrt(2*x^2 - x + 3)*sqrt(13*sqrt(2) + 20) + 253000*sqrt(
31)*sqrt(2)*(28180*x^8 - 254666*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^
4 - 642048*x^3 - 98496*x^2 - sqrt(2)*(8746*x^8 - 102335*x^7 + 396104*x^6 -
783113*x^5 + 1320710*x^4 - 752088*x^3 + 396144*x^2 + 546048*x - 539136) + 1
154304*x - 456192) - 2*sqrt(10)*(sqrt(5)*(4*6050^(3/4)*sqrt(31)*(15454*x^7
- 22399*x^6 + 73509*x^5 - 37360*x^4 + 52200*x^3 + 13824*x^2 - sqrt(2)*(1543
8*x^7 - 22007*x^6 + 69837*x^5 - 21232*x^4 + 19368*x^3 + 44928*x^2 - 44928*x
) - 13824*x) + 5*6050^(1/4)*sqrt(31)*(77254*x^7 - 1000024*x^6 + 3868360*x^5
 - 5120640*x^4 + 7012800*x^3 + 2405376*x^2 - sqrt(2)*(69479*x^7 - 898236*x^
6 + 3454740*x^5 - 4394304*x^4 + 5347296*x^3 + 4478976*x^2 - 4478976*x) - 24
05376*x))*sqrt(2*x^2 - x + 3)*sqrt(13*sqrt(2) + 20) + 550*sqrt(31)*sqrt(2)*
(123408*x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 + 396480*x^4 + 798336*
x^3 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 118051*x^7 + 244047*x^6 - 707374*x
^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*x) + 3276288*x) + 25
*sqrt(31)*(254591*x^8 - 4815126*x^7 + 32303580*x^6 - 90866808*x^5 + 1087819
20*x^4 - 74219328*x^3 - 168956928*x^2 - 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517
*x^6 - 1536*x^5 + 2385*x^4 - 3618*x^3 + 2268*x^2 - 1944*x) + 144820224*x))*
sqrt(-(6050^(1/4)*sqrt(5)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(3*x + 5) - 8*x + 2)
*sqrt(13*sqrt(2) + 20) - 245*x^2 - 220*sqrt(2)*(2*x^2 - x + 3) + 755*x - 10
00)/x^2) + 2875*sqrt(31)*(2828123*x^8 - 9696916*x^7 + 53385560*x^6 - 142835
344*x^5 + 254146592*x^4 - 249300096*x^3 + 37981440*x^2 - 7744*sqrt(2)*(1348
*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^4 - 5568*x^3 + 1080*x^2 +
4320*x - 5184) + 223064064*x - 94887936))/(2585191*x^8 - 4661200*x^7 + 1419
1920*x^6 + 490880*x^5 - 13562944*x^4 + 44249088*x^3 - 34615296*x^2 - 247726
08*x + 18579456)) + 1/1550*6050^(1/4)*sqrt(31)*sqrt(5)*sqrt(2)*sqrt(13*sqrt
(2) + 20)*arctan(1/89125*(460*sqrt(5)*(4*6050^(3/4)*sqrt(31)*(4702*x^7 - 19
541*x^6 + 40352*x^5 - 68777*x^4 + 35480*x^3 - 19080*x^2 - sqrt(2)*(4028*x^7
 - 14488*x^6 + 30919*x^5 - 46671*x^4 + 22688*x^3 - 9144*x^2 - 27648*x + 172
80) - 34560*x + 27648) + 5*6050^(1/4)*sqrt(31)*(22836*x^7 - 355266*x^6 + 19
```

```
14360*x^5 - 4475096*x^4 + 5840640*x^3 - 4011840*x^2 - sqrt(2)*(18463*x^7 -
280047*x^6 + 1453472*x^5 - 3238500*x^4 + 4140576*x^3 - 2378592*x^2 - 306892
8*x + 1990656) - 3981312*x + 3068928))*sqrt(2*x^2 - x + 3)*sqrt(13*sqrt(2)
+ 20) - 253000*sqrt(31)*sqrt(2)*(28180*x^8 - 254666*x^7 + 704270*x^6 - 1385
256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - sqrt(2)*(8746*x^8 - 102335
*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^4 - 752088*x^3 + 396144*x^2 + 54
6048*x - 539136) + 1154304*x - 456192) - 2*sqrt(10)*(sqrt(5)*(4*6050^(3/4)*
sqrt(31)*(15454*x^7 - 22399*x^6 + 73509*x^5 - 37360*x^4 + 52200*x^3 + 13824
*x^2 - sqrt(2)*(15438*x^7 - 22007*x^6 + 69837*x^5 - 21232*x^4 + 19368*x^3 +
 44928*x^2 - 44928*x) - 13824*x) + 5*6050^(1/4)*sqrt(31)*(77254*x^7 - 10000
24*x^6 + 3868360*x^5 - 5120640*x^4 + 7012800*x^3 + 2405376*x^2 - sqrt(2)*(6
9479*x^7 - 898236*x^6 + 3454740*x^5 - 4394304*x^4 + 5347296*x^3 + 4478976*x
^2 - 4478976*x) - 2405376*x))*sqrt(2*x^2 - x + 3)*sqrt(13*sqrt(2) + 20) - 5
50*sqrt(31)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 +
396480*x^4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 118051*x^7 + 2
44047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*
x) + 3276288*x) - 25*sqrt(31)*(254591*x^8 - 4815126*x^7 + 32303580*x^6 - 90
866808*x^5 + 108781920*x^4 - 74219328*x^3 - 168956928*x^2 - 15488*sqrt(2)*(
4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3618*x^3 + 2268*x^2 - 1944
*x) + 144820224*x))*sqrt((6050^(1/4)*sqrt(5)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(
3*x + 5) - 8*x + 2)*sqrt(13*sqrt(2) + 20) + 245*x^2 + 220*sqrt(2)*(2*x^2 -
x + 3) - 755*x + 1000)/x^2) - 2875*sqrt(31)*(2828123*x^8 - 9696916*x^7 + 53
385560*x^6 - 142835344*x^5 + 254146592*x^4 - 249300096*x^3 + 37981440*x^2 -
 7744*sqrt(2)*(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^4 - 556
8*x^3 + 1080*x^2 + 4320*x - 5184) + 223064064*x - 94887936))/(2585191*x^8 -
 4661200*x^7 + 14191920*x^6 + 490880*x^5 - 13562944*x^4 + 44249088*x^3 - 34
615296*x^2 - 24772608*x + 18579456)) - 1/6200*6050^(1/4)*sqrt(5)*sqrt(13*sq
rt(2) + 20)*(13*sqrt(2) - 20)*log(40*(6050^(1/4)*sqrt(5)*sqrt(2*x^2 - x + 3
)*(sqrt(2)*(3*x + 5) - 8*x + 2)*sqrt(13*sqrt(2) + 20) + 245*x^2 + 220*sqrt(
2)*(2*x^2 - x + 3) - 755*x + 1000)/x^2) + 1/6200*6050^(1/4)*sqrt(5)*sqrt(13
*sqrt(2) + 20)*(13*sqrt(2) - 20)*log(-40*(6050^(1/4)*sqrt(5)*sqrt(2*x^2 - x
 + 3)*(sqrt(2)*(3*x + 5) - 8*x + 2)*sqrt(13*sqrt(2) + 20) - 245*x^2 - 220*s
qrt(2)*(2*x^2 - x + 3) + 755*x - 1000)/x^2) + 1/10*sqrt(2)*log(-4*sqrt(2)*s
qrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(1/2)/(5*x^2+3*x+2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Francis algorithm failure for[-1.0,infinity,
infinity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,infi
```

```
nity]Francis algorithm failure for[-1.0,infinity,infinity,infinity,infinity
]proot error [1.0,infinity,infinity,infinity,infinity]Evaluation time: 5.57
Done
```

maple [B] time = 0.15, size = 2065, normalized size = 11.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(1/2)/(5*x^2+3*x+2),x)
```

```
[Out] 1/5*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))-1/52855*(8*(2^(1/2)-1+x)^2/(2^(1
/2)+1-x)^2+3*2^(1/2)*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+8-3*2^(1/2))^(1/2)*2^(
1/2)*(285*2^(1/2)*(-8866+6820*2^(1/2))^(1/2)*arctan(1/11692487*(-775687+549
362*2^(1/2))^(1/2)*(-23*(8+3*2^(1/2))*(-23*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+
24*2^(1/2)-41))^(1/2)*(6485*2^(1/2)*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+10368*(
2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+22379*2^(1/2)+32016)/(23*(2^(1/2)-1+x)^4/(2^
(1/2)+1-x)^4+82*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+23)*(2^(1/2)-1+x)/(2^(1/2)+
1-x)*(8+3*2^(1/2)))*(-775687+549362*2^(1/2))^(1/2)+386*(-8866+6820*2^(1/2))
^(1/2)*arctan(1/11692487*(-775687+549362*2^(1/2))^(1/2)*(-23*(8+3*2^(1/2))*
(-23*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+24*2^(1/2)-41))^(1/2)*(6485*2^(1/2)*(2
^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+10368*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+22379*2
^(1/2)+32016)/(23*(2^(1/2)-1+x)^4/(2^(1/2)+1-x)^4+82*(2^(1/2)-1+x)^2/(2^(1/
2)+1-x)^2+23)*(2^(1/2)-1+x)/(2^(1/2)+1-x)*(8+3*2^(1/2)))*(-775687+549362*2^
(1/2))^(1/2)-274846*arctanh(31/2*(8*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+3*2^(1/
2)*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+8-3*2^(1/2))^(1/2)/(-8866+6820*2^(1/2))^
(1/2))*2^(1/2)-1543366*arctanh(31/2*(8*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+3*2^
(1/2)*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+8-3*2^(1/2))^(1/2)/(-8866+6820*2^(1/2
))^(1/2)))/((8*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+3*2^(1/2)*(2^(1/2)-1+x)^2/(2
^(1/2)+1-x)^2+8-3*2^(1/2))/(1+(2^(1/2)-1+x)/(2^(1/2)+1-x))^2)^(1/2)/(1+(2^(
1/2)-1+x)/(2^(1/2)+1-x))/(8+3*2^(1/2))/(-8866+6820*2^(1/2))^(1/2)+1/21142*(
8*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+3*2^(1/2)*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2
+8-3*2^(1/2))^(1/2)*2^(1/2)*(151*2^(1/2)*(-8866+6820*2^(1/2))^(1/2)*arctan(
1/11692487*(-775687+549362*2^(1/2))^(1/2)*(-23*(8+3*2^(1/2))*(-23*(2^(1/2)-
1+x)^2/(2^(1/2)+1-x)^2+24*2^(1/2)-41))^(1/2)*(6485*2^(1/2)*(2^(1/2)-1+x)^2/
(2^(1/2)+1-x)^2+10368*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+22379*2^(1/2)+32016)/
(23*(2^(1/2)-1+x)^4/(2^(1/2)+1-x)^4+82*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+23)*
(2^(1/2)-1+x)/(2^(1/2)+1-x)*(8+3*2^(1/2)))*(-775687+549362*2^(1/2))^(1/2)+2
18*(-8866+6820*2^(1/2))^(1/2)*arctan(1/11692487*(-775687+549362*2^(1/2))^(1
/2)*(-23*(8+3*2^(1/2))*(-23*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+24*2^(1/2)-41))
^(1/2)*(6485*2^(1/2)*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+10368*(2^(1/2)-1+x)^2/
(2^(1/2)+1-x)^2+22379*2^(1/2)+32016)/(23*(2^(1/2)-1+x)^4/(2^(1/2)+1-x)^4+82
*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+23)*(2^(1/2)-1+x)/(2^(1/2)+1-x)*(8+3*2^(1/
2)))*(-775687+549362*2^(1/2))^(1/2)+401698*arctanh(31/2*(8*(2^(1/2)-1+x)^2/
(2^(1/2)+1-x)^2+3*2^(1/2)*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+8-3*2^(1/2))^(1/2
```

```
)/(-8866+6820*2^(1/2))^(1/2))*2^(1/2)-63426*arctanh(31/2*(8*(2^(1/2)-1+x)^2
/(2^(1/2)+1-x)^2+3*2^(1/2)*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+8-3*2^(1/2))^(1/
2)/(-8866+6820*2^(1/2))^(1/2)))/((8*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+3*2^(1/
2)*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+8-3*2^(1/2))/(1+(2^(1/2)-1+x)/(2^(1/2)+1
-x))^2)^(1/2)/(1+(2^(1/2)-1+x)/(2^(1/2)+1-x))/(8+3*2^(1/2))/(-8866+6820*2^(
1/2))^(1/2)+3/21142*(8*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+3*2^(1/2)*(2^(1/2)-1
+x)^2/(2^(1/2)+1-x)^2+8-3*2^(1/2))^(1/2)*2^(1/2)*(369*2^(1/2)*(-8866+6820*2
^(1/2))^(1/2)*arctan(1/11692487*(-775687+549362*2^(1/2))^(1/2)*(-23*(8+3*2^
(1/2))*(-23*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+24*2^(1/2)-41))^(1/2)*(6485*2^(
1/2)*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+10368*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+
22379*2^(1/2)+32016)/(23*(2^(1/2)-1+x)^4/(2^(1/2)+1-x)^4+82*(2^(1/2)-1+x)^2
/(2^(1/2)+1-x)^2+23)*(2^(1/2)-1+x)/(2^(1/2)+1-x)*(8+3*2^(1/2)))*(-775687+54
9362*2^(1/2))^(1/2)+520*(-8866+6820*2^(1/2))^(1/2)*arctan(1/11692487*(-7756
87+549362*2^(1/2))^(1/2)*(-23*(8+3*2^(1/2))*(-23*(2^(1/2)-1+x)^2/(2^(1/2)+1
-x)^2+24*2^(1/2)-41))^(1/2)*(6485*2^(1/2)*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+1
0368*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+22379*2^(1/2)+32016)/(23*(2^(1/2)-1+x)
^4/(2^(1/2)+1-x)^4+82*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+23)*(2^(1/2)-1+x)/(2^
(1/2)+1-x)*(8+3*2^(1/2)))*(-775687+549362*2^(1/2))^(1/2)+465124*arctanh(31/
2*(8*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+3*2^(1/2)*(2^(1/2)-1+x)^2/(2^(1/2)+1-x
)^2+8-3*2^(1/2))^(1/2)/(-8866+6820*2^(1/2))^(1/2))*2^(1/2)-866822*arctanh(3
1/2*(8*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+3*2^(1/2)*(2^(1/2)-1+x)^2/(2^(1/2)+1
-x)^2+8-3*2^(1/2))^(1/2)/(-8866+6820*2^(1/2))^(1/2)))/((8*(2^(1/2)-1+x)^2/(
2^(1/2)+1-x)^2+3*2^(1/2)*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+8-3*2^(1/2))/(1+(2
^(1/2)-1+x)/(2^(1/2)+1-x))^2)^(1/2)/(1+(2^(1/2)-1+x)/(2^(1/2)+1-x))/(8+3*2^
(1/2))/(-8866+6820*2^(1/2))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2\,x^2 - x + 3}}{5\,x^2 + 3\,x + 2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(1/2)/(5*x^2+3*x+2),x, algorithm="maxima")
[Out] integrate(sqrt(2*x^2 - x + 3)/(5*x^2 + 3*x + 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{2\,x^2 - x + 3}}{5\,x^2 + 3\,x + 2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^(1/2)/(3*x + 5*x^2 + 2),x)
[Out] int((2*x^2 - x + 3)^(1/2)/(3*x + 5*x^2 + 2), x)
```

<u>**sympy**</u> **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2x^2 - x + 3}}{5x^2 + 3x + 2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(1/2)/(5*x**2+3*x+2),x)
```

```
[Out] Integral(sqrt(2*x**2 - x + 3)/(5*x**2 + 3*x + 2), x)
```

3.60 $\int \frac{\sqrt{3-x+2x^2}}{\left(2+3x+5x^2\right)^2}\,dx$

Optimal. Leaf size=188

$$\frac{\sqrt{2x^2-x+3}\,(10x+3)}{31\left(5x^2+3x+2\right)}+\frac{1}{62}\sqrt{\frac{1}{682}\left(70517+49942\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{11}{31\left(70517+49942\sqrt{2}\right)}}\left(\left(973+696\sqrt{2}\right)x+277\sqrt{}\right.}{\sqrt{2x^2-x+3}}\right.$$

Rubi [A] time = 0.39, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {971, 1035, 1029, 206, 204}

$$\frac{\sqrt{2x^2-x+3}\,(10x+3)}{31\left(5x^2+3x+2\right)}+\frac{1}{62}\sqrt{\frac{1}{682}\left(70517+49942\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{11}{31\left(70517+49942\sqrt{2}\right)}}\left(\left(973+696\sqrt{2}\right)x+277\sqrt{2}+419\right)}{\sqrt{2x^2-x+3}}\right)-\frac{1}{62}\sqrt{\frac{1}{682}\left(49942\sqrt{2}-70517\right)}\tanh^{-1}\left(\frac{\sqrt{\frac{11}{31\left(49942\sqrt{2}-70517\right)}}\left(\left(973-696\sqrt{2}\right)x-277\sqrt{2}+419\right)}{\sqrt{2x^2-x+3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3 - x + 2*x^2]/(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] ((3 + 10*x)*Sqrt[3 - x + 2*x^2])/(31*(2 + 3*x + 5*x^2)) + (Sqrt[(70517 + 49
942*Sqrt[2])/682]*ArcTan[(Sqrt[11/(31*(70517 + 49942*Sqrt[2]))]*(419 + 277*
Sqrt[2] + (973 + 696*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]])/62 - (Sqrt[(-70517
+ 49942*Sqrt[2])/682]*ArcTanh[(Sqrt[11/(31*(-70517 + 49942*Sqrt[2]))]*(419
- 277*Sqrt[2] + (973 - 696*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]])/62
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 971

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((b + 2*c*x)*(a + b*x + c*x^2)^(p + 1)*(d + e
*x + f*x^2)^q)/((b^2 - 4*a*c)*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*(p + 1))
, Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*Simp[2*c*d*(2*p +
```

```
 3) + b*e*q + (2*b*f*q + 2*c*e*(2*p + q + 3))*x + 2*c*f*(2*p + 2*q + 3)*x^2
, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
e^2 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0] &&  !IGtQ[q, 0]
```

Rule 1029

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
 c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 1035

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

Rubi steps

$$\int \frac{\sqrt{3-x+2x^2}}{\left(2+3x+5x^2\right)^2}\,dx = \frac{(3+10x)\sqrt{3-x+2x^2}}{31\left(2+3x+5x^2\right)} - \frac{1}{31}\int \frac{-\frac{63}{2}+11x}{\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)}\,dx$$

$$= \frac{(3+10x)\sqrt{3-x+2x^2}}{31\left(2+3x+5x^2\right)} - \frac{\int \frac{\frac{11}{2}\left(85-63\sqrt{2}\right)-\frac{11}{2}\left(41-22\sqrt{2}\right)x}{\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)}\,dx}{682\sqrt{2}} + \frac{\int \frac{\frac{11}{2}\left(85+63\sqrt{2}\right)-\frac{11}{2}\left(41+22\sqrt{2}\right)x}{\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)}}{682\sqrt{2}}$$

$$= \frac{(3+10x)\sqrt{3-x+2x^2}}{31\left(2+3x+5x^2\right)} - \frac{1}{248}\left(11\left(99884-70517\sqrt{2}\right)\right)\mathrm{Subst}\left(\int \frac{1}{-\frac{3751}{4}\left(70517-499\right.}\right.$$

$$= \frac{(3+10x)\sqrt{3-x+2x^2}}{31\left(2+3x+5x^2\right)} + \frac{1}{62}\sqrt{\frac{1}{682}\left(70517+49942\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{11}{31\left(70517+49942\sqrt{2}\right)}}\left(\right.}{}\right.$$

Mathematica [C] time = 1.04, size = 214, normalized size = 1.14

$$\frac{\frac{27280\sqrt{2x^2-x+3}\,(10x+3)}{5x^2+3x+2} + i\sqrt{286-22i\sqrt{31}}\left(973\sqrt{31}+1271i\right)\tanh^{-1}\left(\frac{4i\sqrt{31}x-22x-i\sqrt{31}+63}{2\sqrt{286-22i\sqrt{31}}\sqrt{2x^2-x+3}}\right) - i\sqrt{286+22i\sqrt{31}}\left(973\sqrt{31}-1271i\right)\tanh^{-1}\left(\frac{\left(-22-4i\sqrt{31}\right)x+i\sqrt{31}+63}{2\sqrt{286+22i\sqrt{31}}\sqrt{2x^2-x+3}}\right)}{845680}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3 - x + 2*x^2]/(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] ((27280*(3 + 10*x)*Sqrt[3 - x + 2*x^2])/(2 + 3*x + 5*x^2) + I*Sqrt[286 - (2
2*I)*Sqrt[31]]*(1271*I + 973*Sqrt[31])*ArcTanh[(63 - I*Sqrt[31] - 22*x + (4
*I)*Sqrt[31]*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])] - I*Sq
rt[286 + (22*I)*Sqrt[31]]*(-1271*I + 973*Sqrt[31])*ArcTanh[(63 + I*Sqrt[31]
 + (-22 - (4*I)*Sqrt[31])*x)/(2*Sqrt[286 + (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*
x^2])])/845680
```

IntegrateAlgebraic [C] time = 0.70, size = 423, normalized size = 2.25

$$-\frac{6151\,\mathrm{RootSum}\left[-5\#1^4+6\sqrt{2}\#1^3+17\#1^2-26\sqrt{2}\#1-56\&,\frac{\log\left(-\#1+\sqrt{2x^2-x+3}-\sqrt{2}x\right)}{-10\#1^3+9\sqrt{2}\#1^2+17\#1-13\sqrt{2}}\&\right]}{1550}+\frac{2}{25}\mathrm{RootSum}\left[-5\#1^4+6\sqrt{2}\#1^3+17\#1^2-26\sqrt{2}\#1-56\&,\frac{10\sqrt{2}\#1\log\left(-\#1+\sqrt{2x^2-x+3}-\sqrt{2}x\right)+49\log\left(-\#1+\sqrt{2x^2-x+3}-\sqrt{2}x\right)}{-10\#1^3+9\sqrt{2}\#1^2+17\#1-13\sqrt{2}}\&\right]-\frac{1}{155}\mathrm{RootSum}\left[-5\#1^4+6\sqrt{2}\#1^3+17\#1^2-26\sqrt{2}\#1-56\&,\frac{-55\#1^2\log\left(-\#1+\sqrt{2x^2-x+3}-\sqrt{2}x\right)-191\sqrt{2}\#1\log\left(-\#1+\sqrt{2x^2-x+3}-\sqrt{2}x\right)}{-10\#1^3+9\sqrt{2}\#1^2+17\#1-13\sqrt{2}}\&\right]+\frac{\sqrt{2x^2-x+3}\,(10x+3)}{31\left(5x^2+3x+2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[3 - x + 2*x^2]/(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] ((3 + 10*x)*Sqrt[3 - x + 2*x^2])/(31*(2 + 3*x + 5*x^2)) - (6151*RootSum[-56
 - 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3 - 5*#1^4 & , Log[-(Sqrt[2]*x) +
 Sqrt[3 - x + 2*x^2] - #1]/(-13*Sqrt[2] + 17*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3)
```

```
 & ])/1550 + (2*RootSum[-56 - 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3 - 5*
#1^4 & , (49*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1] + 10*Sqrt[2]*Log[
-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1)/(-13*Sqrt[2] + 17*#1 + 9*Sqrt[
2]*#1^2 - 10*#1^3) & ])/25 - RootSum[-56 - 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt
[2]*#1^3 - 5*#1^4 & , (-191*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2]
- #1]*#1 - 55*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1^2)/(-13*Sqrt[
2] + 17*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3) & ]/155
```

fricas [B] time = 1.19, size = 2102, normalized size = 11.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(1/2)/(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

```
[Out] -1/186703822445536*(88412*4988406728^(1/4)*sqrt(24971)*sqrt(341)*sqrt(2)*(5
*x^2 + 3*x + 2)*sqrt(70517*sqrt(2) + 99884)*arctan(1/10668926457462302923*(
3096404*sqrt(24971)*(11*4988406728^(3/4)*sqrt(341)*(537184*x^7 - 2047820*x^
6 + 4310846*x^5 - 6853210*x^4 + 3421536*x^3 - 1589328*x^2 - sqrt(2)*(370014
*x^7 - 1438653*x^6 + 3014868*x^5 - 4873381*x^4 + 2452952*x^3 - 1184616*x^2
- 2633472*x + 1893888) - 3787776*x + 2633472) + 774101*4988406728^(1/4)*sqr
t(341)*(40625*x^7 - 622509*x^6 + 3280912*x^5 - 7459052*x^4 + 9621216*x^3 -
5992992*x^2 - sqrt(2)*(28204*x^7 - 433677*x^6 + 2297444*x^5 - 5257628*x^4 +
 6800832*x^3 - 4341024*x^2 - 4810752*x + 3442176) - 6884352*x + 4810752))*s
qrt(2*x^2 - x + 3)*sqrt(70517*sqrt(2) + 99884) + 3028598478247363410 4*sqrt(
31)*sqrt(2)*(28180*x^8 - 254666*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^
4 - 642048*x^3 - 98496*x^2 - sqrt(2)*(8746*x^8 - 102335*x^7 + 396104*x^6 -
783113*x^5 + 1320710*x^4 - 752088*x^3 + 396144*x^2 + 546048*x - 539136) + 1
154304*x - 456192) - 2*sqrt(49942)*(sqrt(24971)*(11*4988406728^(3/4)*sqrt(3
41)*(84604*x^7 - 121310*x^6 + 389610*x^5 - 147168*x^4 + 168912*x^3 + 186624
*x^2 - sqrt(2)*(57082*x^7 - 82029*x^6 + 264639*x^5 - 107216*x^4 + 130104*x^
3 + 110592*x^2 - 110592*x) - 186624*x) + 774101*4988406728^(1/4)*sqrt(341)*
(6379*x^7 - 82508*x^6 + 318020*x^5 - 410688*x^4 + 523872*x^3 + 331776*x^2 -
 sqrt(2)*(4365*x^7 - 56468*x^6 + 217820*x^5 - 282816*x^4 + 366624*x^3 + 207
360*x^2 - 207360*x) - 331776*x))*sqrt(2*x^2 - x + 3)*sqrt(70517*sqrt(2) + 9
9884) + 425261673562*sqrt(31)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578888*x^
6 - 3293072*x^5 + 396480*x^4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15550*x^
8 - 118051*x^7 + 244047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209
600*x^2 - 1036800*x) + 3276288*x) + 19330076071*sqrt(31)*(254591*x^8 - 4815
126*x^7 + 32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 74219328*x^3 - 1689
56928*x^2 - 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 -
 3618*x^3 + 2268*x^2 - 1944*x) + 144820224*x))*sqrt(-(4988406728^(1/4)*sqrt
(24971)*sqrt(341)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(10*x + 3) - 13*x -
 7)*sqrt(70517*sqrt(2) + 99884) - 1175859419*x^2 - 1055873764*sqrt(2)*(2*x^
2 - x + 3) + 3623566781*x - 4799426200)/x^2) + 344158917982654933*sqrt(31)*
(2828123*x^8 - 9696916*x^7 + 53385560*x^6 - 142835344*x^5 + 254146592*x^4 -
```

249300096*x^3 + 37981440*x^2 - 7744*sqrt(2)*(1348*x^8 - 2692*x^7 + 9789*x^
6 - 10070*x^5 + 15569*x^4 - 5568*x^3 + 1080*x^2 + 4320*x - 5184) + 22306406
4*x - 94887936))/(2585191*x^8 - 4661200*x^7 + 14191920*x^6 + 490880*x^5 - 1
3562944*x^4 + 44249088*x^3 - 34615296*x^2 - 24772608*x + 18579456)) + 88412
*4988406728^(1/4)*sqrt(24971)*sqrt(341)*sqrt(2)*(5*x^2 + 3*x + 2)*sqrt(7051
7*sqrt(2) + 99884)*arctan(1/10668926457462302923*(3096404*sqrt(24971)*(11*4
988406728^(3/4)*sqrt(341)*(537184*x^7 - 2047820*x^6 + 4310846*x^5 - 6853210
*x^4 + 3421536*x^3 - 1589328*x^2 - sqrt(2)*(370014*x^7 - 1438653*x^6 + 3014
868*x^5 - 4873381*x^4 + 2452952*x^3 - 1184616*x^2 - 2633472*x + 1893888) -
3787776*x + 2633472) + 774101*4988406728^(1/4)*sqrt(341)*(40625*x^7 - 62250
9*x^6 + 3280912*x^5 - 7459052*x^4 + 9621216*x^3 - 5992992*x^2 - sqrt(2)*(28
204*x^7 - 433677*x^6 + 2297444*x^5 - 5257628*x^4 + 6800832*x^3 - 4341024*x^
2 - 4810752*x + 3442176) - 6884352*x + 4810752))*sqrt(2*x^2 - x + 3)*sqrt(7
0517*sqrt(2) + 99884) - 30285984782473634104*sqrt(31)*sqrt(2)*(28180*x^8 -
254666*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^
2 - sqrt(2)*(8746*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^4
- 752088*x^3 + 396144*x^2 + 546048*x - 539136) + 1154304*x - 456192) - 2*sq
rt(49942)*(sqrt(24971)*(11*4988406728^(3/4)*sqrt(341)*(84604*x^7 - 121310*x
^6 + 389610*x^5 - 147168*x^4 + 168912*x^3 + 186624*x^2 - sqrt(2)*(57082*x^7
- 82029*x^6 + 264639*x^5 - 107216*x^4 + 130104*x^3 + 110592*x^2 - 110592*x
) - 186624*x) + 774101*4988406728^(1/4)*sqrt(341)*(6379*x^7 - 82508*x^6 + 3
18020*x^5 - 410688*x^4 + 523872*x^3 + 331776*x^2 - sqrt(2)*(4365*x^7 - 5646
8*x^6 + 217820*x^5 - 282816*x^4 + 366624*x^3 + 207360*x^2 - 207360*x) - 331
776*x))*sqrt(2*x^2 - x + 3)*sqrt(70517*sqrt(2) + 99884) - 425261673562*sqrt
(31)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 + 396480*
x^4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 118051*x^7 + 244047*x
^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*x) + 32
76288*x) - 19330076071*sqrt(31)*(254591*x^8 - 4815126*x^7 + 32303580*x^6 -
90866808*x^5 + 108781920*x^4 - 74219328*x^3 - 168956928*x^2 - 15488*sqrt(2)
*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3618*x^3 + 2268*x^2 - 19
44*x) + 144820224*x))*sqrt((4988406728^(1/4)*sqrt(24971)*sqrt(341)*sqrt(31)
*sqrt(2*x^2 - x + 3)*(sqrt(2)*(10*x + 3) - 13*x - 7)*sqrt(70517*sqrt(2) + 9
9884) + 1175859419*x^2 + 1055873764*sqrt(2)*(2*x^2 - x + 3) - 3623566781*x
+ 4799426200)/x^2) - 344158917982654933*sqrt(31)*(2828123*x^8 - 9696916*x^7
+ 53385560*x^6 - 142835344*x^5 + 254146592*x^4 - 249300096*x^3 + 37981440*
x^2 - 7744*sqrt(2)*(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^4
- 5568*x^3 + 1080*x^2 + 4320*x - 5184) + 223064064*x - 94887936))/(2585191*
x^8 - 4661200*x^7 + 14191920*x^6 + 490880*x^5 - 13562944*x^4 + 44249088*x^3
- 34615296*x^2 - 24772608*x + 18579456)) - 4988406728^(1/4)*sqrt(24971)*sq
rt(341)*sqrt(31)*(499420*x^2 - 70517*sqrt(2)*(5*x^2 + 3*x + 2) + 299652*x +
199768)*sqrt(70517*sqrt(2) + 99884)*log(199768*(4988406728^(1/4)*sqrt(2497
1)*sqrt(341)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(10*x + 3) - 13*x - 7)*s
qrt(70517*sqrt(2) + 99884) + 1175859419*x^2 + 1055873764*sqrt(2)*(2*x^2 - x
+ 3) - 3623566781*x + 4799426200)/x^2) + 4988406728^(1/4)*sqrt(24971)*sqrt
(341)*sqrt(31)*(499420*x^2 - 70517*sqrt(2)*(5*x^2 + 3*x + 2) + 299652*x + 1

```
99768)*sqrt(70517*sqrt(2) + 99884)*log(-199768*(4988406728^(1/4)*sqrt(24971
)*sqrt(341)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(10*x + 3) - 13*x - 7)*sq
rt(70517*sqrt(2) + 99884) - 1175859419*x^2 - 1055873764*sqrt(2)*(2*x^2 - x
+ 3) + 3623566781*x - 4799426200)/x^2) - 6022703949856*sqrt(2*x^2 - x + 3)*
(10*x + 3))/(5*x^2 + 3*x + 2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(1/2)/(5*x^2+3*x+2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Francis algorithm failure for[-1.0,infinity,
infinity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,infi
nity]Francis algorithm failure for[-1.0,infinity,infinity,infinity,infinity
]proot error [1.0,infinity,infinity,infinity,infinity]Francis algorithm fai
lure for[-1.0,infinity,infinity,infinity,infinity]proot error [1.0,infinity
,infinity,infinity,infinity]Francis algorithm failure for[-1.0,infinity,inf
inity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,infinit
y]Evaluation time: 17.67Done
```

maple [B] time = 0.23, size = 16357, normalized size = 87.01

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(1/2)/(5*x^2+3*x+2)^2,x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2\,x^2 - x + 3}}{\left(5\,x^2 + 3\,x + 2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(1/2)/(5*x^2+3*x+2)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(2*x^2 - x + 3)/(5*x^2 + 3*x + 2)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{2\,x^2-x+3}}{{\left(5\,x^2+3\,x+2\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^(1/2)/(3*x + 5*x^2 + 2)^2,x)
[Out] int((2*x^2 - x + 3)^(1/2)/(3*x + 5*x^2 + 2)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2x^2 - x + 3}}{\left(5x^2 + 3x + 2\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(1/2)/(5*x**2+3*x+2)**2,x)
[Out] Integral(sqrt(2*x**2 - x + 3)/(5*x**2 + 3*x + 2)**2, x)
```

3.61 $\int \frac{\sqrt{3-x+2x^2}}{\left(2+3x+5x^2\right)^3}\,dx$

Optimal. Leaf size=223

$$\frac{\sqrt{2x^2-x+3}\,(10x+3)}{62\left(5x^2+3x+2\right)^2}+\frac{(13665x+3464)\sqrt{2x^2-x+3}}{84568\left(5x^2+3x+2\right)}+\frac{\sqrt{\frac{1}{682}\left(112285869463+79399380740\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{}{31(}}}{}\right)}{16}$$

Rubi [A] time = 0.46, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {971, 1060, 1035, 1029, 206, 204}

$$\frac{\sqrt{2x^2-x+3}\,(10x+3)}{62\left(5x^2+3x+2\right)^2}+\frac{(13665x+3464)\sqrt{2x^2-x+3}}{84568\left(5x^2+3x+2\right)}+\frac{\sqrt{\frac{1}{682}\left(112285869463+79399380740\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{11}{31\left(112285869463+79399380740\sqrt{2}\right)}}\left(\left(1235163+872375\sqrt{2}\right)x+362788\sqrt{2}+509587\right)}{\sqrt{2x^2-x+3}}\right)}{169136}-\frac{\sqrt{\frac{1}{682}\left(79399380740\sqrt{2}-112285869463\right)}\tanh^{-1}\left(\frac{\sqrt{\frac{11}{31\left(79399380740\sqrt{2}-112285869463\right)}}\left(\left(1235163-872375\sqrt{2}\right)x-362788\sqrt{2}+509587\right)}{\sqrt{2x^2-x+3}}\right)}{169136}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3 - x + 2*x^2]/(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] ((3 + 10*x)*Sqrt[3 - x + 2*x^2])/(62*(2 + 3*x + 5*x^2)^2) + ((3464 + 13665*
x)*Sqrt[3 - x + 2*x^2])/(84568*(2 + 3*x + 5*x^2)) + (Sqrt[(112285869463 + 7
9399380740*Sqrt[2])/682]*ArcTan[(Sqrt[11/(31*(112285869463 + 79399380740*Sq
rt[2]))]*(509587 + 362788*Sqrt[2] + (1235163 + 872375*Sqrt[2])*x))/Sqrt[3 -
 x + 2*x^2]])/169136 - (Sqrt[(-112285869463 + 79399380740*Sqrt[2])/682]*Arc
Tanh[(Sqrt[11/(31*(-112285869463 + 79399380740*Sqrt[2]))]*(509587 - 362788*
Sqrt[2] + (1235163 - 872375*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]])/169136
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 971

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((b + 2*c*x)*(a + b*x + c*x^2)^(p + 1)*(d + e
```

```
*x + f*x^2)^q)/((b^2 - 4*a*c)*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*(p + 1))
, Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*Simp[2*c*d*(2*p +
 3) + b*e*q + (2*b*f*q + 2*c*e*(2*p + q + 3))*x + 2*c*f*(2*p + 2*q + 3)*x^2
, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
e^2 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0] &&  !IGtQ[q, 0]
```

Rule 1029

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
 c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 1035

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

Rule 1060

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x +
 c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
2*a*(c*d - a*f)))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
```

```
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] && !( !IntegerQ[p] && ILtQ[q, -1]) && !
IGtQ[q, 0]
```

Rubi steps

$$\int \frac{\sqrt{3-x+2x^2}}{\left(2+3x+5x^2\right)^3}\,dx = \frac{(3+10x)\sqrt{3-x+2x^2}}{62\left(2+3x+5x^2\right)^2} - \frac{1}{62}\int \frac{-\frac{183}{2}+31x-40x^2}{\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^2}\,dx$$

$$= \frac{(3+10x)\sqrt{3-x+2x^2}}{62\left(2+3x+5x^2\right)^2} + \frac{(3464+13665x)\sqrt{3-x+2x^2}}{84568\left(2+3x+5x^2\right)} - \frac{\int \frac{-213004+\frac{358655x}{4}}{\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)}\,dx}{465124}$$

$$= \frac{(3+10x)\sqrt{3-x+2x^2}}{62\left(2+3x+5x^2\right)^2} + \frac{(3464+13665x)\sqrt{3-x+2x^2}}{84568\left(2+3x+5x^2\right)} - \frac{\int \frac{\frac{121}{4}\left(110061-77456\sqrt{2}\right)-\frac{121}{4}\left(448\right.}{\sqrt{3-x+2x^2}\left(2+3x+5\right.}}{10232728\sqrt{2}}$$

$$= \frac{(3+10x)\sqrt{3-x+2x^2}}{62\left(2+3x+5x^2\right)^2} + \frac{(3464+13665x)\sqrt{3-x+2x^2}}{84568\left(2+3x+5x^2\right)} - \frac{\left(11\left(158798761480-11228\right.\right.}{}$$

$$= \frac{(3+10x)\sqrt{3-x+2x^2}}{62\left(2+3x+5x^2\right)^2} + \frac{(3464+13665x)\sqrt{3-x+2x^2}}{84568\left(2+3x+5x^2\right)} + \frac{\sqrt{\frac{1}{682}\left(112285869463+793\right.}}{}$$

Mathematica [C] time = 2.08, size = 299, normalized size = 1.34

$$\frac{5\left(\frac{i\sqrt{286+22i\sqrt{31}}\left(258253\sqrt{31}+1004586i\right)\tanh^{-1}\left(\frac{\left(-22-4i\sqrt{31}\right)x+i\sqrt{31}+63}{2\sqrt{286+22i\sqrt{31}}\sqrt{2x^2-x+3}}\right)}{\left(\sqrt{31}-13i\right)^2}+\frac{2000\left(1364\left(\sqrt{31}+13i\right)\sqrt{2x^2-x+3}\left(68325x^3+58315x^2+51362x+11020\right)-5\sqrt{286-22i\sqrt{31}}\left(174475\sqrt{31}-202151i\right)\left(5x^2+3x+2\right)^2\tanh^{-1}\left(\frac{\left(-22+4i\sqrt{31}\right)x-i\sqrt{31}+63}{2\sqrt{286-22i\sqrt{31}}\sqrt{2x^2-x+3}}\right)\right)}{\left(\sqrt{31}-13i\right)\left(\sqrt{31}+13i\right)^2\left(-10ix+\sqrt{31}-3i\right)^2\left(10ix+\sqrt{31}+3i\right)^2}\right)}{14418844}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3 - x + 2*x^2]/(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] (5*((I*Sqrt[286 + (22*I)*Sqrt[31]]*(1004586*I + 258253*Sqrt[31])*ArcTanh[(6
3 + I*Sqrt[31] + (-22 - (4*I)*Sqrt[31])*x)/(2*Sqrt[286 + (22*I)*Sqrt[31]]*S
```

```
qrt[3 - x + 2*x^2])])/(-13*I + Sqrt[31])^2 + (2000*(1364*(13*I + Sqrt[31])*
Sqrt[3 - x + 2*x^2]*(11020 + 51362*x + 58315*x^2 + 68325*x^3) - 5*Sqrt[286
- (22*I)*Sqrt[31]]*(-202151*I + 174475*Sqrt[31])*(2 + 3*x + 5*x^2)^2*ArcTan
h[(63 - I*Sqrt[31] + (-22 + (4*I)*Sqrt[31])*x)/(2*Sqrt[286 - (22*I)*Sqrt[31
]]*Sqrt[3 - x + 2*x^2])]))/((-13*I + Sqrt[31])*(13*I + Sqrt[31])^2*(-3*I +
Sqrt[31] - (10*I)*x)^2*(3*I + Sqrt[31] + (10*I)*x)^2)))/14418844
```

IntegrateAlgebraic [C] time = 0.82, size = 396, normalized size = 1.78

$$\frac{\text{RootSum}\left[-5\#1^4+6\sqrt{2}\#1^3+17\#1^2-26\sqrt{2}\#1-56\&,\frac{-45923442075\#1^2\log\left(-\#1+\sqrt{2x^2-x+3}-\sqrt{2}x\right)+120146195680\sqrt{2}\#1\log\left(-\#1+\sqrt{2x^2-x+3}-\sqrt{2}x\right)-537295920831\log\left(-\#1+\sqrt{2x^2-x+3}-\sqrt{2}x\right)}{-10\#1^3+9\sqrt{2}\#1^2+17\#1-13\sqrt{2}}\&\right]}{55920590000}-\frac{\text{RootSum}\left[-5\#1^4+6\sqrt{2}\#1^3+17\#1^2-26\sqrt{2}\#1-56\&,\frac{-228643025\#1^2\log\left(-\#1+\sqrt{2x^2-x+3}-\sqrt{2}x\right)+277937160\sqrt{2}\#1\log\left(-\#1+\sqrt{2x^2-x+3}-\sqrt{2}x\right)-2139373897\log\left(-\#1+\sqrt{2x^2-x+3}-\sqrt{2}x\right)}{-10\#1^3+9\sqrt{2}\#1^2+17\#1-13\sqrt{2}}\&\right]}{225486250}+\frac{\sqrt{2x^2-x+3}\left(68325x^3+58315x^2+51362x+11020\right)}{84568\left(5x^2+3x+2\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[3 - x + 2*x^2]/(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] (Sqrt[3 - x + 2*x^2]*(11020 + 51362*x + 58315*x^2 + 68325*x^3))/(84568*(2 +
 3*x + 5*x^2)^2) + RootSum[-56 - 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3 -
 5*#1^4 & , (-537295920831*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1] + 1
20146195680*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1 - 45923
442075*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1^2)/(-13*Sqrt[2] + 17
*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3) & ]/55920590000 - RootSum[-56 - 26*Sqrt[2]*
#1 + 17*#1^2 + 6*Sqrt[2]*#1^3 - 5*#1^4 & , (-2139373897*Log[-(Sqrt[2]*x) +
Sqrt[3 - x + 2*x^2] - #1] + 277937160*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x
 + 2*x^2] - #1]*#1 - 228643025*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]
*#1^2)/(-13*Sqrt[2] + 17*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3) & ]/225486250
```

fricas [B] time = 1.44, size = 2182, normalized size = 9.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(1/2)/(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

```
[Out] 1/6505215189695292642599671424O*(14205421276*788032707736935368450^(1/4)*sq
rt(39699690370)*sqrt(341)*sqrt(2)*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)*sqr
t(112285869463*sqrt(2) + 158798761480)*arctan(1/861047662213971287591057659
551879544939625*(2461380802940*sqrt(39699690370)*(22*788032707736935368450^
(3/4)*sqrt(341)*(667937076*x^7 - 2573871186*x^6 + 5404850058*x^5 - 86714302
12*x^4 + 4348809776*x^3 - 2064441888*x^2 - sqrt(2)*(473555282*x^7 - 1821195
871*x^6 + 3826055542*x^5 - 6128133137*x^4 + 3070797960*x^3 - 1452037320*x^2
 - 3352976640*x + 2366869248) - 4733738496*x + 3352976640) + 615345200735*7
88032707736935368450^(1/4)*sqrt(341)*(50730703*x^7 - 778833417*x^6 + 411636
7112*x^5 - 9392273180*x^4 + 12133646496*x^3 - 7660912032*x^2 - sqrt(2)*(359
38543*x^7 - 551546778*x^6 + 2913578540*x^5 - 6643469608*x^4 + 8580088800*x^
3 - 5403919680*x^2 - 6107913216*x + 4313793024) - 8627586048*x + 6107913216
))*sqrt(2*x^2 - x + 3)*sqrt(112285869463*sqrt(2) + 158798761480) + 24442643
```

31446112042193970130340819353377000*sqrt(31)*sqrt(2)*(28180*x^8 - 254666*x^
7 + 704270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - sqrt(
2)*(8746*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^4 - 752088*
x^3 + 396144*x^2 + 546048*x - 539136) + 1154304*x - 456192) - 2*sqrt(396996
90370/160673)*(sqrt(39699690370)*(22*788032707736935368450^(3/4)*sqrt(341)*
(104024992*x^7 - 149335248*x^6 + 480784368*x^5 - 188730368*x^4 + 223535232*
x^3 + 214417152*x^2 - sqrt(2)*(73906058*x^7 - 106073653*x^6 + 341348823*x^5
 - 133050960*x^4 + 156704760*x^3 + 154338048*x^2 - 154338048*x) - 214417152
*x) + 615345200735*788032707736935368450^(1/4)*sqrt(341)*(7903323*x^7 - 102
233612*x^6 + 394216580*x^5 - 510585408*x^4 + 657060192*x^3 + 391744512*x^2
- 4*sqrt(2)*(1401761*x^7 - 18132196*x^6 + 69912940*x^5 - 90501120*x^4 + 116
274240*x^3 + 70118784*x^2 - 70118784*x) - 391744512*x))*sqrt(2*x^2 - x + 3)
*sqrt(112285869463*sqrt(2) + 158798761480) + 431759125243238662114369585 0*
sqrt(31)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 + 396
480*x^4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 118051*x^7 + 2440
47*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*x)
+ 3276288*x) + 19625414783783575550519861 75*sqrt(31)*(254591*x^8 - 4815126*
x^7 + 32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 74219328*x^3 - 16895692
8*x^2 - 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 361
8*x^3 + 2268*x^2 - 1944*x) + 144820224*x))*sqrt(-(788032707736935368450^(1/
4)*sqrt(39699690370)*sqrt(341)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(12053
*x + 5138) - 17191*x - 6915)*sqrt(112285869463*sqrt(2) + 158798761480) - 15
0182556985858180945*x^2 - 134857806273015509420*sqrt(2)*(2*x^2 - x + 3) + 4
62807471527848680055*x - 612990028513706861000)/x^2) + 2777573103916036411 5
840569662963856288375*sqrt(31)*(2828123*x^8 - 9696916*x^7 + 53385560*x^6 -
142835344*x^5 + 254146592*x^4 - 249300096*x^3 + 37981440*x^2 - 7744*sqrt(2)
*(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^4 - 5568*x^3 + 1080*
x^2 + 4320*x - 5184) + 223064064*x - 94887936))/(2585191*x^8 - 4661200*x^7
+ 14191920*x^6 + 490880*x^5 - 13562944*x^4 + 44249088*x^3 - 34615296*x^2 -
24772608*x + 18579456)) + 14205421276*788032707736935368450^(1/4)*sqrt(3969
9690370)*sqrt(341)*sqrt(2)*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)*sqrt(11228
5869463*sqrt(2) + 158798761480)*arctan(1/86104766221397128759105765955187 95
44939625*(2461380802940*sqrt(39699690370)*(22*788032707736935368450^(3/4)*s
qrt(341)*(667937076*x^7 - 2573871186*x^6 + 5404850058*x^5 - 8671430212*x^4
+ 4348809776*x^3 - 2064441888*x^2 - sqrt(2)*(473555282*x^7 - 1821195871*x^6
 + 3826055542*x^5 - 6128133137*x^4 + 3070797960*x^3 - 1452037320*x^2 - 3352
976640*x + 2366869248) - 4733738496*x + 3352976640) + 615345200735*78803270
7736935368450^(1/4)*sqrt(341)*(50730703*x^7 - 778833417*x^6 + 4116367112*x^
5 - 9392273180*x^4 + 12133646496*x^3 - 7660912032*x^2 - sqrt(2)*(35938543*x
^7 - 551546778*x^6 + 2913578540*x^5 - 6643469608*x^4 + 8580088800*x^3 - 540
3919680*x^2 - 6107913216*x + 4313793024) - 8627586048*x + 6107913216))*sqrt
(2*x^2 - x + 3)*sqrt(112285869463*sqrt(2) + 158798761480) - 244426433144611
20421939701303408193533770 00*sqrt(31)*sqrt(2)*(28180*x^8 - 254666*x^7 + 704
270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - sqrt(2)*(874
6*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^4 - 752088*x^3 + 3

96144*x^2 + 546048*x - 539136) + 1154304*x - 456192) - 2*sqrt(39699690370/1
60673)*(sqrt(39699690370)*(22*788032707736935368450^(3/4)*sqrt(341)*(104024
992*x^7 - 149335248*x^6 + 480784368*x^5 - 188730368*x^4 + 223535232*x^3 + 2
14417152*x^2 - sqrt(2)*(73906058*x^7 - 106073653*x^6 + 341348823*x^5 - 1330
50960*x^4 + 156704760*x^3 + 154338048*x^2 - 154338048*x) - 214417152*x) + 6
15345200735*788032707736935368450^(1/4)*sqrt(341)*(7903323*x^7 - 102233612*
x^6 + 394216580*x^5 - 510585408*x^4 + 657060192*x^3 + 391744512*x^2 - 4*sqr
t(2)*(1401761*x^7 - 18132196*x^6 + 69912940*x^5 - 90501120*x^4 + 116274240*
x^3 + 70118784*x^2 - 70118784*x) - 391744512*x))*sqrt(2*x^2 - x + 3)*sqrt(1
12285869463*sqrt(2) + 158798761480) - 431759125243238662111436958 50*sqrt(31
)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 + 396480*x^4
 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 118051*x^7 + 244047*x^6
- 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*x) + 32762
88*x) - 19625414783783575550519861 75*sqrt(31)*(254591*x^8 - 4815126*x^7 + 3
2303580*x^6 - 90866808*x^5 + 108781920*x^4 - 74219328*x^3 - 168956928*x^2 -
 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3618*x^3 +
 2268*x^2 - 1944*x) + 144820224*x))*sqrt((788032707736935368450^(1/4)*sqrt(
39699690370)*sqrt(341)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(12053*x + 513
8) - 17191*x - 6915)*sqrt(112285869463*sqrt(2) + 158798761480) + 1501825569
85858180945*x^2 + 134857806273015509420*sqrt(2)*(2*x^2 - x + 3) - 462807471
527848680055*x + 612990028513706861000)/x^2) - 2777573103916036411584056966
2963856288375*sqrt(31)*(2828123*x^8 - 9696916*x^7 + 53385560*x^6 - 14283534
4*x^5 + 254146592*x^4 - 249300096*x^3 + 37981440*x^2 - 7744*sqrt(2)*(1348*x
^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^4 - 5568*x^3 + 1080*x^2 + 43
20*x - 5184) + 223064064*x - 94887936))/(2585191*x^8 - 4661200*x^7 + 141919
20*x^6 + 490880*x^5 - 13562944*x^4 + 44249088*x^3 - 34615296*x^2 - 24772608
*x + 18579456)) + 788032707736935368450^(1/4)*sqrt(39699690370)*sqrt(341)*s
qrt(31)*(3969969037000*x^4 + 4763962844400*x^3 + 4605164082920*x^2 - 112285
869463*sqrt(2)*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4) + 1905585137760*x + 63
5195045920)*sqrt(112285869463*sqrt(2) + 158798761480)*log(635195045920/1606
73*(788032707736935368450^(1/4)*sqrt(39699690370)*sqrt(341)*sqrt(31)*sqrt(2
x^2 - x + 3)(sqrt(2)*(12053*x + 5138) - 17191*x - 6915)*sqrt(112285869463
*sqrt(2) + 158798761480) + 150182556985858180945*x^2 + 13485780627301550942
0*sqrt(2)*(2*x^2 - x + 3) - 462807471527848680055*x + 612990028513706861000
)/x^2) - 788032707736935368450^(1/4)*sqrt(39699690370)*sqrt(341)*sqrt(31)*(
3969969037000*x^4 + 4763962844400*x^3 + 4605164082920*x^2 - 112285869463*sq
rt(2)*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4) + 1905585137760*x + 63519504592
0)*sqrt(112285869463*sqrt(2) + 158798761480)*log(-635195045920/160673*(7880
32707736935368450^(1/4)*sqrt(39699690370)*sqrt(341)*sqrt(31)*sqrt(2*x^2 - x
 + 3)*(sqrt(2)*(12053*x + 5138) - 17191*x - 6915)*sqrt(112285869463*sqrt(2)
 + 158798761480) - 150182556985858180945*x^2 - 134857806273015509420*sqrt(2
)*(2*x^2 - x + 3) + 462807471527848680055*x - 612990028513706861000)/x^2) +
 7692289269812804657316 80*(68325*x^3 + 58315*x^2 + 51362*x + 11020)*sqrt(2*
x^2 - x + 3))/(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(1/2)/(5*x^2+3*x+2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Francis algorithm failure for[-1.0,infinity,
infinity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,infi
nity]Francis algorithm failure for[-1.0,infinity,infinity,infinity,infinity
]proot error [1.0,infinity,infinity,infinity,infinity]Francis algorithm fai
lure for[-1.0,infinity,infinity,infinity,infinity]proot error [1.0,infinity
,infinity,infinity,infinity]Francis algorithm failure for[-1.0,infinity,inf
inity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,infinit
y]Evaluation time: 48.06Done
```

maple [B] time = 0.37, size = 44343, normalized size = 198.85

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(1/2)/(5*x^2+3*x+2)^3,x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2\,x^2 - x + 3}}{\left(5\,x^2 + 3\,x + 2\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(1/2)/(5*x^2+3*x+2)^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(2*x^2 - x + 3)/(5*x^2 + 3*x + 2)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{2\,x^2 - x + 3}}{\left(5\,x^2 + 3\,x + 2\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^(1/2)/(3*x + 5*x^2 + 2)^3,x)
```

```
[Out] int((2*x^2 - x + 3)^(1/2)/(3*x + 5*x^2 + 2)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2x^2 - x + 3}}{\left(5x^2 + 3x + 2\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(1/2)/(5*x**2+3*x+2)**3,x)
```

```
[Out] Integral(sqrt(2*x**2 - x + 3)/(5*x**2 + 3*x + 2)**3, x)
```

3.62 $\int \left(3-x+2x^2\right)^{3/2} \left(2+3x+5x^2\right)^4 \, dx$

Optimal. Leaf size=231

$$-\frac{56422489 \left(2x^2-x+3\right)^{5/2} x^2}{8257536}+\frac{48669967 \left(2x^2-x+3\right)^{5/2} x}{22020096}+\frac{2124689283 \left(2x^2-x+3\right)^{5/2}}{146800640}-\frac{382121949(1-4x)}{134217}$$

Rubi [A] time = 0.34, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1661, 640, 612, 619, 215}

$$\frac{625}{24} \left(2x^2-x+3\right)^{5/2} x^7+\frac{7625}{96} \left(2x^2-x+3\right)^{5/2} x^6+\frac{95165}{768} \left(2x^2-x+3\right)^{5/2} x^5+\frac{941905 \left(2x^2-x+3\right)^{5/2} x^4}{9216}+\frac{10444117 \left(2x^2-x+3\right)^{5/2} x^3}{294912}-\frac{56422489 \left(2x^2-x+3\right)^{5/2} x^2}{8257536}+\frac{48669967 \left(2x^2-x+3\right)^{5/2} x}{22020096}+\frac{2124689283 \left(2x^2-x+3\right)^{5/2}}{146800640}-\frac{382121949(1-4x) \left(2x^2-x+3\right)^{3/2}}{134217728}-\frac{26366414481(1-4x)\sqrt{2x^2-x+3}}{2147483648}-\frac{606427533063 \sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{4294967296\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)^4,x]
```

```
[Out] (-26366414481*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/2147483648 - (382121949*(1 - 4
*x)*(3 - x + 2*x^2)^(3/2))/134217728 + (2124689283*(3 - x + 2*x^2)^(5/2))/1
46800640 + (48669967*x*(3 - x + 2*x^2)^(5/2))/22020096 - (56422489*x^2*(3 -
 x + 2*x^2)^(5/2))/8257536 + (10444117*x^3*(3 - x + 2*x^2)^(5/2))/294912 +
(941905*x^4*(3 - x + 2*x^2)^(5/2))/9216 + (95165*x^5*(3 - x + 2*x^2)^(5/2))
/768 + (7625*x^6*(3 - x + 2*x^2)^(5/2))/96 + (625*x^7*(3 - x + 2*x^2)^(5/2)
)/24 - (606427533063*ArcSinh[(1 - 4*x)/Sqrt[23]])/(4294967296*Sqrt[2])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \left(3-x+2x^2\right)^{3/2}\left(2+3x+5x^2\right)^4\,dx = \frac{625}{24}x^7\left(3-x+2x^2\right)^{5/2}+\frac{1}{24}\int\left(3-x+2x^2\right)^{3/2}\Big(384+2304x+9024x^2$$

$$= \frac{7625}{96}x^6\left(3-x+2x^2\right)^{5/2}+\frac{625}{24}x^7\left(3-x+2x^2\right)^{5/2}+\frac{1}{528}\int\left(3-x+2x^2\right.$$

$$= \frac{95165}{768}x^5\left(3-x+2x^2\right)^{5/2}+\frac{7625}{96}x^6\left(3-x+2x^2\right)^{5/2}+\frac{625}{24}x^7\left(3-x+2\right.$$

$$= \frac{941905x^4\left(3-x+2x^2\right)^{5/2}}{9216}+\frac{95165}{768}x^5\left(3-x+2x^2\right)^{5/2}+\frac{7625}{96}x^6\left(3-x\right.$$

$$= \frac{10444117x^3\left(3-x+2x^2\right)^{5/2}}{294912}+\frac{941905x^4\left(3-x+2x^2\right)^{5/2}}{9216}+\frac{95165}{768}x^5\left(3\right.$$

$$= -\frac{56422489x^2\left(3-x+2x^2\right)^{5/2}}{8257536}+\frac{10444117x^3\left(3-x+2x^2\right)^{5/2}}{294912}+\frac{941905}{}$$

$$= \frac{48669967x\left(3-x+2x^2\right)^{5/2}}{22020096}-\frac{56422489x^2\left(3-x+2x^2\right)^{5/2}}{8257536}+\frac{1044411}{}$$

$$= \frac{2124689283\left(3-x+2x^2\right)^{5/2}}{146800640}+\frac{48669967x\left(3-x+2x^2\right)^{5/2}}{22020096}-\frac{5642248}{}$$

$$= -\frac{382121949(1-4x)\left(3-x+2x^2\right)^{3/2}}{134217728}+\frac{2124689283\left(3-x+2x^2\right)^{5/2}}{146800640}+$$

$$= -\frac{26366414481(1-4x)\sqrt{3-x+2x^2}}{2147483648}-\frac{382121949(1-4x)\left(3-x+2x^2\right.}{134217728}$$

$$= -\frac{26366414481(1-4x)\sqrt{3-x+2x^2}}{2147483648}-\frac{382121949(1-4x)\left(3-x+2x^2\right.}{134217728}$$

$$= -\frac{26366414481(1-4x)\sqrt{3-x+2x^2}}{2147483648}-\frac{382121949(1-4x)\left(3-x+2x^2\right.}{134217728}$$

Mathematica [A] time = 0.37, size = 95, normalized size = 0.41

$$\frac{4\sqrt{2x^2-x+3}\left(70464307200000x^{11}+144451829760000x^{10}+349379651174400x^9+534038708224000x^8+745133229998080x^7+765087080448000x^6+675479464714240x^5+451581382260736x^4+239021184223104x^3+65151998063712x^2+12971175524316x+74032009514181\right)-191024672914845\sqrt{2}\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{2705829396480}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)^4,x]
```

```
[Out] (4*Sqrt[3 - x + 2*x^2]*(74032009514181 + 12971175524316*x + 65151998063712*
x^2 + 239021184223104*x^3 + 451581382260736*x^4 + 675479464714240*x^5 + 765
087080448000*x^6 + 745133229998080*x^7 + 534038708224000*x^8 + 349379651174
400*x^9 + 144451829760000*x^10 + 70464307200000*x^11) - 191024672914845*Sqr
t[2]*ArcSinh[(1 - 4*x)/Sqrt[23]])/2705829396480
```

IntegrateAlgebraic [A] time = 1.37, size = 110, normalized size = 0.48

$$\frac{\sqrt{2x^2-x+3}\left(70464307200000x^{11}+144451829760000x^{10}+349379651174400x^9+534038708224000x^8+745133229998080x^7+765087080448000x^6+675479464714240x^5+451581382260736x^4+239021184223104x^3+65151998063712x^2+12971175524316x+74032009514181\right)}{676457349120}-\frac{606427533063\log\left(2\sqrt{2}\sqrt{2x^2-x+3}-4x+1\right)}{4294967296\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)^4,x]
```

```
[Out] (Sqrt[3 - x + 2*x^2]*(74032009514181 + 12971175524316*x + 65151998063712*x^
2 + 239021184223104*x^3 + 451581382260736*x^4 + 675479464714240*x^5 + 76508
7080448000*x^6 + 745133229998080*x^7 + 534038708224000*x^8 + 34937965117440
0*x^9 + 144451829760000*x^10 + 70464307200000*x^11))/676457349120 - (606427
533063*Log[1 - 4*x + 2*Sqrt[2]*Sqrt[3 - x + 2*x^2]])/(4294967296*Sqrt[2])
```

fricas [A] time = 0.48, size = 108, normalized size = 0.47

$$\frac{1}{676457349120}\left(70464307200000\,x^{11}+144451829760000\,x^{10}+349379651174400\,x^9+534038708224000\,x^8+745133229998080\,x^7+765087080448000\,x^6+675479464714240\,x^5+451581382260736\,x^4+239021184223104\,x^3+65151998063712\,x^2+12971175524316\,x+74032009514181\right)\sqrt{2\,x^2-x+3}+\frac{606427533063}{17179869184}\,\sqrt{2}\,\log\left(-4\,\sqrt{2}\,\sqrt{2\,x^2-x+3}\,(4\,x-1)-32\,x^2+16\,x-25\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2)^4,x, algorithm="fricas")
```

```
[Out] 1/676457349120*(70464307200000*x^11 + 144451829760000*x^10 + 34937965117440
0*x^9 + 534038708224000*x^8 + 745133229998080*x^7 + 765087080448000*x^6 + 6
75479464714240*x^5 + 451581382260736*x^4 + 239021184223104*x^3 + 6515199806
3712*x^2 + 12971175524316*x + 74032009514181)*sqrt(2*x^2 - x + 3) + 6064275
33063/17179869184*sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32
*x^2 + 16*x - 25)
```

giac [A] time = 0.25, size = 103, normalized size = 0.45

$$\frac{1}{676457349120}\,(4\,(8\,(4\,(16\,(20\,(8\,(28\,(160\,(12\,(200\,(20\,x+41)x+19833)x+363785)x+81213077)x+2334860475)x+16491197869)x+220498721807)x+1867353001743)x+2035999939491)x+3242793881079)x+74032009514181)\sqrt{2\,x^2-x+3}-\frac{606427533063}{8589934592}\,\sqrt{2}\,\log\left(-2\,\sqrt{2}\left(\sqrt{2}\,x-\sqrt{2\,x^2-x+3}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2)^4,x, algorithm="giac")
```

```
[Out] 1/676457349120*(4*(8*(4*(16*(20*(8*(28*(160*(12*(200*(20*x + 41)*x + 19833)
*x + 363785)*x + 81213077)*x + 2334860475)*x + 16491197869)*x + 22049872180
7)*x + 1867353001743)*x + 2035999939491)*x + 3242793881079)*x + 74032009514
```

```
181)*sqrt(2*x^2 - x + 3) - 606427533063/8589934592*sqrt(2)*log(-2*sqrt(2)*(
sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1)
```

maple [A] time = 0.04, size = 185, normalized size = 0.80

$$\frac{625\left(2x^2-x+3\right)^{\frac{5}{2}}x^7}{24}+\frac{7625\left(2x^2-x+3\right)^{\frac{5}{2}}x^6}{96}+\frac{95165\left(2x^2-x+3\right)^{\frac{5}{2}}x^5}{768}+\frac{941905\left(2x^2-x+3\right)^{\frac{5}{2}}x^4}{9216}+\frac{10444117\left(2x^2-x+3\right)^{\frac{5}{2}}x^3}{294912}-\frac{56422489\left(2x^2-x+3\right)^{\frac{5}{2}}x^2}{8257536}+\frac{48669967\left(2x^2-x+3\right)^{\frac{5}{2}}x}{22020096}+\frac{606427533063\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{8589934592}+\frac{2124689283\left(2x^2-x+3\right)^{\frac{5}{2}}}{146800640}+\frac{26366414481\,(4x-1)\sqrt{2x^2-x+3}}{2147483648}+\frac{382121949\,(4x-1)\left(2x^2-x+3\right)^{\frac{3}{2}}}{134217728}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2)^4,x)
```

```
[Out] 625/24*x^7*(2*x^2-x+3)^(5/2)+7625/96*x^6*(2*x^2-x+3)^(5/2)+2124689283/14680
0640*(2*x^2-x+3)^(5/2)+95165/768*x^5*(2*x^2-x+3)^(5/2)+941905/9216*x^4*(2*x
^2-x+3)^(5/2)+10444117/294912*x^3*(2*x^2-x+3)^(5/2)-56422489/8257536*x^2*(2
*x^2-x+3)^(5/2)+48669967/22020096*x*(2*x^2-x+3)^(5/2)+606427533063/85899345
92*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))+26366414481/2147483648*(4*x-1)*(2
*x^2-x+3)^(1/2)+382121949/134217728*(4*x-1)*(2*x^2-x+3)^(3/2)
```

maxima [A] time = 1.02, size = 206, normalized size = 0.89

$$\frac{625}{24}\left(2x^2-x+3\right)^{\frac{5}{2}}x^7+\frac{7625}{96}\left(2x^2-x+3\right)^{\frac{5}{2}}x^6+\frac{95165}{768}\left(2x^2-x+3\right)^{\frac{5}{2}}x^5+\frac{941905}{9216}\left(2x^2-x+3\right)^{\frac{5}{2}}x^4+\frac{10444117}{294912}\left(2x^2-x+3\right)^{\frac{5}{2}}x^3-\frac{56422489}{8257536}\left(2x^2-x+3\right)^{\frac{5}{2}}x^2+\frac{48669967}{22020096}\left(2x^2-x+3\right)^{\frac{5}{2}}x+\frac{2124689283}{146800640}\left(2x^2-x+3\right)^{\frac{5}{2}}+\frac{382121949}{33554432}\left(2x^2-x+3\right)^{\frac{3}{2}}x-\frac{382121949}{134217728}\left(2x^2-x+3\right)^{\frac{3}{2}}+\frac{26366414481}{536870912}\sqrt{2x^2-x+3}\,x+\frac{606427533063}{8589934592}\sqrt{2}\,\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}\,(4x-1)\right)-\frac{26366414481}{2147483648}\sqrt{2x^2-x+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2)^4,x, algorithm="maxima")
```

```
[Out] 625/24*(2*x^2 - x + 3)^(5/2)*x^7 + 7625/96*(2*x^2 - x + 3)^(5/2)*x^6 + 9516
5/768*(2*x^2 - x + 3)^(5/2)*x^5 + 941905/9216*(2*x^2 - x + 3)^(5/2)*x^4 + 1
0444117/294912*(2*x^2 - x + 3)^(5/2)*x^3 - 56422489/8257536*(2*x^2 - x + 3)
^(5/2)*x^2 + 48669967/22020096*(2*x^2 - x + 3)^(5/2)*x + 2124689283/1468006
40*(2*x^2 - x + 3)^(5/2) + 382121949/33554432*(2*x^2 - x + 3)^(3/2)*x - 382
121949/134217728*(2*x^2 - x + 3)^(3/2) + 26366414481/536870912*sqrt(2*x^2 -
 x + 3)*x + 606427533063/8589934592*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)
) - 26366414481/2147483648*sqrt(2*x^2 - x + 3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(2\,x^2-x+3\right)^{3/2}\,\left(5\,x^2+3\,x+2\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^(3/2)*(3*x + 5*x^2 + 2)^4,x)
```

```
[Out] int((2*x^2 - x + 3)^(3/2)*(3*x + 5*x^2 + 2)^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(2x^2-x+3\right)^{\frac{3}{2}}\left(5x^2+3x+2\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(3/2)*(5*x**2+3*x+2)**4,x)
```

```
[Out] Integral((2*x**2 - x + 3)**(3/2)*(5*x**2 + 3*x + 2)**4, x)
```

3.63 $\int \left(3-x+2x^2\right)^{3/2} \left(2+3x+5x^2\right)^3 \, dx$

Optimal. Leaf size=189

$$\frac{384739\left(2x^2-x+3\right)^{5/2}x^2}{43008}-\frac{81685\left(2x^2-x+3\right)^{5/2}x}{114688}-\frac{4625907\left(2x^2-x+3\right)^{5/2}}{2293760}-\frac{667795(1-4x)\left(2x^2-x+3\right)^{3/2}}{2097152}$$

Rubi [A] time = 0.19, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1661, 640, 612, 619, 215}

$$\frac{25}{4}\left(2x^2-x+3\right)^{5/2}x^5+\frac{725}{48}\left(2x^2-x+3\right)^{5/2}x^4+\frac{27785\left(2x^2-x+3\right)^{5/2}x^3}{1536}+\frac{384739\left(2x^2-x+3\right)^{5/2}x^2}{43008}-\frac{81685\left(2x^2-x+3\right)^{5/2}x}{114688}-\frac{4625907\left(2x^2-x+3\right)^{5/2}}{2293760}-\frac{667795(1-4x)\left(2x^2-x+3\right)^{3/2}}{2097152}-\frac{46077855(1-4x)\sqrt{2x^2-x+3}}{33554432}-\frac{1059790665\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{67108864\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] (-46077855*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/33554432 - (667795*(1 - 4*x)*(3 -
 x + 2*x^2)^(3/2))/2097152 - (4625907*(3 - x + 2*x^2)^(5/2))/2293760 - (816
85*x*(3 - x + 2*x^2)^(5/2))/114688 + (384739*x^2*(3 - x + 2*x^2)^(5/2))/430
08 + (27785*x^3*(3 - x + 2*x^2)^(5/2))/1536 + (725*x^4*(3 - x + 2*x^2)^(5/2
))/48 + (25*x^5*(3 - x + 2*x^2)^(5/2))/4 - (1059790665*ArcSinh[(1 - 4*x)/Sq
rt[23]])/(67108864*Sqrt[2])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \left(3-x+2x^2\right)^{3/2} \left(2+3x+5x^2\right)^3 \, dx = \frac{25}{4}x^5 \left(3-x+2x^2\right)^{5/2} + \frac{1}{20}\int \left(3-x+2x^2\right)^{3/2} \Big(160+720x+2280x^2+$$

$$= \frac{725}{48}x^4 \left(3-x+2x^2\right)^{5/2} + \frac{25}{4}x^5 \left(3-x+2x^2\right)^{5/2} + \frac{1}{360}\int \left(3-x+2x^2\right)^{3}$$

$$= \frac{27785x^3 \left(3-x+2x^2\right)^{5/2}}{1536} + \frac{725}{48}x^4 \left(3-x+2x^2\right)^{5/2} + \frac{25}{4}x^5 \left(3-x+2x^2\right.$$

$$= \frac{384739x^2 \left(3-x+2x^2\right)^{5/2}}{43008} + \frac{27785x^3 \left(3-x+2x^2\right)^{5/2}}{1536} + \frac{725}{48}x^4 \left(3-x\right.$$

$$= -\frac{81685x \left(3-x+2x^2\right)^{5/2}}{114688} + \frac{384739x^2 \left(3-x+2x^2\right)^{5/2}}{43008} + \frac{27785x^3 \left(3-\right.}{153}$$

$$= -\frac{4625907 \left(3-x+2x^2\right)^{5/2}}{2293760} - \frac{81685x \left(3-x+2x^2\right)^{5/2}}{114688} + \frac{384739x^2 \left(3-\right.}{4300}$$

$$= -\frac{667795(1-4x) \left(3-x+2x^2\right)^{3/2}}{2097152} - \frac{4625907 \left(3-x+2x^2\right)^{5/2}}{2293760} - \frac{81685x}{}$$

$$= -\frac{46077855(1-4x)\sqrt{3-x+2x^2}}{33554432} - \frac{667795(1-4x) \left(3-x+2x^2\right)^{3/2}}{2097152} - $$

$$= -\frac{46077855(1-4x)\sqrt{3-x+2x^2}}{33554432} - \frac{667795(1-4x) \left(3-x+2x^2\right)^{3/2}}{2097152} - $$

$$= -\frac{46077855(1-4x)\sqrt{3-x+2x^2}}{33554432} - \frac{667795(1-4x) \left(3-x+2x^2\right)^{3/2}}{2097152} - $$

Mathematica [A] time = 0.23, size = 85, normalized size = 0.45

$$\frac{4\sqrt{2x^2-x+3}\left(88080384000x^9+124780544000x^8+328328806400x^7+430820229120x^6+571298324480x^5+487891884032x^4+389257196928x^3+199615064544x^2+53985432012x-72152399943\right)-111278019825\sqrt{2}\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{14092861440}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] (4*Sqrt[3 - x + 2*x^2]*(-72152399943 + 53985432012*x + 199615064544*x^2 + 3
89257196928*x^3 + 487891884032*x^4 + 571298324480*x^5 + 430820229120*x^6 +
328328806400*x^7 + 124780544000*x^8 + 88080384000*x^9) - 111278019825*Sqrt[
2]*ArcSinh[(1 - 4*x)/Sqrt[23]])/14092861440
```

IntegrateAlgebraic [A] time = 1.06, size = 100, normalized size = 0.53

$$\frac{\sqrt{2x^2-x+3}\left(88080384000x^9+124780544000x^8+328328806400x^7+430820229120x^6+571298324480x^5+487891884032x^4+389257196928x^3+199615064544x^2+53985432012x-72152399943\right)}{3523215360}-\frac{1059790665\log\left(2\sqrt{2}\sqrt{2x^2-x+3}-4x+1\right)}{67108864\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] (Sqrt[3 - x + 2*x^2]*(-72152399943 + 53985432012*x + 199615064544*x^2 + 389
257196928*x^3 + 487891884032*x^4 + 571298324480*x^5 + 430820229120*x^6 + 32
8328806400*x^7 + 124780544000*x^8 + 88080384000*x^9))/3523215360 - (1059790
665*Log[1 - 4*x + 2*Sqrt[2]*Sqrt[3 - x + 2*x^2]])/(67108864*Sqrt[2])
```

fricas [A] time = 0.46, size = 98, normalized size = 0.52

$$\frac{1}{3523215360}\left(88080384000x^9+124780544000x^8+328328806400x^7+430820229120x^6+571298324480x^5+487891884032x^4+389257196928x^3+199615064544x^2+53985432012x-72152399943\right)\sqrt{2x^2-x+3}+\frac{1059790665}{268435456}\sqrt{2}\log\left(-4\sqrt{2}\sqrt{2x^2-x+3}(4x-1)-32x^2+16x-25\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

```
[Out] 1/3523215360*(88080384000*x^9 + 124780544000*x^8 + 328328806400*x^7 + 43082
0229120*x^6 + 571298324480*x^5 + 487891884032*x^4 + 389257196928*x^3 + 1996
15064544*x^2 + 53985432012*x - 72152399943)*sqrt(2*x^2 - x + 3) + 105979066
5/268435456*sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 +
 16*x - 25)
```

giac [A] time = 0.26, size = 93, normalized size = 0.49

$$\frac{1}{3523215360}\left(4\left(8\left(4\left(16\left(20\left(8\left(140\left(160\left(12x+17\right)x+7157\right)x+1314759\right)x+13947713\right)x+238228459\right)x+3041071851\right)x+6237970767\right)x+13496358003\right)x-72152399943\right)\sqrt{2x^2-x+3}-\frac{1059790665}{134217728}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2)^3,x, algorithm="giac")
```

```
[Out] 1/3523215360*(4*(8*(4*(16*(20*(8*(140*(160*(12*x + 17)*x + 7157)*x + 131475
9)*x + 13947713)*x + 238228459)*x + 3041071851)*x + 6237970767)*x + 1349635
8003)*x - 72152399943)*sqrt(2*x^2 - x + 3) - 1059790665/134217728*sqrt(2)*l
og(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1)
```

maple [A] time = 0.01, size = 151, normalized size = 0.80

$$\frac{25\left(2x^2-x+3\right)^{\frac{5}{2}}x^5}{4}+\frac{725\left(2x^2-x+3\right)^{\frac{5}{2}}x^4}{48}+\frac{27785\left(2x^2-x+3\right)^{\frac{5}{2}}x^3}{1536}+\frac{384739\left(2x^2-x+3\right)^{\frac{5}{2}}x^2}{43008}-\frac{81685\left(2x^2-x+3\right)^{\frac{5}{2}}x}{114688}+\frac{1059790665\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{134217728}-\frac{4625907\left(2x^2-x+3\right)^{\frac{5}{2}}}{2293760}+\frac{46077855(4x-1)\sqrt{2x^2-x+3}}{33554432}+\frac{667795(4x-1)\left(2x^2-x+3\right)^{\frac{3}{2}}}{2097152}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2)^3,x)
```

```
[Out] -4625907/2293760*(2*x^2-x+3)^(5/2)+25/4*(2*x^2-x+3)^(5/2)*x^5+725/48*(2*x^2
-x+3)^(5/2)*x^4+27785/1536*(2*x^2-x+3)^(5/2)*x^3+384739/43008*(2*x^2-x+3)^(
5/2)*x^2-81685/114688*(2*x^2-x+3)^(5/2)*x+1059790665/134217728*2^(1/2)*arcs
inh(4/23*23^(1/2)*(x-1/4))+46077855/33554432*(4*x-1)*(2*x^2-x+3)^(1/2)+6677
95/2097152*(4*x-1)*(2*x^2-x+3)^(3/2)
```

maxima [A] time = 0.99, size = 172, normalized size = 0.91

$$\frac{25}{4}\left(2x^2-x+3\right)^{\frac{5}{2}}x^5+\frac{725}{48}\left(2x^2-x+3\right)^{\frac{5}{2}}x^4+\frac{27785}{1536}\left(2x^2-x+3\right)^{\frac{5}{2}}x^3+\frac{384739}{43008}\left(2x^2-x+3\right)^{\frac{5}{2}}x^2-\frac{81685}{114688}\left(2x^2-x+3\right)^{\frac{5}{2}}x-\frac{4625907}{2293760}\left(2x^2-x+3\right)^{\frac{5}{2}}+\frac{667795}{524288}\left(2x^2-x+3\right)^{\frac{3}{2}}x-\frac{667795}{2097152}\left(2x^2-x+3\right)^{\frac{3}{2}}+\frac{46077855}{8388608}\sqrt{2x^2-x+3}\,x+\frac{1059790665}{134217728}\sqrt{2}\,\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}\,(4x-1)\right)-\frac{46077855}{33554432}\sqrt{2x^2-x+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2)^3,x, algorithm="maxima")
```

```
[Out] 25/4*(2*x^2 - x + 3)^(5/2)*x^5 + 725/48*(2*x^2 - x + 3)^(5/2)*x^4 + 27785/1
536*(2*x^2 - x + 3)^(5/2)*x^3 + 384739/43008*(2*x^2 - x + 3)^(5/2)*x^2 - 81
685/114688*(2*x^2 - x + 3)^(5/2)*x - 4625907/2293760*(2*x^2 - x + 3)^(5/2)
+ 667795/524288*(2*x^2 - x + 3)^(3/2)*x - 667795/2097152*(2*x^2 - x + 3)^(3
/2) + 46077855/8388608*sqrt(2*x^2 - x + 3)*x + 1059790665/134217728*sqrt(2)
*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 46077855/33554432*sqrt(2*x^2 - x + 3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(2x^2-x+3\right)^{3/2}\left(5x^2+3x+2\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^(3/2)*(3*x + 5*x^2 + 2)^3,x)
```

```
[Out] int((2*x^2 - x + 3)^(3/2)*(3*x + 5*x^2 + 2)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(2x^2-x+3\right)^{\frac{3}{2}}\left(5x^2+3x+2\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(3/2)*(5*x**2+3*x+2)**3,x)
```

```
[Out] Integral((2*x**2 - x + 3)**(3/2)*(5*x**2 + 3*x + 2)**3, x)
```

3.64 $\int \left(3-x+2x^2\right)^{3/2} \left(2+3x+5x^2\right)^2 \, dx$

Optimal. Leaf size=147

$$\frac{1235}{448}\left(2x^2-x+3\right)^{5/2}x^2+\frac{24499\left(2x^2-x+3\right)^{5/2}x}{10752}+\frac{73861\left(2x^2-x+3\right)^{5/2}}{215040}+\frac{24293(1-4x)\left(2x^2-x+3\right)^{3/2}}{196608}+\frac{55}{}$$

Rubi [A] time = 0.12, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1661, 640, 612, 619, 215}

$$\frac{25}{16}\left(2x^2-x+3\right)^{5/2}x^3+\frac{1235}{448}\left(2x^2-x+3\right)^{5/2}x^2+\frac{24499\left(2x^2-x+3\right)^{5/2}x}{10752}+\frac{73861\left(2x^2-x+3\right)^{5/2}}{215040}+\frac{24293(1-4x)\left(2x^2-x+3\right)^{3/2}}{196608}+\frac{558739(1-4x)\sqrt{2x^2-x+3}}{1048576}+\frac{12850997\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{2097152\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] (558739*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/1048576 + (24293*(1 - 4*x)*(3 - x +
2*x^2)^(3/2))/196608 + (73861*(3 - x + 2*x^2)^(5/2))/215040 + (24499*x*(3 -
 x + 2*x^2)^(5/2))/10752 + (1235*x^2*(3 - x + 2*x^2)^(5/2))/448 + (25*x^3*(
3 - x + 2*x^2)^(5/2))/16 + (12850997*ArcSinh[(1 - 4*x)/Sqrt[23]])/(2097152*
Sqrt[2])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \left(3-x+2x^2\right)^{3/2}\left(2+3x+5x^2\right)^2\,dx = \frac{25}{16}x^3\left(3-x+2x^2\right)^{5/2} + \frac{1}{16}\int \left(3-x+2x^2\right)^{3/2}\left(64+192x+239x^2+ \cdots\right.$$

$$= \frac{1235}{448}x^2\left(3-x+2x^2\right)^{5/2} + \frac{25}{16}x^3\left(3-x+2x^2\right)^{5/2} + \frac{1}{224}\int \left(3-x+2x^2\right) \cdots$$

$$= \frac{24499x\left(3-x+2x^2\right)^{5/2}}{10752} + \frac{1235}{448}x^2\left(3-x+2x^2\right)^{5/2} + \frac{25}{16}x^3\left(3-x+2x^2 \cdots\right.$$

$$= \frac{73861\left(3-x+2x^2\right)^{5/2}}{215040} + \frac{24499x\left(3-x+2x^2\right)^{5/2}}{10752} + \frac{1235}{448}x^2\left(3-x+2x \cdots\right.$$

$$= \frac{24293(1-4x)\left(3-x+2x^2\right)^{3/2}}{196608} + \frac{73861\left(3-x+2x^2\right)^{5/2}}{215040} + \frac{24499x\left(3- \cdots\right.}{107 \cdots}$$

$$= \frac{558739(1-4x)\sqrt{3-x+2x^2}}{1048576} + \frac{24293(1-4x)\left(3-x+2x^2\right)^{3/2}}{196608} + \frac{7386 \cdots}{\cdots}$$

$$= \frac{558739(1-4x)\sqrt{3-x+2x^2}}{1048576} + \frac{24293(1-4x)\left(3-x+2x^2\right)^{3/2}}{196608} + \frac{7386 \cdots}{\cdots}$$

$$= \frac{558739(1-4x)\sqrt{3-x+2x^2}}{1048576} + \frac{24293(1-4x)\left(3-x+2x^2\right)^{3/2}}{196608} + \frac{7386 \cdots}{\cdots}$$

Mathematica [A] time = 0.14, size = 75, normalized size = 0.51

$$\frac{4\sqrt{2x^2-x+3}\left(688128000x^7+525926400x^6+2025840640x^5+2061273088x^4+2728413312x^3+1799647136x^2+1619403428x+439831323\right)+1349354685\sqrt{2}\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{440401920}$$

Antiderivative was successfully verified.

[In] Integrate[(3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)^2,x]

[Out] (4*Sqrt[3 - x + 2*x^2]*(439831323 + 1619403428*x + 1799647136*x^2 + 2728413312*x^3 + 2061273088*x^4 + 2025840640*x^5 + 525926400*x^6 + 688128000*x^7) + 1349354685*Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]])/440401920

IntegrateAlgebraic [A] time = 0.82, size = 90, normalized size = 0.61

$$\frac{12850997\log\left(2\sqrt{2}\sqrt{2x^2-x+3}-4x+1\right)}{2097152\sqrt{2}}+\frac{\sqrt{2x^2-x+3}\left(688128000x^7+525926400x^6+2025840640x^5+2061273088x^4+2728413312x^3+1799647136x^2+1619403428x+439831323\right)}{110100480}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)^2,x]

[Out] (Sqrt[3 - x + 2*x^2]*(439831323 + 1619403428*x + 1799647136*x^2 + 2728413312*x^3 + 2061273088*x^4 + 2025840640*x^5 + 525926400*x^6 + 688128000*x^7))/110100480 + (12850997*Log[1 - 4*x + 2*Sqrt[2]*Sqrt[3 - x + 2*x^2]])/(2097152*Sqrt[2])

fricas [A] time = 0.45, size = 88, normalized size = 0.60

$$\frac{1}{110100480}\left(688128000x^7+525926400x^6+2025840640x^5+2061273088x^4+2728413312x^3+1799647136x^2+1619403428x+439831323\right)\sqrt{2x^2-x+3}+\frac{12850997}{8388608}\sqrt{2}\log\left(4\sqrt{2}\sqrt{2x^2-x+3}(4x-1)-32x^2+16x-25\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2)^2,x, algorithm="fricas")

[Out] 1/110100480*(688128000*x^7 + 525926400*x^6 + 2025840640*x^5 + 2061273088*x^4 + 2728413312*x^3 + 1799647136*x^2 + 1619403428*x + 439831323)*sqrt(2*x^2 - x + 3) + 12850997/8388608*sqrt(2)*log(4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25)

giac [A] time = 0.26, size = 83, normalized size = 0.56

$$\frac{1}{110100480}(4(8(4(16(20(120(140x+107)x+49459)x+1006481)x+21315729)x+56238973)x+404850857)x+439831323)\sqrt{2x^2-x+3}+\frac{12850997}{4194304}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2)^2,x, algorithm="giac")

[Out] 1/110100480*(4*(8*(4*(16*(20*(120*(140*x + 107)*x + 49459)*x + 1006481)*x + 21315729)*x + 56238973)*x + 404850857)*x + 439831323)*sqrt(2*x^2 - x + 3)

```
+ 12850997/4194304*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))
 + 1)
```

maple [A] time = 0.01, size = 117, normalized size = 0.80

$$\frac{25\left(2x^{2}-x+3\right)^{\frac{5}{2}}x^{3}}{16}+\frac{1235\left(2x^{2}-x+3\right)^{\frac{5}{2}}x^{2}}{448}+\frac{24499\left(2x^{2}-x+3\right)^{\frac{5}{2}}x}{10752}-\frac{12850997\sqrt{2}\ \operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{4194304}+\frac{73861\left(2x^{2}-x+3\right)^{\frac{5}{2}}}{215040}-\frac{558739\left(4x-1\right)\sqrt{2x^{2}-x+3}}{1048576}-\frac{24293\left(4x-1\right)\left(2x^{2}-x+3\right)^{\frac{3}{2}}}{196608}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2)^2,x)
```

```
[Out] 73861/215040*(2*x^2-x+3)^(5/2)+25/16*(2*x^2-x+3)^(5/2)*x^3+1235/448*(2*x^2-
x+3)^(5/2)*x^2+24499/10752*(2*x^2-x+3)^(5/2)*x-12850997/4194304*2^(1/2)*arc
sinh(4/23*23^(1/2)*(x-1/4))-558739/1048576*(4*x-1)*(2*x^2-x+3)^(1/2)-24293/
196608*(4*x-1)*(2*x^2-x+3)^(3/2)
```

maxima [A] time = 0.98, size = 138, normalized size = 0.94

$$\frac{25}{16}\left(2x^{2}-x+3\right)^{\frac{5}{2}}x^{3}+\frac{1235}{448}\left(2x^{2}-x+3\right)^{\frac{5}{2}}x^{2}+\frac{24499}{10752}\left(2x^{2}-x+3\right)^{\frac{5}{2}}x+\frac{73861}{215040}\left(2x^{2}-x+3\right)^{\frac{5}{2}}-\frac{24293}{49152}\left(2x^{2}-x+3\right)^{\frac{3}{2}}x+\frac{24293}{196608}\left(2x^{2}-x+3\right)^{\frac{3}{2}}-\frac{558739}{262144}\sqrt{2x^{2}-x+3}\,x-\frac{12850997}{4194304}\sqrt{2}\ \operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}\left(4x-1\right)\right)+\frac{558739}{1048576}\sqrt{2x^{2}-x+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2)^2,x, algorithm="maxima")
```

```
[Out] 25/16*(2*x^2 - x + 3)^(5/2)*x^3 + 1235/448*(2*x^2 - x + 3)^(5/2)*x^2 + 2449
9/10752*(2*x^2 - x + 3)^(5/2)*x + 73861/215040*(2*x^2 - x + 3)^(5/2) - 2429
3/49152*(2*x^2 - x + 3)^(3/2)*x + 24293/196608*(2*x^2 - x + 3)^(3/2) - 5587
39/262144*sqrt(2*x^2 - x + 3)*x - 12850997/4194304*sqrt(2)*arcsinh(1/23*sqr
t(23)*(4*x - 1)) + 558739/1048576*sqrt(2*x^2 - x + 3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(2\,x^2-x+3\right)}^{3/2}\,{\left(5\,x^2+3\,x+2\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^(3/2)*(3*x + 5*x^2 + 2)^2,x)
```

```
[Out] int((2*x^2 - x + 3)^(3/2)*(3*x + 5*x^2 + 2)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(2x^{2}-x+3\right)^{\frac{3}{2}}\left(5x^{2}+3x+2\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(3/2)*(5*x**2+3*x+2)**2,x)
```

```
[Out] Integral((2*x**2 - x + 3)**(3/2)*(5*x**2 + 3*x + 2)**2, x)
```

3.65 $\int \left(3-x+2x^2\right)^{3/2} \left(2+3x+5x^2\right) dx$

Optimal. Leaf size=105

$$\frac{5}{12}x\left(2x^2-x+3\right)^{5/2}+\frac{107}{240}\left(2x^2-x+3\right)^{5/2}-\frac{179(1-4x)\left(2x^2-x+3\right)^{3/2}}{1536}-\frac{4117(1-4x)\sqrt{2x^2-x+3}}{8192}-\frac{94691\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{16384\sqrt{2}}$$

Rubi [A] time = 0.05, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1661, 640, 612, 619, 215}

$$\frac{5}{12}x\left(2x^2-x+3\right)^{5/2}+\frac{107}{240}\left(2x^2-x+3\right)^{5/2}-\frac{179(1-4x)\left(2x^2-x+3\right)^{3/2}}{1536}-\frac{4117(1-4x)\sqrt{2x^2-x+3}}{8192}-\frac{94691\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{16384\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2),x]
```

```
[Out] (-4117*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/8192 - (179*(1 - 4*x)*(3 - x + 2*x^2)
^(3/2))/1536 + (107*(3 - x + 2*x^2)^(5/2))/240 + (5*x*(3 - x + 2*x^2)^(5/2)
)/12 - (94691*ArcSinh[(1 - 4*x)/Sqrt[23]])/(16384*Sqrt[2])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
```

```
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \left(3-x+2x^2\right)^{3/2}\left(2+3x+5x^2\right)dx = \frac{5}{12}x\left(3-x+2x^2\right)^{5/2}+\frac{1}{12}\int\left(9+\frac{107x}{2}\right)\left(3-x+2x^2\right)^{3/2}dx$$

$$= \frac{107}{240}\left(3-x+2x^2\right)^{5/2}+\frac{5}{12}x\left(3-x+2x^2\right)^{5/2}+\frac{179}{96}\int\left(3-x+2x^2\right)^{3/2}dx$$

$$= -\frac{179(1-4x)\left(3-x+2x^2\right)^{3/2}}{1536}+\frac{107}{240}\left(3-x+2x^2\right)^{5/2}+\frac{5}{12}x\left(3-x+2x^2\right.$$

$$= -\frac{4117(1-4x)\sqrt{3-x+2x^2}}{8192}-\frac{179(1-4x)\left(3-x+2x^2\right)^{3/2}}{1536}+\frac{107}{240}\left(3-x\right.$$

$$= -\frac{4117(1-4x)\sqrt{3-x+2x^2}}{8192}-\frac{179(1-4x)\left(3-x+2x^2\right)^{3/2}}{1536}+\frac{107}{240}\left(3-x\right.$$

$$= -\frac{4117(1-4x)\sqrt{3-x+2x^2}}{8192}-\frac{179(1-4x)\left(3-x+2x^2\right)^{3/2}}{1536}+\frac{107}{240}\left(3-x\right.$$

Mathematica [A] time = 0.08, size = 65, normalized size = 0.62

$$\frac{4\sqrt{2x^2-x+3}\left(204800x^5+14336x^4+561024x^3+319072x^2+565276x+388341\right)-1420365\sqrt{2}\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{491520}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2),x]
```

[Out] (4*Sqrt[3 - x + 2*x^2]*(388341 + 565276*x + 319072*x^2 + 561024*x^3 + 14336 *x^4 + 204800*x^5) - 1420365*Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]])/491520

IntegrateAlgebraic [A] time = 0.58, size = 80, normalized size = 0.76

$$\frac{\sqrt{2x^2-x+3}\left(204800x^5+14336x^4+561024x^3+319072x^2+565276x+388341\right)}{122880}-\frac{94691\log\left(2\sqrt{2}\sqrt{2x^2-x+3}-4x+1\right)}{16384\sqrt{2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2),x]

[Out] (Sqrt[3 - x + 2*x^2]*(388341 + 565276*x + 319072*x^2 + 561024*x^3 + 14336*x ^4 + 204800*x^5))/122880 - (94691*Log[1 - 4*x + 2*Sqrt[2]*Sqrt[3 - x + 2*x^ 2]])/(16384*Sqrt[2])

fricas [A] time = 0.41, size = 78, normalized size = 0.74

$$\frac{1}{122880}\left(204800\,x^5+14336\,x^4+561024\,x^3+319072\,x^2+565276\,x+388341\right)\sqrt{2\,x^2-x+3}+\frac{94691}{65536}\sqrt{2}\log\left(-4\,\sqrt{2}\sqrt{2\,x^2-x+3}(4\,x-1)-32\,x^2+16\,x-25\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2),x, algorithm="fricas")

[Out] 1/122880*(204800*x^5 + 14336*x^4 + 561024*x^3 + 319072*x^2 + 565276*x + 388 341)*sqrt(2*x^2 - x + 3) + 94691/65536*sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25)

giac [A] time = 0.23, size = 73, normalized size = 0.70

$$\frac{1}{122880}(4(8(4(16(100x+7)x+4383)x+9971)x+141319)x+388341)\sqrt{2x^2-x+3}-\frac{94691}{32768}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2),x, algorithm="giac")

[Out] 1/122880*(4*(8*(4*(16*(100*x + 7)*x + 4383)*x + 9971)*x + 141319)*x + 38834 1)*sqrt(2*x^2 - x + 3) - 94691/32768*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sq rt(2*x^2 - x + 3)) + 1)

maple [A] time = 0.01, size = 83, normalized size = 0.79

$$\frac{5\left(2x^2-x+3\right)^{\frac{5}{2}}x}{12}+\frac{94691\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{32768}+\frac{107\left(2x^2-x+3\right)^{\frac{5}{2}}}{240}+\frac{179(4x-1)\left(2x^2-x+3\right)^{\frac{3}{2}}}{1536}+\frac{4117(4x-1)\sqrt{2x^2-x+3}}{8192}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2),x)
```

```
[Out] 5/12*(2*x^2-x+3)^(5/2)*x+107/240*(2*x^2-x+3)^(5/2)+179/1536*(4*x-1)*(2*x^2-
x+3)^(3/2)+4117/8192*(4*x-1)*(2*x^2-x+3)^(1/2)+94691/32768*2^(1/2)*arcsinh(
4/23*23^(1/2)*(x-1/4))
```

maxima [A] time = 0.96, size = 104, normalized size = 0.99

$$\frac{5}{12}\left(2x^2-x+3\right)^{\frac{5}{2}}x+\frac{107}{240}\left(2x^2-x+3\right)^{\frac{5}{2}}+\frac{179}{384}\left(2x^2-x+3\right)^{\frac{3}{2}}x-\frac{179}{1536}\left(2x^2-x+3\right)^{\frac{3}{2}}+\frac{4117}{2048}\sqrt{2x^2-x+3}\,x+\frac{94691}{32768}\sqrt{2}\,\mathrm{arsinh}\left(\frac{1}{23}\sqrt{23}\,(4x-1)\right)-\frac{4117}{8192}\sqrt{2x^2-x+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2),x, algorithm="maxima")
```

```
[Out] 5/12*(2*x^2 - x + 3)^(5/2)*x + 107/240*(2*x^2 - x + 3)^(5/2) + 179/384*(2*x
^2 - x + 3)^(3/2)*x - 179/1536*(2*x^2 - x + 3)^(3/2) + 4117/2048*sqrt(2*x^2
 - x + 3)*x + 94691/32768*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 4117/8
192*sqrt(2*x^2 - x + 3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(2\,x^2-x+3\right)^{3/2}\,\left(5\,x^2+3\,x+2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^(3/2)*(3*x + 5*x^2 + 2),x)
```

```
[Out] int((2*x^2 - x + 3)^(3/2)*(3*x + 5*x^2 + 2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(2x^2-x+3\right)^{\frac{3}{2}}\left(5x^2+3x+2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(3/2)*(5*x**2+3*x+2),x)
```

```
[Out] Integral((2*x**2 - x + 3)**(3/2)*(5*x**2 + 3*x + 2), x)
```

3.66 $\int \frac{\left(3-x+2x^2\right)^{3/2}}{2+3x+5x^2}\,dx$

Optimal. Leaf size=197

$$-\frac{1}{100}\sqrt{2x^2-x+3}\,(49-20x)+\frac{11}{125}\sqrt{\frac{11}{31}\left(247+500\sqrt{2}\right)}\,\tan^{-1}\left(\frac{\sqrt{\frac{11}{62\left(247+500\sqrt{2}\right)}}\left(\left(130+69\sqrt{2}\right)x+61\sqrt{2}+8\right)}{\sqrt{2x^2-x+3}}\right.$$

Rubi [A] time = 0.49, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {977, 1076, 619, 215, 1035, 1029, 206, 204}

$$-\frac{1}{100}\sqrt{2x^2-x+3}\,(49-20x)+\frac{11}{125}\sqrt{\frac{11}{31}\left(247+500\sqrt{2}\right)}\,\tan^{-1}\left(\frac{\sqrt{\frac{11}{62\left(247+500\sqrt{2}\right)}}\left(\left(130+69\sqrt{2}\right)x+61\sqrt{2}+8\right)}{\sqrt{2x^2-x+3}}\right)-\frac{11}{125}\sqrt{\frac{11}{31}\left(500\sqrt{2}-247\right)}\,\tanh^{-1}\left(\frac{\sqrt{\frac{11}{62\left(500\sqrt{2}-247\right)}}\left(\left(130-69\sqrt{2}\right)x-61\sqrt{2}+8\right)}{\sqrt{2x^2-x+3}}\right)-\frac{2203\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{1000\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^(3/2)/(2 + 3*x + 5*x^2),x]
```

```
[Out] -((49 - 20*x)*Sqrt[3 - x + 2*x^2])/100 - (2203*ArcSinh[(1 - 4*x)/Sqrt[23]])
/(1000*Sqrt[2]) + (11*Sqrt[(11*(247 + 500*Sqrt[2]))/31]*ArcTan[(Sqrt[11/(62
*(247 + 500*Sqrt[2]))]*(8 + 61*Sqrt[2] + (130 + 69*Sqrt[2])*x))/Sqrt[3 - x
+ 2*x^2]])/125 - (11*Sqrt[(11*(-247 + 500*Sqrt[2]))/31]*ArcTanh[(Sqrt[11/(6
2*(-247 + 500*Sqrt[2]))]*(8 - 61*Sqrt[2] + (130 - 69*Sqrt[2])*x))/Sqrt[3 -
x + 2*x^2]])/125
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 977

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((b*f*(3*p + 2*q) - c*e*(2*p + q) + 2*c*f*(p
+ q)*x)*(a + b*x + c*x^2)^(p - 1)*(d + e*x + f*x^2)^(q + 1))/(2*f^2*(p + q)
*(2*p + 2*q + 1)), x] - Dist[1/(2*f^2*(p + q)*(2*p + 2*q + 1)), Int[(a + b*
x + c*x^2)^(p - 2)*(d + e*x + f*x^2)^q*Simp[(b*d - a*e)*(c*e - b*f)*(1 - p)
*(2*p + q) - (p + q)*(b^2*d*f*(1 - p) - a*(f*(b*e - 2*a*f)*(2*p + 2*q + 1)
+ c*(2*d*f - e^2*(2*p + q)))) + (2*(c*d - a*f)*(c*e - b*f)*(1 - p)*(2*p + q
) - (p + q)*((b^2 - 4*a*c)*e*f*(1 - p) + b*(c*(e^2 - 4*d*f)*(2*p + q) + f*(
2*c*d - b*e + 2*a*f)*(2*p + 2*q + 1))))*x + ((c*e - b*f)^2*(1 - p)*p + c*(p
 + q)*(f*(b*e - 2*a*f)*(4*p + 2*q - 1) - c*(2*d*f*(1 - 2*p) + e^2*(3*p + q
- 1))))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a
*c, 0] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 1] && NeQ[p + q, 0] && NeQ[2*p + 2*
q + 1, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

Rule 1029

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
 c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 1035

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

Rule 1076

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqr
t[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$\int \frac{\left(3-x+2x^2\right)^{3/2}}{2+3x+5x^2}\,dx = -\frac{1}{100}(49-20x)\sqrt{3-x+2x^2} - \frac{1}{50}\int \frac{-\frac{731}{2}+\frac{1195x}{4}-\frac{2203x^2}{4}}{\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)}\,dx$$

$$= -\frac{1}{100}(49-20x)\sqrt{3-x+2x^2} - \frac{1}{250}\int \frac{-726+3146x}{\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)}\,dx + \frac{2203\int \frac{}{\sqrt{3-}}}{100}$$

$$= -\frac{1}{100}(49-20x)\sqrt{3-x+2x^2} + \frac{\int \frac{2662\left(16+3\sqrt{2}\right)+2662\left(10-13\sqrt{2}\right)x}{\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)}\,dx}{5500\sqrt{2}} - \frac{\int \frac{2662\left(16-3\sqrt{2}\right)+266}{\sqrt{3-x+2x^2}\left(2+\right.}}{5500\sqrt{}}$$

$$= -\frac{1}{100}(49-20x)\sqrt{3-x+2x^2} - \frac{2203\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{1000\sqrt{2}} - \frac{1}{125}\left(322102\left(1000-247\sqrt{2}\right)\right)$$

$$= -\frac{1}{100}(49-20x)\sqrt{3-x+2x^2} - \frac{2203\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{1000\sqrt{2}} + \frac{11}{125}\sqrt{\frac{11}{31}\left(247+500\sqrt{2}\right)}\tan^{-}$$

Mathematica [C] time = 0.69, size = 310, normalized size = 1.57

$$\frac{400\sqrt{31}\sqrt{2x^2-x+3}\,x-980\sqrt{31}\sqrt{2x^2-x+3}+44\sqrt{286+22i\sqrt{31}}\left(\sqrt{31}-13i\right)\tanh^{-1}\left(\frac{-4i\sqrt{31}x-22x+i\sqrt{31}+63}{2\sqrt{286+22i\sqrt{31}}\sqrt{2x^2-x+3}}\right)+572i\sqrt{286-22i\sqrt{31}}\tanh^{-1}\left(\frac{4i\sqrt{31}x-22x-i\sqrt{31}+63}{2\sqrt{286-22i\sqrt{31}}\sqrt{2x^2-x+3}}\right)+44\sqrt{682\left(13-i\sqrt{31}\right)}\tanh^{-1}\left(\frac{4i\sqrt{31}x-22x-i\sqrt{31}+63}{2\sqrt{286-22i\sqrt{31}}\sqrt{2x^2-x+3}}\right)-2203\sqrt{62}\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{2000\sqrt{31}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^(3/2)/(2 + 3*x + 5*x^2),x]
```

```
[Out] (-980*Sqrt[31]*Sqrt[3 - x + 2*x^2] + 400*Sqrt[31]*x*Sqrt[3 - x + 2*x^2] - 2
203*Sqrt[62]*ArcSinh[(1 - 4*x)/Sqrt[23]] + 44*Sqrt[286 + (22*I)*Sqrt[31]]*(
-13*I + Sqrt[31])*ArcTanh[(63 + I*Sqrt[31] - 22*x - (4*I)*Sqrt[31]*x)/(2*Sq
rt[286 + (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])] + 44*Sqrt[682*(13 - I*Sqrt[
31])]*ArcTanh[(63 - I*Sqrt[31] - 22*x + (4*I)*Sqrt[31]*x)/(2*Sqrt[286 - (22
```

```
*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])] + (572*I)*Sqrt[286 - (22*I)*Sqrt[31]]*A
rcTanh[(63 - I*Sqrt[31] - 22*x + (4*I)*Sqrt[31]*x)/(2*Sqrt[286 - (22*I)*Sqr
t[31]]*Sqrt[3 - x + 2*x^2])])/(2000*Sqrt[31])
```

IntegrateAlgebraic [C] time = 0.53, size = 235, normalized size = 1.19

$$\frac{121}{125}\text{RootSum}\left[-5\#1^4+6\sqrt{2}\#1^3+17\#1^2-26\sqrt{2}\#1-56\&,\frac{13\#1^2\log\left(-\#1+\sqrt{2x^2-x+3}-\sqrt{2}x\right)+6\sqrt{2}\#1\log\left(-\#1+\sqrt{2x^2-x+3}-\sqrt{2}x\right)-36\log\left(-\#1+\sqrt{2x^2-x+3}-\sqrt{2}x\right)}{-10\#1^3+9\sqrt{2}\#1^2+17\#1-13\sqrt{2}}\&\right]+\frac{1}{100}\sqrt{2x^2-x+3}(20x-49)-\frac{2203\log\left(2\sqrt{2}\sqrt{2x^2-x+3}-4x+1\right)}{1000\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(3 - x + 2*x^2)^(3/2)/(2 + 3*x + 5*x^2),x]
```

```
[Out] ((-49 + 20*x)*Sqrt[3 - x + 2*x^2])/100 - (2203*Log[1 - 4*x + 2*Sqrt[2]*Sqrt
[3 - x + 2*x^2]])/(1000*Sqrt[2]) + (121*RootSum[-56 - 26*Sqrt[2]*#1 + 17*#1
^2 + 6*Sqrt[2]*#1^3 - 5*#1^4 & , (-36*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2
] - #1] + 6*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1 + 13*Lo
g[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1^2)/(-13*Sqrt[2] + 17*#1 + 9*S
qrt[2]*#1^2 - 10*#1^3) & ])/125
```

fricas [B] time = 0.99, size = 2027, normalized size = 10.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)/(5*x^2+3*x+2),x, algorithm="fricas")
```

```
[Out] 11/77500*24200^(1/4)*sqrt(31)*sqrt(10)*sqrt(2)*sqrt(247*sqrt(2) + 1000)*arc
tan(1/10605875*(230*sqrt(10)*(2*24200^(3/4)*sqrt(31)*(20846*x^7 - 109153*x^
6 + 215386*x^5 - 427391*x^4 + 234360*x^3 - 156600*x^2 - sqrt(2)*(28854*x^7
- 90639*x^6 + 200187*x^5 - 262838*x^4 + 117544*x^3 - 23472*x^2 - 186624*x +
 86400) - 172800*x + 186624) + 5*24200^(1/4)*sqrt(31)*(112238*x^7 - 1817988
*x^6 + 10351960*x^5 - 25791248*x^4 + 34522560*x^3 - 28368000*x^2 - sqrt(2)*
(125839*x^7 - 1864281*x^6 + 9323336*x^5 - 19725020*x^4 + 24624288*x^3 - 108
62496*x^2 - 19989504*x + 10533888) - 21067776*x + 19989504))*sqrt(2*x^2 - x
 + 3)*sqrt(247*sqrt(2) + 1000) + 30107000*sqrt(31)*sqrt(2)*(28180*x^8 - 254
666*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 -
 sqrt(2)*(8746*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^4 - 7
52088*x^3 + 396144*x^2 + 546048*x - 539136) + 1154304*x - 456192) - sqrt(5/
119)*(sqrt(10)*(2*24200^(3/4)*sqrt(31)*(46522*x^7 - 71117*x^6 + 257247*x^5
- 273360*x^4 + 484920*x^3 - 269568*x^2 - 16*sqrt(2)*(7714*x^7 - 10881*x^6 +
 33771*x^5 - 5576*x^4 - 576*x^3 + 32184*x^2 - 32184*x) + 269568*x) + 5*2420
0^(1/4)*sqrt(31)*(309512*x^7 - 4017952*x^6 + 15741280*x^5 - 22625280*x^4 +
37693440*x^3 - 13519872*x^2 - sqrt(2)*(516957*x^7 - 6676948*x^6 + 25569820*
x^5 - 31522752*x^4 + 34450848*x^3 + 46199808*x^2 - 46199808*x) + 13519872*x
))*sqrt(2*x^2 - x + 3)*sqrt(247*sqrt(2) + 1000) + 130900*sqrt(31)*sqrt(2)*(
123408*x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 + 396480*x^4 + 798336*x
```

^3 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 118051*x^7 + 244047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*x) + 3276288*x) + 5950*sqrt(31)*(254591*x^8 - 4815126*x^7 + 32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 74219328*x^3 - 168956928*x^2 - 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3618*x^3 + 2268*x^2 - 1944*x) + 144820224*x))*sqrt((24200^(1/4)*sqrt(10)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(x - 75) + 74*x - 76)*sqrt(247*sqrt(2) + 1000) + 58310*x^2 + 52360*sqrt(2)*(2*x^2 - x + 3) - 179690*x + 238000)/x^2) + 342125*sqrt(31)*(2828123*x^8 - 9696916*x^7 + 53385560*x^6 - 142835344*x^5 + 254146592*x^4 - 249300096*x^3 + 37981440*x^2 - 7744*sqrt(2)*(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^4 - 5568*x^3 + 1080*x^2 + 4320*x - 5184) + 223064064*x - 94887936))/(2585191*x^8 - 4661200*x^7 + 14191920*x^6 + 490880*x^5 - 13562944*x^4 + 44249088*x^3 - 34615296*x^2 - 24772608*x + 18579456)) + 11/77500*24200^(1/4)*sqrt(31)*sqrt(10)*sqrt(2)*sqrt(247*sqrt(2) + 1000)*arctan(1/10605875*(230*sqrt(10)*(2*24200^(3/4)*sqrt(31)*(20846*x^7 - 109153*x^6 + 215386*x^5 - 427391*x^4 + 234360*x^3 - 156600*x^2 - sqrt(2)*(28854*x^7 - 90639*x^6 + 200187*x^5 - 262838*x^4 + 117544*x^3 - 23472*x^2 - 186624*x + 86400) - 172800*x + 186624) + 5*24200^(1/4)*sqrt(31)*(112238*x^7 - 1817988*x^6 + 10351960*x^5 - 25791248*x^4 + 34522560*x^3 - 28368000*x^2 - sqrt(2)*(125839*x^7 - 1864281*x^6 + 9323336*x^5 - 19725020*x^4 + 24624288*x^3 - 10862496*x^2 - 19989504*x + 10533888) - 21067776*x + 19989504))*sqrt(2*x^2 - x + 3)*sqrt(247*sqrt(2) + 1000) - 30107000*sqrt(31)*sqrt(2)*(28180*x^8 - 254666*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - sqrt(2)*(8746*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^4 - 752088*x^3 + 396144*x^2 + 546048*x - 539136) + 1154304*x - 456192) - sqrt(5/119)*(sqrt(10)*(2*24200^(3/4)*sqrt(31)*(46522*x^7 - 71117*x^6 + 257247*x^5 - 273360*x^4 + 484920*x^3 - 269568*x^2 - 16*sqrt(2)*(7714*x^7 - 10881*x^6 + 33771*x^5 - 5576*x^4 - 576*x^3 + 32184*x^2 - 32184*x) + 269568*x) + 5*24200^(1/4)*sqrt(31)*(309512*x^7 - 4017952*x^6 + 15741280*x^5 - 22625280*x^4 + 37693440*x^3 - 13519872*x^2 - sqrt(2)*(516957*x^7 - 6676948*x^6 + 25569820*x^5 - 31522752*x^4 + 34450848*x^3 + 46199808*x^2 - 46199808*x) + 13519872*x))*sqrt(2*x^2 - x + 3)*sqrt(247*sqrt(2) + 1000) - 130900*sqrt(31)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 + 396480*x^4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 118051*x^7 + 244047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*x) + 3276288*x) - 5950*sqrt(31)*(254591*x^8 - 4815126*x^7 + 32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 74219328*x^3 - 168956928*x^2 - 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3618*x^3 + 2268*x^2 - 1944*x) + 144820224*x))*sqrt(-(24200^(1/4)*sqrt(10)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(x - 75) + 74*x - 76)*sqrt(247*sqrt(2) + 1000) - 58310*x^2 - 52360*sqrt(2)*(2*x^2 - x + 3) + 179690*x - 238000)/x^2) - 342125*sqrt(31)*(2828123*x^8 - 9696916*x^7 + 53385560*x^6 - 142835344*x^5 + 254146592*x^4 - 249300096*x^3 + 37981440*x^2 - 7744*sqrt(2)*(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^4 - 5568*x^3 + 1080*x^2 + 4320*x - 5184) + 223064064*x - 94887936))/(2585191*x^8 - 4661200*x^7 + 14191920*x^6 + 490880*x^5 - 13562944*x^4 + 44249088*x^3 - 34615296*x^2 - 24772608*x + 18579456)) +

```
11/36890000*24200^(1/4)*sqrt(10)*sqrt(247*sqrt(2) + 1000)*(247*sqrt(2) - 10
00)*log(1512500/119*(24200^(1/4)*sqrt(10)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(x -
 75) + 74*x - 76)*sqrt(247*sqrt(2) + 1000) + 58310*x^2 + 52360*sqrt(2)*(2*x
^2 - x + 3) - 179690*x + 238000)/x^2) - 11/36890000*24200^(1/4)*sqrt(10)*sq
rt(247*sqrt(2) + 1000)*(247*sqrt(2) - 1000)*log(-1512500/119*(24200^(1/4)*s
qrt(10)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(x - 75) + 74*x - 76)*sqrt(247*sqrt(2)
 + 1000) - 58310*x^2 - 52360*sqrt(2)*(2*x^2 - x + 3) + 179690*x - 238000)/x
^2) + 1/100*sqrt(2*x^2 - x + 3)*(20*x - 49) + 2203/4000*sqrt(2)*log(-4*sqrt
(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)/(5*x^2+3*x+2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Francis algorithm failure for[-1.0,infinity,
infinity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,infi
nity]Francis algorithm failure for[-1.0,infinity,infinity,infinity,infinity
]proot error [1.0,infinity,infinity,infinity,infinity]Francis algorithm fai
lure for[-1.0,infinity,infinity,infinity,infinity]proot error [1.0,infinity
,infinity,infinity,infinity]Francis algorithm failure for[-1.0,infinity,inf
inity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,infinit
y]Evaluation time: 10.3Done
```

maple [B] time = 0.05, size = 3460, normalized size = 17.56

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(3/2)/(5*x^2+3*x+2),x)
```

```
[Out] 1/5*x*(2*x^2-x+3)^(1/2)-49/100*(2*x^2-x+3)^(1/2)+2203/2000*2^(1/2)*arcsinh(
4/23*23^(1/2)*(x-1/4))-2/1321375*(8*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+3*2^(1
/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+8-3*2^(1/2))^(1/2)*2^(1/2)*(4245*2^(1/
2)*(-8866+6820*2^(1/2))^(1/2)*(-775687+549362*2^(1/2))^(1/2)*arctan(1/11692
487*(-775687+549362*2^(1/2))^(1/2)*(-23*(8+3*2^(1/2))*(-23*(x+2^(1/2)-1)^2/
(-x+2^(1/2)+1)^2+24*2^(1/2)-41))^(1/2)*(6485*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^
(1/2)+1)^2+10368*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+22379*2^(1/2)+32016)/(23*
(x+2^(1/2)-1)^4/(-x+2^(1/2)+1)^4+82*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+23)*(x
+2^(1/2)-1)/(-x+2^(1/2)+1)*(8+3*2^(1/2)))+6154*(-8866+6820*2^(1/2))^(1/2)*(
-775687+549362*2^(1/2))^(1/2)*arctan(1/11692487*(-775687+549362*2^(1/2))^(1
```

```
/2)*(-23*(8+3*2^(1/2))*(-23*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+24*2^(1/2)-41)
)^(1/2)*(6485*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+10368*(x+2^(1/2)-1)^
2/(-x+2^(1/2)+1)^2+22379*2^(1/2)+32016)/(23*(x+2^(1/2)-1)^4/(-x+2^(1/2)+1)^
4+82*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+23)*(x+2^(1/2)-1)/(-x+2^(1/2)+1)*(8+3
*2^(1/2)))+12325786*arctanh(31/2*(8*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+3*2^(1
/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+8-3*2^(1/2))^(1/2)/(-8866+6820*2^(1/2)
)^(1/2))*2^(1/2)-359414*arctanh(31/2*(8*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+3*
2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+8-3*2^(1/2))^(1/2)/(-8866+6820*2^(
1/2))^(1/2)))/((8*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+3*2^(1/2)*(x+2^(1/2)-1)^
2/(-x+2^(1/2)+1)^2+8-3*2^(1/2))/(1+(x+2^(1/2)-1)/(-x+2^(1/2)+1))^2)^(1/2)/(
1+(x+2^(1/2)-1)/(-x+2^(1/2)+1))/(8+3*2^(1/2))/(-8866+6820*2^(1/2))^(1/2)-2/
264275*(8*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+3*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^
(1/2)+1)^2+8-3*2^(1/2))^(1/2)*2^(1/2)*(2365*2^(1/2)*(-8866+6820*2^(1/2))^(1
/2)*(-775687+549362*2^(1/2))^(1/2)*arctan(1/11692487*(-775687+549362*2^(1/2
))^(1/2)*(-23*(8+3*2^(1/2))*(-23*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+24*2^(1/2
)-41))^(1/2)*(6485*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+10368*(x+2^(1/2
)-1)^2/(-x+2^(1/2)+1)^2+22379*2^(1/2)+32016)/(23*(x+2^(1/2)-1)^4/(-x+2^(1/2
)+1)^4+82*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+23)*(x+2^(1/2)-1)/(-x+2^(1/2)+1)
*(8+3*2^(1/2)))+3338*(-8866+6820*2^(1/2))^(1/2)*(-775687+549362*2^(1/2))^(1
/2)*arctan(1/11692487*(-775687+549362*2^(1/2))^(1/2)*(-23*(8+3*2^(1/2))*(-2
3*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+24*2^(1/2)-41))^(1/2)*(6485*2^(1/2)*(x+2
^(1/2)-1)^2/(-x+2^(1/2)+1)^2+10368*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+22379*2
^(1/2)+32016)/(23*(x+2^(1/2)-1)^4/(-x+2^(1/2)+1)^4+82*(x+2^(1/2)-1)^2/(-x+2
^(1/2)+1)^2+23)*(x+2^(1/2)-1)/(-x+2^(1/2)+1)*(8+3*2^(1/2)))+3192442*arctanh
(31/2*(8*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+3*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(
1/2)+1)^2+8-3*2^(1/2))^(1/2)/(-8866+6820*2^(1/2))^(1/2))*2^(1/2)-5264358*ar
ctanh(31/2*(8*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+3*2^(1/2)*(x+2^(1/2)-1)^2/(-
x+2^(1/2)+1)^2+8-3*2^(1/2))^(1/2)/(-8866+6820*2^(1/2))^(1/2)))/((8*(x+2^(1/
2)-1)^2/(-x+2^(1/2)+1)^2+3*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+8-3*2^(
1/2))/(1+(x+2^(1/2)-1)/(-x+2^(1/2)+1))^2)^(1/2)/(1+(x+2^(1/2)-1)/(-x+2^(1/2
)+1))/(8+3*2^(1/2))/(-8866+6820*2^(1/2))^(1/2)-13/105710*(8*(x+2^(1/2)-1)^2
/(-x+2^(1/2)+1)^2+3*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+8-3*2^(1/2))^(
1/2)*2^(1/2)*(285*2^(1/2)*(-8866+6820*2^(1/2))^(1/2)*(-775687+549362*2^(1/2
))^(1/2)*arctan(1/11692487*(-775687+549362*2^(1/2))^(1/2)*(-23*(8+3*2^(1/2)
)*(-23*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+24*2^(1/2)-41))^(1/2)*(6485*2^(1/2)
*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+10368*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+22
379*2^(1/2)+32016)/(23*(x+2^(1/2)-1)^4/(-x+2^(1/2)+1)^4+82*(x+2^(1/2)-1)^2/
(-x+2^(1/2)+1)^2+23)*(x+2^(1/2)-1)/(-x+2^(1/2)+1)*(8+3*2^(1/2)))+386*(-8866
+6820*2^(1/2))^(1/2)*(-775687+549362*2^(1/2))^(1/2)*arctan(1/11692487*(-775
687+549362*2^(1/2))^(1/2)*(-23*(8+3*2^(1/2))*(-23*(x+2^(1/2)-1)^2/(-x+2^(1/
2)+1)^2+24*2^(1/2)-41))^(1/2)*(6485*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^
2+10368*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+22379*2^(1/2)+32016)/(23*(x+2^(1/2
)-1)^4/(-x+2^(1/2)+1)^4+82*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+23)*(x+2^(1/2)-
1)/(-x+2^(1/2)+1)*(8+3*2^(1/2)))-274846*arctanh(31/2*(8*(x+2^(1/2)-1)^2/(-x
+2^(1/2)+1)^2+3*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+8-3*2^(1/2))^(1/2)
```

/(-8866+6820*2^(1/2))^(1/2))*2^(1/2)-1543366*arctanh(31/2*(8*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+3*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+8-3*2^(1/2))^(1/2)/(-8866+6820*2^(1/2))^(1/2)))/((8*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+3*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+8-3*2^(1/2))/(1+(x+2^(1/2)-1)/(-x+2^(1/2)+1))^2)^(1/2)/(1+(x+2^(1/2)-1)/(-x+2^(1/2)+1))/(8+3*2^(1/2))/(-8866+6820*2^(1/2))^(1/2)+3/10571*(8*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+3*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+8-3*2^(1/2))^(1/2)*2^(1/2)*(151*2^(1/2)*(-8866+6820*2^(1/2))^(1/2)*(-775687+549362*2^(1/2))^(1/2)*arctan(1/11692487*(-775687+549362*2^(1/2))^(1/2)*(-23*(8+3*2^(1/2))*(-23*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+24*2^(1/2)-41))^(1/2)*(6485*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+10368*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+22379*2^(1/2)+32016)/(23*(x+2^(1/2)-1)^4/(-x+2^(1/2)+1)^4+82*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+23)*(x+2^(1/2)-1)/(-x+2^(1/2)+1)*(8+3*2^(1/2)))+218*(-8866+6820*2^(1/2))^(1/2)*(-775687+549362*2^(1/2))^(1/2)*arctan(1/11692487*(-775687+549362*2^(1/2))^(1/2)*(-23*(8+3*2^(1/2))*(-23*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+24*2^(1/2)-41))^(1/2)*(6485*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+10368*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+22379*2^(1/2)+32016)/(23*(x+2^(1/2)-1)^4/(-x+2^(1/2)+1)^4+82*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+23)*(x+2^(1/2)-1)/(-x+2^(1/2)+1)*(8+3*2^(1/2)))+401698*arctanh(31/2*(8*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+3*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+8-3*2^(1/2))^(1/2)/(-8866+6820*2^(1/2))^(1/2))*2^(1/2)-63426*arctanh(31/2*(8*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+3*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+8-3*2^(1/2))^(1/2)/(-8866+6820*2^(1/2))^(1/2)))/((8*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+3*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+8-3*2^(1/2))/(1+(x+2^(1/2)-1)/(-x+2^(1/2)+1))^2)^(1/2)/(1+(x+2^(1/2)-1)/(-x+2^(1/2)+1))/(8+3*2^(1/2))/(-8866+6820*2^(1/2))^(1/2)+9/21142*(8*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+3*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+8-3*2^(1/2))^(1/2)*2^(1/2)*(369*2^(1/2)*(-8866+6820*2^(1/2))^(1/2)*(-775687+549362*2^(1/2))^(1/2)*arctan(1/11692487*(-775687+549362*2^(1/2))^(1/2)*(-23*(8+3*2^(1/2))*(-23*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+24*2^(1/2)-41))^(1/2)*(6485*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+10368*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+22379*2^(1/2)+32016)/(23*(x+2^(1/2)-1)^4/(-x+2^(1/2)+1)^4+82*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+23)*(x+2^(1/2)-1)/(-x+2^(1/2)+1)*(8+3*2^(1/2)))+520*(-8866+6820*2^(1/2))^(1/2)*(-775687+549362*2^(1/2))^(1/2)*arctan(1/11692487*(-775687+549362*2^(1/2))^(1/2)*(-23*(8+3*2^(1/2))*(-23*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+24*2^(1/2)-41))^(1/2)*(6485*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+10368*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+22379*2^(1/2)+32016)/(23*(x+2^(1/2)-1)^4/(-x+2^(1/2)+1)^4+82*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+23)*(x+2^(1/2)-1)/(-x+2^(1/2)+1)*(8+3*2^(1/2)))+465124*arctanh(31/2*(8*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+3*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+8-3*2^(1/2))^(1/2)/(-8866+6820*2^(1/2))^(1/2))*2^(1/2)-866822*arctanh(31/2*(8*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+3*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+8-3*2^(1/2))^(1/2)/(-8866+6820*2^(1/2))^(1/2)))/((8*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+3*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+8-3*2^(1/2))/(1+(x+2^(1/2)-1)/(-x+2^(1/2)+1))^2)^(1/2)/(1+(x+2^(1/2)-1)/(-x+2^(1/2)+1))/(8+3*2^(1/2))/(-8866+6820*2^(1/2))^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\,x^2 - x + 3\right)^{\frac{3}{2}}}{5\,x^2 + 3\,x + 2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)/(5*x^2+3*x+2),x, algorithm="maxima")
[Out] integrate((2*x^2 - x + 3)^(3/2)/(5*x^2 + 3*x + 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(2\,x^2 - x + 3\right)^{3/2}}{5\,x^2 + 3\,x + 2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^(3/2)/(3*x + 5*x^2 + 2),x)
[Out] int((2*x^2 - x + 3)^(3/2)/(3*x + 5*x^2 + 2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2x^2 - x + 3\right)^{\frac{3}{2}}}{5x^2 + 3x + 2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(3/2)/(5*x**2+3*x+2),x)
[Out] Integral((2*x**2 - x + 3)**(3/2)/(5*x**2 + 3*x + 2), x)
```

3.67 $\int \frac{(3-x+2x^2)^{3/2}}{(2+3x+5x^2)^2} \, dx$

Optimal. Leaf size=232

$$\frac{(10x+3)\left(2x^2-x+3\right)^{3/2}}{31\left(5x^2+3x+2\right)}+\frac{4}{155}(4-5x)\sqrt{2x^2-x+3}+\frac{\sqrt{\frac{11}{31}\left(3169333+2265350\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{11}{62\left(3169333+2265350\sqrt{2}\right)}}\left(\cdots\right.}{\cdots}\right)}{1550}$$

Rubi [A] time = 0.58, antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {971, 1066, 1076, 619, 215, 1035, 1029, 206, 204}

$$\frac{(10x+3)\left(2x^2-x+3\right)^{3/2}}{31\left(5x^2+3x+2\right)}+\frac{4}{155}(4-5x)\sqrt{2x^2-x+3}+\frac{\sqrt{\frac{11}{31}\left(3169333+2265350\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{11}{62\left(3169333+2265350\sqrt{2}\right)}}\left(\left(9440+6477\sqrt{2}\right)x+2963\sqrt{2}+3514\right)}{\sqrt{2x^2-x+3}}\right)}{1550}-\frac{\sqrt{\frac{11}{31}\left(2265350\sqrt{2}-3169333\right)}\tanh^{-1}\left(\frac{\sqrt{\frac{11}{62\left(2265350\sqrt{2}-3169333\right)}}\left(\left(9440-6477\sqrt{2}\right)x-2963\sqrt{2}+3514\right)}{\sqrt{2x^2-x+3}}\right)}{1550}-\frac{2}{25}\sqrt{2}\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^(3/2)/(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] (4*(4 - 5*x)*Sqrt[3 - x + 2*x^2])/155 + ((3 + 10*x)*(3 - x + 2*x^2)^(3/2))/
(31*(2 + 3*x + 5*x^2)) - (2*Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]])/25 + (Sqrt
[(11*(3169333 + 2265350*Sqrt[2]))/31]*ArcTan[(Sqrt[11/(62*(3169333 + 226535
0*Sqrt[2]))]*(3514 + 2963*Sqrt[2] + (9440 + 6477*Sqrt[2])*x))/Sqrt[3 - x +
2*x^2]])/1550 - (Sqrt[(11*(-3169333 + 2265350*Sqrt[2]))/31]*ArcTanh[(Sqrt[1
1/(62*(-3169333 + 2265350*Sqrt[2]))]*(3514 - 2963*Sqrt[2] + (9440 - 6477*Sq
rt[2])*x))/Sqrt[3 - x + 2*x^2]])/1550
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 971

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((b + 2*c*x)*(a + b*x + c*x^2)^(p + 1)*(d + e
*x + f*x^2)^q)/((b^2 - 4*a*c)*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*(p + 1))
, Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*Simp[2*c*d*(2*p +
 3) + b*e*q + (2*b*f*q + 2*c*e*(2*p + q + 3))*x + 2*c*f*(2*p + 2*q + 3)*x^2
, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
e^2 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0] &&  !IGtQ[q, 0]
```

Rule 1029

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
 c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 1035

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

Rule 1066

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((B*c*f*(2*
```

```
p + 2*q + 3) + C*(b*f*p - c*e*(2*p + q + 2)) + 2*c*C*f*(p + q + 1)*x)*(a +
b*x + c*x^2)^p*(d + e*x + f*x^2)^(q + 1))/(2*c*f^2*(p + q + 1)*(2*p + 2*q +
 3)), x] - Dist[1/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), Int[(a + b*x + c*x
^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[p*(b*d - a*e)*(C*(c*e - b*f)*(q + 1) -
 c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(b^2*C*d*f*p + a*c*(C*(2*d*f
- e^2*(2*p + q + 2)) + f*(B*e - 2*A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*f)
*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*e
*f*p*(b^2 - 4*a*c) - b*c*(C*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e +
2*A*f)*(2*p + 2*q + 3))))*x + (p*(c*e - b*f)*(C*(c*e - b*f)*(q + 1) - c*(C*
e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(b^2 - 4*a*c) - c^2*(C*(e^
2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3))))*x^2,
x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*
q + 3, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

Rule 1076

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqr
t[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$\int \frac{\left(3-x+2x^2\right)^{3/2}}{\left(2+3x+5x^2\right)^2}\,dx = \frac{(3+10x)\left(3-x+2x^2\right)^{3/2}}{31\left(2+3x+5x^2\right)} - \frac{1}{31}\int \frac{\sqrt{3-x+2x^2}\left(-\frac{69}{2}+13x+40x^2\right)}{2+3x+5x^2}\,dx$$

$$= \frac{4}{155}(4-5x)\sqrt{3-x+2x^2} + \frac{(3+10x)\left(3-x+2x^2\right)^{3/2}}{31\left(2+3x+5x^2\right)} + \frac{\int \frac{13070-5750x+2480x^2}{\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)}\,dx}{3100}$$

$$= \frac{4}{155}(4-5x)\sqrt{3-x+2x^2} + \frac{(3+10x)\left(3-x+2x^2\right)^{3/2}}{31\left(2+3x+5x^2\right)} + \frac{\int \frac{60390-36190x}{\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)}\,dx}{15500} + \frac{4}{25}$$

$$= \frac{4}{155}(4-5x)\sqrt{3-x+2x^2} + \frac{(3+10x)\left(3-x+2x^2\right)^{3/2}}{31\left(2+3x+5x^2\right)} + \frac{1}{25}\left(2\sqrt{\frac{2}{23}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+}}\right.$$

$$= \frac{4}{155}(4-5x)\sqrt{3-x+2x^2} + \frac{(3+10x)\left(3-x+2x^2\right)^{3/2}}{31\left(2+3x+5x^2\right)} - \frac{2}{25}\sqrt{2}\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right) + \frac{1}{155}$$

$$= \frac{4}{155}(4-5x)\sqrt{3-x+2x^2} + \frac{(3+10x)\left(3-x+2x^2\right)^{3/2}}{31\left(2+3x+5x^2\right)} - \frac{2}{25}\sqrt{2}\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right) + \frac{\sqrt{}}{}$$

Mathematica [C] time = 2.56, size = 530, normalized size = 2.28

$$\frac{\frac{62000\sqrt{2x^2-x+3}\,x^2}{10x-i\sqrt{31}+3}+\frac{62000\sqrt{2x^2-x+3}\,x^2}{10x+i\sqrt{31}+3}-\frac{31000\sqrt{2x^2-x+3}\,x}{10x-i\sqrt{31}+3}-\frac{31000\sqrt{2x^2-x+3}\,x}{10x+i\sqrt{31}+3}-12400\sqrt{2x^2-x+3}\,x+\frac{93000\sqrt{2x^2-x+3}}{10x-i\sqrt{31}+3}+\frac{93000\sqrt{2x^2-x+3}}{10x+i\sqrt{31}+3}+9920\sqrt{2x^2-x+3}-\frac{\sqrt{286+22i\sqrt{31}}\left(6477\sqrt{31}+10199i\right)\tanh^{-1}\left(\frac{-4i\sqrt{31}x-22x+i\sqrt{31}+63}{2\sqrt{286+22i\sqrt{31}}\sqrt{2x^2-x+3}}\right)}{\sqrt{31}-13i}+\frac{10199i\sqrt{286-22i\sqrt{31}}\tanh^{-1}\left(\frac{4i\sqrt{31}x-22x-i\sqrt{31}+63}{2\sqrt{286-22i\sqrt{31}}\sqrt{2x^2-x+3}}\right)}{\sqrt{31}+13i}-\frac{6477\sqrt{682\left(13-i\sqrt{31}\right)}\tanh^{-1}\left(\frac{4i\sqrt{31}x-22x-i\sqrt{31}+63}{2\sqrt{286-22i\sqrt{31}}\sqrt{2x^2-x+3}}\right)}{\sqrt{31}+13i}-7688\sqrt{2}\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{96100}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^(3/2)/(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] (9920*Sqrt[3 - x + 2*x^2] - 12400*x*Sqrt[3 - x + 2*x^2] + (93000*Sqrt[3 - x
 + 2*x^2])/(3 - I*Sqrt[31] + 10*x) - (31000*x*Sqrt[3 - x + 2*x^2])/(3 - I*S
qrt[31] + 10*x) + (62000*x^2*Sqrt[3 - x + 2*x^2])/(3 - I*Sqrt[31] + 10*x) +
 (93000*Sqrt[3 - x + 2*x^2])/(3 + I*Sqrt[31] + 10*x) - (31000*x*Sqrt[3 - x
+ 2*x^2])/(3 + I*Sqrt[31] + 10*x) + (62000*x^2*Sqrt[3 - x + 2*x^2])/(3 + I*
Sqrt[31] + 10*x) - 7688*Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]] - (Sqrt[286 + (
22*I)*Sqrt[31]]*(10199*I + 6477*Sqrt[31])*ArcTanh[(63 + I*Sqrt[31] - 22*x -
 (4*I)*Sqrt[31]*x)/(2*Sqrt[286 + (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])])/(-
13*I + Sqrt[31]) - (6477*Sqrt[682*(13 - I*Sqrt[31])]*ArcTanh[(63 - I*Sqrt[3
1] - 22*x + (4*I)*Sqrt[31]*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 - x + 2
```

```
*x^2])])/(13*I + Sqrt[31]) + ((10199*I)*Sqrt[286 - (22*I)*Sqrt[31]]*ArcTanh
[(63 - I*Sqrt[31] - 22*x + (4*I)*Sqrt[31]*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]
*Sqrt[3 - x + 2*x^2])])/(13*I + Sqrt[31]))/96100
```

IntegrateAlgebraic [C] time = 0.68, size = 422, normalized size = 1.82

$$\frac{11}{625}\text{RootSum}\left[-5\#1^4+6\sqrt{2}\#1^3+17\#1^2-26\sqrt{2}\#1-56\&,\frac{100\#1^2\log\left(-\#1+\sqrt{2x^2-x+3}-\sqrt{2}x\right)+310\sqrt{2}\#1\log\left(-\#1+\sqrt{2x^2-x+3}-\sqrt{2}x\right)+999\log\left(-\#1+\sqrt{2x^2-x+3}-\sqrt{2}x\right)}{-10\#1^3+9\sqrt{2}\#1^2+17\#1-13\sqrt{2}}\&\right]+\frac{11\text{RootSum}\left[-5\#1^4+6\sqrt{2}\#1^3+17\#1^2-26\sqrt{2}\#1-56\&,\frac{2025\#1^2\log\left(-\#1+\sqrt{2x^2-x+3}-\sqrt{2}x\right)+8230\sqrt{2}\#1\log\left(-\#1+\sqrt{2x^2-x+3}-\sqrt{2}x\right)-72888\log\left(-\#1+\sqrt{2x^2-x+3}-\sqrt{2}x\right)}{-10\#1^3+9\sqrt{2}\#1^2+17\#1-13\sqrt{2}}\&\right]}{38750}+\frac{11\sqrt{2x^2-x+3}(13x+7)}{155\left(5x^2+3x+2\right)}-\frac{2}{25}\sqrt{2}\log\left(2\sqrt{2}\sqrt{2x^2-x+3}-4x+1\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(3 - x + 2*x^2)^(3/2)/(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] (11*(7 + 13*x)*Sqrt[3 - x + 2*x^2])/(155*(2 + 3*x + 5*x^2)) - (2*Sqrt[2]*Lo
g[1 - 4*x + 2*Sqrt[2]*Sqrt[3 - x + 2*x^2]])/25 + (11*RootSum[-56 - 26*Sqrt[
2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3 - 5*#1^4 & , (999*Log[-(Sqrt[2]*x) + Sqrt[
3 - x + 2*x^2] - #1] + 310*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] -
 #1]*#1 + 100*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1^2)/(-13*Sqrt[
2] + 17*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3) & ])/625 + (11*RootSum[-56 - 26*Sqrt
[2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3 - 5*#1^4 & , (-72888*Log[-(Sqrt[2]*x) + S
qrt[3 - x + 2*x^2] - #1] + 8230*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x
^2] - #1]*#1 + 2025*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1^2)/(-13
*Sqrt[2] + 17*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3) & ])/38750
```

fricas [B] time = 1.29, size = 2150, normalized size = 9.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

```
[Out] 1/907468557458536000*(10421084*1987037073032^(1/4)*sqrt(45307)*sqrt(62)*sqrt
(2)*(5*x^2 + 3*x + 2)*sqrt(3169333*sqrt(2) + 4530700)*arctan(1/172758074198
807633719789*(64607782*sqrt(45307)*(2*1987037073032^(3/4)*sqrt(62)*(2433118
*x^7 - 9616349*x^6 + 20077988*x^5 - 32895253*x^4 + 16664280*x^3 - 8289000*x
^2 - sqrt(2)*(1842432*x^7 - 6916062*x^6 + 14611071*x^5 - 22920229*x^4 + 113
67152*x^3 - 5107176*x^2 - 12897792*x + 8726400) - 17452800*x + 12897792) +
1404517*1987037073032^(1/4)*sqrt(62)*(373384*x^7 - 5757834*x^6 + 30631880*x
^5 - 70476664*x^4 + 91370880*x^3 - 59457600*x^2 - sqrt(2)*(276977*x^7 - 423
2733*x^6 + 22218448*x^5 - 50249260*x^4 + 64668384*x^3 - 39479328*x^2 - 4669
7472*x + 32016384) - 64032768*x + 46697472))*sqrt(2*x^2 - x + 3)*sqrt(31693
33*sqrt(2) + 4530700) + 490410017080486186043272*sqrt(31)*sqrt(2)*(28180*x^
8 - 254666*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 9849
6*x^2 - sqrt(2)*(8746*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*
x^4 - 752088*x^3 + 396144*x^2 + 546048*x - 539136) + 1154304*x - 456192) -
sqrt(45307/2711)*(sqrt(45307)*(2*1987037073032^(3/4)*sqrt(62)*(8480726*x^7
- 12210811*x^6 + 39548601*x^5 - 16962480*x^4 + 21434760*x^3 + 14432256*x^2
- sqrt(2)*(6779042*x^7 - 9704193*x^6 + 31062363*x^5 - 11094928*x^4 + 121140
```

```
72*x^3 + 16301952*x^2 - 16301952*x) - 14432256*x) + 1404517*1987037073032^(
1/4)*sqrt(62)*(1312966*x^7 - 16987736*x^6 + 65572040*x^5 - 85530240*x^4 + 1
12374720*x^3 + 57314304*x^2 - sqrt(2)*(1011501*x^7 - 13081364*x^6 + 5039126
0*x^5 - 64806336*x^4 + 81634464*x^3 + 56070144*x^2 - 56070144*x) - 57314304
*x))*sqrt(2*x^2 - x + 3)*sqrt(3169333*sqrt(2) + 4530700) + 7590571938849196
*sqrt(31)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 + 39
6480*x^4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 118051*x^7 + 244
047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*x)
 + 3276288*x) + 345025997220418*sqrt(31)*(254591*x^8 - 4815126*x^7 + 323035
80*x^6 - 90866808*x^5 + 108781920*x^4 - 74219328*x^3 - 168956928*x^2 - 1548
8*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3618*x^3 + 2268
*x^2 - 1944*x) + 144820224*x))*sqrt(-(1987037073032^(1/4)*sqrt(45307)*sqrt(
62)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(1867*x + 1425) - 3292*x - 442)*s
qrt(3169333*sqrt(2) + 4530700) - 11567627293306*x^2 - 10387257161336*sqrt(2
)*(2*x^2 - x + 3) + 35647177985494*x - 47214805278800)/x^2) + 5572841103187
343023219*sqrt(31)*(2828123*x^8 - 9696916*x^7 + 53385560*x^6 - 142835344*x^
5 + 254146592*x^4 - 249300096*x^3 + 37981440*x^2 - 7744*sqrt(2)*(1348*x^8 -
 2692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^4 - 5568*x^3 + 1080*x^2 + 4320*x
 - 5184) + 223064064*x - 94887936))/(2585191*x^8 - 4661200*x^7 + 14191920*x
^6 + 490880*x^5 - 13562944*x^4 + 44249088*x^3 - 34615296*x^2 - 24772608*x +
 18579456)) + 10421084*1987037073032^(1/4)*sqrt(45307)*sqrt(62)*sqrt(2)*(5*
x^2 + 3*x + 2)*sqrt(3169333*sqrt(2) + 4530700)*arctan(1/1727580741988076337
19789*(64607782*sqrt(45307)*(2*1987037073032^(3/4)*sqrt(62)*(2433118*x^7 -
9616349*x^6 + 20077988*x^5 - 32895253*x^4 + 16664280*x^3 - 8289000*x^2 - sq
rt(2)*(1842432*x^7 - 6916062*x^6 + 14611071*x^5 - 22920229*x^4 + 11367152*x
^3 - 5107176*x^2 - 12897792*x + 8726400) - 17452800*x + 12897792) + 1404517
*1987037073032^(1/4)*sqrt(62)*(373384*x^7 - 5757834*x^6 + 30631880*x^5 - 70
476664*x^4 + 91370880*x^3 - 59457600*x^2 - sqrt(2)*(276977*x^7 - 4232733*x^
6 + 22218448*x^5 - 50249260*x^4 + 64668384*x^3 - 39479328*x^2 - 46697472*x
+ 32016384) - 64032768*x + 46697472))*sqrt(2*x^2 - x + 3)*sqrt(3169333*sqrt
(2) + 4530700) - 4904100170804861860432 72*sqrt(31)*sqrt(2)*(28180*x^8 - 254
666*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 -
 sqrt(2)*(8746*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^4 - 7
52088*x^3 + 396144*x^2 + 546048*x - 539136) + 1154304*x - 456192) - sqrt(45
307/2711)*(sqrt(45307)*(2*1987037073032^(3/4)*sqrt(62)*(8480726*x^7 - 12210
811*x^6 + 39548601*x^5 - 16962480*x^4 + 21434760*x^3 + 14432256*x^2 - sqrt(
2)*(6779042*x^7 - 9704193*x^6 + 31062363*x^5 - 11094928*x^4 + 12114072*x^3
+ 16301952*x^2 - 16301952*x) - 14432256*x) + 1404517*1987037073032^(1/4)*sq
rt(62)*(1312966*x^7 - 16987736*x^6 + 65572040*x^5 - 85530240*x^4 + 11237472
0*x^3 + 57314304*x^2 - sqrt(2)*(1011501*x^7 - 13081364*x^6 + 50391260*x^5 -
 64806336*x^4 + 81634464*x^3 + 56070144*x^2 - 56070144*x) - 57314304*x))*sq
rt(2*x^2 - x + 3)*sqrt(3169333*sqrt(2) + 4530700) - 7590571938849196*sqrt(3
1)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 + 396480*x^
4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 118051*x^7 + 244047*x^6
 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*x) + 3276
```

```
288*x) - 345025997220418*sqrt(31)*(254591*x^8 - 4815126*x^7 + 32303580*x^6
- 90866808*x^5 + 108781920*x^4 - 74219328*x^3 - 168956928*x^2 - 15488*sqrt(
2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3618*x^3 + 2268*x^2 -
1944*x) + 144820224*x))*sqrt((1987037073032^(1/4)*sqrt(45307)*sqrt(62)*sqrt
(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(1867*x + 1425) - 3292*x - 442)*sqrt(3169
333*sqrt(2) + 4530700) + 11567627293306*x^2 + 10387257161336*sqrt(2)*(2*x^2
 - x + 3) - 35647177985494*x + 47214805278800)/x^2) - 557284110318734302321
9*sqrt(31)*(2828123*x^8 - 9696916*x^7 + 53385560*x^6 - 142835344*x^5 + 2541
46592*x^4 - 249300096*x^3 + 37981440*x^2 - 7744*sqrt(2)*(1348*x^8 - 2692*x^
7 + 9789*x^6 - 10070*x^5 + 15569*x^4 - 5568*x^3 + 1080*x^2 + 4320*x - 5184)
 + 223064064*x - 94887936))/(2585191*x^8 - 4661200*x^7 + 14191920*x^6 + 490
880*x^5 - 13562944*x^4 + 44249088*x^3 - 34615296*x^2 - 24772608*x + 1857945
6)) + 1987037073032^(1/4)*sqrt(45307)*sqrt(62)*sqrt(31)*(22653500*x^2 - 316
9333*sqrt(2)*(5*x^2 + 3*x + 2) + 13592100*x + 9061400)*sqrt(3169333*sqrt(2)
 + 4530700)*log(113267500/2711*(1987037073032^(1/4)*sqrt(45307)*sqrt(62)*sq
rt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(1867*x + 1425) - 3292*x - 442)*sqrt(31
69333*sqrt(2) + 4530700) + 11567627293306*x^2 + 10387257161336*sqrt(2)*(2*x
^2 - x + 3) - 35647177985494*x + 47214805278800)/x^2) - 1987037073032^(1/4)
*sqrt(45307)*sqrt(62)*sqrt(31)*(22653500*x^2 - 3169333*sqrt(2)*(5*x^2 + 3*x
 + 2) + 13592100*x + 9061400)*sqrt(3169333*sqrt(2) + 4530700)*log(-11326750
0/2711*(1987037073032^(1/4)*sqrt(45307)*sqrt(62)*sqrt(31)*sqrt(2*x^2 - x +
3)*(sqrt(2)*(1867*x + 1425) - 3292*x - 442)*sqrt(3169333*sqrt(2) + 4530700)
 - 11567627293306*x^2 - 10387257161336*sqrt(2)*(2*x^2 - x + 3) + 3564717798
5494*x - 47214805278800)/x^2) + 3629874229834144*sqrt(2)*(5*x^2 + 3*x + 2)*
log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 644009
9440028320*sqrt(2*x^2 - x + 3)*(13*x + 7))/(5*x^2 + 3*x + 2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{174900625,[8]%%%}+%%%{%%{[-419761500,0]:[1,0,-2]%%},[7]%%%}+%%%{-685
610450,[6]%%%}+%%%{%%{[3246155600,0]:[1,0,-2]%%},[5]%%%}+%%%{1574105625,[4]
%%%}+%%%{%%{[-10885814900,0]:[1,0,-2]%%},[3]%%%}+%%%{-3861805800,[2]%%%}+%%
%{%%{[20372424800,0]:[1,0,-2]%%},[1]%%%}+%%%{21939534400,[0]%%%} / %%%{50,[
8]%%%}+%%%{%%{poly1[-120,0]:[1,0,-2]%%},[7]%%%}+%%%{-196,[6]%%%}+%%%{%%{pol
y1[928,0]:[1,0,-2]%%},[5]%%%}+%%%{450,[4]%%%}+%%%{%%{poly1[-3112,0]:[1,0,-2
]%%},[3]%%%}+%%%{-1104,[2]%%%}+%%%{%%{poly1[5824,0]:[1,0,-2]%%},[1]%%%}+%%%
{6272,[0]%%%} Error: Bad Argument Value
```

maple [B] time = 0.16, size = 28185, normalized size = 121.49

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^2,x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\,x^2-x+3\right)^{\frac{3}{2}}}{\left(5\,x^2+3\,x+2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^2,x, algorithm="maxima")
[Out] integrate((2*x^2 - x + 3)^(3/2)/(5*x^2 + 3*x + 2)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(2\,x^2-x+3\right)^{3/2}}{\left(5\,x^2+3\,x+2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^(3/2)/(3*x + 5*x^2 + 2)^2,x)
[Out] int((2*x^2 - x + 3)^(3/2)/(3*x + 5*x^2 + 2)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2x^2-x+3\right)^{\frac{3}{2}}}{\left(5x^2+3x+2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(3/2)/(5*x**2+3*x+2)**2,x)
[Out] Integral((2*x**2 - x + 3)**(3/2)/(5*x**2 + 3*x + 2)**2, x)
```

3.68 $\int \frac{(3-x+2x^2)^{3/2}}{(2+3x+5x^2)^3} \, dx$

Optimal. Leaf size=223

$$\frac{(10x+3)\left(2x^2-x+3\right)^{3/2}}{62\left(5x^2+3x+2\right)^2}+\frac{3(696x+277)\sqrt{2x^2-x+3}}{3844\left(5x^2+3x+2\right)}+\frac{3\sqrt{\frac{1}{682}\left(366990269+259509026\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{31(366990}}{}\right.}{7688}$$

Rubi [A] time = 0.43, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {971, 1013, 1035, 1029, 206, 204}

$$\frac{(10x+3)\left(2x^2-x+3\right)^{3/2}}{62\left(5x^2+3x+2\right)^2}+\frac{3(696x+277)\sqrt{2x^2-x+3}}{3844\left(5x^2+3x+2\right)}+\frac{3\sqrt{\frac{1}{682}\left(366990269+259509026\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{11}{31\left(366990269+259509026\sqrt{2}\right)}}\left(\left(70517+49942\sqrt{2}\right)x+20575\sqrt{2}+29367\right)}{\sqrt{2x^2-x+3}}\right)}{7688}-\frac{3\sqrt{\frac{1}{682}\left(259509026\sqrt{2}-366990269\right)}\tanh^{-1}\left(\frac{\sqrt{\frac{11}{31\left(259509026\sqrt{2}-366990269\right)}}\left(\left(70517-49942\sqrt{2}\right)x-20575\sqrt{2}+29367\right)}{\sqrt{2x^2-x+3}}\right)}{7688}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^(3/2)/(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] ((3 + 10*x)*(3 - x + 2*x^2)^(3/2))/(62*(2 + 3*x + 5*x^2)^2) + (3*(277 + 696
*x)*Sqrt[3 - x + 2*x^2])/(3844*(2 + 3*x + 5*x^2)) + (3*Sqrt[(366990269 + 25
9509026*Sqrt[2])/682]*ArcTan[(Sqrt[11/(31*(366990269 + 259509026*Sqrt[2]))]
*(29367 + 20575*Sqrt[2] + (70517 + 49942*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]])
/7688 - (3*Sqrt[(-366990269 + 259509026*Sqrt[2])/682]*ArcTanh[(Sqrt[11/(31*
(-366990269 + 259509026*Sqrt[2]))]*(29367 - 20575*Sqrt[2] + (70517 - 49942*
Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]])/7688
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 971

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((b + 2*c*x)*(a + b*x + c*x^2)^(p + 1)*(d + e
```

```
*x + f*x^2)^q)/((b^2 - 4*a*c)*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*(p + 1))
, Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*Simp[2*c*d*(2*p +
 3) + b*e*q + (2*b*f*q + 2*c*e*(2*p + q + 3))*x + 2*c*f*(2*p + 2*q + 3)*x^2
, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
e^2 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0] &&  !IGtQ[q, 0]
```

Rule 1013

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e
_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((g*b - 2*a*h - (b*h - 2*g
*c)*x)*(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q)/((b^2 - 4*a*c)*(p + 1
)), x] - Dist[1/((b^2 - 4*a*c)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d +
 e*x + f*x^2)^(q - 1)*Simp[e*q*(g*b - 2*a*h) - d*(b*h - 2*g*c)*(2*p + 3) +
(2*f*q*(g*b - 2*a*h) - e*(b*h - 2*g*c)*(2*p + q + 3))*x - f*(b*h - 2*g*c)*(
2*p + 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 1029

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
 c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 1035

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

Rubi steps

$$\int \frac{\left(3-x+2x^2\right)^{3/2}}{\left(2+3x+5x^2\right)^3}\,dx = \frac{(3+10x)\left(3-x+2x^2\right)^{3/2}}{62\left(2+3x+5x^2\right)^2} - \frac{1}{62}\int \frac{\left(-\frac{189}{2}+33x\right)\sqrt{3-x+2x^2}}{\left(2+3x+5x^2\right)^2}\,dx$$

$$= \frac{(3+10x)\left(3-x+2x^2\right)^{3/2}}{62\left(2+3x+5x^2\right)^2} + \frac{3(277+696x)\sqrt{3-x+2x^2}}{3844\left(2+3x+5x^2\right)} + \frac{\int \frac{\frac{13359}{4}-1353x}{\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)}\,dx}{1922}$$

$$= \frac{(3+10x)\left(3-x+2x^2\right)^{3/2}}{62\left(2+3x+5x^2\right)^2} + \frac{3(277+696x)\sqrt{3-x+2x^2}}{3844\left(2+3x+5x^2\right)} + \frac{\int \frac{-\frac{33}{4}\left(6257-4453\sqrt{2}\right)+\frac{33}{4}\left(2649-1\cdots\right.}{\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)\cdots}}{42284\sqrt{2}}$$

$$= \frac{(3+10x)\left(3-x+2x^2\right)^{3/2}}{62\left(2+3x+5x^2\right)^2} + \frac{3(277+696x)\sqrt{3-x+2x^2}}{3844\left(2+3x+5x^2\right)} + \frac{\left(99\left(519018052-3669902\cdots\right.\right.}{\cdots}$$

$$= \frac{(3+10x)\left(3-x+2x^2\right)^{3/2}}{62\left(2+3x+5x^2\right)^2} + \frac{3(277+696x)\sqrt{3-x+2x^2}}{3844\left(2+3x+5x^2\right)} + \frac{3\sqrt{\frac{1}{682}\left(366990269+2595\cdots\right.}}{\cdots}$$

<u>Mathematica</u> [C]   time = 5.34, size = 1262, normalized size = 5.66

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^(3/2)/(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] (((248000*I)*Sqrt[31]*(3 - x + 2*x^2)^(3/2))/(3*I + Sqrt[31] + (10*I)*x)^2
+ (744000*(3 - x + 2*x^2)^(3/2))/(3 - I*Sqrt[31] + 10*x) + ((248000*I)*Sqrt
[31]*(3 - x + 2*x^2)^(3/2))/(3 + I*Sqrt[31] + 10*x)^2 + (744000*(3 - x + 2*
x^2)^(3/2))/(3 + I*Sqrt[31] + 10*x) + (3*I)*Sqrt[31]*(20*(1199 + (98*I)*Sqr
t[31] - 20*(11 + (2*I)*Sqrt[31])*x)*Sqrt[3 - x + 2*x^2] + Sqrt[2]*(13453 +
(4406*I)*Sqrt[31])*ArcSinh[(1 - 4*x)/Sqrt[23]] - (352*Sqrt[286 + (22*I)*Sqr
t[31]]*(-69*I + 13*Sqrt[31])*ArcTanh[(63 + I*Sqrt[31] + (-22 - (4*I)*Sqrt[3
1])*x)/(2*Sqrt[286 + (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])])/(-13*I + Sqrt[
31])) + 558*(20*(27 + (4*I)*Sqrt[31] - 20*x)*Sqrt[3 - x + 2*x^2] + Sqrt[2]*
(569 + (88*I)*Sqrt[31])*ArcSinh[(1 - 4*x)/Sqrt[23]] - (4*Sqrt[286 + (22*I)*
Sqrt[31]]*(-81*I + 37*Sqrt[31])*ArcTanh[(63 + I*Sqrt[31] + (-22 - (4*I)*Sqr
t[31])*x)/(2*Sqrt[286 + (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])])/(-13*I + Sq
rt[31])) + (744*Sqrt[31]*(220*(-439 + (497*I)*Sqrt[31] + 20*(69 + (13*I)*Sq
```

```
rt[31])*x)*Sqrt[3 - x + 2*x^2] + 88*Sqrt[2]*(4426 - (398*I)*Sqrt[31] + 5*(4
7 - (281*I)*Sqrt[31])*x)*ArcSinh[(-1 + 4*x)/Sqrt[23]] + Sqrt[286 + (22*I)*S
qrt[31]]*(19548 - (4904*I)*Sqrt[31] + (-23345 - (8565*I)*Sqrt[31])*x)*ArcTa
nh[(63 + I*Sqrt[31] + (-22 - (4*I)*Sqrt[31])*x)/(2*Sqrt[286 + (22*I)*Sqrt[3
1]]*Sqrt[3 - x + 2*x^2])]))/(11*(-13*I + Sqrt[31])^2*(-3*I + Sqrt[31] - (10
*I)*x)) + (744*Sqrt[31]*(220*(-439 - (497*I)*Sqrt[31] + 20*(69 - (13*I)*Sqr
t[31])*x)*Sqrt[3 - x + 2*x^2] + 88*Sqrt[2]*(4426 + (398*I)*Sqrt[31] + 5*(47
 + (281*I)*Sqrt[31])*x)*ArcSinh[(-1 + 4*x)/Sqrt[23]] + Sqrt[286 - (22*I)*Sq
rt[31]]*(-19548 - (4904*I)*Sqrt[31] + 5*(4669 - (1713*I)*Sqrt[31])*x)*ArcTa
nh[(-63 + I*Sqrt[31] + (22 - (4*I)*Sqrt[31])*x)/(2*Sqrt[286 - (22*I)*Sqrt[3
1]]*Sqrt[3 - x + 2*x^2])]))/(11*(13*I + Sqrt[31])^2*(3*I + Sqrt[31] + (10*I
)*x)) + (3*Sqrt[31]*(20*(12549 - (2473*I)*Sqrt[31] + (20*I)*(81*I + 37*Sqrt
[31])*x)*Sqrt[3 - x + 2*x^2] + Sqrt[2]*(38303 - (70731*I)*Sqrt[31])*ArcSinh
[(1 - 4*x)/Sqrt[23]] + (352*I)*Sqrt[286 - (22*I)*Sqrt[31]]*(69*I + 13*Sqrt[
31])*ArcTanh[(63 - I*Sqrt[31] + (-22 + (4*I)*Sqrt[31])*x)/(2*Sqrt[286 - (22
*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])]))/(13*I + Sqrt[31]) + 558*(-20*(-27 + (
4*I)*Sqrt[31] + 20*x)*Sqrt[3 - x + 2*x^2] + Sqrt[2]*(569 - (88*I)*Sqrt[31])
*ArcSinh[(1 - 4*x)/Sqrt[23]] - (4*Sqrt[286 - (22*I)*Sqrt[31]]*(81*I + 37*Sq
rt[31])*ArcTanh[(63 - I*Sqrt[31] + (-22 + (4*I)*Sqrt[31])*x)/(2*Sqrt[286 -
(22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])])/(13*I + Sqrt[31])))/4766560
```

IntegrateAlgebraic [C] time = 1.06, size = 578, normalized size = 2.59

$$-\frac{3219561\text{RootSum}\left[-5\#1^4+6\sqrt{2}\#1^3+17\#1^2-26\sqrt{2}\#1-56\&,\frac{\log\left(-\#1+\sqrt{2x^2-x+3}-\sqrt{2}x\right)}{-10\#1^3+9\sqrt{2}\#1^2+17\#1-13\sqrt{2}}\&\right]}{961000}+\frac{4}{125}\text{RootSum}\left[-5\#1^4+6\sqrt{2}\#1^3+17\#1^2-26\sqrt{2}\#1-56\&,\frac{10\sqrt{2}\#1\log\left(-\#1+\sqrt{2x^2-x+3}-\sqrt{2}x\right)+93\log\left(-\#1+\sqrt{2x^2-x+3}-\sqrt{2}x\right)}{-10\#1^3+9\sqrt{2}\#1^2+17\#1-13\sqrt{2}}\&\right]+\frac{7\text{RootSum}\left[-5\#1^4+6\sqrt{2}\#1^3+17\#1^2-26\sqrt{2}\#1-56\&,\frac{4926449381\sqrt{2}\#1\log\left(-\#1+\sqrt{2x^2-x+3}-\sqrt{2}x\right)-2660991465\#1^2\log\left(-\#1+\sqrt{2x^2-x+3}-\sqrt{2}x\right)}{-10\#1^3+9\sqrt{2}\#1^2+17\#1-13\sqrt{2}}\&\right]}{6354612500}-\frac{3\text{RootSum}\left[-5\#1^4+6\sqrt{2}\#1^3+17\#1^2-26\sqrt{2}\#1-56\&,\frac{155209944\sqrt{2}\#1\log\left(-\#1+\sqrt{2x^2-x+3}-\sqrt{2}x\right)-248390285\#1^2\log\left(-\#1+\sqrt{2x^2-x+3}-\sqrt{2}x\right)}{-10\#1^3+9\sqrt{2}\#1^2+17\#1-13\sqrt{2}}\&\right]}{204987500}+\frac{\sqrt{2x^2-x+3}\left(11680x^3+10171x^2+8343x+2220\right)}{3844\left(5x^2+3x+2\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(3 - x + 2*x^2)^(3/2)/(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] (Sqrt[3 - x + 2*x^2]*(2220 + 8343*x + 10171*x^2 + 11680*x^3))/(3844*(2 + 3*
x + 5*x^2)^2) - (3219561*RootSum[-56 - 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[2]*
#1^3 - 5*#1^4 & , Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]/(-13*Sqrt[2]
 + 17*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3) & ])/961000 + (4*RootSum[-56 - 26*Sqrt
[2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3 - 5*#1^4 & , (93*Log[-(Sqrt[2]*x) + Sqrt[
3 - x + 2*x^2] - #1] + 10*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] -
#1]*#1)/(-13*Sqrt[2] + 17*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3) & ])/125 + (7*Root
Sum[-56 - 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3 - 5*#1^4 & , (4926449381
*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1 - 2660991465*Log[-
(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1^2)/(-13*Sqrt[2] + 17*#1 + 9*Sqrt
[2]*#1^2 - 10*#1^3) & ])/6354612500 - (3*RootSum[-56 - 26*Sqrt[2]*#1 + 17*#
1^2 + 6*Sqrt[2]*#1^3 - 5*#1^4 & , (155209944*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqr
t[3 - x + 2*x^2] - #1]*#1 - 248390285*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2
] - #1]*#1^2)/(-13*Sqrt[2] + 17*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3) & ])/2049875
00
```

fricas [B] time = 1.27, size = 2183, normalized size = 9.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

```
[Out] -1/8577307141769792410969 6*(189113268*134689869150937352^(1/4)*sqrt(1297545
13)*sqrt(341)*sqrt(2)*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)*sqrt(366990269*
sqrt(2) + 519018052)*arctan(1/1067259092343193675559267622545473*(160895596
12*sqrt(129754513)*(11*134689869150937352^(3/4)*sqrt(341)*(38305160*x^7 - 1
47261352*x^6 + 309398878*x^5 - 495410374*x^4 + 248212864*x^3 - 117285552*x^
2 - sqrt(2)*(26988622*x^7 - 104036813*x^6 + 218448200*x^5 - 350579241*x^4 +
 175844824*x^3 - 83534472*x^2 - 191303424*x + 135585792) - 271171584*x + 19
1303424) + 4022389903*134689869150937352^(1/4)*sqrt(341)*(2906601*x^7 - 446
04657*x^6 + 235604928*x^5 - 537156764*x^4 + 693706464*x^3 - 436717728*x^2 -
 sqrt(2)*(2050114*x^7 - 31475955*x^6 + 166375268*x^5 - 379661892*x^4 + 4905
00864*x^3 - 309827808*x^2 - 348696576*x + 246965760) - 493931520*x + 348696
576))*sqrt(2*x^2 - x + 3)*sqrt(366990269*sqrt(2) + 519018052) + 30296387137
484207564263080898065504*sqrt(31)*sqrt(2)*(28180*x^8 - 254666*x^7 + 704270*x
^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - sqrt(2)*(8746*x^8
 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^4 - 752088*x^3 + 396144
*x^2 + 546048*x - 539136) + 1154304*x - 456192) - 2*sqrt(259509026/713)*(sq
rt(129754513)*(11*134689869150937352^(3/4)*sqrt(341)*(5980372*x^7 - 8582986
*x^6 + 27618126*x^5 - 10751392*x^4 + 12649968*x^3 + 12517632*x^2 - sqrt(2)*
(4201650*x^7 - 6032009*x^6 + 19421619*x^5 - 7633552*x^4 + 9050328*x^3 + 864
0000*x^2 - 8640000*x) - 12517632*x) + 4022389903*134689869150937352^(1/4)*s
qrt(341)*(453599*x^7 - 5867420*x^6 + 22622900*x^5 - 29282112*x^4 + 37610208
*x^3 + 22726656*x^2 - sqrt(2)*(319303*x^7 - 4130364*x^6 + 15927060*x^5 - 20
630592*x^4 + 26556768*x^3 + 15800832*x^2 - 15800832*x) - 22726656*x))*sqrt(
2*x^2 - x + 3)*sqrt(366990269*sqrt(2) + 519018052) + 818688798906871280095 4
*sqrt(31)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 + 39
6480*x^4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 118051*x^7 + 244
047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*x)
 + 3276288*x) + 3721312722303960364 07*sqrt(31)*(254591*x^8 - 4815126*x^7 +
32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 74219328*x^3 - 168956928*x^2
- 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3618*x^3
+ 2268*x^2 - 1944*x) + 144820224*x))*sqrt(-(134689869150937352^(1/4)*sqrt(1
29754513)*sqrt(341)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(696*x + 277) - 9
73*x - 419)*sqrt(366990269*sqrt(2) + 519018052) - 4356437317274441*x^2 - 39
11902897144396*sqrt(2)*(2*x^2 - x + 3) + 13424939487927359*x - 177813768052
01800)/x^2) + 34427712656232054050298955565983*sqrt(31)*(2828123*x^8 - 9696
916*x^7 + 53385560*x^6 - 142835344*x^5 + 254146592*x^4 - 249300096*x^3 + 37
981440*x^2 - 7744*sqrt(2)*(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 155
69*x^4 - 5568*x^3 + 1080*x^2 + 4320*x - 5184) + 223064064*x - 94887936))/(2
585191*x^8 - 4661200*x^7 + 14191920*x^6 + 490880*x^5 - 13562944*x^4 + 44249
088*x^3 - 34615296*x^2 - 24772608*x + 18579456)) + 189113268*13468986915093
```

7352^(1/4)*sqrt(129754513)*sqrt(341)*sqrt(2)*(25*x^4 + 30*x^3 + 29*x^2 + 12
*x + 4)*sqrt(366990269*sqrt(2) + 519018052)*arctan(1/1067259092343193675559
267622545473*(16089559612*sqrt(129754513)*(11*134689869150937352^(3/4)*sqrt
(341)*(38305160*x^7 - 147261352*x^6 + 309398878*x^5 - 495410374*x^4 + 24821
2864*x^3 - 117285552*x^2 - sqrt(2)*(26988622*x^7 - 104036813*x^6 + 21844820
0*x^5 - 350579241*x^4 + 175844824*x^3 - 83534472*x^2 - 191303424*x + 135585
792) - 271171584*x + 191303424) + 4022389903*134689869150937352^(1/4)*sqrt(
341)*(2906601*x^7 - 44604657*x^6 + 235604928*x^5 - 537156764*x^4 + 69370646
4*x^3 - 436717728*x^2 - sqrt(2)*(2050114*x^7 - 31475955*x^6 + 166375268*x^5
 - 379661892*x^4 + 490500864*x^3 - 309827808*x^2 - 348696576*x + 246965760)
 - 493931520*x + 348696576))*sqrt(2*x^2 - x + 3)*sqrt(366990269*sqrt(2) + 5
19018052) - 30296387137484207564263080898065O4*sqrt(31)*sqrt(2)*(28180*x^8
- 254666*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*
x^2 - sqrt(2)*(8746*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^
4 - 752088*x^3 + 396144*x^2 + 546048*x - 539136) + 1154304*x - 456192) - 2*
sqrt(259509026/713)*(sqrt(129754513)*(11*134689869150937352^(3/4)*sqrt(341)
*(5980372*x^7 - 8582986*x^6 + 27618126*x^5 - 10751392*x^4 + 12649968*x^3 +
12517632*x^2 - sqrt(2)*(4201650*x^7 - 6032009*x^6 + 19421619*x^5 - 7633552*
x^4 + 9050328*x^3 + 8640000*x^2 - 8640000*x) - 12517632*x) + 4022389903*134
689869150937352^(1/4)*sqrt(341)*(453599*x^7 - 5867420*x^6 + 22622900*x^5 -
29282112*x^4 + 37610208*x^3 + 22726656*x^2 - sqrt(2)*(319303*x^7 - 4130364*
x^6 + 15927060*x^5 - 20630592*x^4 + 26556768*x^3 + 15800832*x^2 - 15800832*
x) - 22726656*x))*sqrt(2*x^2 - x + 3)*sqrt(366990269*sqrt(2) + 519018052) -
 8186887989068712800954*sqrt(31)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578888
*x^6 - 3293072*x^5 + 396480*x^4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15550
*x^8 - 118051*x^7 + 244047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1
209600*x^2 - 1036800*x) + 3276288*x) - 3721312722303960364O7*sqrt(31)*(2545
91*x^8 - 4815126*x^7 + 32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 742193
28*x^3 - 168956928*x^2 - 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5
 + 2385*x^4 - 3618*x^3 + 2268*x^2 - 1944*x) + 144820224*x))*sqrt((134689869
150937352^(1/4)*sqrt(129754513)*sqrt(341)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqr
t(2)*(696*x + 277) - 973*x - 419)*sqrt(366990269*sqrt(2) + 519018052) + 435
6437317274441*x^2 + 3911902897144396*sqrt(2)*(2*x^2 - x + 3) - 134249394879
27359*x + 17781376805201800)/x^2) - 34427712656232054050298955565983*sqrt(3
1)*(2828123*x^8 - 9696916*x^7 + 53385560*x^6 - 142835344*x^5 + 254146592*x^
4 - 249300096*x^3 + 37981440*x^2 - 7744*sqrt(2)*(1348*x^8 - 2692*x^7 + 9789
*x^6 - 10070*x^5 + 15569*x^4 - 5568*x^3 + 1080*x^2 + 4320*x - 5184) + 22306
4064*x - 94887936))/(2585191*x^8 - 4661200*x^7 + 14191920*x^6 + 490880*x^5
- 13562944*x^4 + 44249088*x^3 - 34615296*x^2 - 24772608*x + 18579456)) - 3*
134689869150937352^(1/4)*sqrt(129754513)*sqrt(341)*sqrt(31)*(12975451300*x^
4 + 15570541560*x^3 + 15051523508*x^2 - 366990269*sqrt(2)*(25*x^4 + 30*x^3
+ 29*x^2 + 12*x + 4) + 6228216624*x + 2076072208)*sqrt(366990269*sqrt(2) +
519018052)*log(9342324936/713*(134689869150937352^(1/4)*sqrt(129754513)*sqr
t(341)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(696*x + 277) - 973*x - 419)*s
qrt(366990269*sqrt(2) + 519018052) + 4356437317274441*x^2 + 391190289714439

```
6*sqrt(2)*(2*x^2 - x + 3) - 13424939487927359*x + 17781376805201800)/x^2) +
 3*134689869150937352^(1/4)*sqrt(129754513)*sqrt(341)*sqrt(31)*(12975451300
*x^4 + 15570541560*x^3 + 15051523508*x^2 - 366990269*sqrt(2)*(25*x^4 + 30*x
^3 + 29*x^2 + 12*x + 4) + 6228216624*x + 2076072208)*sqrt(366990269*sqrt(2)
 + 519018052)*log(-9342324936/713*(134689869150937352^(1/4)*sqrt(129754513)
*sqrt(341)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(696*x + 277) - 973*x - 41
9)*sqrt(366990269*sqrt(2) + 519018052) - 4356437317274441*x^2 - 39119028971
44396*sqrt(2)*(2*x^2 - x + 3) + 13424939487927359*x - 17781376805201800)/x^
2) - 22313494125311634784*(11680*x^3 + 10171*x^2 + 8343*x + 2220)*sqrt(2*x^
2 - x + 3))/(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Francis algorithm failure for[-1.0,infinity,
infinity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,infi
nity]Francis algorithm failure for[-1.0,infinity,infinity,infinity,infinity
]proot error [1.0,infinity,infinity,infinity,infinity]Francis algorithm fai
lure for[-1.0,infinity,infinity,infinity,infinity]proot error [1.0,infinity
,infinity,infinity,infinity]Francis algorithm failure for[-1.0,infinity,inf
inity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,infinit
y]Evaluation time: 36.71Done
```

maple [B] time = 0.35, size = 81552, normalized size = 365.70

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^3,x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\,x^{2}-x+3\right)^{\frac{3}{2}}}{\left(5\,x^{2}+3\,x+2\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^3,x, algorithm="maxima")
[Out] integrate((2*x^2 - x + 3)^(3/2)/(5*x^2 + 3*x + 2)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(2\,x^2-x+3\right)^{3/2}}{\left(5\,x^2+3\,x+2\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^(3/2)/(3*x + 5*x^2 + 2)^3,x)
[Out] int((2*x^2 - x + 3)^(3/2)/(3*x + 5*x^2 + 2)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2x^{2}-x+3\right)^{\frac{3}{2}}}{\left(5x^{2}+3x+2\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(3/2)/(5*x**2+3*x+2)**3,x)
[Out] Integral((2*x**2 - x + 3)**(3/2)/(5*x**2 + 3*x + 2)**3, x)
```

3.69 $\int \left(3-x+2x^2\right)^{5/2} \left(2+3x+5x^2\right)^4 \, dx$

Optimal. Leaf size=254

$$\frac{122595067\left(2x^2-x+3\right)^{7/2}x^2}{19169280}+\frac{112244125\left(2x^2-x+3\right)^{7/2}x}{122683392}+\frac{25250178739\left(2x^2-x+3\right)^{7/2}}{5725224960}-\frac{401135647(1-4x}{33554}$$

Rubi [A] time = 0.37, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1661, 640, 612, 619, 215}

$$\frac{625}{28}\left(2x^2-x+3\right)^{7/2}x^7+\frac{13875}{208}\left(2x^2-x+3\right)^{7/2}x^6+\frac{1046225\left(2x^2-x+3\right)^{7/2}x^5}{9984}+\frac{3684995\left(2x^2-x+3\right)^{7/2}x^4}{39936}+\frac{23460839\left(2x^2-x+3\right)^{7/2}x^3}{532480}+\frac{122595067\left(2x^2-x+3\right)^{7/2}x^2}{19169280}+\frac{112244125\left(2x^2-x+3\right)^{7/2}x}{122683392}+\frac{25250178739\left(2x^2-x+3\right)^{7/2}}{5725224960}-\frac{401135647(1-4x)\left(2x^2-x+3\right)^{5/2}}{335544320}-\frac{9226119881(1-4x)\left(2x^2-x+3\right)^{3/2}}{2147483648}-\frac{636602271789(1-4x)\sqrt{2x^2-x+3}}{34359738368}-\frac{14641852251147\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{68719476736\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^(5/2)*(2 + 3*x + 5*x^2)^4,x]
```

```
[Out] (-636602271789*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/34359738368 - (9226119881*(1
 - 4*x)*(3 - x + 2*x^2)^(3/2))/2147483648 - (401135647*(1 - 4*x)*(3 - x + 2*
x^2)^(5/2))/335544320 + (25250178739*(3 - x + 2*x^2)^(7/2))/5725224960 + (1
12244125*x*(3 - x + 2*x^2)^(7/2))/122683392 + (122595067*x^2*(3 - x + 2*x^2
)^(7/2))/19169280 + (23460839*x^3*(3 - x + 2*x^2)^(7/2))/532480 + (3684995*
x^4*(3 - x + 2*x^2)^(7/2))/39936 + (1046225*x^5*(3 - x + 2*x^2)^(7/2))/9984
 + (13875*x^6*(3 - x + 2*x^2)^(7/2))/208 + (625*x^7*(3 - x + 2*x^2)^(7/2))/
28 - (14641852251147*ArcSinh[(1 - 4*x)/Sqrt[23]])/(68719476736*Sqrt[2])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \left(3-x+2x^2\right)^{5/2}\left(2+3x+5x^2\right)^4\,dx = \frac{625}{28}x^7\left(3-x+2x^2\right)^{7/2}+\frac{1}{28}\int \left(3-x+2x^2\right)^{5/2}\Big(448+2688x+10528x$$

$$= \frac{13875}{208}x^6\left(3-x+2x^2\right)^{7/2}+\frac{625}{28}x^7\left(3-x+2x^2\right)^{7/2}+\frac{1}{728}\int \left(3-x+2x\right.$$

$$= \frac{1046225x^5\left(3-x+2x^2\right)^{7/2}}{9984}+\frac{13875}{208}x^6\left(3-x+2x^2\right)^{7/2}+\frac{625}{28}x^7\left(3-x\right.$$

$$= \frac{3684995x^4\left(3-x+2x^2\right)^{7/2}}{39936}+\frac{1046225x^5\left(3-x+2x^2\right)^{7/2}}{9984}+\frac{13875}{208}x^6\left(\right.$$

$$= \frac{23460839x^3\left(3-x+2x^2\right)^{7/2}}{532480}+\frac{3684995x^4\left(3-x+2x^2\right)^{7/2}}{39936}+\frac{1046225x}{}$$

$$= \frac{122595067x^2\left(3-x+2x^2\right)^{7/2}}{19169280}+\frac{23460839x^3\left(3-x+2x^2\right)^{7/2}}{532480}+\frac{368499}{}$$

$$= \frac{112244125x\left(3-x+2x^2\right)^{7/2}}{122683392}+\frac{122595067x^2\left(3-x+2x^2\right)^{7/2}}{19169280}+\frac{234608}{}$$

$$= \frac{25250178739\left(3-x+2x^2\right)^{7/2}}{5725224960}+\frac{112244125x\left(3-x+2x^2\right)^{7/2}}{122683392}+\frac{122595}{}$$

$$= -\frac{401135647(1-4x)\left(3-x+2x^2\right)^{5/2}}{335544320}+\frac{25250178739\left(3-x+2x^2\right)^{7/2}}{5725224960}$$

$$= -\frac{9226119881(1-4x)\left(3-x+2x^2\right)^{3/2}}{2147483648}-\frac{401135647(1-4x)\left(3-x+2x\right.}{335544320}$$

$$= -\frac{636602271789(1-4x)\sqrt{3-x+2x^2}}{34359738368}-\frac{9226119881(1-4x)\left(3-x+2\right.}{2147483648}$$

$$= -\frac{636602271789(1-4x)\sqrt{3-x+2x^2}}{34359738368}-\frac{9226119881(1-4x)\left(3-x+2\right.}{2147483648}$$

$$= -\frac{636602271789(1-4x)\sqrt{3-x+2x^2}}{34359738368}-\frac{9226119881(1-4x)\left(3-x+2\right.}{2147483648}$$

Mathematica [A] time = 0.45, size = 105, normalized size = 0.41

$$\frac{4\sqrt{2x^2-x+3}\left(25125558681600000x^{13}+37398427729920000x^{12}+137233466130432000x^{11}+204932411660697600x^{10}+363646430503501824x^9+439064558846345216x^8+530502956133122048x^7+485091164642279424x^6+405468382284161024x^5+257786732552566784x^4+142490931553577856x^3+50064174038215008x^2+12071614275862524x+10820567498568669\right)-59958384968446965\sqrt{2}\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{562812514467840}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^(5/2)*(2 + 3*x + 5*x^2)^4,x]
```

```
[Out] (4*Sqrt[3 - x + 2*x^2]*(10820567498568669 + 12071614275862524*x + 500641740
38215008*x^2 + 142490931553577856*x^3 + 257786732552566784*x^4 + 4054683822
84161024*x^5 + 485091164642279424*x^6 + 530502956133122048*x^7 + 4390645588
46345216*x^8 + 363646430503501824*x^9 + 204932411660697600*x^10 + 137233466
130432000*x^11 + 37398427729920000*x^12 + 25125558681600000*x^13) - 5995838
4968446965*Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]])/562812514467840
```

IntegrateAlgebraic [A] time = 1.83, size = 120, normalized size = 0.47

$$\frac{\sqrt{2x^2-x+3}\,\left(25125558681600000x^{13}+37398427729920000x^{12}+137233466130432000x^{11}+204932411660697600x^{10}+363646430503501824x^9+439064558846345216x^8+530502956133122048x^7+485091164642279424x^6+405468382284161024x^5+257786732552566784x^4+142490931553577856x^3+50064174038215008x^2+12071614275862524x+10820567498568669\right)}{140703128616960}-\frac{14641852251147\log\left(2\sqrt{2}\sqrt{2x^2-x+3}-4x+1\right)}{68719476736\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(3 - x + 2*x^2)^(5/2)*(2 + 3*x + 5*x^2)^4,x]
```

```
[Out] (Sqrt[3 - x + 2*x^2]*(10820567498568669 + 12071614275862524*x + 50064174038
215008*x^2 + 142490931553577856*x^3 + 257786732552566784*x^4 + 405468382284
161024*x^5 + 485091164642279424*x^6 + 530502956133122048*x^7 + 439064558846
345216*x^8 + 363646430503501824*x^9 + 204932411660697600*x^10 + 13723346613
0432000*x^11 + 37398427729920000*x^12 + 25125558681600000*x^13))/1407031286
16960 - (14641852251147*Log[1 - 4*x + 2*Sqrt[2]*Sqrt[3 - x + 2*x^2]])/(6871
9476736*Sqrt[2])
```

fricas [A] time = 0.42, size = 118, normalized size = 0.46

$$\frac{1}{140703128616960}\,\left(25125558681600000\,x^{13}+37398427729920000\,x^{12}+137233466130432000\,x^{11}+204932411660697600\,x^{10}+363646430503501824\,x^9+439064558846345216\,x^8+530502956133122048\,x^7+485091164642279424\,x^6+405468382284161024\,x^5+257786732552566784\,x^4+142490931553577856\,x^3+50064174038215008\,x^2+12071614275862524\,x+10820567498568669\right)\sqrt{2\,x^2-x+3}+\frac{14641852251147}{274877906944}\,\sqrt{2}\,\log\left(-4\,\sqrt{2}\,\sqrt{2\,x^2-x+3}\,(4\,x-1)-32\,x^2+16\,x-25\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2)^4,x, algorithm="fricas")
```

```
[Out] 1/140703128616960*(25125558681600000*x^13 + 37398427729920000*x^12 + 137233
466130432000*x^11 + 204932411660697600*x^10 + 363646430503501824*x^9 + 4390
64558846345216*x^8 + 530502956133122048*x^7 + 485091164642279424*x^6 + 4054
68382284161024*x^5 + 257786732552566784*x^4 + 142490931553577856*x^3 + 5006
4174038215008*x^2 + 12071614275862524*x + 10820567498568669)*sqrt(2*x^2 - x
 + 3) + 14641852251147/274877906944*sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2 - x +
 3)*(4*x - 1) - 32*x^2 + 16*x - 25)
```

giac [A] time = 0.54, size = 113, normalized size = 0.44

$$\frac{1}{140703128616960}\,\left(4\,\left(8\,\left(4\,\left(16\,\left(4\,\left(8\,\left(4\,\left(32\,\left(12\,\left(200\,\left(20\,\left(240\,(260\,x+387)x+340823\right)x+10179103\right)x+3612502719\right)x+52340574127\right)x+2023708176167\right)x+7401903757359\right)x+49495652134297\right)x+125872428004183\right)x+1113210402762327\right)x+1564505438694219\right)x+3017903568965631\right)x+10820567498568669\right)\sqrt{2\,x^2-x+3}-\frac{14641852251147}{137438953472}\,\sqrt{2}\,\log\left(-2\,\sqrt{2}\left(\sqrt{2}\,x-\sqrt{2\,x^2-x+3}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2)^4,x, algorithm="giac")
```

```
[Out] 1/140703128616960*(4*(8*(4*(16*(4*(8*(4*(32*(12*(200*(20*(240*(260*x + 387)
*x + 340823)*x + 10179103)*x + 3612502719)*x + 52340574127)*x + 20237081761
67)*x + 7401903757359)*x + 49495652134297)*x + 125872428004183)*x + 1113210
402762327)*x + 1564505438694219)*x + 3017903568965631)*x + 1082056749856866
9)*sqrt(2*x^2 - x + 3) - 14641852251147/137438953472*sqrt(2)*log(-2*sqrt(2)
*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1)
```

maple [A] time = 0.04, size = 204, normalized size = 0.80

$$\frac{625\left(2x^2-x+3\right)^{\frac{7}{2}}x^7}{28}+\frac{13875\left(2x^2-x+3\right)^{\frac{7}{2}}x^6}{208}+\frac{1046225\left(2x^2-x+3\right)^{\frac{7}{2}}x^5}{9984}+\frac{3684995\left(2x^2-x+3\right)^{\frac{7}{2}}x^4}{39936}+\frac{23460839\left(2x^2-x+3\right)^{\frac{7}{2}}x^3}{532480}+\frac{122595067\left(2x^2-x+3\right)^{\frac{7}{2}}x^2}{19169280}+\frac{112244125\left(2x^2-x+3\right)^{\frac{7}{2}}x}{122683392}+\frac{14641852251147\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{137438953472}+\frac{25250178739\left(2x^2-x+3\right)^{\frac{7}{2}}}{5725224960}+\frac{636602271789\left(4x-1\right)\sqrt{2x^2-x+3}}{34359738368}+\frac{401135647\left(4x-1\right)\left(2x^2-x+3\right)^{\frac{5}{2}}}{335544320}+\frac{9226119881\left(4x-1\right)\left(2x^2-x+3\right)^{\frac{3}{2}}}{2147483648}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2)^4,x)
```

```
[Out] 625/28*x^7*(2*x^2-x+3)^(7/2)+13875/208*x^6*(2*x^2-x+3)^(7/2)+25250178739/57
25224960*(2*x^2-x+3)^(7/2)+1046225/9984*x^5*(2*x^2-x+3)^(7/2)+3684995/39936
*x^4*(2*x^2-x+3)^(7/2)+23460839/532480*x^3*(2*x^2-x+3)^(7/2)+122595067/1916
9280*x^2*(2*x^2-x+3)^(7/2)+112244125/122683392*x*(2*x^2-x+3)^(7/2)+14641852
251147/137438953472*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))+636602271789/343
59738368*(4*x-1)*(2*x^2-x+3)^(1/2)+401135647/335544320*(4*x-1)*(2*x^2-x+3)^
(5/2)+9226119881/2147483648*(4*x-1)*(2*x^2-x+3)^(3/2)
```

maxima [A] time = 1.02, size = 235, normalized size = 0.93

$$\frac{625}{28}\left(2x^2-x+3\right)^{\frac{7}{2}}x^7+\frac{13875}{208}\left(2x^2-x+3\right)^{\frac{7}{2}}x^6+\frac{1046225}{9984}\left(2x^2-x+3\right)^{\frac{7}{2}}x^5+\frac{3684995}{39936}\left(2x^2-x+3\right)^{\frac{7}{2}}x^4+\frac{23460839}{532480}\left(2x^2-x+3\right)^{\frac{7}{2}}x^3+\frac{122595067}{19169280}\left(2x^2-x+3\right)^{\frac{7}{2}}x^2+\frac{112244125}{122683392}\left(2x^2-x+3\right)^{\frac{7}{2}}x+\frac{25250178739}{5725224960}\left(2x^2-x+3\right)^{\frac{7}{2}}+\frac{401135647}{83886080}\left(2x^2-x+3\right)^{\frac{5}{2}}x-\frac{401135647}{335544320}\left(2x^2-x+3\right)^{\frac{5}{2}}+\frac{9226119881}{536870912}\left(2x^2-x+3\right)^{\frac{3}{2}}x-\frac{9226119881}{2147483648}\left(2x^2-x+3\right)^{\frac{3}{2}}+\frac{636602271789}{8589934592}\sqrt{2x^2-x+3}\,x+\frac{14641852251147}{137438953472}\sqrt{2}\,\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}\left(4x-1\right)\right)-\frac{636602271789}{34359738368}\sqrt{2x^2-x+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2)^4,x, algorithm="maxima")
```

```
[Out] 625/28*(2*x^2 - x + 3)^(7/2)*x^7 + 13875/208*(2*x^2 - x + 3)^(7/2)*x^6 + 10
46225/9984*(2*x^2 - x + 3)^(7/2)*x^5 + 3684995/39936*(2*x^2 - x + 3)^(7/2)*
x^4 + 23460839/532480*(2*x^2 - x + 3)^(7/2)*x^3 + 122595067/19169280*(2*x^2
 - x + 3)^(7/2)*x^2 + 112244125/122683392*(2*x^2 - x + 3)^(7/2)*x + 2525017
8739/5725224960*(2*x^2 - x + 3)^(7/2) + 401135647/83886080*(2*x^2 - x + 3)^
(5/2)*x - 401135647/335544320*(2*x^2 - x + 3)^(5/2) + 9226119881/536870912*
(2*x^2 - x + 3)^(3/2)*x - 9226119881/2147483648*(2*x^2 - x + 3)^(3/2) + 636
602271789/8589934592*sqrt(2*x^2 - x + 3)*x + 14641852251147/137438953472*sq
rt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 636602271789/34359738368*sqrt(2*x^
2 - x + 3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(2\,x^2-x+3\right)^{5/2}\,\left(5\,x^2+3\,x+2\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^(5/2)*(3*x + 5*x^2 + 2)^4,x)
[Out] int((2*x^2 - x + 3)^(5/2)*(3*x + 5*x^2 + 2)^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(2x^2 - x + 3\right)^{\frac{5}{2}} \left(5x^2 + 3x + 2\right)^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(5/2)*(5*x**2+3*x+2)**4,x)
[Out] Integral((2*x**2 - x + 3)**(5/2)*(5*x**2 + 3*x + 2)**4, x)
```

3.70 $\int \left(3-x+2x^2\right)^{5/2} \left(2+3x+5x^2\right)^3 \, dx$

Optimal. Leaf size=212

$$\frac{80483\left(2x^2-x+3\right)^{7/2}x^2}{9216}+\frac{509257\left(2x^2-x+3\right)^{7/2}x}{294912}-\frac{1696165\left(2x^2-x+3\right)^{7/2}}{2752512}-\frac{57915(1-4x)\left(2x^2-x+3\right)^{5/2}}{2097152}-$$

Rubi [A] time = 0.22, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1661, 640, 612, 619, 215}

$$\frac{125}{24}\left(2x^2-x+3\right)^{7/2}x^5+\frac{1175}{96}\left(2x^2-x+3\right)^{7/2}x^4+\frac{3823}{256}\left(2x^2-x+3\right)^{7/2}x^3+\frac{80483\left(2x^2-x+3\right)^{7/2}x^2}{9216}+\frac{509257\left(2x^2-x+3\right)^{7/2}x}{294912}-\frac{1696165\left(2x^2-x+3\right)^{7/2}}{2752512}-\frac{57915(1-4x)\left(2x^2-x+3\right)^{5/2}}{2097152}-\frac{6660225(1-4x)\left(2x^2-x+3\right)^{3/2}}{67108864}-\frac{459555525(1-4x)\sqrt{2x^2-x+3}}{1073741824}-\frac{10569777075\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{2147483648\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^(5/2)*(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] (-459555525*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/1073741824 - (6660225*(1 - 4*x)*
(3 - x + 2*x^2)^(3/2))/67108864 - (57915*(1 - 4*x)*(3 - x + 2*x^2)^(5/2))/2
097152 - (1696165*(3 - x + 2*x^2)^(7/2))/2752512 + (509257*x*(3 - x + 2*x^2
)^(7/2))/294912 + (80483*x^2*(3 - x + 2*x^2)^(7/2))/9216 + (3823*x^3*(3 - x
 + 2*x^2)^(7/2))/256 + (1175*x^4*(3 - x + 2*x^2)^(7/2))/96 + (125*x^5*(3 -
x + 2*x^2)^(7/2))/24 - (10569777075*ArcSinh[(1 - 4*x)/Sqrt[23]])/(214748364
8*Sqrt[2])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \left(3-x+2x^2\right)^{5/2} \left(2+3x+5x^2\right)^3 \, dx &= \frac{125}{24} x^5 \left(3-x+2x^2\right)^{7/2} + \frac{1}{24} \int \left(3-x+2x^2\right)^{5/2} \Big(192 + 864x + 2736x^2 \\
&= \frac{1175}{96} x^4 \left(3-x+2x^2\right)^{7/2} + \frac{125}{24} x^5 \left(3-x+2x^2\right)^{7/2} + \frac{1}{528} \int \Big(3-x+2x^2 \\
&= \frac{3823}{256} x^3 \left(3-x+2x^2\right)^{7/2} + \frac{1175}{96} x^4 \left(3-x+2x^2\right)^{7/2} + \frac{125}{24} x^5 \Big(3-x+2x \\
&= \frac{80483x^2 \left(3-x+2x^2\right)^{7/2}}{9216} + \frac{3823}{256} x^3 \left(3-x+2x^2\right)^{7/2} + \frac{1175}{96} x^4 \Big(3-x+ \\
&= \frac{509257x \left(3-x+2x^2\right)^{7/2}}{294912} + \frac{80483x^2 \left(3-x+2x^2\right)^{7/2}}{9216} + \frac{3823}{256} x^3 \Big(3-x \\
&= -\frac{1696165 \left(3-x+2x^2\right)^{7/2}}{2752512} + \frac{509257x \left(3-x+2x^2\right)^{7/2}}{294912} + \frac{80483x^2 \Big(3-}{921} \\
&= -\frac{57915(1-4x) \left(3-x+2x^2\right)^{5/2}}{2097152} - \frac{1696165 \left(3-x+2x^2\right)^{7/2}}{2752512} + \frac{509257x}{} \\
&= -\frac{6660225(1-4x) \left(3-x+2x^2\right)^{3/2}}{67108864} - \frac{57915(1-4x) \left(3-x+2x^2\right)^{5/2}}{2097152} - \\
&= -\frac{459555525(1-4x)\sqrt{3-x+2x^2}}{1073741824} - \frac{6660225(1-4x) \left(3-x+2x^2\right)^{3/2}}{67108864} \\
&= -\frac{459555525(1-4x)\sqrt{3-x+2x^2}}{1073741824} - \frac{6660225(1-4x) \left(3-x+2x^2\right)^{3/2}}{67108864} \\
&= -\frac{459555525(1-4x)\sqrt{3-x+2x^2}}{1073741824} - \frac{6660225(1-4x) \left(3-x+2x^2\right)^{3/2}}{67108864}
\end{aligned}
$$

Mathematica [A] time = 0.30, size = 95, normalized size = 0.45

$$
\frac{4\sqrt{2x^2-x+3}\left(2818572288000x^{11}+2395786444800x^{10}+12943588589568x^9+14341894045696x^8+27835561148416x^7+28347538538496x^6+34378613923840x^5+26186527209472x^4+20384824684416x^3+10060731582048x^2+4560943728924x-1191399152715\right)-665895955725\sqrt{2}\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{270582939648}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^(5/2)*(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] (4*Sqrt[3 - x + 2*x^2]*(-1191399152715 + 4560943728924*x + 10060731582048*x
^2 + 20384824684416*x^3 + 26186527209472*x^4 + 34378613923840*x^5 + 2834753
```

```
8538496*x^6 + 27835561148416*x^7 + 14341894045696*x^8 + 12943588589568*x^9
+ 2395786444800*x^10 + 2818572288000*x^11) - 665895955725*Sqrt[2]*ArcSinh[(
1 - 4*x)/Sqrt[23]])/270582939648
```

IntegrateAlgebraic [A] time = 1.34, size = 110, normalized size = 0.52

$$\frac{\sqrt{2x^2-x+3}\left(2818572288000x^{11}+2395786444800x^{10}+12943588589568x^9+14341894045696x^8+27835561148416x^7+28347538538496x^6+34378613923840x^5+26186527209472x^4+20384824684416x^3+10060731582048x^2+4560943728924x-1191399152715\right)}{67645734912}-\frac{10569777075\log\left(2\sqrt{2}\sqrt{2x^2-x+3}-4x+1\right)}{2147483648\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(3 - x + 2*x^2)^(5/2)*(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] (Sqrt[3 - x + 2*x^2]*(-1191399152715 + 4560943728924*x + 10060731582048*x^2
 + 20384824684416*x^3 + 26186527209472*x^4 + 34378613923840*x^5 + 283475385
38496*x^6 + 27835561148416*x^7 + 14341894045696*x^8 + 12943588589568*x^9 +
2395786444800*x^10 + 2818572288000*x^11))/67645734912 - (10569777075*Log[1
- 4*x + 2*Sqrt[2]*Sqrt[3 - x + 2*x^2]])/(2147483648*Sqrt[2])
```

fricas [A] time = 0.42, size = 108, normalized size = 0.51

$$\frac{1}{67645734912}\left(2818572288000\,x^{11}+2395786444800\,x^{10}+12943588589568\,x^9+14341894045696\,x^8+27835561148416\,x^7+28347538538496\,x^6+34378613923840\,x^5+26186527209472\,x^4+20384824684416\,x^3+10060731582048\,x^2+4560943728924\,x-1191399152715\right)\sqrt{2\,x^2-x+3}+\frac{10569777075}{8589934592}\sqrt{2}\log\left(-4\sqrt{2}\sqrt{2\,x^2-x+3}(4\,x-1)-32\,x^2+16\,x-25\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

```
[Out] 1/67645734912*(2818572288000*x^11 + 2395786444800*x^10 + 12943588589568*x^9
 + 14341894045696*x^8 + 27835561148416*x^7 + 28347538538496*x^6 + 343786139
23840*x^5 + 26186527209472*x^4 + 20384824684416*x^3 + 10060731582048*x^2 +
4560943728924*x - 1191399152715)*sqrt(2*x^2 - x + 3) + 10569777075/85899345
92*sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 2
5)
```

giac [A] time = 0.55, size = 103, normalized size = 0.49

$$\frac{1}{67645734912}\left(4\left(8\left(4\left(16\left(4\left(8\left(28\left(32\left(12\left(200\left(20\,x+17\right)x+18369\right)x+244241\right)x+15169177\right)x+432549111\right)x+4196608145\right)x+12786390239\right)x+159256442847\right)x+314397861939\right)x+1140235932231\right)x-1191399152715\right)\sqrt{2\,x^2-x+3}-\frac{10569777075}{4294967296}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}\,x-\sqrt{2\,x^2-x+3}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2)^3,x, algorithm="giac")
```

```
[Out] 1/67645734912*(4*(8*(4*(16*(4*(8*(28*(32*(12*(200*(20*x + 17)*x + 18369)*x
+ 244241)*x + 15169177)*x + 432549111)*x + 4196608145)*x + 12786390239)*x +
 159256442847)*x + 314397861939)*x + 1140235932231)*x - 1191399152715)*sqrt
(2*x^2 - x + 3) - 10569777075/4294967296*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x
- sqrt(2*x^2 - x + 3)) + 1)
```

maple [A] time = 0.01, size = 170, normalized size = 0.80

$$\frac{125\left(2x^2-x+3\right)^{\frac{7}{2}}x^5}{24}+\frac{1175\left(2x^2-x+3\right)^{\frac{7}{2}}x^4}{96}+\frac{3823\left(2x^2-x+3\right)^{\frac{7}{2}}x^3}{256}+\frac{80483\left(2x^2-x+3\right)^{\frac{7}{2}}x^2}{9216}+\frac{509257\left(2x^2-x+3\right)^{\frac{7}{2}}x}{294912}+\frac{10569777075\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{4294967296}-\frac{1696165\left(2x^2-x+3\right)^{\frac{7}{2}}}{2752512}+\frac{459555525\left(4x-1\right)\sqrt{2x^2-x+3}}{1073741824}+\frac{57915\left(4x-1\right)\left(2x^2-x+3\right)^{\frac{5}{2}}}{2097152}+\frac{6660225\left(4x-1\right)\left(2x^2-x+3\right)^{\frac{3}{2}}}{67108864}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2)^3,x)
```

```
[Out] -1696165/2752512*(2*x^2-x+3)^(7/2)+125/24*(2*x^2-x+3)^(7/2)*x^5+1175/96*(2*
x^2-x+3)^(7/2)*x^4+3823/256*(2*x^2-x+3)^(7/2)*x^3+80483/9216*(2*x^2-x+3)^(7
/2)*x^2+509257/294912*(2*x^2-x+3)^(7/2)*x+10569777075/4294967296*2^(1/2)*ar
csinh(4/23*23^(1/2)*(x-1/4))+459555525/1073741824*(4*x-1)*(2*x^2-x+3)^(1/2)
+57915/2097152*(4*x-1)*(2*x^2-x+3)^(5/2)+6660225/67108864*(4*x-1)*(2*x^2-x+
3)^(3/2)
```

maxima [A] time = 1.00, size = 201, normalized size = 0.95

$$\frac{125}{24}\left(2x^2-x+3\right)^{\frac{7}{2}}x^5+\frac{1175}{96}\left(2x^2-x+3\right)^{\frac{7}{2}}x^4+\frac{3823}{256}\left(2x^2-x+3\right)^{\frac{7}{2}}x^3+\frac{80483}{9216}\left(2x^2-x+3\right)^{\frac{7}{2}}x^2+\frac{509257}{294912}\left(2x^2-x+3\right)^{\frac{7}{2}}x-\frac{1696165}{2752512}\left(2x^2-x+3\right)^{\frac{7}{2}}+\frac{57915}{524288}\left(2x^2-x+3\right)^{\frac{5}{2}}x-\frac{57915}{2097152}\left(2x^2-x+3\right)^{\frac{5}{2}}+\frac{6660225}{16777216}\left(2x^2-x+3\right)^{\frac{3}{2}}x-\frac{6660225}{67108864}\left(2x^2-x+3\right)^{\frac{3}{2}}+\frac{459555525}{268435456}\sqrt{2x^2-x+3}\,x+\frac{10569777075}{4294967296}\sqrt{2}\,\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}\left(4x-1\right)\right)-\frac{459555525}{1073741824}\sqrt{2x^2-x+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2)^3,x, algorithm="maxima")
```

```
[Out] 125/24*(2*x^2 - x + 3)^(7/2)*x^5 + 1175/96*(2*x^2 - x + 3)^(7/2)*x^4 + 3823
/256*(2*x^2 - x + 3)^(7/2)*x^3 + 80483/9216*(2*x^2 - x + 3)^(7/2)*x^2 + 509
257/294912*(2*x^2 - x + 3)^(7/2)*x - 1696165/2752512*(2*x^2 - x + 3)^(7/2)
+ 57915/524288*(2*x^2 - x + 3)^(5/2)*x - 57915/2097152*(2*x^2 - x + 3)^(5/2
) + 6660225/16777216*(2*x^2 - x + 3)^(3/2)*x - 6660225/67108864*(2*x^2 - x
+ 3)^(3/2) + 459555525/268435456*sqrt(2*x^2 - x + 3)*x + 10569777075/429496
7296*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 459555525/1073741824*sqrt(2
*x^2 - x + 3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(2\,x^2-x+3\right)^{5/2}\,\left(5\,x^2+3\,x+2\right)^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^(5/2)*(3*x + 5*x^2 + 2)^3,x)
```

```
[Out] int((2*x^2 - x + 3)^(5/2)*(3*x + 5*x^2 + 2)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(2x^{2} - x + 3\right)^{\frac{5}{2}} \left(5x^{2} + 3x + 2\right)^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(5/2)*(5*x**2+3*x+2)**3,x)
```

```
[Out] Integral((2*x**2 - x + 3)**(5/2)*(5*x**2 + 3*x + 2)**3, x)
```

3.71 $\int \left(3-x+2x^2\right)^{5/2} \left(2+3x+5x^2\right)^2 \, dx$

Optimal. Leaf size=170

$$\frac{305}{144}x^2\left(2x^2-x+3\right)^{7/2}+\frac{8467x\left(2x^2-x+3\right)^{7/2}}{4608}+\frac{23225\left(2x^2-x+3\right)^{7/2}}{43008}-\frac{1547(1-4x)\left(2x^2-x+3\right)^{5/2}}{98304}-\frac{177905(}{}$$

Rubi [A] time = 0.13, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1661, 640, 612, 619, 215}

$$\frac{5}{4}x^3\left(2x^2-x+3\right)^{7/2}+\frac{305}{144}x^2\left(2x^2-x+3\right)^{7/2}+\frac{8467x\left(2x^2-x+3\right)^{7/2}}{4608}+\frac{23225\left(2x^2-x+3\right)^{7/2}}{43008}-\frac{1547(1-4x)\left(2x^2-x+3\right)^{5/2}}{98304}-\frac{177905(1-4x)\left(2x^2-x+3\right)^{3/2}}{3145728}-\frac{4091815(1-4x)\sqrt{2x^2-x+3}}{16777216}-\frac{94111745\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{33554432\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^(5/2)*(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] (-4091815*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/16777216 - (177905*(1 - 4*x)*(3 -
x + 2*x^2)^(3/2))/3145728 - (1547*(1 - 4*x)*(3 - x + 2*x^2)^(5/2))/98304 +
(23225*(3 - x + 2*x^2)^(7/2))/43008 + (8467*x*(3 - x + 2*x^2)^(7/2))/4608 +
 (305*x^2*(3 - x + 2*x^2)^(7/2))/144 + (5*x^3*(3 - x + 2*x^2)^(7/2))/4 - (9
4111745*ArcSinh[(1 - 4*x)/Sqrt[23]])/(33554432*Sqrt[2])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
```

```
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \left(3-x+2x^2\right)^{5/2}\left(2+3x+5x^2\right)^2\,dx = \frac{5}{4}x^3\left(3-x+2x^2\right)^{7/2}+\frac{1}{20}\int\left(3-x+2x^2\right)^{5/2}\left(80+240x+355x^2+\right.$$

$$= \frac{305}{144}x^2\left(3-x+2x^2\right)^{7/2}+\frac{5}{4}x^3\left(3-x+2x^2\right)^{7/2}+\frac{1}{360}\int\left(3-x+2x^2\right)$$

$$= \frac{8467x\left(3-x+2x^2\right)^{7/2}}{4608}+\frac{305}{144}x^2\left(3-x+2x^2\right)^{7/2}+\frac{5}{4}x^3\left(3-x+2x^2\right)$$

$$= \frac{23225\left(3-x+2x^2\right)^{7/2}}{43008}+\frac{8467x\left(3-x+2x^2\right)^{7/2}}{4608}+\frac{305}{144}x^2\left(3-x+2x\right.$$

$$= -\frac{1547(1-4x)\left(3-x+2x^2\right)^{5/2}}{98304}+\frac{23225\left(3-x+2x^2\right)^{7/2}}{43008}+\frac{8467x\left(3-\right.}{}$$

$$= -\frac{177905(1-4x)\left(3-x+2x^2\right)^{3/2}}{3145728}-\frac{1547(1-4x)\left(3-x+2x^2\right)^{5/2}}{98304}+$$

$$= -\frac{4091815(1-4x)\sqrt{3-x+2x^2}}{16777216}-\frac{177905(1-4x)\left(3-x+2x^2\right)^{3/2}}{3145728}-$$

$$= -\frac{4091815(1-4x)\sqrt{3-x+2x^2}}{16777216}-\frac{177905(1-4x)\left(3-x+2x^2\right)^{3/2}}{3145728}-$$

$$= -\frac{4091815(1-4x)\sqrt{3-x+2x^2}}{16777216}-\frac{177905(1-4x)\left(3-x+2x^2\right)^{3/2}}{3145728}-$$

Mathematica [A] time = 0.18, size = 85, normalized size = 0.50

$$\frac{4\sqrt{2x^2-x+3}\left(10569646080x^9+2055208960x^8+44163137536x^7+26401898496x^6+75389820928x^5+57147467776x^4+77872272000x^3+42992644128x^2+39533249652x+14824182519\right)-5929039935\sqrt{2}\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{4227858432}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^(5/2)*(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] (4*Sqrt[3 - x + 2*x^2]*(14824182519 + 39533249652*x + 42992644128*x^2 + 778
72272000*x^3 + 57147467776*x^4 + 75389820928*x^5 + 26401898496*x^6 + 441631
37536*x^7 + 2055208960*x^8 + 10569646080*x^9) - 5929039935*Sqrt[2]*ArcSinh[
(1 - 4*x)/Sqrt[23]])/4227858432
```

IntegrateAlgebraic [A] time = 1.05, size = 100, normalized size = 0.59

$$\frac{\sqrt{2x^2-x+3}\left(10569646080x^9+2055208960x^8+44163137536x^7+26401898496x^6+75389820928x^5+57147467776x^4+77872272000x^3+42992644128x^2+39533249652x+14824182519\right)}{1056964608}-\frac{94111745\log\left(2\sqrt{2}\sqrt{2x^2-x+3}-4x+1\right)}{33554432\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(3 - x + 2*x^2)^(5/2)*(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] (Sqrt[3 - x + 2*x^2]*(14824182519 + 39533249652*x + 42992644128*x^2 + 77872
272000*x^3 + 57147467776*x^4 + 75389820928*x^5 + 26401898496*x^6 + 44163137
536*x^7 + 2055208960*x^8 + 10569646080*x^9))/1056964608 - (94111745*Log[1 -
 4*x + 2*Sqrt[2]*Sqrt[3 - x + 2*x^2]])/(33554432*Sqrt[2])
```

fricas [A] time = 0.41, size = 98, normalized size = 0.58

$$\frac{1}{1056964608}\left(10569646080\,x^9+2055208960\,x^8+44163137536\,x^7+26401898496\,x^6+75389820928\,x^5+57147467776\,x^4+77872272000\,x^3+42992644128\,x^2+39533249652\,x+14824182519\right)\sqrt{2\,x^2-x+3}+\frac{94111745}{134217728}\sqrt{2}\log\left(-4\sqrt{2}\sqrt{2\,x^2-x+3}(4\,x-1)-32\,x^2+16\,x-25\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

```
[Out] 1/1056964608*(10569646080*x^9 + 2055208960*x^8 + 44163137536*x^7 + 26401898
496*x^6 + 75389820928*x^5 + 57147467776*x^4 + 77872272000*x^3 + 42992644128
*x^2 + 39533249652*x + 14824182519)*sqrt(2*x^2 - x + 3) + 94111745/13421772
8*sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25
)
```

giac [A] time = 0.52, size = 93, normalized size = 0.55

$$\frac{1}{1056964608}\,(4\,(8\,(4\,(16\,(4\,(8\,(28\,(160\,(36\,x+7)x+24067)x+402861)x+9202859)x+27904037)x+608377125)x+1343520129)x+9883312413)x+14824182519)\sqrt{2\,x^2-x+3}-\frac{94111745}{67108864}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}\,x-\sqrt{2\,x^2-x+3}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2)^2,x, algorithm="giac")
```

```
[Out] 1/1056964608*(4*(8*(4*(16*(4*(8*(28*(160*(36*x + 7)*x + 24067)*x + 402861)*
x + 9202859)*x + 27904037)*x + 608377125)*x + 1343520129)*x + 9883312413)*x
 + 14824182519)*sqrt(2*x^2 - x + 3) - 94111745/67108864*sqrt(2)*log(-2*sqrt
(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1)
```

maple [A] time = 0.01, size = 136, normalized size = 0.80

$$\frac{5\left(2x^{2}-x+3\right)^{\frac{7}{2}}x^{3}}{4}+\frac{305\left(2x^{2}-x+3\right)^{\frac{7}{2}}x^{2}}{144}+\frac{8467\left(2x^{2}-x+3\right)^{\frac{7}{2}}x}{4608}+\frac{94111745\sqrt{2}\ \mathrm{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{67108864}+\frac{23225\left(2x^{2}-x+3\right)^{\frac{7}{2}}}{43008}+\frac{4091815\left(4x-1\right)\sqrt{2x^{2}-x+3}}{16777216}+\frac{1547\left(4x-1\right)\left(2x^{2}-x+3\right)^{\frac{5}{2}}}{98304}+\frac{177905\left(4x-1\right)\left(2x^{2}-x+3\right)^{\frac{3}{2}}}{3145728}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2)^2,x)
```

```
[Out] 23225/43008*(2*x^2-x+3)^(7/2)+5/4*(2*x^2-x+3)^(7/2)*x^3+305/144*(2*x^2-x+3)
^(7/2)*x^2+8467/4608*(2*x^2-x+3)^(7/2)*x+94111745/67108864*2^(1/2)*arcsinh(
4/23*23^(1/2)*(x-1/4))+4091815/16777216*(4*x-1)*(2*x^2-x+3)^(1/2)+1547/9830
4*(4*x-1)*(2*x^2-x+3)^(5/2)+177905/3145728*(4*x-1)*(2*x^2-x+3)^(3/2)
```

maxima [A] time = 1.03, size = 167, normalized size = 0.98

$$\frac{5}{4}\left(2\,x^{2}-x+3\right)^{\frac{7}{2}}x^{3}+\frac{305}{144}\left(2\,x^{2}-x+3\right)^{\frac{7}{2}}x^{2}+\frac{8467}{4608}\left(2\,x^{2}-x+3\right)^{\frac{7}{2}}x+\frac{23225}{43008}\left(2\,x^{2}-x+3\right)^{\frac{7}{2}}+\frac{1547}{24576}\left(2\,x^{2}-x+3\right)^{\frac{5}{2}}x-\frac{1547}{98304}\left(2\,x^{2}-x+3\right)^{\frac{5}{2}}+\frac{177905}{786432}\left(2\,x^{2}-x+3\right)^{\frac{3}{2}}x-\frac{177905}{3145728}\left(2\,x^{2}-x+3\right)^{\frac{3}{2}}+\frac{4091815}{4194304}\sqrt{2\,x^{2}-x+3}\,x+\frac{94111745}{67108864}\sqrt{2}\ \mathrm{arsinh}\left(\frac{1}{23}\sqrt{23}\left(4\,x-1\right)\right)-\frac{4091815}{16777216}\sqrt{2\,x^{2}-x+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2)^2,x, algorithm="maxima")
```

```
[Out] 5/4*(2*x^2 - x + 3)^(7/2)*x^3 + 305/144*(2*x^2 - x + 3)^(7/2)*x^2 + 8467/46
08*(2*x^2 - x + 3)^(7/2)*x + 23225/43008*(2*x^2 - x + 3)^(7/2) + 1547/24576
*(2*x^2 - x + 3)^(5/2)*x - 1547/98304*(2*x^2 - x + 3)^(5/2) + 177905/786432
*(2*x^2 - x + 3)^(3/2)*x - 177905/3145728*(2*x^2 - x + 3)^(3/2) + 4091815/4
194304*sqrt(2*x^2 - x + 3)*x + 94111745/67108864*sqrt(2)*arcsinh(1/23*sqrt(
23)*(4*x - 1)) - 4091815/16777216*sqrt(2*x^2 - x + 3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(2\,x^2-x+3\right)}^{5/2}\,{\left(5\,x^2+3\,x+2\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^(5/2)*(3*x + 5*x^2 + 2)^2,x)
```

```
[Out] int((2*x^2 - x + 3)^(5/2)*(3*x + 5*x^2 + 2)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(2x^{2}-x+3\right)^{\frac{5}{2}}\left(5x^{2}+3x+2\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(5/2)*(5*x**2+3*x+2)**2,x)
```

```
[Out] Integral((2*x**2 - x + 3)**(5/2)*(5*x**2 + 3*x + 2)**2, x)
```

3.72 $\int \left(3-x+2x^2\right)^{5/2} \left(2+3x+5x^2\right) dx$

Optimal. Leaf size=128

$$\frac{5}{16}x\left(2x^2-x+3\right)^{7/2}+\frac{141}{448}\left(2x^2-x+3\right)^{7/2}-\frac{277(1-4x)\left(2x^2-x+3\right)^{5/2}}{3072}-\frac{31855(1-4x)\left(2x^2-x+3\right)^{3/2}}{98304}-\frac{7326}{}$$

Rubi [A] time = 0.06, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1661, 640, 612, 619, 215}

$$\frac{5}{16}x\left(2x^2-x+3\right)^{7/2}+\frac{141}{448}\left(2x^2-x+3\right)^{7/2}-\frac{277(1-4x)\left(2x^2-x+3\right)^{5/2}}{3072}-\frac{31855(1-4x)\left(2x^2-x+3\right)^{3/2}}{98304}-\frac{732665(1-4x)\sqrt{2x^2-x+3}}{524288}-\frac{16851295\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{1048576\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^(5/2)*(2 + 3*x + 5*x^2),x]
```

```
[Out] (-732665*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/524288 - (31855*(1 - 4*x)*(3 - x +
2*x^2)^(3/2))/98304 - (277*(1 - 4*x)*(3 - x + 2*x^2)^(5/2))/3072 + (141*(3
- x + 2*x^2)^(7/2))/448 + (5*x*(3 - x + 2*x^2)^(7/2))/16 - (16851295*ArcSin
h[(1 - 4*x)/Sqrt[23]])/(1048576*Sqrt[2])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
```

```
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \left(3-x+2x^2\right)^{5/2}\left(2+3x+5x^2\right)dx = \frac{5}{16}x\left(3-x+2x^2\right)^{7/2}+\frac{1}{16}\int\left(17+\frac{141x}{2}\right)\left(3-x+2x^2\right)^{5/2}dx$$

$$= \frac{141}{448}\left(3-x+2x^2\right)^{7/2}+\frac{5}{16}x\left(3-x+2x^2\right)^{7/2}+\frac{277}{128}\int\left(3-x+2x^2\right)^{5/2}dx$$

$$= -\frac{277(1-4x)\left(3-x+2x^2\right)^{5/2}}{3072}+\frac{141}{448}\left(3-x+2x^2\right)^{7/2}+\frac{5}{16}x\left(3-x+2x^2\right.$$

$$= -\frac{31855(1-4x)\left(3-x+2x^2\right)^{3/2}}{98304}-\frac{277(1-4x)\left(3-x+2x^2\right)^{5/2}}{3072}+\frac{141}{448}\left(3\right.$$

$$= -\frac{732665(1-4x)\sqrt{3-x+2x^2}}{524288}-\frac{31855(1-4x)\left(3-x+2x^2\right)^{3/2}}{98304}-\frac{277(1}{}$$

$$= -\frac{732665(1-4x)\sqrt{3-x+2x^2}}{524288}-\frac{31855(1-4x)\left(3-x+2x^2\right)^{3/2}}{98304}-\frac{277(1}{}$$

$$= -\frac{732665(1-4x)\sqrt{3-x+2x^2}}{524288}-\frac{31855(1-4x)\left(3-x+2x^2\right)^{3/2}}{98304}-\frac{277(1}{}$$

Mathematica [A] time = 0.11, size = 75, normalized size = 0.59

$$\frac{4\sqrt{2x^2-x+3}\left(27525120x^7-13565952x^6+118808576x^5-1619968x^4+172684416x^3+67272352x^2+148957444x+58536675\right)-353877195\sqrt{2}\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{44040192}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^(5/2)*(2 + 3*x + 5*x^2),x]
```

```
[Out] (4*Sqrt[3 - x + 2*x^2]*(58536675 + 148957444*x + 67272352*x^2 + 172684416*x
^3 - 1619968*x^4 + 118808576*x^5 - 13565952*x^6 + 27525120*x^7) - 353877195
*Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]])/44040192
```

IntegrateAlgebraic [A] time = 0.84, size = 90, normalized size = 0.70

$$\frac{\sqrt{2x^2-x+3}\left(27525120x^7-13565952x^6+118808576x^5-1619968x^4+172684416x^3+67272352x^2+148957444x+58536675\right)}{11010048}-\frac{16851295\log\left(2\sqrt{2}\sqrt{2x^2-x+3}-4x+1\right)}{1048576\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(3 - x + 2*x^2)^(5/2)*(2 + 3*x + 5*x^2),x]
```

```
[Out] (Sqrt[3 - x + 2*x^2]*(58536675 + 148957444*x + 67272352*x^2 + 172684416*x^3
 - 1619968*x^4 + 118808576*x^5 - 13565952*x^6 + 27525120*x^7))/11010048 - (
16851295*Log[1 - 4*x + 2*Sqrt[2]*Sqrt[3 - x + 2*x^2]])/(1048576*Sqrt[2])
```

fricas [A] time = 0.41, size = 88, normalized size = 0.69

$$\frac{1}{11010048}\left(27525120\,x^7-13565952\,x^6+118808576\,x^5-1619968\,x^4+172684416\,x^3+67272352\,x^2+148957444\,x+58536675\right)\sqrt{2\,x^2-x+3}+\frac{16851295}{4194304}\sqrt{2}\,\log\left(-4\,\sqrt{2}\,\sqrt{2\,x^2-x+3}\,(4\,x-1)-32\,x^2+16\,x-25\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2),x, algorithm="fricas")
```

```
[Out] 1/11010048*(27525120*x^7 - 13565952*x^6 + 118808576*x^5 - 1619968*x^4 + 172
684416*x^3 + 67272352*x^2 + 148957444*x + 58536675)*sqrt(2*x^2 - x + 3) + 1
6851295/4194304*sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x
^2 + 16*x - 25)
```

giac [A] time = 0.39, size = 83, normalized size = 0.65

$$\frac{1}{11010048}\,(4\,(8\,(4\,(16\,(4\,(24\,(140\,x-69)x+14503)x-791)x+1349097)x+2102261)x+37239361)x+58536675)\sqrt{2\,x^2-x+3}-\frac{16851295}{2097152}\sqrt{2}\,\log\left(-2\,\sqrt{2}\left(\sqrt{2}\,x-\sqrt{2\,x^2-x+3}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2),x, algorithm="giac")
```

```
[Out] 1/11010048*(4*(8*(4*(16*(4*(24*(140*x - 69)*x + 14503)*x - 791)*x + 1349097
)*x + 2102261)*x + 37239361)*x + 58536675)*sqrt(2*x^2 - x + 3) - 16851295/2
097152*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1)
```

maple [A] time = 0.01, size = 102, normalized size = 0.80

$$\frac{5\left(2x^2-x+3\right)^{\frac{7}{2}}x}{16}+\frac{16851295\sqrt{2}\ \operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{2097152}+\frac{141\left(2x^2-x+3\right)^{\frac{7}{2}}}{448}+\frac{277\,(4x-1)\left(2x^2-x+3\right)^{\frac{5}{2}}}{3072}+\frac{31855\,(4x-1)\left(2x^2-x+3\right)^{\frac{3}{2}}}{98304}+\frac{732665\,(4x-1)\sqrt{2x^2-x+3}}{524288}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2),x)
```

```
[Out] 5/16*(2*x^2-x+3)^(7/2)*x+141/448*(2*x^2-x+3)^(7/2)+277/3072*(4*x-1)*(2*x^2-
x+3)^(5/2)+31855/98304*(4*x-1)*(2*x^2-x+3)^(3/2)+732665/524288*(4*x-1)*(2*x
^2-x+3)^(1/2)+16851295/2097152*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))
```

maxima [A] time = 0.97, size = 133, normalized size = 1.04

$$\frac{5}{16}\left(2x^2-x+3\right)^{\frac{7}{2}}x+\frac{141}{448}\left(2x^2-x+3\right)^{\frac{7}{2}}+\frac{277}{768}\left(2x^2-x+3\right)^{\frac{5}{2}}x-\frac{277}{3072}\left(2x^2-x+3\right)^{\frac{5}{2}}+\frac{31855}{24576}\left(2x^2-x+3\right)^{\frac{3}{2}}x-\frac{31855}{98304}\left(2x^2-x+3\right)^{\frac{3}{2}}+\frac{732665}{131072}\sqrt{2x^2-x+3}\,x+\frac{16851295}{2097152}\sqrt{2}\,\mathrm{arsinh}\left(\frac{1}{23}\sqrt{23}\,(4x-1)\right)-\frac{732665}{524288}\sqrt{2x^2-x+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2),x, algorithm="maxima")
```

```
[Out] 5/16*(2*x^2 - x + 3)^(7/2)*x + 141/448*(2*x^2 - x + 3)^(7/2) + 277/768*(2*x
^2 - x + 3)^(5/2)*x - 277/3072*(2*x^2 - x + 3)^(5/2) + 31855/24576*(2*x^2 -
 x + 3)^(3/2)*x - 31855/98304*(2*x^2 - x + 3)^(3/2) + 732665/131072*sqrt(2*
x^2 - x + 3)*x + 16851295/2097152*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1))
- 732665/524288*sqrt(2*x^2 - x + 3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(2\,x^2-x+3\right)^{5/2}\,\left(5\,x^2+3\,x+2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^(5/2)*(3*x + 5*x^2 + 2),x)
```

```
[Out] int((2*x^2 - x + 3)^(5/2)*(3*x + 5*x^2 + 2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(2x^2-x+3\right)^{\frac{5}{2}}\left(5x^2+3x+2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(5/2)*(5*x**2+3*x+2),x)
```

```
[Out] Integral((2*x**2 - x + 3)**(5/2)*(5*x**2 + 3*x + 2), x)
```

3.73 $\int \frac{\left(3-x+2x^2\right)^{5/2}}{2+3x+5x^2}\,dx$

Optimal. Leaf size=222

$$-\frac{1}{600}(103-60x)\left(2x^2-x+3\right)^{3/2}-\frac{(226249-99620x)\sqrt{2x^2-x+3}}{80000}-\frac{121\sqrt{\frac{11}{31}\left(25000\sqrt{2}-15457\right)}\tan^{-1}\left(\sqrt{\frac{}{62(2}}\right.}{3125}$$

Rubi [A] time = 0.54, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {977, 1066, 1076, 619, 215, 1035, 1029, 206, 204}

$$-\frac{1}{600}(103-60x)\left(2x^2-x+3\right)^{3/2}-\frac{(226249-99620x)\sqrt{2x^2-x+3}}{80000}-\frac{121\sqrt{\frac{11}{31}\left(25000\sqrt{2}-15457\right)}\tan^{-1}\left(\frac{\sqrt{\frac{11}{62\left(25000\sqrt{2}-15457\right)}}\left(-\left(690+247\sqrt{2}\right)x-443\sqrt{2}+196\right)}{\sqrt{2x^2-x+3}}\right)}{3125}+\frac{121\sqrt{\frac{11}{31}\left(15457+25000\sqrt{2}\right)}\tanh^{-1}\left(\frac{\sqrt{\frac{11}{62\left(15457+25000\sqrt{2}\right)}}\left(-\left(690-247\sqrt{2}\right)x+443\sqrt{2}+196\right)}{\sqrt{2x^2-x+3}}\right)}{3125}-\frac{7216203\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{800000\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^(5/2)/(2 + 3*x + 5*x^2),x]
```

```
[Out] -((226249 - 99620*x)*Sqrt[3 - x + 2*x^2])/80000 - ((103 - 60*x)*(3 - x + 2*
x^2)^(3/2))/600 - (7216203*ArcSinh[(1 - 4*x)/Sqrt[23]])/(800000*Sqrt[2]) -
(121*Sqrt[(11*(-15457 + 25000*Sqrt[2]))/31]*ArcTan[(Sqrt[11/(62*(-15457 + 2
5000*Sqrt[2]))]*(196 - 443*Sqrt[2] - (690 + 247*Sqrt[2])*x))/Sqrt[3 - x + 2
*x^2]])/3125 + (121*Sqrt[(11*(15457 + 25000*Sqrt[2]))/31]*ArcTanh[(Sqrt[11/
(62*(15457 + 25000*Sqrt[2]))]*(196 + 443*Sqrt[2] - (690 - 247*Sqrt[2])*x))/
Sqrt[3 - x + 2*x^2]])/3125
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 977

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((b*f*(3*p + 2*q) - c*e*(2*p + q) + 2*c*f*(p
+ q)*x)*(a + b*x + c*x^2)^(p - 1)*(d + e*x + f*x^2)^(q + 1))/(2*f^2*(p + q)
*(2*p + 2*q + 1)), x] - Dist[1/(2*f^2*(p + q)*(2*p + 2*q + 1)), Int[(a + b*
x + c*x^2)^(p - 2)*(d + e*x + f*x^2)^q*Simp[(b*d - a*e)*(c*e - b*f)*(1 - p)
*(2*p + q) - (p + q)*(b^2*d*f*(1 - p) - a*(f*(b*e - 2*a*f)*(2*p + 2*q + 1)
+ c*(2*d*f - e^2*(2*p + q)))) + (2*(c*d - a*f)*(c*e - b*f)*(1 - p)*(2*p + q
) - (p + q)*((b^2 - 4*a*c)*e*f*(1 - p) + b*(c*(e^2 - 4*d*f)*(2*p + q) + f*(
2*c*d - b*e + 2*a*f)*(2*p + 2*q + 1))))*x + ((c*e - b*f)^2*(1 - p)*p + c*(p
 + q)*(f*(b*e - 2*a*f)*(4*p + 2*q - 1) - c*(2*d*f*(1 - 2*p) + e^2*(3*p + q
- 1))))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a
*c, 0] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 1] && NeQ[p + q, 0] && NeQ[2*p + 2*
q + 1, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

Rule 1029

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
 c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 1035

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

Rule 1066

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((B*c*f*(2*
p + 2*q + 3) + C*(b*f*p - c*e*(2*p + q + 2)) + 2*c*C*f*(p + q + 1)*x)*(a +
b*x + c*x^2)^p*(d + e*x + f*x^2)^(q + 1))/(2*c*f^2*(p + q + 1)*(2*p + 2*q +
 3)), x] - Dist[1/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), Int[(a + b*x + c*x
^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[p*(b*d - a*e)*(C*(c*e - b*f)*(q + 1) -
 c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(b^2*C*d*f*p + a*c*(C*(2*d*f
- e^2*(2*p + q + 2)) + f*(B*e - 2*A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*f)
*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*e
*f*p*(b^2 - 4*a*c) - b*c*(C*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e +
2*A*f)*(2*p + 2*q + 3))))*x + (p*(c*e - b*f)*(C*(c*e - b*f)*(q + 1) - c*(C*
e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(b^2 - 4*a*c) - c^2*(C*(e^
2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3))))*x^2,
x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*
q + 3, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

Rule 1076

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqr
t[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$\int \frac{\left(3-x+2x^2\right)^{5/2}}{2+3x+5x^2}\,dx = -\frac{1}{600}(103-60x)\left(3-x+2x^2\right)^{3/2} - \frac{1}{300}\int \frac{\left(-\frac{4731}{2}+\frac{6135x}{4}-\frac{14943x^2}{4}\right)\sqrt{3-x+2x^2}}{2+3x+5x^2}\,dx$$

$$= -\frac{(226249-99620x)\sqrt{3-x+2x^2}}{80000} - \frac{1}{600}(103-60x)\left(3-x+2x^2\right)^{3/2} + \frac{\int \frac{\frac{3205293}{8}-\frac{1133938\ldots}{16}\ldots}{\sqrt{3-x+2x^2}\,(2\ldots}}{300\ldots}$$

$$= -\frac{(226249-99620x)\sqrt{3-x+2x^2}}{80000} - \frac{1}{600}(103-60x)\left(3-x+2x^2\right)^{3/2} + \frac{\int \frac{-702768-7602\ldots}{\sqrt{3-x+2x^2}\,(2+3\ldots}}{15000\ldots}$$

$$= -\frac{(226249-99620x)\sqrt{3-x+2x^2}}{80000} - \frac{1}{600}(103-60x)\left(3-x+2x^2\right)^{3/2} - \frac{\int \frac{-702768(108-11\ldots}{\sqrt{3-x\ldots}}}{3\ldots}$$

$$= -\frac{(226249-99620x)\sqrt{3-x+2x^2}}{80000} - \frac{1}{600}(103-60x)\left(3-x+2x^2\right)^{3/2} - \frac{7216203\sinh^{-1}\ldots}{800000\sqrt{\ldots}}$$

$$= -\frac{(226249-99620x)\sqrt{3-x+2x^2}}{80000} - \frac{1}{600}(103-60x)\left(3-x+2x^2\right)^{3/2} - \frac{7216203\sinh^{-1}\ldots}{800000\sqrt{\ldots}}$$

Mathematica [C]   time = 1.05, size = 229, normalized size = 1.03

$$\frac{46464\sqrt{286+22i\sqrt{31}}\left(403-69i\sqrt{31}\right)\tanh^{-1}\left(\frac{\left(-22-4i\sqrt{31}\right)x+i\sqrt{31}+63}{2\sqrt{286+22i\sqrt{31}}\sqrt{2x^2-x+3}}\right)-46464i\sqrt{286-22i\sqrt{31}}\left(69\sqrt{31}-403i\right)\tanh^{-1}\left(\frac{\left(22-4i\sqrt{31}\right)x+i\sqrt{31}-63}{2\sqrt{286-22i\sqrt{31}}\sqrt{2x^2-x+3}}\right)+620\sqrt{2x^2-x+3}\left(48000x^3-106400x^2+412060x-802347\right)+671106879\sqrt{2}\sinh^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{148800000}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^(5/2)/(2 + 3*x + 5*x^2),x]
```

```
[Out] (620*Sqrt[3 - x + 2*x^2]*(-802347 + 412060*x - 106400*x^2 + 48000*x^3) + 67
1106879*Sqrt[2]*ArcSinh[(-1 + 4*x)/Sqrt[23]] + 46464*Sqrt[286 + (22*I)*Sqrt
[31]]*(403 - (69*I)*Sqrt[31])*ArcTanh[(63 + I*Sqrt[31] + (-22 - (4*I)*Sqrt[
31])*x)/(2*Sqrt[286 + (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])] - (46464*I)*Sq
rt[286 - (22*I)*Sqrt[31]]*(-403*I + 69*Sqrt[31])*ArcTanh[(-63 + I*Sqrt[31]
+ (22 - (4*I)*Sqrt[31])*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^
2])])/148800000
```

IntegrateAlgebraic [C]   time = 0.76, size = 245, normalized size = 1.10

$$-\frac{1331\,\mathrm{RootSum}\left[-5\#1^4+6\sqrt{2}\#1^3+17\#1^2-26\sqrt{2}\#1-56\&,\frac{-119\#1^2\log\left(-\#1+\sqrt{2x^2-x+3}-\sqrt{2}x\right)+22\sqrt{2}\#1\log\left(-\#1+\sqrt{2x^2-x+3}-\sqrt{2}x\right)+368\log\left(-\#1+\sqrt{2x^2-x+3}-\sqrt{2}x\right)}{-10\#1^3+9\sqrt{2}\#1^2+17\#1-13\sqrt{2}}\&\right]}{3125}-\frac{7216203\log\left(2\sqrt{2}\sqrt{2x^2-x+3}-4x+1\right)}{800000\sqrt{2}}+\frac{\sqrt{2x^2-x+3}\left(48000x^3-106400x^2+412060x-802347\right)}{240000}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(3 - x + 2*x^2)^(5/2)/(2 + 3*x + 5*x^2),x]
```

```
[Out] (Sqrt[3 - x + 2*x^2]*(-802347 + 412060*x - 106400*x^2 + 48000*x^3))/240000
- (7216203*Log[1 - 4*x + 2*Sqrt[2]*Sqrt[3 - x + 2*x^2]])/(800000*Sqrt[2]) -
 (1331*RootSum[-56 - 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3 - 5*#1^4 & ,
(368*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1] + 22*Sqrt[2]*Log[-(Sqrt[2
]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1 - 119*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2
*x^2] - #1]*#1^2)/(-13*Sqrt[2] + 17*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3) & ])/312
5
```

fricas [B] time = 1.04, size = 2010, normalized size = 9.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(5/2)/(5*x^2+3*x+2),x, algorithm="fricas")
```

```
[Out] 121/96875000*6050^(1/4)*sqrt(31)*sqrt(2)*sqrt(-772850000*sqrt(2) + 25000000
00)*arctan(1/254496437500*(722441500000*sqrt(31)*sqrt(2)*(28180*x^8 - 25466
6*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - s
qrt(2)*(8746*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^4 - 752
088*x^3 + 396144*x^2 + 546048*x - 539136) + 1154304*x - 456192) + 2300*(4*6
050^(3/4)*sqrt(31)*(35898*x^7 - 441939*x^6 + 782418*x^5 - 2117233*x^4 + 127
2680*x^3 - 1081800*x^2 - sqrt(2)*(173702*x^7 - 453907*x^6 + 1056481*x^5 - 1
083344*x^4 + 393672*x^3 + 152064*x^2 - 1043712*x + 259200) - 518400*x + 104
3712) + 5*6050^(1/4)*sqrt(31)*(317294*x^7 - 5870544*x^6 + 38857480*x^5 - 11
1531424*x^4 + 156761280*x^3 - 168192000*x^2 - sqrt(2)*(712757*x^7 - 1023330
3*x^6 + 48529768*x^5 - 94500260*x^4 + 113086944*x^3 - 22282848*x^2 - 106417
152*x + 37407744) - 74815488*x + 106417152))*sqrt(2*x^2 - x + 3)*sqrt(-7728
50000*sqrt(2) + 2500000000) - sqrt(10/5711)*(314105000*sqrt(31)*sqrt(2)*(12
3408*x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 + 396480*x^4 + 798336*x^3
 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 118051*x^7 + 244047*x^6 - 707374*x^5
+ 1053960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*x) + 3276288*x) - (4*60
50^(3/4)*sqrt(31)*(167914*x^7 - 195429*x^6 + 331239*x^5 + 1685680*x^4 - 369
3960*x^3 + 4195584*x^2 + 22*sqrt(2)*(37846*x^7 - 52859*x^6 + 160569*x^5 - 4
464*x^4 - 49464*x^3 + 202176*x^2 - 202176*x) - 4195584*x) - 5*6050^(1/4)*sq
rt(31)*(160956*x^7 - 2232176*x^6 + 11218640*x^5 - 38096640*x^4 + 139374720*
x^3 - 296027136*x^2 - sqrt(2)*(3246491*x^7 - 41888524*x^6 + 159670660*x^5 -
 190080576*x^4 + 180496224*x^3 + 376648704*x^2 - 376648704*x) + 296027136*x
))*sqrt(2*x^2 - x + 3)*sqrt(-772850000*sqrt(2) + 2500000000) + 14277500*sqr
t(31)*(254591*x^8 - 4815126*x^7 + 32303580*x^6 - 90866808*x^5 + 108781920*x
^4 - 74219328*x^3 - 168956928*x^2 - 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6
 - 1536*x^5 + 2385*x^4 - 3618*x^3 + 2268*x^2 - 1944*x) + 144820224*x))*sqrt
((6050^(1/4)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(163*x - 725) + 562*x - 888)*sqrt
```

(-772850000*sqrt(2) + 2500000000) + 139919500*x^2 + 125642000*sqrt(2)*(2*x^
2 - x + 3) - 431180500*x + 571100000)/x^2) + 8209562500*sqrt(31)*(2828123*x
^8 - 9696916*x^7 + 53385560*x^6 - 142835344*x^5 + 254146592*x^4 - 249300096
*x^3 + 37981440*x^2 - 7744*sqrt(2)*(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*
x^5 + 15569*x^4 - 5568*x^3 + 1080*x^2 + 4320*x - 5184) + 223064064*x - 9488
7936))/(2585191*x^8 - 4661200*x^7 + 14191920*x^6 + 490880*x^5 - 13562944*x^
4 + 44249088*x^3 - 34615296*x^2 - 24772608*x + 18579456)) + 121/96875000*60
50^(1/4)*sqrt(31)*sqrt(2)*sqrt(-772850000*sqrt(2) + 2500000000)*arctan(-1/2
54496437500*(722441500000*sqrt(31)*sqrt(2)*(28180*x^8 - 254666*x^7 + 704270
*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - sqrt(2)*(8746*x
^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^4 - 752088*x^3 + 3961
44*x^2 + 546048*x - 539136) + 1154304*x - 456192) - 2300*(4*6050^(3/4)*sqrt
(31)*(35898*x^7 - 441939*x^6 + 782418*x^5 - 2117233*x^4 + 1272680*x^3 - 108
1800*x^2 - sqrt(2)*(173702*x^7 - 453907*x^6 + 1056481*x^5 - 1083344*x^4 + 3
93672*x^3 + 152064*x^2 - 1043712*x + 259200) - 518400*x + 1043712) + 5*6050
^(1/4)*sqrt(31)*(317294*x^7 - 5870544*x^6 + 38857480*x^5 - 111531424*x^4 +
156761280*x^3 - 168192000*x^2 - sqrt(2)*(712757*x^7 - 10233303*x^6 + 485297
68*x^5 - 94500260*x^4 + 113086944*x^3 - 22282848*x^2 - 106417152*x + 374077
44) - 74815488*x + 106417152))*sqrt(2*x^2 - x + 3)*sqrt(-772850000*sqrt(2)
+ 2500000000) - sqrt(10/5711)*(314105000*sqrt(31)*sqrt(2)*(123408*x^8 - 914
152*x^7 + 1578888*x^6 - 3293072*x^5 + 396480*x^4 + 798336*x^3 - 3822336*x^2
 - sqrt(2)*(15550*x^8 - 118051*x^7 + 244047*x^6 - 707374*x^5 + 1053960*x^4
- 1667952*x^3 + 1209600*x^2 - 1036800*x) + 3276288*x) + (4*6050^(3/4)*sqrt(
31)*(167914*x^7 - 195429*x^6 + 331239*x^5 + 1685680*x^4 - 3693960*x^3 + 419
5584*x^2 + 22*sqrt(2)*(37846*x^7 - 52859*x^6 + 160569*x^5 - 4464*x^4 - 4946
4*x^3 + 202176*x^2 - 202176*x) - 4195584*x) - 5*6050^(1/4)*sqrt(31)*(160956
*x^7 - 2232176*x^6 + 11218640*x^5 - 38096640*x^4 + 139374720*x^3 - 29602713
6*x^2 - sqrt(2)*(3246491*x^7 - 41888524*x^6 + 159670660*x^5 - 190080576*x^4
 + 180496224*x^3 + 376648704*x^2 - 376648704*x) + 296027136*x))*sqrt(2*x^2
- x + 3)*sqrt(-772850000*sqrt(2) + 2500000000) + 14277500*sqrt(31)*(254591*
x^8 - 4815126*x^7 + 32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 74219328*
x^3 - 168956928*x^2 - 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 +
2385*x^4 - 3618*x^3 + 2268*x^2 - 1944*x) + 144820224*x))*sqrt(-(6050^(1/4)*
sqrt(2*x^2 - x + 3)*(sqrt(2)*(163*x - 725) + 562*x - 888)*sqrt(-772850000*s
qrt(2) + 2500000000) - 139919500*x^2 - 125642000*sqrt(2)*(2*x^2 - x + 3) +
431180500*x - 571100000)/x^2) + 8209562500*sqrt(31)*(2828123*x^8 - 9696916*
x^7 + 53385560*x^6 - 142835344*x^5 + 254146592*x^4 - 249300096*x^3 + 379814
40*x^2 - 7744*sqrt(2)*(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x
^4 - 5568*x^3 + 1080*x^2 + 4320*x - 5184) + 223064064*x - 94887936))/(25851
91*x^8 - 4661200*x^7 + 14191920*x^6 + 490880*x^5 - 13562944*x^4 + 44249088*
x^3 - 34615296*x^2 - 24772608*x + 18579456)) - 121/2213012500000*6050^(1/4)
*(15457*sqrt(2) + 50000)*sqrt(-772850000*sqrt(2) + 2500000000)*log(91506250
00/5711*(6050^(1/4)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(163*x - 725) + 562*x - 88
8)*sqrt(-772850000*sqrt(2) + 2500000000) + 139919500*x^2 + 125642000*sqrt(2
)*(2*x^2 - x + 3) - 431180500*x + 571100000)/x^2) + 121/2213012500000*6050^

```
(1/4)*(15457*sqrt(2) + 50000)*sqrt(-772850000*sqrt(2) + 2500000000)*log(-91
50625000/5711*(6050^(1/4)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(163*x - 725) + 562*
x - 888)*sqrt(-772850000*sqrt(2) + 2500000000) - 139919500*x^2 - 125642000*
sqrt(2)*(2*x^2 - x + 3) + 431180500*x - 571100000)/x^2) + 1/240000*(48000*x
^3 - 106400*x^2 + 412060*x - 802347)*sqrt(2*x^2 - x + 3) + 7216203/3200000*
sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(5/2)/(5*x^2+3*x+2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Francis algorithm failure for[-1.0,infinity,
infinity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,infi
nity]Francis algorithm failure for[-1.0,infinity,infinity,infinity,infinity
]proot error [1.0,infinity,infinity,infinity,infinity]Francis algorithm fai
lure for[-1.0,infinity,infinity,infinity,infinity]proot error [1.0,infinity
,infinity,infinity,infinity]Francis algorithm failure for[-1.0,infinity,inf
inity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,infinit
y]Evaluation time: 15.78Done
```

maple [B] time = 0.05, size = 4860, normalized size = 21.89

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(5/2)/(5*x^2+3*x+2),x)
```

```
[Out] 1/5*x^3*(2*x^2-x+3)^(1/2)-133/300*x^2*(2*x^2-x+3)^(1/2)+20603/12000*x*(2*x^
2-x+3)^(1/2)-267449/80000*(2*x^2-x+3)^(1/2)+7216203/1600000*2^(1/2)*arcsinh
(4/23*23^(1/2)*(x-1/4))+4/33034375*(8*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+3*2^
(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+8-3*2^(1/2))^(1/2)*2^(1/2)*(75195*2^
(1/2)*(-8866+6820*2^(1/2))^(1/2)*(-775687+549362*2^(1/2))^(1/2)*arctan(1/11
692487*(-775687+549362*2^(1/2))^(1/2)*(-23*(8+3*2^(1/2))*(-23*(x+2^(1/2)-1)
^2/(-x+2^(1/2)+1)^2+24*2^(1/2)-41))^(1/2)*(6485*2^(1/2)*(x+2^(1/2)-1)^2/(-x
+2^(1/2)+1)^2+10368*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+22379*2^(1/2)+32016)/(
23*(x+2^(1/2)-1)^4/(-x+2^(1/2)+1)^4+82*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+23)
*(x+2^(1/2)-1)/(-x+2^(1/2)+1)*(8+3*2^(1/2)))+106294*(-8866+6820*2^(1/2))^(1
/2)*(-775687+549362*2^(1/2))^(1/2)*arctan(1/11692487*(-775687+549362*2^(1/2
))^(1/2)*(-23*(8+3*2^(1/2))*(-23*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+24*2^(1/2
)-41))^(1/2)*(6485*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+10368*(x+2^(1/2
```

)-1)^2/(-x+2^(1/2)+1)^2+22379*2^(1/2)+32016)/(23*(x+2^(1/2)-1)^4/(-x+2^(1/2
)+1)^4+82*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+23)*(x+2^(1/2)-1)/(-x+2^(1/2)+1)
*(8+3*2^(1/2)))+108099046*arctanh(31/2*(8*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+
3*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+8-3*2^(1/2))^(1/2)/(-8866+6820*2
^(1/2))^(1/2))*2^(1/2)-158290154*arctanh(31/2*(8*(x+2^(1/2)-1)^2/(-x+2^(1/2
)+1)^2+3*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+8-3*2^(1/2))^(1/2)/(-8866
+6820*2^(1/2))^(1/2)))/((8*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+3*2^(1/2)*(x+2^
(1/2)-1)^2/(-x+2^(1/2)+1)^2+8-3*2^(1/2))/(1+(x+2^(1/2)-1)/(-x+2^(1/2)+1))^2
)^(1/2)/(1+(x+2^(1/2)-1)/(-x+2^(1/2)+1))/(8+3*2^(1/2))/(-8866+6820*2^(1/2))
^(1/2)+6/6606875*(8*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+3*2^(1/2)*(x+2^(1/2)-1
)^2/(-x+2^(1/2)+1)^2+8-3*2^(1/2))^(1/2)*2^(1/2)*(10915*2^(1/2)*(-8866+6820*
2^(1/2))^(1/2)*(-775687+549362*2^(1/2))^(1/2)*arctan(1/11692487*(-775687+54
9362*2^(1/2))^(1/2)*(-23*(8+3*2^(1/2))*(-23*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^
2+24*2^(1/2)-41))^(1/2)*(6485*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+1036
8*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+22379*2^(1/2)+32016)/(23*(x+2^(1/2)-1)^4
/(-x+2^(1/2)+1)^4+82*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+23)*(x+2^(1/2)-1)/(-x
+2^(1/2)+1)*(8+3*2^(1/2)))+14918*(-8866+6820*2^(1/2))^(1/2)*(-775687+549362
*2^(1/2))^(1/2)*arctan(1/11692487*(-775687+549362*2^(1/2))^(1/2)*(-23*(8+3*
2^(1/2))*(-23*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+24*2^(1/2)-41))^(1/2)*(6485*
2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+10368*(x+2^(1/2)-1)^2/(-x+2^(1/2)+
1)^2+22379*2^(1/2)+32016)/(23*(x+2^(1/2)-1)^4/(-x+2^(1/2)+1)^4+82*(x+2^(1/2
)-1)^2/(-x+2^(1/2)+1)^2+23)*(x+2^(1/2)-1)/(-x+2^(1/2)+1)*(8+3*2^(1/2)))-505
2938*arctanh(31/2*(8*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+3*2^(1/2)*(x+2^(1/2)-
1)^2/(-x+2^(1/2)+1)^2+8-3*2^(1/2))^(1/2)/(-8866+6820*2^(1/2))^(1/2))*2^(1/2
)-51565338*arctanh(31/2*(8*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+3*2^(1/2)*(x+2^
(1/2)-1)^2/(-x+2^(1/2)+1)^2+8-3*2^(1/2))^(1/2)/(-8866+6820*2^(1/2))^(1/2)))
/((8*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+3*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)
+1)^2+8-3*2^(1/2))/(1+(x+2^(1/2)-1)/(-x+2^(1/2)+1))^2)^(1/2)/(1+(x+2^(1/2)-
1)/(-x+2^(1/2)+1))/(8+3*2^(1/2))/(-8866+6820*2^(1/2))^(1/2)-21/1321375*(8*(
x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+3*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+
8-3*2^(1/2))^(1/2)*2^(1/2)*(4245*2^(1/2)*(-8866+6820*2^(1/2))^(1/2)*(-77568
7+549362*2^(1/2))^(1/2)*arctan(1/11692487*(-775687+549362*2^(1/2))^(1/2)*(-
23*(8+3*2^(1/2))*(-23*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+24*2^(1/2)-41))^(1/2
)*(6485*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+10368*(x+2^(1/2)-1)^2/(-x+
2^(1/2)+1)^2+22379*2^(1/2)+32016)/(23*(x+2^(1/2)-1)^4/(-x+2^(1/2)+1)^4+82*(
x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+23)*(x+2^(1/2)-1)/(-x+2^(1/2)+1)*(8+3*2^(1/
2)))+6154*(-8866+6820*2^(1/2))^(1/2)*(-775687+549362*2^(1/2))^(1/2)*arctan(
1/11692487*(-775687+549362*2^(1/2))^(1/2)*(-23*(8+3*2^(1/2))*(-23*(x+2^(1/2
)-1)^2/(-x+2^(1/2)+1)^2+24*2^(1/2)-41))^(1/2)*(6485*2^(1/2)*(x+2^(1/2)-1)^2
/(-x+2^(1/2)+1)^2+10368*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+22379*2^(1/2)+3201
6)/(23*(x+2^(1/2)-1)^4/(-x+2^(1/2)+1)^4+82*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2
+23)*(x+2^(1/2)-1)/(-x+2^(1/2)+1)*(8+3*2^(1/2)))+12325786*arctanh(31/2*(8*(
x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+3*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+
8-3*2^(1/2))^(1/2)/(-8866+6820*2^(1/2))^(1/2))*2^(1/2)-359414*arctanh(31/2*
(8*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+3*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1

```
)^2+8-3*2^(1/2))^(1/2)/(-8866+6820*2^(1/2))^(1/2)))/((8*(x+2^(1/2)-1)^2/(-x
+2^(1/2)+1)^2+3*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+8-3*2^(1/2))/(1+(x
+2^(1/2)-1)/(-x+2^(1/2)+1))^2)^(1/2)/(1+(x+2^(1/2)-1)/(-x+2^(1/2)+1))/(8+3*
2^(1/2))/(-8866+6820*2^(1/2))^(1/2)-37/528550*(8*(x+2^(1/2)-1)^2/(-x+2^(1/2
)+1)^2+3*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+8-3*2^(1/2))^(1/2)*2^(1/2
)*(2365*2^(1/2)*(-8866+6820*2^(1/2))^(1/2)*(-775687+549362*2^(1/2))^(1/2)*a
rctan(1/11692487*(-775687+549362*2^(1/2))^(1/2)*(-23*(8+3*2^(1/2))*(-23*(x+
2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+24*2^(1/2)-41))^(1/2)*(6485*2^(1/2)*(x+2^(1/2
)-1)^2/(-x+2^(1/2)+1)^2+10368*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+22379*2^(1/2
)+32016)/(23*(x+2^(1/2)-1)^4/(-x+2^(1/2)+1)^4+82*(x+2^(1/2)-1)^2/(-x+2^(1/2
)+1)^2+23)*(x+2^(1/2)-1)/(-x+2^(1/2)+1)*(8+3*2^(1/2)))+3338*(-8866+6820*2^(
1/2))^(1/2)*(-775687+549362*2^(1/2))^(1/2)*arctan(1/11692487*(-775687+54936
2*2^(1/2))^(1/2)*(-23*(8+3*2^(1/2))*(-23*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+2
4*2^(1/2)-41))^(1/2)*(6485*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+10368*(
x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+22379*2^(1/2)+32016)/(23*(x+2^(1/2)-1)^4/(-
x+2^(1/2)+1)^4+82*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+23)*(x+2^(1/2)-1)/(-x+2^
(1/2)+1)*(8+3*2^(1/2)))+3192442*arctanh(31/2*(8*(x+2^(1/2)-1)^2/(-x+2^(1/2)
+1)^2+3*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+8-3*2^(1/2))^(1/2)/(-8866+
6820*2^(1/2))^(1/2))*2^(1/2)-5264358*arctanh(31/2*(8*(x+2^(1/2)-1)^2/(-x+2^
(1/2)+1)^2+3*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+8-3*2^(1/2))^(1/2)/(-
8866+6820*2^(1/2))^(1/2)))/((8*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+3*2^(1/2)*(
x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+8-3*2^(1/2))/(1+(x+2^(1/2)-1)/(-x+2^(1/2)+1
))^2)^(1/2)/(1+(x+2^(1/2)-1)/(-x+2^(1/2)+1))/(8+3*2^(1/2))/(-8866+6820*2^(1
/2))^(1/2)-63/105710*(8*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+3*2^(1/2)*(x+2^(1/
2)-1)^2/(-x+2^(1/2)+1)^2+8-3*2^(1/2))^(1/2)*2^(1/2)*(285*2^(1/2)*(-8866+682
0*2^(1/2))^(1/2)*(-775687+549362*2^(1/2))^(1/2)*arctan(1/11692487*(-775687+
549362*2^(1/2))^(1/2)*(-23*(8+3*2^(1/2))*(-23*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1
)^2+24*2^(1/2)-41))^(1/2)*(6485*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+10
368*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+22379*2^(1/2)+32016)/(23*(x+2^(1/2)-1)
^4/(-x+2^(1/2)+1)^4+82*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+23)*(x+2^(1/2)-1)/(
-x+2^(1/2)+1)*(8+3*2^(1/2)))+386*(-8866+6820*2^(1/2))^(1/2)*(-775687+549362
*2^(1/2))^(1/2)*arctan(1/11692487*(-775687+549362*2^(1/2))^(1/2)*(-23*(8+3*
2^(1/2))*(-23*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+24*2^(1/2)-41))^(1/2)*(6485*
2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+10368*(x+2^(1/2)-1)^2/(-x+2^(1/2)+
1)^2+22379*2^(1/2)+32016)/(23*(x+2^(1/2)-1)^4/(-x+2^(1/2)+1)^4+82*(x+2^(1/2
)-1)^2/(-x+2^(1/2)+1)^2+23)*(x+2^(1/2)-1)/(-x+2^(1/2)+1)*(8+3*2^(1/2)))-274
846*arctanh(31/2*(8*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+3*2^(1/2)*(x+2^(1/2)-1
)^2/(-x+2^(1/2)+1)^2+8-3*2^(1/2))^(1/2)/(-8866+6820*2^(1/2))^(1/2))*2^(1/2)
-1543366*arctanh(31/2*(8*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+3*2^(1/2)*(x+2^(1
/2)-1)^2/(-x+2^(1/2)+1)^2+8-3*2^(1/2))^(1/2)/(-8866+6820*2^(1/2))^(1/2)))/(
(8*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+3*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1
)^2+8-3*2^(1/2))/(1+(x+2^(1/2)-1)/(-x+2^(1/2)+1))^2)^(1/2)/(1+(x+2^(1/2)-1)
/(-x+2^(1/2)+1))/(8+3*2^(1/2))/(-8866+6820*2^(1/2))^(1/2)+27/21142*(8*(x+2^
(1/2)-1)^2/(-x+2^(1/2)+1)^2+3*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+8-3*
2^(1/2))^(1/2)*2^(1/2)*(151*2^(1/2)*(-8866+6820*2^(1/2))^(1/2)*(-775687+549
```

```
362*2^(1/2))^(1/2)*arctan(1/11692487*(-775687+549362*2^(1/2))^(1/2)*(-23*(8
+3*2^(1/2))*(-23*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+24*2^(1/2)-41))^(1/2)*(64
85*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+10368*(x+2^(1/2)-1)^2/(-x+2^(1/
2)+1)^2+22379*2^(1/2)+32016)/(23*(x+2^(1/2)-1)^4/(-x+2^(1/2)+1)^4+82*(x+2^(
1/2)-1)^2/(-x+2^(1/2)+1)^2+23)*(x+2^(1/2)-1)/(-x+2^(1/2)+1)*(8+3*2^(1/2)))+
218*(-8866+6820*2^(1/2))^(1/2)*(-775687+549362*2^(1/2))^(1/2)*arctan(1/1169
2487*(-775687+549362*2^(1/2))^(1/2)*(-23*(8+3*2^(1/2))*(-23*(x+2^(1/2)-1)^2
/(-x+2^(1/2)+1)^2+24*2^(1/2)-41))^(1/2)*(6485*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2
^(1/2)+1)^2+10368*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+22379*2^(1/2)+32016)/(23
*(x+2^(1/2)-1)^4/(-x+2^(1/2)+1)^4+82*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+23)*(
x+2^(1/2)-1)/(-x+2^(1/2)+1)*(8+3*2^(1/2)))+401698*arctanh(31/2*(8*(x+2^(1/2
)-1)^2/(-x+2^(1/2)+1)^2+3*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+8-3*2^(1
/2))^(1/2)/(-8866+6820*2^(1/2))^(1/2))*2^(1/2)-63426*arctanh(31/2*(8*(x+2^(
1/2)-1)^2/(-x+2^(1/2)+1)^2+3*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+8-3*2
^(1/2))^(1/2)/(-8866+6820*2^(1/2))^(1/2)))/((8*(x+2^(1/2)-1)^2/(-x+2^(1/2)+
1)^2+3*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+8-3*2^(1/2))/(1+(x+2^(1/2)-
1)/(-x+2^(1/2)+1))^2)^(1/2)/(1+(x+2^(1/2)-1)/(-x+2^(1/2)+1))/(8+3*2^(1/2))/
(-8866+6820*2^(1/2))^(1/2)+27/21142*(8*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+3*2
^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+8-3*2^(1/2))^(1/2)*2^(1/2)*(369*2^(
1/2)*(-8866+6820*2^(1/2))^(1/2)*(-775687+549362*2^(1/2))^(1/2)*arctan(1/116
92487*(-775687+549362*2^(1/2))^(1/2)*(-23*(8+3*2^(1/2))*(-23*(x+2^(1/2)-1)^
2/(-x+2^(1/2)+1)^2+24*2^(1/2)-41))^(1/2)*(6485*2^(1/2)*(x+2^(1/2)-1)^2/(-x+
2^(1/2)+1)^2+10368*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+22379*2^(1/2)+32016)/(2
3*(x+2^(1/2)-1)^4/(-x+2^(1/2)+1)^4+82*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+23)*
(x+2^(1/2)-1)/(-x+2^(1/2)+1)*(8+3*2^(1/2)))+520*(-8866+6820*2^(1/2))^(1/2)*
(-775687+549362*2^(1/2))^(1/2)*arctan(1/11692487*(-775687+549362*2^(1/2))^(
1/2)*(-23*(8+3*2^(1/2))*(-23*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+24*2^(1/2)-41
))^(1/2)*(6485*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+10368*(x+2^(1/2)-1)
^2/(-x+2^(1/2)+1)^2+22379*2^(1/2)+32016)/(23*(x+2^(1/2)-1)^4/(-x+2^(1/2)+1)
^4+82*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+23)*(x+2^(1/2)-1)/(-x+2^(1/2)+1)*(8+
3*2^(1/2)))+465124*arctanh(31/2*(8*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+3*2^(1/
2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+8-3*2^(1/2))^(1/2)/(-8866+6820*2^(1/2))
^(1/2))*2^(1/2)-866822*arctanh(31/2*(8*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+3*2
^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+8-3*2^(1/2))^(1/2)/(-8866+6820*2^(1
/2))^(1/2)))/((8*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+3*2^(1/2)*(x+2^(1/2)-1)^2
/(-x+2^(1/2)+1)^2+8-3*2^(1/2))/(1+(x+2^(1/2)-1)/(-x+2^(1/2)+1))^2)^(1/2)/(1
+(x+2^(1/2)-1)/(-x+2^(1/2)+1))/(8+3*2^(1/2))/(-8866+6820*2^(1/2))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\,x^2 - x + 3\right)^{\frac{5}{2}}}{5\,x^2 + 3\,x + 2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(5/2)/(5*x^2+3*x+2),x, algorithm="maxima")
[Out] integrate((2*x^2 - x + 3)^(5/2)/(5*x^2 + 3*x + 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(2\,x^2-x+3\right)^{5/2}}{5\,x^2+3\,x+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^(5/2)/(3*x + 5*x^2 + 2),x)
[Out] int((2*x^2 - x + 3)^(5/2)/(3*x + 5*x^2 + 2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2x^{2} - x + 3\right)^{\frac{5}{2}}}{5x^{2} + 3x + 2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(5/2)/(5*x**2+3*x+2),x)
[Out] Integral((2*x**2 - x + 3)**(5/2)/(5*x**2 + 3*x + 2), x)
```

3.74 $\int \frac{(3-x+2x^2)^{5/2}}{(2+3x+5x^2)^2}\,dx$

Optimal. Leaf size=255

$$\frac{(10x+3)\left(2x^2-x+3\right)^{5/2}}{31\left(5x^2+3x+2\right)}+\frac{4}{155}(4-5x)\left(2x^2-x+3\right)^{3/2}-\frac{(2240x+1277)\sqrt{2x^2-x+3}}{7750}+\frac{11\sqrt{\frac{11}{31}}\left(224510383+194\ldots\right.}{}$$

Rubi [A] time = 0.66, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {971, 1066, 1076, 619, 215, 1035, 1029, 206, 204}

$$\frac{(10x+3)\left(2x^2-x+3\right)^{5/2}}{31\left(5x^2+3x+2\right)}+\frac{4}{155}(4-5x)\left(2x^2-x+3\right)^{3/2}-\frac{(2240x+1277)\sqrt{2x^2-x+3}}{7750}+\frac{11\sqrt{\frac{11}{31}\left(224510383+194487500\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{11}{62\left(224510383+194487500\sqrt{2}\right)}}\left(\left(87710+54423\sqrt{2}\right)x+33287\sqrt{2}+21136\right)}{\sqrt{2x^2-x+3}}\right)}{38750}-\frac{11\sqrt{\frac{11}{31}\left(194487500\sqrt{2}-224510383\right)}\tanh^{-1}\left(\frac{\sqrt{\frac{11}{62\left(194487500\sqrt{2}-224510383\right)}}\left(\left(87710-54423\sqrt{2}\right)x-33287\sqrt{2}+21136\right)}{\sqrt{2x^2-x+3}}\right)}{38750}-\frac{4799\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{2500\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^(5/2)/(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] -((1277 + 2240*x)*Sqrt[3 - x + 2*x^2])/7750 + (4*(4 - 5*x)*(3 - x + 2*x^2)^
(3/2))/155 + ((3 + 10*x)*(3 - x + 2*x^2)^(5/2))/(31*(2 + 3*x + 5*x^2)) - (4
799*ArcSinh[(1 - 4*x)/Sqrt[23]])/(2500*Sqrt[2]) + (11*Sqrt[(11*(224510383 +
 194487500*Sqrt[2]))/31]*ArcTan[(Sqrt[11/(62*(224510383 + 194487500*Sqrt[2]
))]*(21136 + 33287*Sqrt[2] + (87710 + 54423*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2
]])/38750 - (11*Sqrt[(11*(-224510383 + 194487500*Sqrt[2]))/31]*ArcTanh[(Sqr
t[11/(62*(-224510383 + 194487500*Sqrt[2]))]*(21136 - 33287*Sqrt[2] + (87710
 - 54423*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]])/38750
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 971

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((b + 2*c*x)*(a + b*x + c*x^2)^(p + 1)*(d + e
*x + f*x^2)^q)/((b^2 - 4*a*c)*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*(p + 1))
, Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*Simp[2*c*d*(2*p +
 3) + b*e*q + (2*b*f*q + 2*c*e*(2*p + q + 3))*x + 2*c*f*(2*p + 2*q + 3)*x^2
, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
e^2 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0] &&  !IGtQ[q, 0]
```

Rule 1029

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
 c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 1035

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

Rule 1066

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((B*c*f*(2*
```

```
p + 2*q + 3) + C*(b*f*p - c*e*(2*p + q + 2)) + 2*c*C*f*(p + q + 1)*x)*(a +
b*x + c*x^2)^p*(d + e*x + f*x^2)^(q + 1))/(2*c*f^2*(p + q + 1)*(2*p + 2*q +
 3)), x] - Dist[1/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), Int[(a + b*x + c*x
^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[p*(b*d - a*e)*(C*(c*e - b*f)*(q + 1) -
 c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(b^2*C*d*f*p + a*c*(C*(2*d*f
- e^2*(2*p + q + 2)) + f*(B*e - 2*A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*f)
*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*e
*f*p*(b^2 - 4*a*c) - b*c*(C*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e +
2*A*f)*(2*p + 2*q + 3))))*x + (p*(c*e - b*f)*(C*(c*e - b*f)*(q + 1) - c*(C*
e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(b^2 - 4*a*c) - c^2*(C*(e^
2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3))))*x^2,
x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*
q + 3, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

Rule 1076

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqr
t[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$\int \frac{\left(3-x+2x^2\right)^{5/2}}{\left(2+3x+5x^2\right)^2}\,dx = \frac{(3+10x)\left(3-x+2x^2\right)^{5/2}}{31\left(2+3x+5x^2\right)} - \frac{1}{31}\int \frac{\left(3-x+2x^2\right)^{3/2}\left(-\frac{75}{2}+15x+80x^2\right)}{2+3x+5x^2}\,dx$$

$$= \frac{4}{155}(4-5x)\left(3-x+2x^2\right)^{3/2} + \frac{(3+10x)\left(3-x+2x^2\right)^{5/2}}{31\left(2+3x+5x^2\right)} + \frac{\int \frac{\left(87660-54300x-53760x^2\right)\sqrt{3-}}{2+3x+5x^2}}{18600}$$

$$= -\frac{(1277+2240x)\sqrt{3-x+2x^2}}{7750} + \frac{4}{155}(4-5x)\left(3-x+2x^2\right)^{3/2} + \frac{(3+10x)\left(3-x+2x\right.}{31\left(2+3x+5x^2\right.}$$

$$= -\frac{(1277+2240x)\sqrt{3-x+2x^2}}{7750} + \frac{4}{155}(4-5x)\left(3-x+2x^2\right)^{3/2} + \frac{(3+10x)\left(3-x+2x\right.}{31\left(2+3x+5x^2\right.}$$

$$= -\frac{(1277+2240x)\sqrt{3-x+2x^2}}{7750} + \frac{4}{155}(4-5x)\left(3-x+2x^2\right)^{3/2} + \frac{(3+10x)\left(3-x+2x\right.}{31\left(2+3x+5x^2\right.}$$

$$= -\frac{(1277+2240x)\sqrt{3-x+2x^2}}{7750} + \frac{4}{155}(4-5x)\left(3-x+2x^2\right)^{3/2} + \frac{(3+10x)\left(3-x+2x\right.}{31\left(2+3x+5x^2\right.}$$

$$= -\frac{(1277+2240x)\sqrt{3-x+2x^2}}{7750} + \frac{4}{155}(4-5x)\left(3-x+2x^2\right)^{3/2} + \frac{(3+10x)\left(3-x+2x\right.}{31\left(2+3x+5x^2\right.}$$

Mathematica [C] time = 1.65, size = 685, normalized size = 2.69

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^(5/2)/(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] -1/4805000*(-5577520*Sqrt[3 - x + 2*x^2] - 5759180*x*Sqrt[3 - x + 2*x^2] +
7784100*x^2*Sqrt[3 - x + 2*x^2] - 1922000*x^3*Sqrt[3 - x + 2*x^2] - 4611839
*Sqrt[2]*(2 + 3*x + 5*x^2)*ArcSinh[(-1 + 4*x)/Sqrt[23]] + (11*I)*Sqrt[286 +
 (22*I)*Sqrt[31]]*(5177*I + 8771*Sqrt[31])*(2 + 3*x + 5*x^2)*ArcTanh[(63 +
I*Sqrt[31] - 22*x - (4*I)*Sqrt[31]*x)/(2*Sqrt[286 + (22*I)*Sqrt[31]]*Sqrt[3
```

```
 - x + 2*x^2])] + (192962*I)*Sqrt[682*(13 - I*Sqrt[31])]*ArcTanh[(-63 + I*S
qrt[31] + 22*x - (4*I)*Sqrt[31]*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 -
x + 2*x^2])] + 113894*Sqrt[286 - (22*I)*Sqrt[31]]*ArcTanh[(-63 + I*Sqrt[31]
 + 22*x - (4*I)*Sqrt[31]*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x
^2])] + (289443*I)*Sqrt[682*(13 - I*Sqrt[31])]*x*ArcTanh[(-63 + I*Sqrt[31]
+ 22*x - (4*I)*Sqrt[31]*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^
2])] + 170841*Sqrt[286 - (22*I)*Sqrt[31]]*x*ArcTanh[(-63 + I*Sqrt[31] + 22*
x - (4*I)*Sqrt[31]*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])]
+ (482405*I)*Sqrt[682*(13 - I*Sqrt[31])]*x^2*ArcTanh[(-63 + I*Sqrt[31] + 22
*x - (4*I)*Sqrt[31]*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])]
 + 284735*Sqrt[286 - (22*I)*Sqrt[31]]*x^2*ArcTanh[(-63 + I*Sqrt[31] + 22*x
- (4*I)*Sqrt[31]*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])])/(
2 + 3*x + 5*x^2)
```

IntegrateAlgebraic [C] time = 0.97, size = 442, normalized size = 1.73

$$\frac{121\text{RootSum}\left[-5\#1^4+6\sqrt{2}\#1^3+17\#1^2-26\sqrt{2}\#1-56\&,\frac{2225\#1^2\log\left(-\#1+\sqrt{2x^2-x+3}-\sqrt{2}x\right)+2880\sqrt{2}\#1\log\left(-\#1+\sqrt{2x^2-x+3}-\sqrt{2}x\right)+5237\log\left(-\#1+\sqrt{2x^2-x+3}-\sqrt{2}x\right)}{-10\#1^3+9\sqrt{2}\#1^2+17\#1-13\sqrt{2}}\&\right]}{15625}-\frac{121\text{RootSum}\left[-5\#1^4+6\sqrt{2}\#1^3+17\#1^2-26\sqrt{2}\#1-56\&,\frac{1175\sqrt{2}\#1^2\log\left(-\#1+\sqrt{2x^2-x+3}-\sqrt{2}x\right)-22980\#1\log\left(-\#1+\sqrt{2x^2-x+3}-\sqrt{2}x\right)+639994\sqrt{2}\log\left(-\#1+\sqrt{2x^2-x+3}-\sqrt{2}x\right)}{-10\#1^3+9\sqrt{2}\#1^2+17\#1-13\sqrt{2}}\&\right]}{968750\sqrt{2}}-\frac{4799\log\left(2\sqrt{2}\sqrt{2x^2-x+3}-4x+1\right)}{2500\sqrt{2}}+\frac{\sqrt{2x^2-x+3}\left(3100x^3-12555x^2+9289x+8996\right)}{7750\left(5x^2+3x+2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(3 - x + 2*x^2)^(5/2)/(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] (Sqrt[3 - x + 2*x^2]*(8996 + 9289*x - 12555*x^2 + 3100*x^3))/(7750*(2 + 3*x
 + 5*x^2)) - (4799*Log[1 - 4*x + 2*Sqrt[2]*Sqrt[3 - x + 2*x^2]])/(2500*Sqrt
[2]) + (121*RootSum[-56 - 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3 - 5*#1^4
 & , (5237*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1] + 2880*Sqrt[2]*Log[
-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1 + 2225*Log[-(Sqrt[2]*x) + Sqrt[
3 - x + 2*x^2] - #1]*#1^2)/(-13*Sqrt[2] + 17*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3)
 & ])/15625 - (121*RootSum[-56 - 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3 -
 5*#1^4 & , (639994*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1] -
22980*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1 + 1175*Sqrt[2]*Log[-(
Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1^2)/(-13*Sqrt[2] + 17*#1 + 9*Sqrt[
2]*#1^2 - 10*#1^3) & ])/(968750*Sqrt[2])
```

fricas [B] time = 1.32, size = 2161, normalized size = 8.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

```
[Out] 1/132275992243570790000*(38925001324*1464599010050^(1/4)*sqrt(155590)*sqrt
(62)*sqrt(2)*(5*x^2 + 3*x + 2)*sqrt(224510383*sqrt(2) + 388975000)*arctan(1
/296975447063866363819995875*(110935670*sqrt(155590)*(4*1464599010050^(3/4)
*sqrt(62)*(18997882*x^7 - 82713851*x^6 + 169131062*x^5 - 298338397*x^4 + 15
6222120*x^3 - 89116200*x^2 - sqrt(2)*(18111018*x^7 - 62947113*x^6 + 1354639
29*x^5 - 197908246*x^4 + 94500248*x^3 - 34095024*x^2 - 122404608*x + 714528
```

00) - 142905600*x + 122404608) + 2411645*1464599010050^(1/4)*sqrt(62)*(3035
566*x^7 - 47612316*x^6 + 259553720*x^5 - 615321136*x^4 + 807721920*x^3 - 57
9888000*x^2 - sqrt(2)*(2643323*x^7 - 39854517*x^6 + 204950152*x^5 - 4510041
40*x^4 + 573424416*x^3 - 311722272*x^2 - 434377728*x + 268655616) - 5373112
32*x + 434377728))*sqrt(2*x^2 - x + 3)*sqrt(224510383*sqrt(2) + 388975000)
+ 843027075536136774714827000*sqrt(31)*sqrt(2)*(28180*x^8 - 254666*x^7 + 70
4270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - sqrt(2)*(87
46*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^4 - 752088*x^3 +
396144*x^2 + 546048*x - 539136) + 1154304*x - 456192) - sqrt(77795/920561)*
(sqrt(155590)*(4*1464599010050^(3/4)*sqrt(62)*(58767374*x^7 - 85793239*x^6
+ 285539949*x^5 - 168939120*x^4 + 253241640*x^3 + 601344*x^2 - 4*sqrt(2)*(1
7889302*x^7 - 25424283*x^6 + 80174553*x^5 - 21241168*x^4 + 15593832*x^3 + 5
8564512*x^2 - 58564512*x) - 601344*x) + 2411645*1464599010050^(1/4)*sqrt(62
)*(9891184*x^7 - 128099264*x^6 + 496592960*x^5 - 666984960*x^4 + 949582080*
x^3 + 183223296*x^2 - sqrt(2)*(10181049*x^7 - 131588036*x^6 + 505509740*x^5
 - 637596864*x^4 + 754818336*x^3 + 725677056*x^2 - 725677056*x) - 183223296
*x))*sqrt(2*x^2 - x + 3)*sqrt(224510383*sqrt(2) + 388975000) + 759924265600
1778100*sqrt(31)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x
^5 + 396480*x^4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 118051*x^
7 + 244047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^2 - 103
6800*x) + 3276288*x) + 345420120727353550*sqrt(31)*(254591*x^8 - 4815126*x^
7 + 32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 74219328*x^3 - 168956928*
x^2 - 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3618*
x^3 + 2268*x^2 - 1944*x) + 144820224*x))*sqrt(-(1464599010050^(1/4)*sqrt(15
5590)*sqrt(62)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(9733*x + 29025) - 387
58*x + 19292)*sqrt(224510383*sqrt(2) + 388975000) - 6744561519183110*x^2 -
6056340956001160*sqrt(2)*(2*x^2 - x + 3) + 20784261008094890*x - 2752882252
7278000)/x^2) + 957985313109246334903212 5*sqrt(31)*(2828123*x^8 - 9696916*x
^7 + 53385560*x^6 - 142835344*x^5 + 254146592*x^4 - 249300096*x^3 + 3798144
0*x^2 - 7744*sqrt(2)*(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^
4 - 5568*x^3 + 1080*x^2 + 4320*x - 5184) + 223064064*x - 94887936))/(258519
1*x^8 - 4661200*x^7 + 14191920*x^6 + 490880*x^5 - 13562944*x^4 + 44249088*x
^3 - 34615296*x^2 - 24772608*x + 18579456)) + 38925001324*1464599010050^(1/
4)*sqrt(155590)*sqrt(62)*sqrt(2)*(5*x^2 + 3*x + 2)*sqrt(224510383*sqrt(2) +
 388975000)*arctan(1/29697544706386636381999 5875*(110935670*sqrt(155590)*(4
*1464599010050^(3/4)*sqrt(62)*(18997882*x^7 - 82713851*x^6 + 169131062*x^5
- 298338397*x^4 + 156222120*x^3 - 89116200*x^2 - sqrt(2)*(18111018*x^7 - 62
947113*x^6 + 135463929*x^5 - 197908246*x^4 + 94500248*x^3 - 34095024*x^2 -
122404608*x + 71452800) - 142905600*x + 122404608) + 2411645*1464599010050^
(1/4)*sqrt(62)*(3035566*x^7 - 47612316*x^6 + 259553720*x^5 - 615321136*x^4
+ 807721920*x^3 - 579888000*x^2 - sqrt(2)*(2643323*x^7 - 39854517*x^6 + 204
950152*x^5 - 451004140*x^4 + 573424416*x^3 - 311722272*x^2 - 434377728*x +
268655616) - 537311232*x + 434377728))*sqrt(2*x^2 - x + 3)*sqrt(224510383*s
qrt(2) + 388975000) - 843027075536136774714827000*sqrt(31)*sqrt(2)*(28180*x
^8 - 254666*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 984

```
96*x^2 - sqrt(2)*(8746*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710
*x^4 - 752088*x^3 + 396144*x^2 + 546048*x - 539136) + 1154304*x - 456192) -
 sqrt(77795/920561)*(sqrt(155590)*(4*1464599010050^(3/4)*sqrt(62)*(58767374
*x^7 - 85793239*x^6 + 285539949*x^5 - 168939120*x^4 + 253241640*x^3 + 60134
4*x^2 - 4*sqrt(2)*(17889302*x^7 - 25424283*x^6 + 80174553*x^5 - 21241168*x^
4 + 15593832*x^3 + 58564512*x^2 - 58564512*x) - 601344*x) + 2411645*1464599
010050^(1/4)*sqrt(62)*(9891184*x^7 - 128099264*x^6 + 496592960*x^5 - 666984
960*x^4 + 949582080*x^3 + 183223296*x^2 - sqrt(2)*(10181049*x^7 - 131588036
*x^6 + 505509740*x^5 - 637596864*x^4 + 754818336*x^3 + 725677056*x^2 - 7256
77056*x) - 183223296*x))*sqrt(2*x^2 - x + 3)*sqrt(224510383*sqrt(2) + 38897
5000) - 7599242656001778100*sqrt(31)*sqrt(2)*(123408*x^8 - 914152*x^7 + 157
8888*x^6 - 3293072*x^5 + 396480*x^4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(1
5550*x^8 - 118051*x^7 + 244047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3
 + 1209600*x^2 - 1036800*x) + 3276288*x) - 345420120727353550*sqrt(31)*(254
591*x^8 - 4815126*x^7 + 32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 74219
328*x^3 - 168956928*x^2 - 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^
5 + 2385*x^4 - 3618*x^3 + 2268*x^2 - 1944*x) + 144820224*x))*sqrt((14645990
10050^(1/4)*sqrt(155590)*sqrt(62)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(97
33*x + 29025) - 38758*x + 19292)*sqrt(224510383*sqrt(2) + 388975000) + 6744
561519183110*x^2 + 6056340956001160*sqrt(2)*(2*x^2 - x + 3) - 2078426100809
4890*x + 27528822527278000)/x^2) - 9579853131092463349032125*sqrt(31)*(2828
123*x^8 - 9696916*x^7 + 53385560*x^6 - 142835344*x^5 + 254146592*x^4 - 2493
00096*x^3 + 37981440*x^2 - 7744*sqrt(2)*(1348*x^8 - 2692*x^7 + 9789*x^6 - 1
0070*x^5 + 15569*x^4 - 5568*x^3 + 1080*x^2 + 4320*x - 5184) + 223064064*x -
 94887936))/(2585191*x^8 - 4661200*x^7 + 14191920*x^6 + 490880*x^5 - 135629
44*x^4 + 44249088*x^3 - 34615296*x^2 - 24772608*x + 18579456)) + 11*1464599
010050^(1/4)*sqrt(155590)*sqrt(62)*sqrt(31)*(1944875000*x^2 - 224510383*sqr
t(2)*(5*x^2 + 3*x + 2) + 1166925000*x + 777950000)*sqrt(224510383*sqrt(2) +
 388975000)*log(14708117187500/920561*(1464599010050^(1/4)*sqrt(155590)*sqr
t(62)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(9733*x + 29025) - 38758*x + 19
292)*sqrt(224510383*sqrt(2) + 388975000) + 6744561519183110*x^2 + 605634095
6001160*sqrt(2)*(2*x^2 - x + 3) - 20784261008094890*x + 27528822527278000)/
x^2) - 11*1464599010050^(1/4)*sqrt(155590)*sqrt(62)*sqrt(31)*(1944875000*x^
2 - 224510383*sqrt(2)*(5*x^2 + 3*x + 2) + 1166925000*x + 777950000)*sqrt(22
4510383*sqrt(2) + 388975000)*log(-14708117187500/920561*(1464599010050^(1/4
)*sqrt(155590)*sqrt(62)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(9733*x + 290
25) - 38758*x + 19292)*sqrt(224510383*sqrt(2) + 388975000) - 67445615191831
10*x^2 - 6056340956001160*sqrt(2)*(2*x^2 - x + 3) + 20784261008094890*x - 2
7528822527278000)/x^2) + 634792486776896221210*sqrt(2)*(5*x^2 + 3*x + 2)*lo
g(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 17067869
9669123600*(3100*x^3 - 12555*x^2 + 9289*x + 8996)*sqrt(2*x^2 - x + 3))/(5*x
^2 + 3*x + 2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{15625,[8]%%%}+%%%{%%{[-37500,0]:[1,0,-2]%%},[7]%%%}+%%%{-61250,[6]%%
%}+%%%{%%{[290000,0]:[1,0,-2]%%},[5]%%%}+%%%{140625,[4]%%%}+%%%{%%{[-972500
,0]:[1,0,-2]%%},[3]%%%}+%%%{-345000,[2]%%%}+%%%{%%{[1820000,0]:[1,0,-2]%%},
[1]%%%}+%%%{1960000,[0]%%%} / %%%{50,[8]%%%}+%%%{%%{poly1[-120,0]:[1,0,-2]%
%},[7]%%%}+%%%{-196,[6]%%%}+%%%{%%{poly1[928,0]:[1,0,-2]%%},[5]%%%}+%%%{450
,[4]%%%}+%%%{%%{poly1[-3112,0]:[1,0,-2]%%},[3]%%%}+%%%{-1104,[2]%%%}+%%%{%%
{poly1[5824,0]:[1,0,-2]%%},[1]%%%}+%%%{6272,[0]%%%} Error: Bad Argument Val
ue
```

maple [B] time = 0.16, size = 40028, normalized size = 156.97

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^2,x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\,x^2 - x + 3\right)^{\frac{5}{2}}}{\left(5\,x^2 + 3\,x + 2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^2,x, algorithm="maxima")
```

```
[Out] integrate((2*x^2 - x + 3)^(5/2)/(5*x^2 + 3*x + 2)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(2\,x^2 - x + 3\right)^{5/2}}{\left(5\,x^2 + 3\,x + 2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^(5/2)/(3*x + 5*x^2 + 2)^2,x)
```

```
[Out] int((2*x^2 - x + 3)^(5/2)/(3*x + 5*x^2 + 2)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2x^2 - x + 3\right)^{\frac{5}{2}}}{\left(5x^2 + 3x + 2\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(5/2)/(5*x**2+3*x+2)**2,x)
```

```
[Out] Integral((2*x**2 - x + 3)**(5/2)/(5*x**2 + 3*x + 2)**2, x)
```

3.75 $\int \frac{\left(3-x+2x^2\right)^{5/2}}{\left(2+3x+5x^2\right)^3}\,dx$

Optimal. Leaf size=281

$$\frac{(10x+3)\left(2x^2-x+3\right)^{5/2}}{62\left(5x^2+3x+2\right)^2}+\frac{(2336x+769)\left(2x^2-x+3\right)^{3/2}}{3844\left(5x^2+3x+2\right)}+\frac{(11359-12920x)\sqrt{2x^2-x+3}}{48050}+\frac{\sqrt{11\left(1+4\sqrt{2}\right)}\,\left(\cdots\right.}{\cdots}$$

Rubi [A] time = 0.65, antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.370, Rules used = {971, 1054, 1066, 1076, 619, 215, 1035, 1029, 206, 204}

$$\frac{(10x+3)\left(2x^2-x+3\right)^{5/2}}{62\left(5x^2+3x+2\right)^2}+\frac{(2336x+769)\left(2x^2-x+3\right)^{3/2}}{3844\left(5x^2+3x+2\right)}+\frac{(11359-12920x)\sqrt{2x^2-x+3}}{48050}+\frac{\sqrt{11\left(1+4\sqrt{2}\right)}\left(2937349+1978861\sqrt{2}\right)\tan^{-1}\left(\frac{\sqrt{\frac{11}{62\left(3531015707557+2498852071250\sqrt{2}\right)}}\left(\left(9832420+6895071\sqrt{2}\right)x+2937349\sqrt{2}+3957722\right)}{\sqrt{2x^2-x+3}}\right)}{29791000}-\frac{\left(2937349-1978861\sqrt{2}\right)\sqrt{11\left(4\sqrt{2}-1\right)}\tanh^{-1}\left(\frac{\sqrt{\frac{11}{62\left(2498852071250\sqrt{2}-3531015707557\right)}}\left(\left(9832420-6895071\sqrt{2}\right)x-2937349\sqrt{2}+3957722\right)}{\sqrt{2x^2-x+3}}\right)}{29791000}-\frac{4}{125}\sqrt{2}\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^(5/2)/(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] ((11359 - 12920*x)*Sqrt[3 - x + 2*x^2])/48050 + ((3 + 10*x)*(3 - x + 2*x^2)
^(5/2))/(62*(2 + 3*x + 5*x^2)^2) + ((769 + 2336*x)*(3 - x + 2*x^2)^(3/2))/(
3844*(2 + 3*x + 5*x^2)) - (4*Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]])/125 + (Sq
rt[11*(1 + 4*Sqrt[2])]*(2937349 + 1978861*Sqrt[2])*ArcTan[(Sqrt[11/(62*(353
1015707557 + 2498852071250*Sqrt[2]))]*(3957722 + 2937349*Sqrt[2] + (9832420
 + 6895071*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]])/29791000 - ((2937349 - 197886
1*Sqrt[2])*Sqrt[11*(-1 + 4*Sqrt[2])]*ArcTanh[(Sqrt[11/(62*(-3531015707557 +
 2498852071250*Sqrt[2]))]*(3957722 - 2937349*Sqrt[2] + (9832420 - 6895071*S
qrt[2])*x))/Sqrt[3 - x + 2*x^2]])/29791000
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 971

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((b + 2*c*x)*(a + b*x + c*x^2)^(p + 1)*(d + e
*x + f*x^2)^q)/((b^2 - 4*a*c)*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*(p + 1))
, Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*Simp[2*c*d*(2*p +
 3) + b*e*q + (2*b*f*q + 2*c*e*(2*p + q + 3))*x + 2*c*f*(2*p + 2*q + 3)*x^2
, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
e^2 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0] &&  !IGtQ[q, 0]
```

Rule 1029

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
 c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 1035

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

Rule 1054

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((A*b*c - 2
```

```
*a*B*c + a*b*C - (c*(b*B - 2*A*c) - C*(b^2 - 2*a*c))*x)*(a + b*x + c*x^2)^(
p + 1)*(d + e*x + f*x^2)^q)/(c*(b^2 - 4*a*c)*(p + 1)), x] - Dist[1/(c*(b^2
- 4*a*c)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*
Simp[e*q*(A*b*c - 2*a*B*c + a*b*C) - d*(c*(b*B - 2*A*c)*(2*p + 3) + C*(2*a*
c - b^2*(p + 2))) + (2*f*q*(A*b*c - 2*a*B*c + a*b*C) - e*(c*(b*B - 2*A*c)*(
2*p + q + 3) + C*(2*a*c*(q + 1) - b^2*(p + q + 2))))*x - f*(c*(b*B - 2*A*c)
*(2*p + 2*q + 3) + C*(2*a*c*(2*q + 1) - b^2*(p + 2*q + 2)))*x^2, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2
 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0] &&  !IGtQ[q, 0]
```

Rule 1066

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((B*c*f*(2*
p + 2*q + 3) + C*(b*f*p - c*e*(2*p + q + 2)) + 2*c*C*f*(p + q + 1)*x)*(a +
b*x + c*x^2)^p*(d + e*x + f*x^2)^(q + 1))/(2*c*f^2*(p + q + 1)*(2*p + 2*q +
 3)), x] - Dist[1/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), Int[(a + b*x + c*x
^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[p*(b*d - a*e)*(C*(c*e - b*f)*(q + 1) -
 c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(b^2*C*d*f*p + a*c*(C*(2*d*f
- e^2*(2*p + q + 2)) + f*(B*e - 2*A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*f)
*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*e
*f*p*(b^2 - 4*a*c) - b*c*(C*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e +
2*A*f)*(2*p + 2*q + 3))))*x + (p*(c*e - b*f)*(C*(c*e - b*f)*(q + 1) - c*(C*
e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(b^2 - 4*a*c) - c^2*(C*(e^
2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3))))*x^2,
x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*
q + 3, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

Rule 1076

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqr
t[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$\int \frac{\left(3-x+2x^2\right)^{5/2}}{\left(2+3x+5x^2\right)^3}\,dx = \frac{(3+10x)\left(3-x+2x^2\right)^{5/2}}{62\left(2+3x+5x^2\right)^2} - \frac{1}{62}\int \frac{\left(3-x+2x^2\right)^{3/2}\left(-\frac{195}{2}+35x+40x^2\right)}{\left(2+3x+5x^2\right)^2}\,dx$$

$$= \frac{(3+10x)\left(3-x+2x^2\right)^{5/2}}{62\left(2+3x+5x^2\right)^2} + \frac{(769+2336x)\left(3-x+2x^2\right)^{3/2}}{3844\left(2+3x+5x^2\right)} + \frac{\int \frac{\left(\frac{66735}{4}-7375x-25840x^2\right)\sqrt{3-}}{2+3x+5x^2}}{9610}$$

$$= \frac{(11359-12920x)\sqrt{3-x+2x^2}}{48050} + \frac{(3+10x)\left(3-x+2x^2\right)^{5/2}}{62\left(2+3x+5x^2\right)^2} + \frac{(769+2336x)\left(3-x+\right.}{3844\left(2+3x+5x^2\right.}$$

$$= \frac{(11359-12920x)\sqrt{3-x+2x^2}}{48050} + \frac{(3+10x)\left(3-x+2x^2\right)^{5/2}}{62\left(2+3x+5x^2\right)^2} + \frac{(769+2336x)\left(3-x+\right.}{3844\left(2+3x+5x^2\right.}$$

$$= \frac{(11359-12920x)\sqrt{3-x+2x^2}}{48050} + \frac{(3+10x)\left(3-x+2x^2\right)^{5/2}}{62\left(2+3x+5x^2\right)^2} + \frac{(769+2336x)\left(3-x+\right.}{3844\left(2+3x+5x^2\right.}$$

$$= \frac{(11359-12920x)\sqrt{3-x+2x^2}}{48050} + \frac{(3+10x)\left(3-x+2x^2\right)^{5/2}}{62\left(2+3x+5x^2\right)^2} + \frac{(769+2336x)\left(3-x+\right.}{3844\left(2+3x+5x^2\right.}$$

$$= \frac{(11359-12920x)\sqrt{3-x+2x^2}}{48050} + \frac{(3+10x)\left(3-x+2x^2\right)^{5/2}}{62\left(2+3x+5x^2\right)^2} + \frac{(769+2336x)\left(3-x+\right.}{3844\left(2+3x+5x^2\right.}$$

Mathematica [C] time = 2.00, size = 1009, normalized size = 3.59

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^(5/2)/(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] (153804640*Sqrt[3 - x + 2*x^2] + 474815220*x*Sqrt[3 - x + 2*x^2] + 64020704
0*x^2*Sqrt[3 - x + 2*x^2] + 662597100*x^3*Sqrt[3 - x + 2*x^2] + 1906624*Sqr
t[2]*(2 + 3*x + 5*x^2)^2*ArcSinh[(-1 + 4*x)/Sqrt[23]] - I*Sqrt[286 + (22*I)
*Sqrt[31]]*(-503998*I + 491621*Sqrt[31])*(2 + 3*x + 5*x^2)^2*ArcTanh[(63 +
I*Sqrt[31] - 22*x - (4*I)*Sqrt[31]*x)/(2*Sqrt[286 + (22*I)*Sqrt[31]]*Sqrt[3
```

```
 - x + 2*x^2])] - (1966484*I)*Sqrt[682*(13 - I*Sqrt[31])]*ArcTanh[(-63 + I*
Sqrt[31] + 22*x - (4*I)*Sqrt[31]*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 -
 x + 2*x^2])] + 2015992*Sqrt[286 - (22*I)*Sqrt[31]]*ArcTanh[(-63 + I*Sqrt[3
1] + 22*x - (4*I)*Sqrt[31]*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 - x + 2
*x^2])] - (5899452*I)*Sqrt[682*(13 - I*Sqrt[31])]*x*ArcTanh[(-63 + I*Sqrt[3
1] + 22*x - (4*I)*Sqrt[31]*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 - x + 2
*x^2])] + 6047976*Sqrt[286 - (22*I)*Sqrt[31]]*x*ArcTanh[(-63 + I*Sqrt[31] +
 22*x - (4*I)*Sqrt[31]*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2
])] - (14257009*I)*Sqrt[682*(13 - I*Sqrt[31])]*x^2*ArcTanh[(-63 + I*Sqrt[31
] + 22*x - (4*I)*Sqrt[31]*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*
x^2])] + 14615942*Sqrt[286 - (22*I)*Sqrt[31]]*x^2*ArcTanh[(-63 + I*Sqrt[31]
 + 22*x - (4*I)*Sqrt[31]*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x
^2])] - (14748630*I)*Sqrt[682*(13 - I*Sqrt[31])]*x^3*ArcTanh[(-63 + I*Sqrt[
31] + 22*x - (4*I)*Sqrt[31]*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 - x +
2*x^2])] + 15119940*Sqrt[286 - (22*I)*Sqrt[31]]*x^3*ArcTanh[(-63 + I*Sqrt[3
1] + 22*x - (4*I)*Sqrt[31]*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 - x + 2
*x^2])] - (12290525*I)*Sqrt[682*(13 - I*Sqrt[31])]*x^4*ArcTanh[(-63 + I*Sqr
t[31] + 22*x - (4*I)*Sqrt[31]*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 - x
+ 2*x^2])] + 12599950*Sqrt[286 - (22*I)*Sqrt[31]]*x^4*ArcTanh[(-63 + I*Sqrt
[31] + 22*x - (4*I)*Sqrt[31]*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 - x +
 2*x^2])])/(59582000*(2 + 3*x + 5*x^2)^2)
```

IntegrateAlgebraic [C] time = 1.09, size = 627, normalized size = 2.23

$$\frac{22\text{RootSum}\left[-5\#1^4+6\sqrt{2}\#1^3+17\#1^2-26\sqrt{2}\#1-56\&,\frac{150\#1^2\log\left(-\#1+\sqrt{2x^2-x+3}-\sqrt{2}x\right)+630\sqrt{2}\#1\log\left(-\#1+\sqrt{2x^2-x+3}-\sqrt{2}x\right)+3781\log\left(-\#1+\sqrt{2x^2-x+3}-\sqrt{2}x\right)}{-10\#1^3+9\sqrt{2}\#1^2+17\#1-13\sqrt{2}}\&\right]}{3125}+\frac{11\text{RootSum}\left[-5\#1^4+6\sqrt{2}\#1^3+17\#1^2-26\sqrt{2}\#1-56\&,\frac{1110955025\sqrt{2}\#1^2\log\left(-\#1+\sqrt{2x^2-x+3}-\sqrt{2}x\right)-165870920\#1\log\left(-\#1+\sqrt{2x^2-x+3}-\sqrt{2}x\right)+4978708507\sqrt{2}\log\left(-\#1+\sqrt{2x^2-x+3}-\sqrt{2}x\right)}{-10\#1^3+9\sqrt{2}\#1^2+17\#1-13\sqrt{2}}\&\right]}{1114062500\sqrt{2}}-\frac{11\text{RootSum}\left[-5\#1^4+6\sqrt{2}\#1^3+17\#1^2-26\sqrt{2}\#1-56\&,\frac{55365920925\sqrt{2}\#1^2\log\left(-\#1+\sqrt{2x^2-x+3}-\sqrt{2}x\right)-128644699540\#1\log\left(-\#1+\sqrt{2x^2-x+3}-\sqrt{2}x\right)+492740319684\sqrt{2}\log\left(-\#1+\sqrt{2x^2-x+3}-\sqrt{2}x\right)}{-10\#1^3+9\sqrt{2}\#1^2+17\#1-13\sqrt{2}}\&\right]}{69071875000\sqrt{2}}-\frac{4}{125}\sqrt{2}\log\left(2\sqrt{2}\sqrt{2x^2-x+3}-4x+1\right)+\frac{11\sqrt{2x^2-x+3}\left(97155x^3+93872x^2+69621x+22552\right)}{96100\left(5x^2+3x+2\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(3 - x + 2*x^2)^(5/2)/(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] (11*Sqrt[3 - x + 2*x^2]*(22552 + 69621*x + 93872*x^2 + 97155*x^3))/(96100*(
2 + 3*x + 5*x^2)^2) - (4*Sqrt[2]*Log[1 - 4*x + 2*Sqrt[2]*Sqrt[3 - x + 2*x^2
]])/125 + (22*RootSum[-56 - 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3 - 5*#1
^4 & , (3781*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1] + 630*Sqrt[2]*Log
[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1 + 150*Log[-(Sqrt[2]*x) + Sqrt[
3 - x + 2*x^2] - #1]*#1^2)/(-13*Sqrt[2] + 17*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3)
 & ])/3125 + (11*RootSum[-56 - 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3 - 5
*#1^4 & , (4978708507*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]
- 165870920*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1 + 1110955025*Sq
rt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1^2)/(-13*Sqrt[2] + 17*
#1 + 9*Sqrt[2]*#1^2 - 10*#1^3) & ])/(1114062500*Sqrt[2]) - (11*RootSum[-56
- 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3 - 5*#1^4 & , (492740319684*Sqrt[
2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1] - 128644699540*Log[-(Sqrt[2
]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1 + 55365920925*Sqrt[2]*Log[-(Sqrt[2]*x)
+ Sqrt[3 - x + 2*x^2] - #1]*#1^2)/(-13*Sqrt[2] + 17*#1 + 9*Sqrt[2]*#1^2 - 1
0*#1^3) & ])/(69071875000*Sqrt[2])
```

fricas [B] time = 1.48, size = 2240, normalized size = 7.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

```
[Out] 1/758714159921174808909075728000*(3184949732636*3868444992270541948232^(1/4
)*sqrt(1999081657)*sqrt(62)*sqrt(2)*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)*s
qrt(3531015707557*sqrt(2) + 4997704142500)*arctan(1/45354848806294031039917
896246958932041502 31*(2850690442882*sqrt(1999081657)*(2*3868444992270541948
232^(3/4)*sqrt(62)*(2627559914*x^7 - 10187615527*x^6 + 21362956024*x^5 - 34
451465819*x^4 + 17321103240*x^3 - 8320757400*x^2 - sqrt(2)*(1893366636*x^7
- 7237484076*x^6 + 15226003533*x^5 - 24262105817*x^4 + 12127036096*x^3 - 56
64787848*x^2 - 13367586816*x + 9338025600) - 18676051200*x + 13367586816) +
 61971531367*3868444992270541948232^(1/4)*sqrt(62)*(400116332*x^7 - 6149336
082*x^6 + 32552996440*x^5 - 74427496472*x^4 + 96235107840*x^3 - 61219656000
*x^2 - sqrt(2)*(286685371*x^7 - 4395067059*x^6 + 23180544704*x^5 - 52748573
780*x^4 + 68065744032*x^3 - 42544702944*x^2 - 48625837056*x + 34092306432)
- 68184612864*x + 48625837056))*sqrt(2*x^2 - x + 3)*sqrt(3531015707557*sqrt
(2) + 4997704142500) + 12874924822431853972621854418491567805329688*sqrt(31
)*sqrt(2)*(28180*x^8 - 254666*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^4
- 642048*x^3 - 98496*x^2 - sqrt(2)*(8746*x^8 - 102335*x^7 + 396104*x^6 - 78
3113*x^5 + 1320710*x^4 - 752088*x^3 + 396144*x^2 + 546048*x - 539136) + 115
4304*x - 456192) - sqrt(1999081657/828550919)*(sqrt(1999081657)*(2*38684449
92270541948232^(3/4)*sqrt(62)*(9351066298*x^7 - 13433496653*x^6 + 433103458
23*x^5 - 17374572240*x^4 + 20927636280*x^3 + 18483199488*x^2 - sqrt(2)*(683
9273266*x^7 - 9809465289*x^6 + 31524099699*x^5 - 12024617744*x^4 + 13914887
256*x^3 + 14839341696*x^2 - 14839341696*x) - 18483199488*x) + 61971531367*3
868444992270541948232^(1/4)*sqrt(62)*(1427210918*x^7 - 18462714328*x^6 + 71
210222920*x^5 - 92387041920*x^4 + 119489780160*x^3 + 68726817792*x^2 - sqrt
(2)*(1033310523*x^7 - 13365477772*x^6 + 51521534980*x^5 - 66583614528*x^4 +
 85122955872*x^3 + 53108877312*x^2 - 53108877312*x) - 68726817792*x))*sqrt(
2*x^2 - x + 3)*sqrt(3531015707557*sqrt(2) + 4997704142500) + 45164233298567
21284677540671884*sqrt(31)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578888*x^6 -
 3293072*x^5 + 396480*x^4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15550*x^8 -
 118051*x^7 + 244047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600
*x^2 - 1036800*x) + 3276288*x) + 20529196953894187657625184872 2*sqrt(31)*(2
54591*x^8 - 4815126*x^7 + 32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 742
19328*x^3 - 168956928*x^2 - 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*
x^5 + 2385*x^4 - 3618*x^3 + 2268*x^2 - 1944*x) + 144820224*x))*sqrt(-(38684
44992270541948232^(1/4)*sqrt(1999081657)*sqrt(62)*sqrt(31)*sqrt(2*x^2 - x +
 3)*(sqrt(2)*(2141441*x + 1076175) - 3217616*x - 1065266)*sqrt(353101570755
7*sqrt(2) + 4997704142500) - 1559908774300022055173 74*x^2 - 140073440957553
000872744*sqrt(2)*(2*x^2 - x + 3) + 4807065814679659802678 26*x - 6366974588
```

97968185785200)/x^2) + 14630596389127106787070289111922236142420 1*sqrt(31)*
(2828123*x^8 - 9696916*x^7 + 53385560*x^6 - 142835344*x^5 + 254146592*x^4 -
249300096*x^3 + 37981440*x^2 - 7744*sqrt(2)*(1348*x^8 - 2692*x^7 + 9789*x^
6 - 10070*x^5 + 15569*x^4 - 5568*x^3 + 1080*x^2 + 4320*x - 5184) + 22306406
4*x - 94887936))/(2585191*x^8 - 4661200*x^7 + 14191920*x^6 + 490880*x^5 - 1
3562944*x^4 + 44249088*x^3 - 34615296*x^2 - 24772608*x + 18579456)) + 31849
49732636*3868444992270541948232^(1/4)*sqrt(1999081657)*sqrt(62)*sqrt(2)*(25
*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)*sqrt(3531015707557*sqrt(2) + 49977041425
00)*arctan(1/453548488062940310399178962469589320415023 1*(2850690442882*sqr
t(1999081657)*(2*3868444992270541948232^(3/4)*sqrt(62)*(2627559914*x^7 - 10
187615527*x^6 + 21362956024*x^5 - 34451465819*x^4 + 17321103240*x^3 - 83207
57400*x^2 - sqrt(2)*(1893366636*x^7 - 7237484076*x^6 + 15226003533*x^5 - 24
262105817*x^4 + 12127036096*x^3 - 5664787848*x^2 - 13367586816*x + 93380256
00) - 18676051200*x + 13367586816) + 61971531367*3868444992270541948232^(1/
4)*sqrt(62)*(400116332*x^7 - 6149336082*x^6 + 32552996440*x^5 - 74427496472
*x^4 + 96235107840*x^3 - 61219656000*x^2 - sqrt(2)*(286685371*x^7 - 4395067
059*x^6 + 23180544704*x^5 - 52748573780*x^4 + 68065744032*x^3 - 42544702944
*x^2 - 48625837056*x + 34092306432) - 68184612864*x + 48625837056))*sqrt(2*
x^2 - x + 3)*sqrt(3531015707557*sqrt(2) + 4997704142500) - 1287492482243185
3972621854418491567805329688*sqrt(31)*sqrt(2)*(28180*x^8 - 254666*x^7 + 704
270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - sqrt(2)*(874
6*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^4 - 752088*x^3 + 3
96144*x^2 + 546048*x - 539136) + 1154304*x - 456192) - sqrt(1999081657/8285
50919)*(sqrt(1999081657)*(2*3868444992270541948232^(3/4)*sqrt(62)*(93510662
98*x^7 - 13433496653*x^6 + 43310345823*x^5 - 17374572240*x^4 + 20927636280*
x^3 + 18483199488*x^2 - sqrt(2)*(6839273266*x^7 - 9809465289*x^6 + 31524099
699*x^5 - 12024617744*x^4 + 13914887256*x^3 + 14839341696*x^2 - 14839341696
*x) - 18483199488*x) + 61971531367*3868444992270541948232^(1/4)*sqrt(62)*(1
427210918*x^7 - 18462714328*x^6 + 71210222920*x^5 - 92387041920*x^4 + 11948
9780160*x^3 + 68726817792*x^2 - sqrt(2)*(1033310523*x^7 - 13365477772*x^6 +
51521534980*x^5 - 66583614528*x^4 + 85122955872*x^3 + 53108877312*x^2 - 53
108877312*x) - 68726817792*x))*sqrt(2*x^2 - x + 3)*sqrt(3531015707557*sqrt(
2) + 4997704142500) - 4516423329856721284677540671884*sqrt(31)*sqrt(2)*(123
408*x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 + 396480*x^4 + 798336*x^3
- 3822336*x^2 - sqrt(2)*(15550*x^8 - 118051*x^7 + 244047*x^6 - 707374*x^5 +
1053960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*x) + 3276288*x) - 205291
96953894187657625184872 2*sqrt(31)*(254591*x^8 - 4815126*x^7 + 32303580*x^6
- 90866808*x^5 + 108781920*x^4 - 74219328*x^3 - 168956928*x^2 - 15488*sqrt(
2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3618*x^3 + 2268*x^2 -
1944*x) + 144820224*x))*sqrt((3868444992270541948232^(1/4)*sqrt(1999081657)
*sqrt(62)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(2141441*x + 1076175) - 321
7616*x - 1065266)*sqrt(3531015707557*sqrt(2) + 4997704142500) + 15599087743
0002205517374*x^2 + 14007344095755300087274 4*sqrt(2)*(2*x^2 - x + 3) - 4807
06581467965980267826*x + 636697458897968185785200)/x^2) - 14630596389127106
7870702891119222361424201*sqrt(31)*(2828123*x^8 - 9696916*x^7 + 53385560*x^

```
6 - 142835344*x^5 + 254146592*x^4 - 249300096*x^3 + 37981440*x^2 - 7744*sqr
t(2)*(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^4 - 5568*x^3 + 1
080*x^2 + 4320*x - 5184) + 223064064*x - 94887936))/(2585191*x^8 - 4661200*
x^7 + 14191920*x^6 + 490880*x^5 - 13562944*x^4 + 44249088*x^3 - 34615296*x^
2 - 24772608*x + 18579456)) + 3868444992270541948232^(1/4)*sqrt(1999081657)
*sqrt(62)*sqrt(31)*(124942603562500*x^4 + 149931124275000*x^3 + 14493342013
2500*x^2 - 3531015707557*sqrt(2)*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4) + 59
972449710000*x + 19990816570000)*sqrt(3531015707557*sqrt(2) + 4997704142500
)*log(3123565089062500/828550919*(3868444992270541948232^(1/4)*sqrt(1999081
657)*sqrt(62)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(2141441*x + 1076175) -
 3217616*x - 1065266)*sqrt(3531015707557*sqrt(2) + 4997704142500) + 1559908
77430002205517374*x^2 + 1400734409575530008727444*sqrt(2)*(2*x^2 - x + 3) -
4807065814679659802678266*x + 6366974588979681857852000)/x^2) - 3868444992270
541948232^(1/4)*sqrt(1999081657)*sqrt(62)*sqrt(31)*(124942603562500*x^4 + 1
49931124275000*x^3 + 144933420132500*x^2 - 3531015707557*sqrt(2)*(25*x^4 +
30*x^3 + 29*x^2 + 12*x + 4) + 59972449710000*x + 19990816570000)*sqrt(35310
15707557*sqrt(2) + 4997704142500)*log(-3123565089062500/828550919*(38684449
92270541948232^(1/4)*sqrt(1999081657)*sqrt(62)*sqrt(31)*sqrt(2*x^2 - x + 3)
*(sqrt(2)*(2141441*x + 1076175) - 3217616*x - 1065266)*sqrt(3531015707557*s
qrt(2) + 4997704142500) - 155990877430002205517374*x^2 - 140073440957553000
872744*sqrt(2)*(2*x^2 - x + 3) + 480706581467965980267826*x - 6366974588979
68185785200)/x^2) + 12139426558738796942545211648*sqrt(2)*(25*x^4 + 30*x^3
+ 29*x^2 + 12*x + 4)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2
+ 16*x - 25) + 868455333936828605411012 80*(97155*x^3 + 93872*x^2 + 69621*x
+ 22552)*sqrt(2*x^2 - x + 3))/(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Francis algorithm failure for[-1.0,infinity,
infinity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,infi
nity]Francis algorithm failure for[-1.0,infinity,infinity,infinity,infinity
]proot error [1.0,infinity,infinity,infinity,infinity]Francis algorithm fai
lure for[-1.0,infinity,infinity,infinity,infinity]proot error [1.0,infinity
,infinity,infinity,infinity]Francis algorithm failure for[-1.0,infinity,inf
inity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,infinit
y]Evaluation time: 59.56Done
```

maple [B] time = 0.38, size = 119458, normalized size = 425.12

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^3,x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\,x^2-x+3\right)^{\frac{5}{2}}}{\left(5\,x^2+3\,x+2\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^3,x, algorithm="maxima")
```

```
[Out] integrate((2*x^2 - x + 3)^(5/2)/(5*x^2 + 3*x + 2)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(2\,x^2-x+3\right)^{5/2}}{\left(5\,x^2+3\,x+2\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^(5/2)/(3*x + 5*x^2 + 2)^3,x)
```

```
[Out] int((2*x^2 - x + 3)^(5/2)/(3*x + 5*x^2 + 2)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2x^2-x+3\right)^{\frac{5}{2}}}{\left(5x^2+3x+2\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(5/2)/(5*x**2+3*x+2)**3,x)
```

```
[Out] Integral((2*x**2 - x + 3)**(5/2)/(5*x**2 + 3*x + 2)**3, x)
```

3.76 $\int \frac{(2+3x+5x^2)^4}{\sqrt{3-x+2x^2}}\,dx$

Optimal. Leaf size=185

$$-\frac{15428243\sqrt{2x^2-x+3}\,x^2}{131072}+\frac{1572007407\sqrt{2x^2-x+3}\,x}{7340032}+\frac{16493087661\sqrt{2x^2-x+3}}{29360128}+\frac{625}{16}\sqrt{2x^2-x+3}\,x^7+\frac{57\cdots}{4\cdots}$$

Rubi [A] time = 0.31, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1661, 640, 619, 215}

$$\frac{625}{16}\sqrt{2x^2-x+3}\,x^7+\frac{57375}{448}\sqrt{2x^2-x+3}\,x^6+\frac{2116475\sqrt{2x^2-x+3}\,x^5}{10752}+\frac{686531\sqrt{2x^2-x+3}\,x^4}{6144}-\frac{19750457\sqrt{2x^2-x+3}\,x^3}{229376}-\frac{15428243\sqrt{2x^2-x+3}\,x^2}{131072}+\frac{1572007407\sqrt{2x^2-x+3}\,x}{7340032}+\frac{16493087661\sqrt{2x^2-x+3}}{29360128}+\frac{2899366573\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{8388608\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + 5*x^2)^4/Sqrt[3 - x + 2*x^2],x]
```

```
[Out] (16493087661*Sqrt[3 - x + 2*x^2])/29360128 + (1572007407*x*Sqrt[3 - x + 2*x
^2])/7340032 - (15428243*x^2*Sqrt[3 - x + 2*x^2])/131072 - (19750457*x^3*Sq
rt[3 - x + 2*x^2])/229376 + (686531*x^4*Sqrt[3 - x + 2*x^2])/6144 + (211647
5*x^5*Sqrt[3 - x + 2*x^2])/10752 + (57375*x^6*Sqrt[3 - x + 2*x^2])/448 + (6
25*x^7*Sqrt[3 - x + 2*x^2])/16 + (2899366573*ArcSinh[(1 - 4*x)/Sqrt[23]])/(
8388608*Sqrt[2])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(2+3x+5x^2\right)^4}{\sqrt{3-x+2x^2}}\,dx = \frac{625}{16}x^7\sqrt{3-x+2x^2} + \frac{1}{16}\int \frac{256+1536x+6016x^2+14976x^3+28176x^4+37440x^5}{\sqrt{3-x+2x^2}}$$

$$= \frac{57375}{448}x^6\sqrt{3-x+2x^2} + \frac{625}{16}x^7\sqrt{3-x+2x^2} + \frac{1}{224}\int \frac{3584+21504x+84224x^2+2}{}$$

$$= \frac{2116475x^5\sqrt{3-x+2x^2}}{10752} + \frac{57375}{448}x^6\sqrt{3-x+2x^2} + \frac{625}{16}x^7\sqrt{3-x+2x^2} + \frac{\int \frac{43008+25}{}}{}$$

$$= \frac{686531x^4\sqrt{3-x+2x^2}}{6144} + \frac{2116475x^5\sqrt{3-x+2x^2}}{10752} + \frac{57375}{448}x^6\sqrt{3-x+2x^2} + \frac{625}{16}x^7$$

$$= -\frac{19750457x^3\sqrt{3-x+2x^2}}{229376} + \frac{686531x^4\sqrt{3-x+2x^2}}{6144} + \frac{2116475x^5\sqrt{3-x+2x^2}}{10752} + \frac{5}{}$$

$$= -\frac{15428243x^2\sqrt{3-x+2x^2}}{131072} - \frac{19750457x^3\sqrt{3-x+2x^2}}{229376} + \frac{686531x^4\sqrt{3-x+2x^2}}{6144} +$$

$$= \frac{1572007407x\sqrt{3-x+2x^2}}{7340032} - \frac{15428243x^2\sqrt{3-x+2x^2}}{131072} - \frac{19750457x^3\sqrt{3-x+2x^2}}{229376}$$

$$= \frac{16493087661\sqrt{3-x+2x^2}}{29360128} + \frac{1572007407x\sqrt{3-x+2x^2}}{7340032} - \frac{15428243x^2\sqrt{3-x+2x^2}}{131072}$$

$$= \frac{16493087661\sqrt{3-x+2x^2}}{29360128} + \frac{1572007407x\sqrt{3-x+2x^2}}{7340032} - \frac{15428243x^2\sqrt{3-x+2x^2}}{131072}$$

$$= \frac{16493087661\sqrt{3-x+2x^2}}{29360128} + \frac{1572007407x\sqrt{3-x+2x^2}}{7340032} - \frac{15428243x^2\sqrt{3-x+2x^2}}{131072}$$

Mathematica [A] time = 0.24, size = 75, normalized size = 0.41

$$\frac{4\sqrt{2x^2-x+3}\left(3440640000x^7+11280384000x^6+17338163200x^5+9842108416x^4-7584175488x^3-10367779296x^2+18864088884x+49479262983\right)+60886698033\sqrt{2}\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{352321536}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)^4/Sqrt[3 - x + 2*x^2],x]
```

```
[Out] (4*Sqrt[3 - x + 2*x^2]*(49479262983 + 18864088884*x - 10367779296*x^2 - 758
4175488*x^3 + 9842108416*x^4 + 17338163200*x^5 + 11280384000*x^6 + 34406400
00*x^7) + 60886698033*Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]])/352321536
```

IntegrateAlgebraic [A] time = 0.80, size = 90, normalized size = 0.49

$$\frac{2899366573\log\left(2\sqrt{2}\sqrt{2x^2-x+3}-4x+1\right)}{8388608\sqrt{2}}+\frac{\sqrt{2x^2-x+3}\left(3440640000x^7+11280384000x^6+17338163200x^5+9842108416x^4-7584175488x^3-10367779296x^2+18864088884x+49479262983\right)}{88080384}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 + 3*x + 5*x^2)^4/Sqrt[3 - x + 2*x^2],x]
```

```
[Out] (Sqrt[3 - x + 2*x^2]*(49479262983 + 18864088884*x - 10367779296*x^2 - 75841
75488*x^3 + 9842108416*x^4 + 17338163200*x^5 + 11280384000*x^6 + 3440640000
*x^7))/88080384 + (2899366573*Log[1 - 4*x + 2*Sqrt[2]*Sqrt[3 - x + 2*x^2]])
/(8388608*Sqrt[2])
```

fricas [A] time = 0.43, size = 88, normalized size = 0.48

$$\frac{1}{88080384}\left(3440640000\,x^7+11280384000\,x^6+17338163200\,x^5+9842108416\,x^4-7584175488\,x^3-10367779296\,x^2+18864088884\,x+49479262983\right)\sqrt{2\,x^2-x+3}+\frac{2899366573}{33554432}\sqrt{2}\log\left(4\sqrt{2}\sqrt{2\,x^2-x+3}(4\,x-1)-32\,x^2+16\,x-25\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/88080384*(3440640000*x^7 + 11280384000*x^6 + 17338163200*x^5 + 9842108416
*x^4 - 7584175488*x^3 - 10367779296*x^2 + 18864088884*x + 49479262983)*sqrt
(2*x^2 - x + 3) + 2899366573/33554432*sqrt(2)*log(4*sqrt(2)*sqrt(2*x^2 - x
+ 3)*(4*x - 1) - 32*x^2 + 16*x - 25)
```

giac [A] time = 0.50, size = 83, normalized size = 0.45

$$\frac{1}{88080384}\left(4\left(8\left(4\left(16\left(100\left(120\left(140\,x+459\right)x+84659\right)x+4805717\right)x-59251371\right)x-323993103\right)x+4716022221\right)x+49479262983\right)\sqrt{2\,x^2-x+3}+\frac{2899366573}{16777216}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}\,x-\sqrt{2\,x^2-x+3}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/88080384*(4*(8*(4*(16*(100*(120*(140*x + 459)*x + 84659)*x + 4805717)*x -
 59251371)*x - 323993103)*x + 4716022221)*x + 49479262983)*sqrt(2*x^2 - x +
```

```
 3) + 2899366573/16777216*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 -
x + 3)) + 1)
```

maple [A] time = 0.02, size = 147, normalized size = 0.79

$$\frac{625\sqrt{2x^2-x+3}\,x^7}{16}+\frac{57375\sqrt{2x^2-x+3}\,x^6}{448}+\frac{2116475\sqrt{2x^2-x+3}\,x^5}{10752}+\frac{686531\sqrt{2x^2-x+3}\,x^4}{6144}-\frac{19750457\sqrt{2x^2-x+3}\,x^3}{229376}-\frac{15428243\sqrt{2x^2-x+3}\,x^2}{131072}+\frac{1572007407\sqrt{2x^2-x+3}\,x}{7340032}-\frac{2899366573\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{16777216}+\frac{16493087661\sqrt{2x^2-x+3}}{29360128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)^4/(2*x^2-x+3)^(1/2),x)
```

```
[Out] 16493087661/29360128*(2*x^2-x+3)^(1/2)-2899366573/16777216*2^(1/2)*arcsinh(
4/23*23^(1/2)*(x-1/4))+625/16*x^7*(2*x^2-x+3)^(1/2)+57375/448*x^6*(2*x^2-x+
3)^(1/2)+2116475/10752*x^5*(2*x^2-x+3)^(1/2)+686531/6144*x^4*(2*x^2-x+3)^(1
/2)-19750457/229376*x^3*(2*x^2-x+3)^(1/2)-15428243/131072*x^2*(2*x^2-x+3)^(
1/2)+1572007407/7340032*x*(2*x^2-x+3)^(1/2)
```

maxima [A] time = 1.00, size = 148, normalized size = 0.80

$$\frac{625}{16}\sqrt{2x^2-x+3}\,x^7+\frac{57375}{448}\sqrt{2x^2-x+3}\,x^6+\frac{2116475}{10752}\sqrt{2x^2-x+3}\,x^5+\frac{686531}{6144}\sqrt{2x^2-x+3}\,x^4-\frac{19750457}{229376}\sqrt{2x^2-x+3}\,x^3-\frac{15428243}{131072}\sqrt{2x^2-x+3}\,x^2+\frac{1572007407}{7340032}\sqrt{2x^2-x+3}\,x-\frac{2899366573}{16777216}\sqrt{2}\,\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}\,(4x-1)\right)+\frac{16493087661}{29360128}\sqrt{2x^2-x+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^(1/2),x, algorithm="maxima")
```

```
[Out] 625/16*sqrt(2*x^2 - x + 3)*x^7 + 57375/448*sqrt(2*x^2 - x + 3)*x^6 + 211647
5/10752*sqrt(2*x^2 - x + 3)*x^5 + 686531/6144*sqrt(2*x^2 - x + 3)*x^4 - 197
50457/229376*sqrt(2*x^2 - x + 3)*x^3 - 15428243/131072*sqrt(2*x^2 - x + 3)*
x^2 + 1572007407/7340032*sqrt(2*x^2 - x + 3)*x - 2899366573/16777216*sqrt(2
)*arcsinh(1/23*sqrt(23)*(4*x - 1)) + 16493087661/29360128*sqrt(2*x^2 - x +
3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(5\,x^2+3\,x+2\right)^4}{\sqrt{2\,x^2-x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 5*x^2 + 2)^4/(2*x^2 - x + 3)^(1/2),x)
```

```
[Out] int((3*x + 5*x^2 + 2)^4/(2*x^2 - x + 3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(5x^2+3x+2\right)^4}{\sqrt{2x^2-x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)**4/(2*x**2-x+3)**(1/2),x)
[Out] Integral((5*x**2 + 3*x + 2)**4/sqrt(2*x**2 - x + 3), x)
```

3.77 $\int \frac{(2+3x+5x^2)^3}{\sqrt{3-x+2x^2}} \, dx$

Optimal. Leaf size=143

$$-\frac{3387\sqrt{2x^2-x+3}\,x^2}{1024}-\frac{372783\sqrt{2x^2-x+3}\,x}{8192}-\frac{203373\sqrt{2x^2-x+3}}{32768}+\frac{125}{12}\sqrt{2x^2-x+3}\,x^5+\frac{1355}{48}\sqrt{2x^2-x+$$

Rubi [A] time = 0.17, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1661, 640, 619, 215}

$$\frac{125}{12}\sqrt{2x^2-x+3}\,x^5+\frac{1355}{48}\sqrt{2x^2-x+3}\,x^4+\frac{8185}{256}\sqrt{2x^2-x+3}\,x^3-\frac{3387\sqrt{2x^2-x+3}\,x^2}{1024}-\frac{372783\sqrt{2x^2-x+3}\,x}{8192}-\frac{203373\sqrt{2x^2-x+3}}{32768}-\frac{9267707\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{65536\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + 5*x^2)^3/Sqrt[3 - x + 2*x^2],x]
```

```
[Out] (-203373*Sqrt[3 - x + 2*x^2])/32768 - (372783*x*Sqrt[3 - x + 2*x^2])/8192 -
 (3387*x^2*Sqrt[3 - x + 2*x^2])/1024 + (8185*x^3*Sqrt[3 - x + 2*x^2])/256 +
 (1355*x^4*Sqrt[3 - x + 2*x^2])/48 + (125*x^5*Sqrt[3 - x + 2*x^2])/12 - (92
67707*ArcSinh[(1 - 4*x)/Sqrt[23]])/(65536*Sqrt[2])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(2+3x+5x^2\right)^3}{\sqrt{3-x+2x^2}}\,dx = \frac{125}{12}x^5\sqrt{3-x+2x^2} + \frac{1}{12}\int \frac{96+432x+1368x^2+2484x^3+1545x^4+\frac{6775x^5}{2}}{\sqrt{3-x+2x^2}}\,dx$$

$$= \frac{1355}{48}x^4\sqrt{3-x+2x^2} + \frac{125}{12}x^5\sqrt{3-x+2x^2} + \frac{1}{120}\int \frac{960+4320x+13680x^2-15810x}{\sqrt{3-x+2x^2}}$$

$$= \frac{8185}{256}x^3\sqrt{3-x+2x^2} + \frac{1355}{48}x^4\sqrt{3-x+2x^2} + \frac{125}{12}x^5\sqrt{3-x+2x^2} + \frac{1}{960}\int \frac{7680+3}{}$$

$$= -\frac{3387x^2\sqrt{3-x+2x^2}}{1024} + \frac{8185}{256}x^3\sqrt{3-x+2x^2} + \frac{1355}{48}x^4\sqrt{3-x+2x^2} + \frac{125}{12}x^5\sqrt{3-x}$$

$$= -\frac{372783x\sqrt{3-x+2x^2}}{8192} - \frac{3387x^2\sqrt{3-x+2x^2}}{1024} + \frac{8185}{256}x^3\sqrt{3-x+2x^2} + \frac{1355}{48}x^4\sqrt{3-}$$

$$= -\frac{203373\sqrt{3-x+2x^2}}{32768} - \frac{372783x\sqrt{3-x+2x^2}}{8192} - \frac{3387x^2\sqrt{3-x+2x^2}}{1024} + \frac{8185}{256}x^3\sqrt{3-}$$

$$= -\frac{203373\sqrt{3-x+2x^2}}{32768} - \frac{372783x\sqrt{3-x+2x^2}}{8192} - \frac{3387x^2\sqrt{3-x+2x^2}}{1024} + \frac{8185}{256}x^3\sqrt{3-}$$

$$= -\frac{203373\sqrt{3-x+2x^2}}{32768} - \frac{372783x\sqrt{3-x+2x^2}}{8192} - \frac{3387x^2\sqrt{3-x+2x^2}}{1024} + \frac{8185}{256}x^3\sqrt{3-}$$

Mathematica [A] time = 0.13, size = 65, normalized size = 0.45

$$\frac{4\sqrt{2x^2-x+3}\left(1024000x^5+2775040x^4+3143040x^3-325152x^2-4473396x-610119\right)-27803121\sqrt{2}\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{393216}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)^3/Sqrt[3 - x + 2*x^2],x]
```

```
[Out] (4*Sqrt[3 - x + 2*x^2]*(-610119 - 4473396*x - 325152*x^2 + 3143040*x^3 + 27
75040*x^4 + 1024000*x^5) - 27803121*Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]])/39
3216
```

IntegrateAlgebraic [A] time = 0.60, size = 80, normalized size = 0.56

$$\frac{\sqrt{2x^2-x+3}\left(1024000x^5+2775040x^4+3143040x^3-325152x^2-4473396x-610119\right)}{98304}-\frac{9267707\log\left(2\sqrt{2}\sqrt{2x^2-x+3}-4x+1\right)}{65536\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 + 3*x + 5*x^2)^3/Sqrt[3 - x + 2*x^2],x]
```

```
[Out] (Sqrt[3 - x + 2*x^2]*(-610119 - 4473396*x - 325152*x^2 + 3143040*x^3 + 2775
040*x^4 + 1024000*x^5))/98304 - (9267707*Log[1 - 4*x + 2*Sqrt[2]*Sqrt[3 - x
 + 2*x^2]])/(65536*Sqrt[2])
```

fricas [A] time = 0.41, size = 78, normalized size = 0.55

$$\frac{1}{98304}\left(1024000\,x^5+2775040\,x^4+3143040\,x^3-325152\,x^2-4473396\,x-610119\right)\sqrt{2\,x^2-x+3}+\frac{9267707}{262144}\,\sqrt{2}\,\log\left(-4\,\sqrt{2}\,\sqrt{2\,x^2-x+3}\,(4\,x-1)-32\,x^2+16\,x-25\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/98304*(1024000*x^5 + 2775040*x^4 + 3143040*x^3 - 325152*x^2 - 4473396*x -
 610119)*sqrt(2*x^2 - x + 3) + 9267707/262144*sqrt(2)*log(-4*sqrt(2)*sqrt(2
*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25)
```

giac [A] time = 0.52, size = 73, normalized size = 0.51

$$\frac{1}{98304}\,(4\,(8\,(20\,(16\,(100\,x+271)x+4911)x-10161)x-1118349)x-610119)\sqrt{2\,x^2-x+3}-\frac{9267707}{131072}\,\sqrt{2}\,\log\left(-2\,\sqrt{2}\left(\sqrt{2}\,x-\sqrt{2\,x^2-x+3}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/98304*(4*(8*(20*(16*(100*x + 271)*x + 4911)*x - 10161)*x - 1118349)*x - 6
10119)*sqrt(2*x^2 - x + 3) - 9267707/131072*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)
*x - sqrt(2*x^2 - x + 3)) + 1)
```

maple [A] time = 0.01, size = 113, normalized size = 0.79

$$\frac{125\sqrt{2x^2-x+3}\,x^5}{12}+\frac{1355\sqrt{2x^2-x+3}\,x^4}{48}+\frac{8185\sqrt{2x^2-x+3}\,x^3}{256}-\frac{3387\sqrt{2x^2-x+3}\,x^2}{1024}-\frac{372783\sqrt{2x^2-x+3}\,x}{8192}+\frac{9267707\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{131072}-\frac{203373\sqrt{2x^2-x+3}}{32768}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)^3/(2*x^2-x+3)^(1/2),x)
```

```
[Out] -203373/32768*(2*x^2-x+3)^(1/2)+9267707/131072*2^(1/2)*arcsinh(4/23*23^(1/2
)*(x-1/4))+125/12*(2*x^2-x+3)^(1/2)*x^5+1355/48*(2*x^2-x+3)^(1/2)*x^4+8185/
256*(2*x^2-x+3)^(1/2)*x^3-3387/1024*(2*x^2-x+3)^(1/2)*x^2-372783/8192*(2*x^
2-x+3)^(1/2)*x
```

maxima [A] time = 0.98, size = 114, normalized size = 0.80

$$\frac{125}{12}\sqrt{2x^2-x+3}\,x^5+\frac{1355}{48}\sqrt{2x^2-x+3}\,x^4+\frac{8185}{256}\sqrt{2x^2-x+3}\,x^3-\frac{3387}{1024}\sqrt{2x^2-x+3}\,x^2-\frac{372783}{8192}\sqrt{2x^2-x+3}\,x+\frac{9267707}{131072}\sqrt{2}\,\mathrm{arsinh}\left(\frac{1}{23}\sqrt{23}(4x-1)\right)-\frac{203373}{32768}\sqrt{2x^2-x+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^(1/2),x, algorithm="maxima")
```

```
[Out] 125/12*sqrt(2*x^2 - x + 3)*x^5 + 1355/48*sqrt(2*x^2 - x + 3)*x^4 + 8185/256
*sqrt(2*x^2 - x + 3)*x^3 - 3387/1024*sqrt(2*x^2 - x + 3)*x^2 - 372783/8192*
sqrt(2*x^2 - x + 3)*x + 9267707/131072*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x -
 1)) - 203373/32768*sqrt(2*x^2 - x + 3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(5\,x^2+3\,x+2\right)^3}{\sqrt{2\,x^2-x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 5*x^2 + 2)^3/(2*x^2 - x + 3)^(1/2),x)
```

```
[Out] int((3*x + 5*x^2 + 2)^3/(2*x^2 - x + 3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(5x^2+3x+2\right)^3}{\sqrt{2x^2-x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)**3/(2*x**2-x+3)**(1/2),x)
```

```
[Out] Integral((5*x**2 + 3*x + 2)**3/sqrt(2*x**2 - x + 3), x)
```

3.78 $\int \frac{(2+3x+5x^2)^2}{\sqrt{3-x+2x^2}}\,dx$

Optimal. Leaf size=101

$$\frac{655}{96}\sqrt{2x^2-x+3}\,x^2+\frac{3443}{768}\sqrt{2x^2-x+3}\,x-\frac{11373\sqrt{2x^2-x+3}}{1024}+\frac{25}{8}\sqrt{2x^2-x+3}\,x^3+\frac{30725\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{2048\sqrt{2}}$$

Rubi [A] time = 0.09, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1661, 640, 619, 215}

$$\frac{25}{8}\sqrt{2x^2-x+3}\,x^3+\frac{655}{96}\sqrt{2x^2-x+3}\,x^2+\frac{3443}{768}\sqrt{2x^2-x+3}\,x-\frac{11373\sqrt{2x^2-x+3}}{1024}+\frac{30725\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{2048\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + 5*x^2)^2/Sqrt[3 - x + 2*x^2],x]
```

```
[Out] (-11373*Sqrt[3 - x + 2*x^2])/1024 + (3443*x*Sqrt[3 - x + 2*x^2])/768 + (655
*x^2*Sqrt[3 - x + 2*x^2])/96 + (25*x^3*Sqrt[3 - x + 2*x^2])/8 + (30725*ArcS
inh[(1 - 4*x)/Sqrt[23]])/(2048*Sqrt[2])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
```

```
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{(2+3x+5x^2)^2}{\sqrt{3-x+2x^2}}\,dx = \frac{25}{8}x^3\sqrt{3-x+2x^2} + \frac{1}{8}\int \frac{32+96x+7x^2+\frac{655x^3}{2}}{\sqrt{3-x+2x^2}}\,dx$$

$$= \frac{655}{96}x^2\sqrt{3-x+2x^2} + \frac{25}{8}x^3\sqrt{3-x+2x^2} + \frac{1}{48}\int \frac{192-1389x+\frac{3443x^2}{4}}{\sqrt{3-x+2x^2}}\,dx$$

$$= \frac{3443}{768}x\sqrt{3-x+2x^2} + \frac{655}{96}x^2\sqrt{3-x+2x^2} + \frac{25}{8}x^3\sqrt{3-x+2x^2} + \frac{1}{192}\int \frac{-\frac{7257}{4}-\frac{3411}{8}}{\sqrt{3-x+2x}}$$

$$= -\frac{11373\sqrt{3-x+2x^2}}{1024} + \frac{3443}{768}x\sqrt{3-x+2x^2} + \frac{655}{96}x^2\sqrt{3-x+2x^2} + \frac{25}{8}x^3\sqrt{3-x+2x}$$

$$= -\frac{11373\sqrt{3-x+2x^2}}{1024} + \frac{3443}{768}x\sqrt{3-x+2x^2} + \frac{655}{96}x^2\sqrt{3-x+2x^2} + \frac{25}{8}x^3\sqrt{3-x+2x}$$

$$= -\frac{11373\sqrt{3-x+2x^2}}{1024} + \frac{3443}{768}x\sqrt{3-x+2x^2} + \frac{655}{96}x^2\sqrt{3-x+2x^2} + \frac{25}{8}x^3\sqrt{3-x+2x}$$

Mathematica [A] time = 0.08, size = 55, normalized size = 0.54

$$\frac{4\sqrt{2x^2-x+3}\,(9600x^3+20960x^2+13772x-34119)+92175\sqrt{2}\,\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{12288}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)^2/Sqrt[3 - x + 2*x^2],x]
```

```
[Out] (4*Sqrt[3 - x + 2*x^2]*(-34119 + 13772*x + 20960*x^2 + 9600*x^3) + 92175*Sq
rt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]])/12288
```

IntegrateAlgebraic [A] time = 0.49, size = 70, normalized size = 0.69

$$\frac{30725\log\left(2\sqrt{2}\sqrt{2x^2-x+3}-4x+1\right)}{2048\sqrt{2}} + \frac{\sqrt{2x^2-x+3}\,(9600x^3+20960x^2+13772x-34119)}{3072}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 + 3*x + 5*x^2)^2/Sqrt[3 - x + 2*x^2],x]
```

```
[Out] (Sqrt[3 - x + 2*x^2]*(-34119 + 13772*x + 20960*x^2 + 9600*x^3))/3072 + (307
25*Log[1 - 4*x + 2*Sqrt[2]*Sqrt[3 - x + 2*x^2]])/(2048*Sqrt[2])
```

fricas [A] time = 0.41, size = 68, normalized size = 0.67

$$\frac{1}{3072}\left(9600\,x^3+20960\,x^2+13772\,x-34119\right)\sqrt{2\,x^2-x+3}+\frac{30725}{8192}\,\sqrt{2}\,\log\left(4\,\sqrt{2}\,\sqrt{2\,x^2-x+3}\,(4\,x-1)-32\,x^2+16\,x-25\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3072*(9600*x^3 + 20960*x^2 + 13772*x - 34119)*sqrt(2*x^2 - x + 3) + 30725
/8192*sqrt(2)*log(4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x -
 25)
```

giac [A] time = 0.49, size = 63, normalized size = 0.62

$$\frac{1}{3072}\,(4\,(40\,(60\,x+131)x+3443)x-34119)\sqrt{2\,x^2-x+3}+\frac{30725}{4096}\,\sqrt{2}\,\log\left(-2\,\sqrt{2}\left(\sqrt{2}\,x-\sqrt{2\,x^2-x+3}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3072*(4*(40*(60*x + 131)*x + 3443)*x - 34119)*sqrt(2*x^2 - x + 3) + 30725
/4096*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1)
```

maple [A] time = 0.01, size = 79, normalized size = 0.78

$$\frac{25\sqrt{2x^2-x+3}\;x^3}{8}+\frac{655\sqrt{2x^2-x+3}\;x^2}{96}+\frac{3443\sqrt{2x^2-x+3}\;x}{768}-\frac{30725\sqrt{2}\;\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{4096}-\frac{11373\sqrt{2x^2-x+3}}{1024}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)^2/(2*x^2-x+3)^(1/2),x)
```

```
[Out] 25/8*(2*x^2-x+3)^(1/2)*x^3+655/96*(2*x^2-x+3)^(1/2)*x^2+3443/768*(2*x^2-x+3
)^(1/2)*x-11373/1024*(2*x^2-x+3)^(1/2)-30725/4096*2^(1/2)*arcsinh(4/23*23^(
1/2)*(x-1/4))
```

maxima [A] time = 0.97, size = 80, normalized size = 0.79

$$\frac{25}{8}\,\sqrt{2\,x^2-x+3}\,x^3+\frac{655}{96}\,\sqrt{2\,x^2-x+3}\,x^2+\frac{3443}{768}\,\sqrt{2\,x^2-x+3}\,x-\frac{30725}{4096}\,\sqrt{2}\,\operatorname{arsinh}\left(\frac{1}{23}\,\sqrt{23}\,(4\,x-1)\right)-\frac{11373}{1024}\,\sqrt{2\,x^2-x+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^(1/2),x, algorithm="maxima")
```

```
[Out] 25/8*sqrt(2*x^2 - x + 3)*x^3 + 655/96*sqrt(2*x^2 - x + 3)*x^2 + 3443/768*sq
rt(2*x^2 - x + 3)*x - 30725/4096*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) -
 11373/1024*sqrt(2*x^2 - x + 3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(5\,x^2+3\,x+2\right)^2}{\sqrt{2\,x^2-x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 5*x^2 + 2)^2/(2*x^2 - x + 3)^(1/2),x)
```

```
[Out] int((3*x + 5*x^2 + 2)^2/(2*x^2 - x + 3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(5x^2+3x+2\right)^2}{\sqrt{2x^2-x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)**2/(2*x**2-x+3)**(1/2),x)
```

```
[Out] Integral((5*x**2 + 3*x + 2)**2/sqrt(2*x**2 - x + 3), x)
```

3.79 $\int \frac{2+3x+5x^2}{\sqrt{3-x+2x^2}}\,dx$

Optimal. Leaf size=59

$$\frac{5}{4}\sqrt{2x^2-x+3}\,x+\frac{39}{16}\sqrt{2x^2-x+3}+\frac{17\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{32\sqrt{2}}$$

Rubi [A] time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1661, 640, 619, 215}

$$\frac{5}{4}\sqrt{2x^2-x+3}\,x+\frac{39}{16}\sqrt{2x^2-x+3}+\frac{17\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{32\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + 5*x^2)/Sqrt[3 - x + 2*x^2],x]
```

```
[Out] (39*Sqrt[3 - x + 2*x^2])/16 + (5*x*Sqrt[3 - x + 2*x^2])/4 + (17*ArcSinh[(1
- 4*x)/Sqrt[23]])/(32*Sqrt[2])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
```

```
c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{2+3x+5x^2}{\sqrt{3-x+2x^2}}\,dx &= \frac{5}{4}x\sqrt{3-x+2x^2} + \frac{1}{4}\int \frac{-7+\frac{39x}{2}}{\sqrt{3-x+2x^2}}\,dx\\ &= \frac{39}{16}\sqrt{3-x+2x^2} + \frac{5}{4}x\sqrt{3-x+2x^2} - \frac{17}{32}\int \frac{1}{\sqrt{3-x+2x^2}}\,dx\\ &= \frac{39}{16}\sqrt{3-x+2x^2} + \frac{5}{4}x\sqrt{3-x+2x^2} - \frac{17\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{23}}}\,dx, x, -1+4x\right)}{32\sqrt{46}}\\ &= \frac{39}{16}\sqrt{3-x+2x^2} + \frac{5}{4}x\sqrt{3-x+2x^2} + \frac{17\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{32\sqrt{2}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 45, normalized size = 0.76

$$\frac{1}{64}\left(4\sqrt{2x^2-x+3}\,(20x+39)+17\sqrt{2}\,\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)/Sqrt[3 - x + 2*x^2],x]
[Out] (4*(39 + 20*x)*Sqrt[3 - x + 2*x^2] + 17*Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]]
)/64
```

IntegrateAlgebraic [A] time = 0.28, size = 60, normalized size = 1.02

$$\frac{1}{16}\sqrt{2x^2-x+3}\,(20x+39)+\frac{17\log\left(2\sqrt{2}\,\sqrt{2x^2-x+3}-4x+1\right)}{32\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 + 3*x + 5*x^2)/Sqrt[3 - x + 2*x^2],x]
```

```
[Out] ((39 + 20*x)*Sqrt[3 - x + 2*x^2])/16 + (17*Log[1 - 4*x + 2*Sqrt[2]*Sqrt[3 -
 x + 2*x^2]])/(32*Sqrt[2])
```

fricas [A] time = 0.41, size = 58, normalized size = 0.98

$$\frac{1}{16}\sqrt{2x^2-x+3}\,(20x+39)+\frac{17}{128}\sqrt{2}\log\left(4\sqrt{2}\sqrt{2x^2-x+3}\,(4x-1)-32x^2+16x-25\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)/(2*x^2-x+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/16*sqrt(2*x^2 - x + 3)*(20*x + 39) + 17/128*sqrt(2)*log(4*sqrt(2)*sqrt(2*
x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25)
```

giac [A] time = 0.53, size = 53, normalized size = 0.90

$$\frac{1}{16}\sqrt{2x^2-x+3}\,(20x+39)+\frac{17}{64}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)/(2*x^2-x+3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/16*sqrt(2*x^2 - x + 3)*(20*x + 39) + 17/64*sqrt(2)*log(-2*sqrt(2)*(sqrt(2
)*x - sqrt(2*x^2 - x + 3)) + 1)
```

maple [A] time = 0.01, size = 45, normalized size = 0.76

$$\frac{5\sqrt{2x^2-x+3}\,x}{4}-\frac{17\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{64}+\frac{39\sqrt{2x^2-x+3}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)/(2*x^2-x+3)^(1/2),x)
```

```
[Out] 5/4*(2*x^2-x+3)^(1/2)*x+39/16*(2*x^2-x+3)^(1/2)-17/64*2^(1/2)*arcsinh(4/23*
23^(1/2)*(x-1/4))
```

maxima [A] time = 0.96, size = 46, normalized size = 0.78

$$\frac{5}{4}\sqrt{2x^2-x+3}\,x-\frac{17}{64}\sqrt{2}\,\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}\,(4x-1)\right)+\frac{39}{16}\sqrt{2x^2-x+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)/(2*x^2-x+3)^(1/2),x, algorithm="maxima")
```

```
[Out] 5/4*sqrt(2*x^2 - x + 3)*x - 17/64*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1))
+ 39/16*sqrt(2*x^2 - x + 3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{5\,x^2+3\,x+2}{\sqrt{2\,x^2-x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 5*x^2 + 2)/(2*x^2 - x + 3)^(1/2),x)
```

```
[Out] int((3*x + 5*x^2 + 2)/(2*x^2 - x + 3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5x^2+3x+2}{\sqrt{2x^2-x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)/(2*x**2-x+3)**(1/2),x)
```

```
[Out] Integral((5*x**2 + 3*x + 2)/sqrt(2*x**2 - x + 3), x)
```

3.80 $\int \frac{1}{\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)}\,dx$

Optimal. Leaf size=148

$$\sqrt{\frac{1}{682}\left(13+10\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{11}{31\left(13+10\sqrt{2}\right)}}\left(\left(13+10\sqrt{2}\right)x+3\sqrt{2}+7\right)}{\sqrt{2x^2-x+3}}\right)-\sqrt{\frac{1}{682}\left(10\sqrt{2}-13\right)}\tanh^{-1}\left(\sqrt{\frac{11}{31(10}}\right.$$

Rubi [A] time = 0.31, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {986, 1029, 204, 206}

$$\sqrt{\frac{1}{682}\left(13+10\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{11}{31\left(13+10\sqrt{2}\right)}}\left(\left(13+10\sqrt{2}\right)x+3\sqrt{2}+7\right)}{\sqrt{2x^2-x+3}}\right)-\sqrt{\frac{1}{682}\left(10\sqrt{2}-13\right)}\tanh^{-1}\left(\frac{\sqrt{\frac{11}{31\left(10\sqrt{2}-13\right)}}\left(\left(13-10\sqrt{2}\right)x-3\sqrt{2}+7\right)}{\sqrt{2x^2-x+3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)),x]
```

```
[Out] Sqrt[(13 + 10*Sqrt[2])/682]*ArcTan[(Sqrt[11/(31*(13 + 10*Sqrt[2]))]*(7 + 3*
Sqrt[2] + (13 + 10*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]] - Sqrt[(-13 + 10*Sqrt[
2])/682]*ArcTanh[(Sqrt[11/(31*(-13 + 10*Sqrt[2]))]*(7 - 3*Sqrt[2] + (13 - 1
0*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 986

```
Int[1/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*
(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f)
, 2]}, Dist[1/(2*q), Int[(c*d - a*f + q + (c*e - b*f)*x)/((a + b*x + c*x^2)
*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[(c*d - a*f - q + (c*e -
 b*f)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b,
```

```
c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[c*e -
b*f, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1029

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
 c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)}\,dx = -\frac{\int \frac{11-11\sqrt{2}-11x}{\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)}\,dx}{22\sqrt{2}}+\frac{\int \frac{11+11\sqrt{2}-11x}{\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)}\,dx}{22\sqrt{2}}$$

$$= -\left(\frac{1}{2}\left(11\left(20-13\sqrt{2}\right)\right)\text{Subst}\left(\int \frac{1}{-3751\left(13-10\sqrt{2}\right)-11x^2}\,dx, x, \frac{11\left(7-\right.}{}\right.\right.$$

$$= \sqrt{\frac{1}{682}\left(13+10\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{11}{31\left(13+10\sqrt{2}\right)}}\left(7+3\sqrt{2}+\left(13+10\sqrt{2}\right)x\right)}{\sqrt{3-x+2x^2}}\right)-$$

Mathematica [C] time = 0.30, size = 176, normalized size = 1.19

$$-\frac{\sqrt{13+i\sqrt{31}}\left(\sqrt{31}+13i\right)\tanh^{-1}\left(\frac{\left(-22-4i\sqrt{31}\right)x+i\sqrt{31}+63}{2\sqrt{286+22i\sqrt{31}}\sqrt{2x^2-x+3}}\right)+\sqrt{13-i\sqrt{31}}\left(\sqrt{31}-13i\right)\tanh^{-1}\left(\frac{\left(-22+4i\sqrt{31}\right)x-i\sqrt{31}+63}{2\sqrt{286-22i\sqrt{31}}\sqrt{2x^2-x+3}}\right)}{20\sqrt{682}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)),x]
```

```
[Out] -1/20*(Sqrt[13 + I*Sqrt[31]]*(13*I + Sqrt[31])*ArcTanh[(63 + I*Sqrt[31] + (
-22 - (4*I)*Sqrt[31])*x)/(2*Sqrt[286 + (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2]
)] + Sqrt[13 - I*Sqrt[31]]*(-13*I + Sqrt[31])*ArcTanh[(63 - I*Sqrt[31] + (-
22 + (4*I)*Sqrt[31])*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])
])/Sqrt[682]
```

IntegrateAlgebraic [C] time = 0.36, size = 135, normalized size = 0.91

$$\text{RootSum}\left[-5\#1^4+6\sqrt{2}\#1^3+17\#1^2-26\sqrt{2}\#1-56\&,\frac{2\sqrt{2}\#1\log\left(-\#1+\sqrt{2x^2-x+3}-\sqrt{2}x\right)+\log\left(-\#1+\sqrt{2x^2-x+3}-\sqrt{2}x\right)}{-10\#1^3+9\sqrt{2}\#1^2+17\#1-13\sqrt{2}}\&\right]$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)),x]
```

```
[Out] RootSum[-56 - 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3 - 5*#1^4 & , (Log[-(
Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1] + 2*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[
3 - x + 2*x^2] - #1]*#1)/(-13*Sqrt[2] + 17*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3) &
 ]
```

fricas [B] time = 1.13, size = 2002, normalized size = 13.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+3*x+2)/(2*x^2-x+3)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/845680*sqrt(341)*200^(1/4)*sqrt(31)*sqrt(5)*sqrt(13*sqrt(2) + 20)*(13*sq
rt(2) - 20)*log(1240*(sqrt(341)*200^(1/4)*sqrt(31)*sqrt(5)*sqrt(2*x^2 - x +
 3)*(sqrt(2)*(4*x - 1) - 3*x - 5)*sqrt(13*sqrt(2) + 20) + 7595*x^2 + 6820*s
qrt(2)*(2*x^2 - x + 3) - 23405*x + 31000)/x^2) + 1/845680*sqrt(341)*200^(1/
4)*sqrt(31)*sqrt(5)*sqrt(13*sqrt(2) + 20)*(13*sqrt(2) - 20)*log(-1240*(sqrt
(341)*200^(1/4)*sqrt(31)*sqrt(5)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(4*x - 1) - 3
*x - 5)*sqrt(13*sqrt(2) + 20) - 7595*x^2 - 6820*sqrt(2)*(2*x^2 - x + 3) + 2
3405*x - 31000)/x^2) - 1/6820*sqrt(341)*200^(1/4)*sqrt(5)*sqrt(2)*sqrt(13*s
qrt(2) + 20)*arctan(1/2762875*(14260*sqrt(341)*sqrt(5)*sqrt(2*x^2 - x + 3)*
(11*200^(3/4)*(8056*x^7 - 28976*x^6 + 61838*x^5 - 93342*x^4 + 45376*x^3 - 1
8288*x^2 - sqrt(2)*(4702*x^7 - 19541*x^6 + 40352*x^5 - 68777*x^4 + 35480*x^
3 - 19080*x^2 - 34560*x + 27648) - 55296*x + 34560) + 5*200^(1/4)*(18463*x^
7 - 280047*x^6 + 1453472*x^5 - 3238500*x^4 + 4140576*x^3 - 2378592*x^2 - sq
rt(2)*(11418*x^7 - 177633*x^6 + 957180*x^5 - 2237548*x^4 + 2920320*x^3 - 20
05920*x^2 - 1990656*x + 1534464) - 3068928*x + 1990656))*sqrt(13*sqrt(2) +
20) + 7843000*sqrt(31)*sqrt(2)*(28180*x^8 - 254666*x^7 + 704270*x^6 - 13852
56*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - sqrt(2)*(8746*x^8 - 102335*
x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^4 - 752088*x^3 + 396144*x^2 + 546
048*x - 539136) + 1154304*x - 456192) - 2*sqrt(310)*(sqrt(341)*sqrt(5)*sqrt
(2*x^2 - x + 3)*(11*200^(3/4)*(30876*x^7 - 44014*x^6 + 139674*x^5 - 42464*x
^4 + 38736*x^3 + 89856*x^2 - sqrt(2)*(15454*x^7 - 22399*x^6 + 73509*x^5 - 3
7360*x^4 + 52200*x^3 + 13824*x^2 - 13824*x) - 89856*x) + 5*200^(1/4)*(69479
*x^7 - 898236*x^6 + 3454740*x^5 - 4394304*x^4 + 5347296*x^3 + 4478976*x^2 -
 sqrt(2)*(38627*x^7 - 500012*x^6 + 1934180*x^5 - 2560320*x^4 + 3506400*x^3
+ 1202688*x^2 - 1202688*x) - 4478976*x))*sqrt(13*sqrt(2) + 20) + 550*sqrt(3
```

1)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 + 396480*x^
4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 118051*x^7 + 244047*x^6
 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*x) + 3276
288*x) + 25*sqrt(31)*(254591*x^8 - 4815126*x^7 + 32303580*x^6 - 90866808*x^
5 + 108781920*x^4 - 74219328*x^3 - 168956928*x^2 - 15488*sqrt(2)*(4*x^8 - 7
6*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3618*x^3 + 2268*x^2 - 1944*x) + 144
820224*x))*sqrt(-(sqrt(341)*200^(1/4)*sqrt(31)*sqrt(5)*sqrt(2*x^2 - x + 3)*
(sqrt(2)*(4*x - 1) - 3*x - 5)*sqrt(13*sqrt(2) + 20) - 7595*x^2 - 6820*sqrt(
2)*(2*x^2 - x + 3) + 23405*x - 31000)/x^2) + 89125*sqrt(31)*(2828123*x^8 -
9696916*x^7 + 53385560*x^6 - 142835344*x^5 + 254146592*x^4 - 249300096*x^3
+ 37981440*x^2 - 7744*sqrt(2)*(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 +
 15569*x^4 - 5568*x^3 + 1080*x^2 + 4320*x - 5184) + 223064064*x - 94887936)
)/(2585191*x^8 - 4661200*x^7 + 14191920*x^6 + 490880*x^5 - 13562944*x^4 + 4
4249088*x^3 - 34615296*x^2 - 24772608*x + 18579456)) - 1/6820*sqrt(341)*200
^(1/4)*sqrt(5)*sqrt(2)*sqrt(13*sqrt(2) + 20)*arctan(1/2762875*(14260*sqrt(3
41)*sqrt(5)*sqrt(2*x^2 - x + 3)*(11*200^(3/4)*(8056*x^7 - 28976*x^6 + 61838
*x^5 - 93342*x^4 + 45376*x^3 - 18288*x^2 - sqrt(2)*(4702*x^7 - 19541*x^6 +
40352*x^5 - 68777*x^4 + 35480*x^3 - 19080*x^2 - 34560*x + 27648) - 55296*x
+ 34560) + 5*200^(1/4)*(18463*x^7 - 280047*x^6 + 1453472*x^5 - 3238500*x^4
+ 4140576*x^3 - 2378592*x^2 - sqrt(2)*(11418*x^7 - 177633*x^6 + 957180*x^5
- 2237548*x^4 + 2920320*x^3 - 2005920*x^2 - 1990656*x + 1534464) - 3068928*
x + 1990656))*sqrt(13*sqrt(2) + 20) - 7843000*sqrt(31)*sqrt(2)*(28180*x^8 -
 254666*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x
^2 - sqrt(2)*(8746*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^4
 - 752088*x^3 + 396144*x^2 + 546048*x - 539136) + 1154304*x - 456192) - 2*s
qrt(310)*(sqrt(341)*sqrt(5)*sqrt(2*x^2 - x + 3)*(11*200^(3/4)*(30876*x^7 -
44014*x^6 + 139674*x^5 - 42464*x^4 + 38736*x^3 + 89856*x^2 - sqrt(2)*(15454
*x^7 - 22399*x^6 + 73509*x^5 - 37360*x^4 + 52200*x^3 + 13824*x^2 - 13824*x)
 - 89856*x) + 5*200^(1/4)*(69479*x^7 - 898236*x^6 + 3454740*x^5 - 4394304*x
^4 + 5347296*x^3 + 4478976*x^2 - sqrt(2)*(38627*x^7 - 500012*x^6 + 1934180*
x^5 - 2560320*x^4 + 3506400*x^3 + 1202688*x^2 - 1202688*x) - 4478976*x))*sq
rt(13*sqrt(2) + 20) - 550*sqrt(31)*sqrt(2)*(123408*x^8 - 914152*x^7 + 15788
88*x^6 - 3293072*x^5 + 396480*x^4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(155
50*x^8 - 118051*x^7 + 244047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 +
 1209600*x^2 - 1036800*x) + 3276288*x) - 25*sqrt(31)*(254591*x^8 - 4815126*
x^7 + 32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 74219328*x^3 - 16895692
8*x^2 - 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 361
8*x^3 + 2268*x^2 - 1944*x) + 144820224*x))*sqrt((sqrt(341)*200^(1/4)*sqrt(3
1)*sqrt(5)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(4*x - 1) - 3*x - 5)*sqrt(13*sqrt(2
) + 20) + 7595*x^2 + 6820*sqrt(2)*(2*x^2 - x + 3) - 23405*x + 31000)/x^2) -
 89125*sqrt(31)*(2828123*x^8 - 9696916*x^7 + 53385560*x^6 - 142835344*x^5 +
 254146592*x^4 - 249300096*x^3 + 37981440*x^2 - 7744*sqrt(2)*(1348*x^8 - 26
92*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^4 - 5568*x^3 + 1080*x^2 + 4320*x -
5184) + 223064064*x - 94887936))/(2585191*x^8 - 4661200*x^7 + 14191920*x^6
+ 490880*x^5 - 13562944*x^4 + 44249088*x^3 - 34615296*x^2 - 24772608*x + 18

```
579456))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+3*x+2)/(2*x^2-x+3)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Francis algorithm failure for[-1.0,infinity,
infinity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,infi
nity]Francis algorithm failure for[-1.0,infinity,infinity,infinity,infinity
]proot error [1.0,infinity,infinity,infinity,infinity]Evaluation time: 7.57
Done
```

maple [B] time = 0.00, size = 684, normalized size = 4.62

$$\frac{\sqrt{\frac{8(x+\sqrt{2}-1)^2}{(-x+\sqrt{2}+1)^2}+\frac{3\sqrt{2}(x+\sqrt{2}-1)^2}{(-x+\sqrt{2}+1)^2}+8-3\sqrt{2}}\,\sqrt{2}\left(465124\sqrt{2}\,\mathrm{arctanh}\left(\frac{31\sqrt{\frac{8(x+\sqrt{2}-1)^2}{(-x+\sqrt{2}+1)^2}+\frac{3\sqrt{2}(x+\sqrt{2}-1)^2}{(-x+\sqrt{2}+1)^2}+8-3\sqrt{2}}}{2\sqrt{-8866+6820\sqrt{2}}}\right)-866822\,\mathrm{arctanh}\left(\frac{31\sqrt{\frac{8(x+\sqrt{2}-1)^2}{(-x+\sqrt{2}+1)^2}+\frac{3\sqrt{2}(x+\sqrt{2}-1)^2}{(-x+\sqrt{2}+1)^2}+8-3\sqrt{2}}}{2\sqrt{-8866+6820\sqrt{2}}}\right)+369\sqrt{2}\sqrt{-8866+6820\sqrt{2}}\,\sqrt{-775687+549362\sqrt{2}}\,\arctan\left(\frac{\sqrt{-775687+549362\sqrt{2}}\,\sqrt{-23(8+3\sqrt{2})\left(-\frac{23(x+\sqrt{2}-1)^2}{(-x+\sqrt{2}+1)^2}+24\sqrt{2}-41\right)}\left(\frac{6485\sqrt{2}(x+\sqrt{2}-1)^2}{(-x+\sqrt{2}+1)^2}+\frac{10368(x+\sqrt{2}-1)^2}{(-x+\sqrt{2}+1)^2}+22379\sqrt{2}+32016\right)(x+\sqrt{2}-1)(8+3\sqrt{2})}{11692487\left(\frac{23(x+\sqrt{2}-1)^4}{(-x+\sqrt{2}+1)^4}+\frac{82(x+\sqrt{2}-1)^2}{(-x+\sqrt{2}+1)^2}+23\right)(-x+\sqrt{2}+1)}\right)+520\sqrt{-8866+6820\sqrt{2}}\,\sqrt{-775687+549362\sqrt{2}}\,\arctan\left(\frac{\sqrt{-775687+549362\sqrt{2}}\,\sqrt{-23(8+3\sqrt{2})\left(-\frac{23(x+\sqrt{2}-1)^2}{(-x+\sqrt{2}+1)^2}+24\sqrt{2}-41\right)}\left(\frac{6485\sqrt{2}(x+\sqrt{2}-1)^2}{(-x+\sqrt{2}+1)^2}+\frac{10368(x+\sqrt{2}-1)^2}{(-x+\sqrt{2}+1)^2}+22379\sqrt{2}+32016\right)(x+\sqrt{2}-1)(8+3\sqrt{2})}{11692487\left(\frac{23(x+\sqrt{2}-1)^4}{(-x+\sqrt{2}+1)^4}+\frac{82(x+\sqrt{2}-1)^2}{(-x+\sqrt{2}+1)^2}+23\right)(-x+\sqrt{2}+1)}\right)\right)}{21142\sqrt{\frac{\frac{8(x+\sqrt{2}-1)^2}{(-x+\sqrt{2}+1)^2}+\frac{3\sqrt{2}(x+\sqrt{2}-1)^2}{(-x+\sqrt{2}+1)^2}+8-3\sqrt{2}}{\left(1+\frac{x+\sqrt{2}-1}{-x+\sqrt{2}+1}\right)^2}}\left(1+\frac{x+\sqrt{2}-1}{-x+\sqrt{2}+1}\right)(8+3\sqrt{2})\sqrt{-8866+6820\sqrt{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2+3*x+2)/(2*x^2-x+3)^(1/2),x)
```

```
[Out] 1/21142*(8*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+3*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2
^(1/2)+1)^2+8-3*2^(1/2))^(1/2)*2^(1/2)*(369*2^(1/2)*(-8866+6820*2^(1/2))^(1
/2)*(-775687+549362*2^(1/2))^(1/2)*arctan(1/11692487*(-775687+549362*2^(1/2
))^(1/2)*(-23*(8+3*2^(1/2))*(-23*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+24*2^(1/2
)-41))^(1/2)*(6485*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+10368*(x+2^(1/2
)-1)^2/(-x+2^(1/2)+1)^2+22379*2^(1/2)+32016)/(23*(x+2^(1/2)-1)^4/(-x+2^(1/2
)+1)^4+82*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+23)*(x+2^(1/2)-1)/(-x+2^(1/2)+1)
*(8+3*2^(1/2)))+520*(-8866+6820*2^(1/2))^(1/2)*(-775687+549362*2^(1/2))^(1/
2)*arctan(1/11692487*(-775687+549362*2^(1/2))^(1/2)*(-23*(8+3*2^(1/2))*(-23
*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+24*2^(1/2)-41))^(1/2)*(6485*2^(1/2)*(x+2^
(1/2)-1)^2/(-x+2^(1/2)+1)^2+10368*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+22379*2^
(1/2)+32016)/(23*(x+2^(1/2)-1)^4/(-x+2^(1/2)+1)^4+82*(x+2^(1/2)-1)^2/(-x+2^
(1/2)+1)^2+23)*(x+2^(1/2)-1)/(-x+2^(1/2)+1)*(8+3*2^(1/2)))+465124*arctanh(3
1/2*(8*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+3*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/
2)+1)^2+8-3*2^(1/2))^(1/2)/(-8866+6820*2^(1/2))^(1/2))*2^(1/2)-866822*arcta
nh(31/2*(8*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+3*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2
^(1/2)+1)^2+8-3*2^(1/2))^(1/2)/(-8866+6820*2^(1/2))^(1/2)))/((8*(x+2^(1/2)-
1)^2/(-x+2^(1/2)+1)^2+3*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+8-3*2^(1/2
))/(1+(x+2^(1/2)-1)/(-x+2^(1/2)+1))^2)^(1/2)/(1+(x+2^(1/2)-1)/(-x+2^(1/2)+1
))/(8+3*2^(1/2))/(-8866+6820*2^(1/2))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(5\,x^2+3\,x+2\right)\sqrt{2\,x^2-x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+3*x+2)/(2*x^2-x+3)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((5*x^2 + 3*x + 2)*sqrt(2*x^2 - x + 3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{2\,x^2-x+3}\,\left(5\,x^2+3\,x+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*x^2 - x + 3)^(1/2)*(3*x + 5*x^2 + 2)),x)
[Out] int(1/((2*x^2 - x + 3)^(1/2)*(3*x + 5*x^2 + 2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2x^2-x+3}\,\left(5x^2+3x+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x**2+3*x+2)/(2*x**2-x+3)**(1/2),x)
[Out] Integral(1/(sqrt(2*x**2 - x + 3)*(5*x**2 + 3*x + 2)), x)
```

3.81 $\int \frac{1}{\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^2} \, dx$

Optimal. Leaf size=188

$$\frac{\sqrt{2x^2-x+3}\,(65x+4)}{682\left(5x^2+3x+2\right)}+\frac{\sqrt{\frac{1}{682}\left(2343727+1678700\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{11}{31\left(2343727+1678700\sqrt{2}\right)}}\left(\left(5751+3935\sqrt{2}\right)x+1816\sqrt{2}+2119\right)}{\sqrt{2x^2-x+3}}\right)}{1364}$$

Rubi [A] time = 0.43, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {974, 1035, 1029, 206, 204}

$$\frac{\sqrt{2x^2-x+3}\,(65x+4)}{682\left(5x^2+3x+2\right)}+\frac{\sqrt{\frac{1}{682}\left(2343727+1678700\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{11}{31\left(2343727+1678700\sqrt{2}\right)}}\left(\left(5751+3935\sqrt{2}\right)x+1816\sqrt{2}+2119\right)}{\sqrt{2x^2-x+3}}\right)}{1364}-\frac{\sqrt{\frac{1}{682}\left(1678700\sqrt{2}-2343727\right)}\tanh^{-1}\left(\frac{\sqrt{\frac{11}{31\left(1678700\sqrt{2}-2343727\right)}}\left(\left(5751-3935\sqrt{2}\right)x-1816\sqrt{2}+2119\right)}{\sqrt{2x^2-x+3}}\right)}{1364}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)^2),x]
```

```
[Out] ((4 + 65*x)*Sqrt[3 - x + 2*x^2])/(682*(2 + 3*x + 5*x^2)) + (Sqrt[(2343727 +
 1678700*Sqrt[2])/682]*ArcTan[(Sqrt[11/(31*(2343727 + 1678700*Sqrt[2]))]*(2
119 + 1816*Sqrt[2] + (5751 + 3935*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]])/1364 -
 (Sqrt[(-2343727 + 1678700*Sqrt[2])/682]*ArcTanh[(Sqrt[11/(31*(-2343727 + 1
678700*Sqrt[2]))]*(2119 - 1816*Sqrt[2] + (5751 - 3935*Sqrt[2])*x))/Sqrt[3 -
 x + 2*x^2]])/1364
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 974

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*
a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*(
```

```
d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q,
 0]
```

Rule 1029

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
 c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 1035

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

Rubi steps

$$\int \frac{1}{\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^2}\,dx = \frac{(4+65x)\sqrt{3-x+2x^2}}{682\left(2+3x+5x^2\right)} - \frac{\int \frac{-1826+\frac{2255x}{2}}{\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)}\,dx}{7502}$$

$$= \frac{(4+65x)\sqrt{3-x+2x^2}}{682\left(2+3x+5x^2\right)} - \frac{\int \frac{\frac{121}{2}\left(537-332\sqrt{2}\right)-\frac{121}{2}\left(127-205\sqrt{2}\right)x}{\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)}\,dx}{165044\sqrt{2}} + \frac{\int \frac{\frac{121}{2}(53}{}}{}$$

$$= \frac{(4+65x)\sqrt{3-x+2x^2}}{682\left(2+3x+5x^2\right)} - \frac{1}{496}\left(11\left(3357400-2343727\sqrt{2}\right)\right)\text{Subst}\left(\int\right.$$

$$= \frac{(4+65x)\sqrt{3-x+2x^2}}{682\left(2+3x+5x^2\right)} + \frac{\sqrt{\frac{1}{682}\left(2343727+1678700\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{31(23}}{}\right.}{1364}$$

Mathematica [C] time = 1.00, size = 287, normalized size = 1.53

$$\frac{25\left(\frac{i\sqrt{286+22i\sqrt{31}}\left(224\sqrt{31}+1023i\right)\tanh^{-1}\left(\frac{\left(-22-4i\sqrt{31}\right)x+i\sqrt{31}+63}{2\sqrt{286+22i\sqrt{31}}\sqrt{2x^2-x+3}}\right)}{\left(\sqrt{31}-13i\right)^2}+\frac{10i\left(1364\left(\sqrt{31}+13i\right)(65x+4)\sqrt{2x^2-x+3}-5\sqrt{286-22i\sqrt{31}}\left(787\sqrt{31}-1271i\right)\left(5x^2+3x+2\right)\tanh^{-1}\left(\frac{\left(-22+4i\sqrt{31}\right)x-i\sqrt{31}+63}{2\sqrt{286-22i\sqrt{31}}\sqrt{2x^2-x+3}}\right)\right)}{\left(\sqrt{31}+13i\right)^2\left(10ix+\sqrt{31}+3i\right)\left(5\left(\sqrt{31}-13i\right)x+8\sqrt{31}-4i\right)}\right)}{116281}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)^2),x]
```

```
[Out] (25*((I*Sqrt[286 + (22*I)*Sqrt[31]]*(1023*I + 224*Sqrt[31])*ArcTanh[(63 + I
*Sqrt[31] + (-22 - (4*I)*Sqrt[31])*x)/(2*Sqrt[286 + (22*I)*Sqrt[31]]*Sqrt[3
 - x + 2*x^2])])/(-13*I + Sqrt[31])^2 + ((10*I)*(1364*(13*I + Sqrt[31])*(4
+ 65*x)*Sqrt[3 - x + 2*x^2] - 5*Sqrt[286 - (22*I)*Sqrt[31]]*(-1271*I + 787*
Sqrt[31])*(2 + 3*x + 5*x^2)*ArcTanh[(63 - I*Sqrt[31] + (-22 + (4*I)*Sqrt[31
])*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])]))/((13*I + Sqrt[
31])^2*(3*I + Sqrt[31] + (10*I)*x)*(-4*I + 8*Sqrt[31] + 5*(-13*I + Sqrt[31]
)*x))))/116281
```

IntegrateAlgebraic [C] time = 0.54, size = 230, normalized size = 1.22

$$\frac{\text{RootSum}\left[-5\#1^4+44\sqrt{2}\#1^3+68\#1^2-2024\sqrt{2}\#1-10580\&,\frac{205\sqrt{2}\#1^2\log\left(\#1-4\sqrt{2x^2-x+3}+\sqrt{2}(4x-1)\right)+4492\#1\log\left(\#1-4\sqrt{2x^2-x+3}+\sqrt{2}(4x-1)\right)-9430\sqrt{2}\log\left(\#1-4\sqrt{2x^2-x+3}+\sqrt{2}(4x-1)\right)}{-5\#1^3+33\sqrt{2}\#1^2+34\#1-506\sqrt{2}}\&\right]}{682\sqrt{2}}+\frac{\sqrt{2x^2-x+3}(65x+4)}{682\left(5x^2+3x+2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)^2),x]
```

```
[Out] ((4 + 65*x)*Sqrt[3 - x + 2*x^2])/(682*(2 + 3*x + 5*x^2)) + RootSum[-10580 -
 2024*Sqrt[2]*#1 + 68*#1^2 + 44*Sqrt[2]*#1^3 - 5*#1^4 & , (-9430*Sqrt[2]*Lo
g[Sqrt[2]*(-1 + 4*x) - 4*Sqrt[3 - x + 2*x^2] + #1] + 4492*Log[Sqrt[2]*(-1 +
 4*x) - 4*Sqrt[3 - x + 2*x^2] + #1]*#1 + 205*Sqrt[2]*Log[Sqrt[2]*(-1 + 4*x)
 - 4*Sqrt[3 - x + 2*x^2] + #1]*#1^2)/(-506*Sqrt[2] + 34*#1 + 33*Sqrt[2]*#1^
2 - 5*#1^3) & ]/(682*Sqrt[2])
```

fricas [B] time = 1.25, size = 2102, normalized size = 11.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+3*x+2)^2/(2*x^2-x+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/263043507934399808*(8422204*563606738^(1/4)*sqrt(33574)*sqrt(341)*sqrt(2)
*(5*x^2 + 3*x + 2)*sqrt(2343727*sqrt(2) + 3357400)*arctan(1/710190022151725
4683789*(47876524*sqrt(33574)*(22*563606738^(3/4)*sqrt(341)*(2950932*x^7 -
11691762*x^6 + 24397746*x^5 - 40053004*x^4 + 20309552*x^3 - 10145376*x^2 -
sqrt(2)*(2248634*x^7 - 8421787*x^6 + 17801494*x^5 - 27869789*x^4 + 13808040
*x^3 - 6172200*x^2 - 15724800*x + 10596096) - 21192192*x + 15724800) + 5203
97*563606738^(1/4)*sqrt(341)*(226651*x^7 - 3496629*x^6 + 18614024*x^5 - 428
60780*x^4 + 55586592*x^3 - 36274464*x^2 - sqrt(2)*(168871*x^7 - 2579646*x^6
 + 13533020*x^5 - 30582616*x^4 + 39345120*x^3 - 23947200*x^2 - 28449792*x +
 19450368) - 38900736*x + 28449792))*sqrt(2*x^2 - x + 3)*sqrt(2343727*sqrt(
2) + 3357400) + 20160232886887690715272*sqrt(31)*sqrt(2)*(28180*x^8 - 25466
6*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - s
qrt(2)*(8746*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^4 - 752
088*x^3 + 396144*x^2 + 546048*x - 539136) + 1154304*x - 456192) - 2*sqrt(33
574/2191)*(sqrt(33574)*(22*563606738^(3/4)*sqrt(341)*(10257392*x^7 - 147733
68*x^6 + 47877288*x^5 - 20710528*x^4 + 26321472*x^3 + 17079552*x^2 - sqrt(2
)*(8292238*x^7 - 11867543*x^6 + 37968813*x^5 - 13449840*x^4 + 14570280*x^3
+ 20176128*x^2 - 20176128*x) - 17079552*x) + 520397*563606738^(1/4)*sqrt(34
1)*(795513*x^7 - 10292932*x^6 + 39734380*x^5 - 51864768*x^4 + 68281632*x^3
+ 34255872*x^2 - 8*sqrt(2)*(77213*x^7 - 998548*x^6 + 3846220*x^5 - 4943520*
x^4 + 6215760*x^3 + 4318272*x^2 - 4318272*x) - 34255872*x))*sqrt(2*x^2 - x
+ 3)*sqrt(2343727*sqrt(2) + 3357400) + 421088065768678*sqrt(31)*sqrt(2)*(12
3408*x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 + 396480*x^4 + 798336*x^3
 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 118051*x^7 + 244047*x^6 - 707374*x^5
+ 1053960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*x) + 3276288*x) + 19140
366625849*sqrt(31)*(254591*x^8 - 4815126*x^7 + 32303580*x^6 - 90866808*x^5
+ 108781920*x^4 - 74219328*x^3 - 168956928*x^2 - 15488*sqrt(2)*(4*x^8 - 76*
x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3618*x^3 + 2268*x^2 - 1944*x) + 14482
0224*x))*sqrt(-(563606738^(1/4)*sqrt(33574)*sqrt(341)*sqrt(31)*sqrt(2*x^2 -
 x + 3)*(sqrt(2)*(1123*x + 898) - 2021*x - 225)*sqrt(2343727*sqrt(2) + 3357
```

400) - 1731948347213*x^2 - 1555218924028*sqrt(2)*(2*x^2 - x + 3) + 53372285
80187*x - 7069176927400)/x^2) + 229093555532814667219*sqrt(31)*(2828123*x^8
 - 9696916*x^7 + 53385560*x^6 - 142835344*x^5 + 254146592*x^4 - 249300096*x
^3 + 37981440*x^2 - 7744*sqrt(2)*(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^
5 + 15569*x^4 - 5568*x^3 + 1080*x^2 + 4320*x - 5184) + 223064064*x - 948879
36))/(2585191*x^8 - 4661200*x^7 + 14191920*x^6 + 490880*x^5 - 13562944*x^4
+ 44249088*x^3 - 34615296*x^2 - 24772608*x + 18579456)) + 8422204*563606738
^(1/4)*sqrt(33574)*sqrt(341)*sqrt(2)*(5*x^2 + 3*x + 2)*sqrt(2343727*sqrt(2)
 + 3357400)*arctan(1/710190022151725468378 9*(47876524*sqrt(33574)*(22*56360
6738^(3/4)*sqrt(341)*(2950932*x^7 - 11691762*x^6 + 24397746*x^5 - 40053004*
x^4 + 20309552*x^3 - 10145376*x^2 - sqrt(2)*(2248634*x^7 - 8421787*x^6 + 17
801494*x^5 - 27869789*x^4 + 13808040*x^3 - 6172200*x^2 - 15724800*x + 10596
096) - 21192192*x + 15724800) + 520397*563606738^(1/4)*sqrt(341)*(226651*x^
7 - 3496629*x^6 + 18614024*x^5 - 42860780*x^4 + 55586592*x^3 - 36274464*x^2
 - sqrt(2)*(168871*x^7 - 2579646*x^6 + 13533020*x^5 - 30582616*x^4 + 393451
20*x^3 - 23947200*x^2 - 28449792*x + 19450368) - 38900736*x + 28449792))*sq
rt(2*x^2 - x + 3)*sqrt(2343727*sqrt(2) + 3357400) - 2016023288688769071 5272
*sqrt(31)*sqrt(2)*(28180*x^8 - 254666*x^7 + 704270*x^6 - 1385256*x^5 + 1549
144*x^4 - 642048*x^3 - 98496*x^2 - sqrt(2)*(8746*x^8 - 102335*x^7 + 396104*
x^6 - 783113*x^5 + 1320710*x^4 - 752088*x^3 + 396144*x^2 + 546048*x - 53913
6) + 1154304*x - 456192) - 2*sqrt(33574/2191)*(sqrt(33574)*(22*563606738^(3
/4)*sqrt(341)*(10257392*x^7 - 14773368*x^6 + 47877288*x^5 - 20710528*x^4 +
26321472*x^3 + 17079552*x^2 - sqrt(2)*(8292238*x^7 - 11867543*x^6 + 3796881
3*x^5 - 13449840*x^4 + 14570280*x^3 + 20176128*x^2 - 20176128*x) - 17079552
*x) + 520397*563606738^(1/4)*sqrt(341)*(795513*x^7 - 10292932*x^6 + 3973438
0*x^5 - 51864768*x^4 + 68281632*x^3 + 34255872*x^2 - 8*sqrt(2)*(77213*x^7 -
 998548*x^6 + 3846220*x^5 - 4943520*x^4 + 6215760*x^3 + 4318272*x^2 - 43182
72*x) - 34255872*x))*sqrt(2*x^2 - x + 3)*sqrt(2343727*sqrt(2) + 3357400) -
421088065768678*sqrt(31)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578888*x^6 - 3
293072*x^5 + 396480*x^4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 1
18051*x^7 + 244047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x
^2 - 1036800*x) + 3276288*x) - 19140366625849*sqrt(31)*(254591*x^8 - 481512
6*x^7 + 32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 74219328*x^3 - 168956
928*x^2 - 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3
618*x^3 + 2268*x^2 - 1944*x) + 144820224*x))*sqrt((563606738^(1/4)*sqrt(335
74)*sqrt(341)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(1123*x + 898) - 2021*x
 - 225)*sqrt(2343727*sqrt(2) + 3357400) + 1731948347213*x^2 + 1555218924028
sqrt(2)(2*x^2 - x + 3) - 5337228580187*x + 7069176927400)/x^2) - 22909355
5532814667219*sqrt(31)*(2828123*x^8 - 9696916*x^7 + 53385560*x^6 - 14283534
4*x^5 + 254146592*x^4 - 249300096*x^3 + 37981440*x^2 - 7744*sqrt(2)*(1348*x
^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^4 - 5568*x^3 + 1080*x^2 + 43
20*x - 5184) + 223064064*x - 94887936))/(2585191*x^8 - 4661200*x^7 + 141919
20*x^6 + 490880*x^5 - 13562944*x^4 + 44249088*x^3 - 34615296*x^2 - 24772608
*x + 18579456)) + 563606738^(1/4)*sqrt(33574)*sqrt(341)*sqrt(31)*(16787000*
x^2 - 2343727*sqrt(2)*(5*x^2 + 3*x + 2) + 10072200*x + 6714800)*sqrt(234372

```
7*sqrt(2) + 3357400)*log(335740000/2191*(563606738^(1/4)*sqrt(33574)*sqrt(3
41)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(1123*x + 898) - 2021*x - 225)*sq
rt(2343727*sqrt(2) + 3357400) + 1731948347213*x^2 + 1555218924028*sqrt(2)*(
2*x^2 - x + 3) - 5337228580187*x + 7069176927400)/x^2) - 563606738^(1/4)*sq
rt(33574)*sqrt(341)*sqrt(31)*(16787000*x^2 - 2343727*sqrt(2)*(5*x^2 + 3*x +
 2) + 10072200*x + 6714800)*sqrt(2343727*sqrt(2) + 3357400)*log(-335740000/
2191*(563606738^(1/4)*sqrt(33574)*sqrt(341)*sqrt(31)*sqrt(2*x^2 - x + 3)*(s
qrt(2)*(1123*x + 898) - 2021*x - 225)*sqrt(2343727*sqrt(2) + 3357400) - 173
1948347213*x^2 - 1555218924028*sqrt(2)*(2*x^2 - x + 3) + 5337228580187*x -
7069176927400)/x^2) + 385694293158944*sqrt(2*x^2 - x + 3)*(65*x + 4))/(5*x^
2 + 3*x + 2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+3*x+2)^2/(2*x^2-x+3)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Francis algorithm failure for[-1.0,infinity,
infinity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,infi
nity]Francis algorithm failure for[-1.0,infinity,infinity,infinity,infinity
]proot error [1.0,infinity,infinity,infinity,infinity]Francis algorithm fai
lure for[-1.0,infinity,infinity,infinity,infinity]proot error [1.0,infinity
,infinity,infinity,infinity]Francis algorithm failure for[-1.0,infinity,inf
inity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,infinit
y]Evaluation time: 27.71Done
```

maple [B] time = 0.01, size = 5225, normalized size = 27.79

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2+3*x+2)^2/(2*x^2-x+3)^(1/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(5\,x^2+3\,x+2\right)^2\sqrt{2\,x^2-x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+3*x+2)^2/(2*x^2-x+3)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((5*x^2 + 3*x + 2)^2*sqrt(2*x^2 - x + 3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{2\,x^2-x+3}\,{\left(5\,x^2+3\,x+2\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*x^2 - x + 3)^(1/2)*(3*x + 5*x^2 + 2)^2),x)
[Out] int(1/((2*x^2 - x + 3)^(1/2)*(3*x + 5*x^2 + 2)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2x^{2} - x + 3} \left(5x^{2} + 3x + 2\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x**2+3*x+2)**2/(2*x**2-x+3)**(1/2),x)
[Out] Integral(1/(sqrt(2*x**2 - x + 3)*(5*x**2 + 3*x + 2)**2), x)
```

3.82 $\int \frac{1}{\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^3}\,dx$

Optimal. Leaf size=223

$$\frac{\sqrt{2x^2-x+3}\,(65x+4)}{1364\left(5x^2+3x+2\right)^2}+\frac{(86265x+26794)\sqrt{2x^2-x+3}}{1860496\left(5x^2+3x+2\right)}+\frac{25\sqrt{\frac{1}{682}\left(6414867847+4536374600\sqrt{2}\right)}\tan^{-1}\left(\sqrt{\frac{\ldots}{31\ldots}}\ldots\right.}{372\ldots}$$

Rubi [A] time = 0.47, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {974, 1060, 1035, 1029, 206, 204}

$$\frac{\sqrt{2x^2-x+3}\,(65x+4)}{1364\left(5x^2+3x+2\right)^2}+\frac{(86265x+26794)\sqrt{2x^2-x+3}}{1860496\left(5x^2+3x+2\right)}+\frac{25\sqrt{\frac{1}{682}\left(6414867847+4536374600\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{11}{31\left(6414867847+4536374600\sqrt{2}\right)}}\left(\left(294669+208915\sqrt{2}\right)x+85754\sqrt{2}+123161\right)}{\sqrt{2x^2-x+3}}\right)}{3720992}-\frac{25\sqrt{\frac{1}{682}\left(4536374600\sqrt{2}-6414867847\right)}\tanh^{-1}\left(\frac{\sqrt{\frac{11}{31\left(4536374600\sqrt{2}-6414867847\right)}}\left(\left(294669-208915\sqrt{2}\right)x-85754\sqrt{2}+123161\right)}{\sqrt{2x^2-x+3}}\right)}{3720992}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)^3),x]
```

```
[Out] ((4 + 65*x)*Sqrt[3 - x + 2*x^2])/(1364*(2 + 3*x + 5*x^2)^2) + ((26794 + 862
65*x)*Sqrt[3 - x + 2*x^2])/(1860496*(2 + 3*x + 5*x^2)) + (25*Sqrt[(64148678
47 + 4536374600*Sqrt[2])/682]*ArcTan[(Sqrt[11/(31*(6414867847 + 4536374600*
Sqrt[2]))]*(123161 + 85754*Sqrt[2] + (294669 + 208915*Sqrt[2])*x))/Sqrt[3 -
 x + 2*x^2]])/3720992 - (25*Sqrt[(-6414867847 + 4536374600*Sqrt[2])/682]*Ar
cTanh[(Sqrt[11/(31*(-6414867847 + 4536374600*Sqrt[2]))]*(123161 - 85754*Sqr
t[2] + (294669 - 208915*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]])/3720992
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 974

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*
```

```
a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*(
d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q,
 0]
```

Rule 1029

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
 c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 1035

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

Rule 1060

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x +
 c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
2*a*(c*d - a*f)))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
```

```
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !
IGtQ[q, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^3}dx = \frac{(4+65x)\sqrt{3-x+2x^2}}{1364\left(2+3x+5x^2\right)^2} - \frac{\int \frac{-5775+\frac{6479x}{2}-2860x^2}{\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^2}dx}{15004}$$

$$= \frac{(4+65x)\sqrt{3-x+2x^2}}{1364\left(2+3x+5x^2\right)^2} + \frac{(26794+86265x)\sqrt{3-x+2x^2}}{1860496\left(2+3x+5x^2\right)} - \frac{\int \frac{-\frac{28220225}{2}+ \cdots}{\sqrt{3-x+2x^2}\left(\cdots\right.}}{11256\cdots}$$

$$= \frac{(4+65x)\sqrt{3-x+2x^2}}{1364\left(2+3x+5x^2\right)^2} + \frac{(26794+86265x)\sqrt{3-x+2x^2}}{1860496\left(2+3x+5x^2\right)} - \frac{\int \frac{33275}{4}\left(26103\cdots\right.}{\cdots}$$

$$= \frac{(4+65x)\sqrt{3-x+2x^2}}{1364\left(2+3x+5x^2\right)^2} + \frac{(26794+86265x)\sqrt{3-x+2x^2}}{1860496\left(2+3x+5x^2\right)} - \frac{\left(6875\left(9072\cdots\right.\right.}{\cdots}$$

$$= \frac{(4+65x)\sqrt{3-x+2x^2}}{1364\left(2+3x+5x^2\right)^2} + \frac{(26794+86265x)\sqrt{3-x+2x^2}}{1860496\left(2+3x+5x^2\right)} + \frac{25\sqrt{\frac{1}{682}\left(64\cdots\right.}}{\cdots}$$

Mathematica [C] time = 6.23, size = 1277, normalized size = 5.73

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)^3),x]
```

```
[Out] (2500*Sqrt[3 - x + 2*x^2])/(341*Sqrt[31]*(13*I + Sqrt[31])*(3 - I*Sqrt[31]
+ 10*x)^2) + (7500*Sqrt[3 - x + 2*x^2])/(10571*(13 - I*Sqrt[31])*(3 - I*Sqr
t[31] + 10*x)) - (2500*Sqrt[3 - x + 2*x^2])/(341*Sqrt[31]*(13*I - Sqrt[31])
*(3 + I*Sqrt[31] + 10*x)^2) + (7500*Sqrt[3 - x + 2*x^2])/(10571*(13 + I*Sqr
t[31])*(3 + I*Sqrt[31] + 10*x)) - (375*Sqrt[(2*(13 - I*Sqrt[31]))/11]*(11 -
 (2*I)*Sqrt[31])*ArcTanh[(63 - I*Sqrt[31] - 2*(11 - (2*I)*Sqrt[31])*x)/(2*S
qrt[22*(13 - I*Sqrt[31])]*Sqrt[3 - x + 2*x^2])])/(10571*(13*I + Sqrt[31])^2
) - (750*Sqrt[(2*(13 - I*Sqrt[31]))/341]*ArcTanh[(63 - I*Sqrt[31] - 2*(11 -
 (2*I)*Sqrt[31])*x)/(2*Sqrt[22*(13 - I*Sqrt[31])]*Sqrt[3 - x + 2*x^2])])/(9
61*(13*I + Sqrt[31])) + (750*Sqrt[(2*(13 + I*Sqrt[31]))/341]*ArcTanh[(63 +
I*Sqrt[31] - 2*(11 + (2*I)*Sqrt[31])*x)/(2*Sqrt[22*(13 + I*Sqrt[31])]*Sqrt[
3 - x + 2*x^2])])/(961*(13*I - Sqrt[31])) - (375*Sqrt[(2*(13 + I*Sqrt[31]))
/11]*(11 + (2*I)*Sqrt[31])*ArcTanh[(63 + I*Sqrt[31] - 2*(11 + (2*I)*Sqrt[31
])*x)/(2*Sqrt[22*(13 + I*Sqrt[31])]*Sqrt[3 - x + 2*x^2])])/(10571*(13*I - S
qrt[31])^2) + (((500*I)/31)*(((-20*(-3 + I*Sqrt[31]) + 10*(27 - (4*I)*Sqrt[
31]))*Sqrt[3 - x + 2*x^2])/((300 - 10*(-3 + I*Sqrt[31]) + 2*(-3 + I*Sqrt[31
])^2)*(-3 + I*Sqrt[31] - 10*x)) + (2*Sqrt[22*(13 - I*Sqrt[31])]*(20*(-3 + I
*Sqrt[31]) + 10*(27 - (4*I)*Sqrt[31]) - 2*(600 + 2*(-3 + I*Sqrt[31])*(27 -
(4*I)*Sqrt[31])))*ArcTanh[(-63 + I*Sqrt[31] - (-10 + 4*(-3 + I*Sqrt[31]))*x
)/(2*Sqrt[22*(13 - I*Sqrt[31])]*Sqrt[3 - x + 2*x^2])])/((300 - 10*(-3 + I*S
qrt[31]) + 2*(-3 + I*Sqrt[31])^2)*(1200 - 40*(-3 + I*Sqrt[31]) + 8*(-3 + I*
Sqrt[31])^2))))/(Sqrt[31]*(300 - 10*(-3 + I*Sqrt[31]) + 2*(-3 + I*Sqrt[31])
^2)) + (((500*I)/31)*(((20*(3 + I*Sqrt[31]) - 10*(-27 - (4*I)*Sqrt[31]))*Sq
rt[3 - x + 2*x^2])/((300 + 10*(3 + I*Sqrt[31]) + 2*(3 + I*Sqrt[31])^2)*(3 +
 I*Sqrt[31] + 10*x)) + (2*Sqrt[22*(13 + I*Sqrt[31])]*(-20*(3 + I*Sqrt[31])
- 10*(-27 - (4*I)*Sqrt[31]) - 2*(600 + 2*(3 + I*Sqrt[31])*(-27 - (4*I)*Sqrt
[31])))*ArcTanh[(63 + I*Sqrt[31] - (10 + 4*(3 + I*Sqrt[31]))*x)/(2*Sqrt[22*
(13 + I*Sqrt[31])]*Sqrt[3 - x + 2*x^2])])/((300 + 10*(3 + I*Sqrt[31]) + 2*(
3 + I*Sqrt[31])^2)*(1200 + 40*(3 + I*Sqrt[31]) + 8*(3 + I*Sqrt[31])^2))))/(
Sqrt[31]*(300 + 10*(3 + I*Sqrt[31]) + 2*(3 + I*Sqrt[31])^2))
```

IntegrateAlgebraic [C] time = 0.84, size = 396, normalized size = 1.78

$$\frac{3\text{RootSum}\left[-5\#1^4+6\sqrt{2}\#1^3+17\#1^2-26\sqrt{2}\#1-56\&,\frac{-2992879225\#1^2\log\left(-\#1+\sqrt{2x^2-x+3}-\sqrt{2}x\right)+11629301740\sqrt{2}\#1\log\left(-\#1+\sqrt{2x^2-x+3}-\sqrt{2}x\right)-42330420383\log\left(-\#1+\sqrt{2x^2-x+3}-\sqrt{2}x\right)}{-10\#1^3+9\sqrt{2}\#1^2+17\#1-13\sqrt{2}}\&\right]}{49210119200}-\frac{16\text{RootSum}\left[-5\#1^4+6\sqrt{2}\#1^3+17\#1^2-26\sqrt{2}\#1-56\&,\frac{-65525\#1^2\log\left(-\#1+\sqrt{2x^2-x+3}-\sqrt{2}x\right)+129160\sqrt{2}\#1\log\left(-\#1+\sqrt{2x^2-x+3}-\sqrt{2}x\right)-720397\log\left(-\#1+\sqrt{2x^2-x+3}-\sqrt{2}x\right)}{-10\#1^3+9\sqrt{2}\#1^2+17\#1-13\sqrt{2}}\&\right]}{4509725}+\frac{\sqrt{2x^2-x+3}\left(431325x^3+392765x^2+341572x+59044\right)}{1860496\left(5x^2+3x+2\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)^3),x]
```

```
[Out] (Sqrt[3 - x + 2*x^2]*(59044 + 341572*x + 392765*x^2 + 431325*x^3))/(1860496
*(2 + 3*x + 5*x^2)^2) + (3*RootSum[-56 - 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[2
]*#1^3 - 5*#1^4 & , (-42330420383*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] -
```

```
#1] + 11629301740*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1 -
 2992879225*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1^2)/(-13*Sqrt[2]
 + 17*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3) & ])/49210119200 - (16*RootSum[-56 - 2
6*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3 - 5*#1^4 & , (-720397*Log[-(Sqrt[2]
*x) + Sqrt[3 - x + 2*x^2] - #1] + 129160*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3
- x + 2*x^2] - #1]*#1 - 65525*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*
#1^2)/(-13*Sqrt[2] + 17*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3) & ])/4509725
```

fricas [B] time = 1.28, size = 2183, normalized size = 9.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+3*x+2)^3/(2*x^2-x+3)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/21234400002747742634682214 4*(46113488900*4115738902305032^(1/4)*sqrt(226
81873)*sqrt(341)*sqrt(2)*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)*sqrt(6414867
847*sqrt(2) + 9072749200)*arctan(1/3836668309294009530058322373948769*(6468
8701796*sqrt(22681873)*(11*4115738902305032^(3/4)*sqrt(341)*(160344708*x^7
- 615873378*x^6 + 1294230774*x^5 - 2070733376*x^4 + 1037098288*x^3 - 489164
544*x^2 - sqrt(2)*(112700446*x^7 - 434839553*x^6 + 912850886*x^5 - 14661276
91*x^4 + 735661560*x^3 - 350098200*x^2 - 799200000*x + 567316224) - 1134632
448*x + 799200000) + 703138063*4115738902305032^(1/4)*sqrt(341)*(12162569*x
^7 - 186616851*x^6 + 985490056*x^5 - 2246141620*x^4 + 2900382048*x^3 - 1823
848416*x^2 - sqrt(2)*(8564099*x^7 - 131508024*x^6 + 695288980*x^5 - 1587105
104*x^4 + 2050714080*x^3 - 1296806400*x^2 - 1457077248*x + 1033108992) - 20
66217984*x + 1457077248))*sqrt(2*x^2 - x + 3)*sqrt(6414867847*sqrt(2) + 907
2749200) + 10891187458641059311133302222822312*sqrt(31)*sqrt(2)*(28180*x^8
- 254666*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*
x^2 - sqrt(2)*(8746*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^
4 - 752088*x^3 + 396144*x^2 + 546048*x - 539136) + 1154304*x - 456192) - 2*
sqrt(45363746/479849)*(sqrt(22681873)*(11*4115738902305032^(3/4)*sqrt(341)*
(576322648*x^7 - 827050092*x^6 + 2660713572*x^5 - 1032439232*x^4 + 12116047
68*x^3 + 1213394688*x^2 - sqrt(2)*(403157522*x^7 - 578844217*x^6 + 18641293
47*x^5 - 735062160*x^4 + 873708120*x^3 + 823986432*x^2 - 823986432*x) - 121
3394688*x) + 703138063*4115738902305032^(1/4)*sqrt(341)*(43684647*x^7 - 565
067708*x^6 + 2178643220*x^5 - 2819241792*x^4 + 3618371808*x^3 + 2197767168*
x^2 - 2*sqrt(2)*(15328963*x^7 - 198290348*x^6 + 764653220*x^5 - 990717120*x
^4 + 1276256160*x^3 + 755350272*x^2 - 755350272*x) - 2197767168*x))*sqrt(2*
x^2 - x + 3)*sqrt(6414867847*sqrt(2) + 9072749200) + 168363055004367262339 3
22*sqrt(31)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 +
396480*x^4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 118051*x^7 + 2
44047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*
x) + 3276288*x) + 76528661365621482881 51*sqrt(31)*(254591*x^8 - 4815126*x^7
 + 32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 74219328*x^3 - 168956928*x
^2 - 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3618*x
```

^3 + 2268*x^2 - 1944*x) + 144820224*x))*sqrt(-(4115738902305032^(1/4)*sqrt(
22681873)*sqrt(341)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(67187*x + 26012)
 - 93199*x - 41175)*sqrt(6414867847*sqrt(2) + 9072749200) - 512510746420187
753*x^2 - 460213731479352268*sqrt(2)*(2*x^2 - x + 3) + 1579369851213231647*
x - 2091880597633419400)/x^2) + 12376349384819385580833279804799*sqrt(31)*
(2828123*x^8 - 9696916*x^7 + 53385560*x^6 - 142835344*x^5 + 254146592*x^4 -
 249300096*x^3 + 37981440*x^2 - 7744*sqrt(2)*(1348*x^8 - 2692*x^7 + 9789*x^
6 - 10070*x^5 + 15569*x^4 - 5568*x^3 + 1080*x^2 + 4320*x - 5184) + 22306406
4*x - 94887936))/(2585191*x^8 - 4661200*x^7 + 14191920*x^6 + 490880*x^5 - 1
3562944*x^4 + 44249088*x^3 - 34615296*x^2 - 24772608*x + 18579456)) + 46113
488900*4115738902305032^(1/4)*sqrt(22681873)*sqrt(341)*sqrt(2)*(25*x^4 + 30
*x^3 + 29*x^2 + 12*x + 4)*sqrt(6414867847*sqrt(2) + 9072749200)*arctan(1/38
366683092940095300583223739487 69*(64688701796*sqrt(22681873)*(11*4115738902
305032^(3/4)*sqrt(341)*(160344708*x^7 - 615873378*x^6 + 1294230774*x^5 - 20
70733376*x^4 + 1037098288*x^3 - 489164544*x^2 - sqrt(2)*(112700446*x^7 - 43
4839553*x^6 + 912850886*x^5 - 1466127691*x^4 + 735661560*x^3 - 350098200*x^
2 - 799200000*x + 567316224) - 1134632448*x + 799200000) + 703138063*411573
8902305032^(1/4)*sqrt(341)*(12162569*x^7 - 186616851*x^6 + 985490056*x^5 -
2246141620*x^4 + 2900382048*x^3 - 1823848416*x^2 - sqrt(2)*(8564099*x^7 - 1
31508024*x^6 + 695288980*x^5 - 1587105104*x^4 + 2050714080*x^3 - 1296806400
*x^2 - 1457077248*x + 1033108992) - 2066217984*x + 1457077248))*sqrt(2*x^2
- x + 3)*sqrt(6414867847*sqrt(2) + 9072749200) - 108911874586410593111333O2
222822312*sqrt(31)*sqrt(2)*(28180*x^8 - 254666*x^7 + 704270*x^6 - 1385256*x
^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - sqrt(2)*(8746*x^8 - 102335*x^7
+ 396104*x^6 - 783113*x^5 + 1320710*x^4 - 752088*x^3 + 396144*x^2 + 546048*
x - 539136) + 1154304*x - 456192) - 2*sqrt(45363746/479849)*(sqrt(22681873)
*(11*4115738902305032^(3/4)*sqrt(341)*(576322648*x^7 - 827050092*x^6 + 2660
713572*x^5 - 1032439232*x^4 + 1211604768*x^3 + 1213394688*x^2 - sqrt(2)*(40
3157522*x^7 - 578844217*x^6 + 1864129347*x^5 - 735062160*x^4 + 873708120*x^
3 + 823986432*x^2 - 823986432*x) - 1213394688*x) + 703138063*41157389023050
32^(1/4)*sqrt(341)*(43684647*x^7 - 565067708*x^6 + 2178643220*x^5 - 2819241
792*x^4 + 3618371808*x^3 + 2197767168*x^2 - 2*sqrt(2)*(15328963*x^7 - 19829
0348*x^6 + 764653220*x^5 - 990717120*x^4 + 1276256160*x^3 + 755350272*x^2 -
 755350272*x) - 2197767168*x))*sqrt(2*x^2 - x + 3)*sqrt(6414867847*sqrt(2)
+ 9072749200) - 16836305500436726233932 2*sqrt(31)*sqrt(2)*(123408*x^8 - 914
152*x^7 + 1578888*x^6 - 3293072*x^5 + 396480*x^4 + 798336*x^3 - 3822336*x^2
 - sqrt(2)*(15550*x^8 - 118051*x^7 + 244047*x^6 - 707374*x^5 + 1053960*x^4
- 1667952*x^3 + 1209600*x^2 - 1036800*x) + 3276288*x) - 7652866136562148288
151*sqrt(31)*(254591*x^8 - 4815126*x^7 + 32303580*x^6 - 90866808*x^5 + 1087
81920*x^4 - 74219328*x^3 - 168956928*x^2 - 15488*sqrt(2)*(4*x^8 - 76*x^7 +
517*x^6 - 1536*x^5 + 2385*x^4 - 3618*x^3 + 2268*x^2 - 1944*x) + 144820224*x
))*sqrt((4115738902305032^(1/4)*sqrt(22681873)*sqrt(341)*sqrt(31)*sqrt(2*x^
2 - x + 3)*(sqrt(2)*(67187*x + 26012) - 93199*x - 41175)*sqrt(6414867847*sq
rt(2) + 9072749200) + 512510746420187753*x^2 + 460213731479352268*sqrt(2)*(
2*x^2 - x + 3) - 1579369851213231647*x + 2091880597633419400)/x^2) - 123763

```
4938481938558083329798O4799*sqrt(31)*(2828123*x^8 - 9696916*x^7 + 53385560*
x^6 - 142835344*x^5 + 254146592*x^4 - 249300096*x^3 + 37981440*x^2 - 7744*s
qrt(2)*(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^4 - 5568*x^3 +
 1080*x^2 + 4320*x - 5184) + 223064064*x - 94887936))/(2585191*x^8 - 466120
0*x^7 + 14191920*x^6 + 490880*x^5 - 13562944*x^4 + 44249088*x^3 - 34615296*
x^2 - 24772608*x + 18579456)) - 25*4115738902305032^(1/4)*sqrt(22681873)*sq
rt(341)*sqrt(31)*(226818730000*x^4 + 272182476000*x^3 + 263109726800*x^2 -
6414867847*sqrt(2)*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4) + 108872990400*x +
 36290996800)*sqrt(6414867847*sqrt(2) + 9072749200)*log(1134093650000000/47
9849*(4115738902305032^(1/4)*sqrt(22681873)*sqrt(341)*sqrt(31)*sqrt(2*x^2 -
 x + 3)*(sqrt(2)*(67187*x + 26012) - 93199*x - 41175)*sqrt(6414867847*sqrt(
2) + 9072749200) + 512510746420187753*x^2 + 460213731479352268*sqrt(2)*(2*x
^2 - x + 3) - 1579369851213231647*x + 2091880597633419400)/x^2) + 25*411573
8902305032^(1/4)*sqrt(22681873)*sqrt(341)*sqrt(31)*(226818730000*x^4 + 2721
82476000*x^3 + 263109726800*x^2 - 6414867847*sqrt(2)*(25*x^4 + 30*x^3 + 29*
x^2 + 12*x + 4) + 108872990400*x + 36290996800)*sqrt(6414867847*sqrt(2) + 9
072749200)*log(-1134093650000000/479849*(4115738902305032^(1/4)*sqrt(226818
73)*sqrt(341)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(67187*x + 26012) - 931
99*x - 41175)*sqrt(6414867847*sqrt(2) + 9072749200) - 512510746420187753*x^
2 - 460213731479352268*sqrt(2)*(2*x^2 - x + 3) + 1579369851213231647*x - 20
91880597633419400)/x^2) - 114133005406879362464*(431325*x^3 + 392765*x^2 +
341572*x + 59044)*sqrt(2*x^2 - x + 3))/(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4
)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+3*x+2)^3/(2*x^2-x+3)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Francis algorithm failure for[-1.0,infinity,
infinity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,infi
nity]Francis algorithm failure for[-1.0,infinity,infinity,infinity,infinity
]proot error [1.0,infinity,infinity,infinity,infinity]Francis algorithm fai
lure for[-1.0,infinity,infinity,infinity,infinity]proot error [1.0,infinity
,infinity,infinity,infinity]Francis algorithm failure for[-1.0,infinity,inf
inity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,infinit
y]Evaluation time: 42.09Done
```

maple [B] time = 0.01, size = 13040, normalized size = 58.48

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2+3*x+2)^3/(2*x^2-x+3)^(1/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(5\,x^2+3\,x+2\right)^3\,\sqrt{2\,x^2-x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+3*x+2)^3/(2*x^2-x+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((5*x^2 + 3*x + 2)^3*sqrt(2*x^2 - x + 3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{2\,x^2-x+3}\,\left(5\,x^2+3\,x+2\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*x^2 - x + 3)^(1/2)*(3*x + 5*x^2 + 2)^3),x)
```

```
[Out] int(1/((2*x^2 - x + 3)^(1/2)*(3*x + 5*x^2 + 2)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2x^2-x+3}\,\left(5x^2+3x+2\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x**2+3*x+2)**3/(2*x**2-x+3)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(2*x**2 - x + 3)*(5*x**2 + 3*x + 2)**3), x)
```

3.83 $\int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^{3/2}}\,dx$

Optimal. Leaf size=166

$$-\frac{111315\sqrt{2x^2-x+3}\,x^2}{2048}-\frac{8992487\sqrt{2x^2-x+3}\,x}{16384}-\frac{31009685\sqrt{2x^2-x+3}}{65536}-\frac{14641(79x+101)}{1472\sqrt{2x^2-x+3}}+\frac{625}{24}\sqrt{2x^2-x}$$

Rubi [A] time = 0.20, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1660, 1661, 640, 619, 215}

$$\frac{625}{24}\sqrt{2x^2-x+3}\,x^5+\frac{10075}{96}\sqrt{2x^2-x+3}\,x^4+\frac{79425}{512}\sqrt{2x^2-x+3}\,x^3-\frac{111315\sqrt{2x^2-x+3}\,x^2}{2048}-\frac{8992487\sqrt{2x^2-x+3}\,x}{16384}-\frac{31009685\sqrt{2x^2-x+3}}{65536}-\frac{14641(79x+101)}{1472\sqrt{2x^2-x+3}}-\frac{310445587\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{131072\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + 5*x^2)^4/(3 - x + 2*x^2)^(3/2),x]
```

```
[Out] (-14641*(101 + 79*x))/(1472*Sqrt[3 - x + 2*x^2]) - (31009685*Sqrt[3 - x + 2
*x^2])/65536 - (8992487*x*Sqrt[3 - x + 2*x^2])/16384 - (111315*x^2*Sqrt[3 -
 x + 2*x^2])/2048 + (79425*x^3*Sqrt[3 - x + 2*x^2])/512 + (10075*x^4*Sqrt[3
 - x + 2*x^2])/96 + (625*x^5*Sqrt[3 - x + 2*x^2])/24 - (310445587*ArcSinh[(
1 - 4*x)/Sqrt[23]])/(131072*Sqrt[2])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(2+3x+5x^2\right)^4}{\left(3-x+2x^2\right)^{3/2}}\,dx = -\frac{14641(101+79x)}{1472\sqrt{3-x+2x^2}} + \frac{2}{23}\int \frac{\frac{2821893}{256} - \frac{661181x}{128} - \frac{488267x^2}{64} + \frac{143635x^3}{32} + \frac{213325x^4}{16} + \frac{83375x^5}{8}}{\sqrt{3-x+2x^2}}$$

$$= -\frac{14641(101+79x)}{1472\sqrt{3-x+2x^2}} + \frac{625}{24}x^5\sqrt{3-x+2x^2} + \frac{1}{138}\int \frac{\frac{8465679}{64} - \frac{1983543x}{32} - \frac{1464801x^2}{16} + 43}{\sqrt{3-x+}}$$

$$= -\frac{14641(101+79x)}{1472\sqrt{3-x+2x^2}} + \frac{10075}{96}x^4\sqrt{3-x+2x^2} + \frac{625}{24}x^5\sqrt{3-x+2x^2} + \frac{\int \frac{\frac{42328395}{32} - \frac{9917715}{16}}{}}{}$$

$$= -\frac{14641(101+79x)}{1472\sqrt{3-x+2x^2}} + \frac{79425}{512}x^3\sqrt{3-x+2x^2} + \frac{10075}{96}x^4\sqrt{3-x+2x^2} + \frac{625}{24}x^5\sqrt{3-x}$$

$$= -\frac{14641(101+79x)}{1472\sqrt{3-x+2x^2}} - \frac{111315x^2\sqrt{3-x+2x^2}}{2048} + \frac{79425}{512}x^3\sqrt{3-x+2x^2} + \frac{10075}{96}x^4\sqrt{3}$$

$$= -\frac{14641(101+79x)}{1472\sqrt{3-x+2x^2}} - \frac{8992487x\sqrt{3-x+2x^2}}{16384} - \frac{111315x^2\sqrt{3-x+2x^2}}{2048} + \frac{79425}{512}x^3\sqrt{3}$$

$$= -\frac{14641(101+79x)}{1472\sqrt{3-x+2x^2}} - \frac{31009685\sqrt{3-x+2x^2}}{65536} - \frac{8992487x\sqrt{3-x+2x^2}}{16384} - \frac{111315x^2\sqrt{}}{20}$$

$$= -\frac{14641(101+79x)}{1472\sqrt{3-x+2x^2}} - \frac{31009685\sqrt{3-x+2x^2}}{65536} - \frac{8992487x\sqrt{3-x+2x^2}}{16384} - \frac{111315x^2\sqrt{}}{20}$$

$$= -\frac{14641(101+79x)}{1472\sqrt{3-x+2x^2}} - \frac{31009685\sqrt{3-x+2x^2}}{65536} - \frac{8992487x\sqrt{3-x+2x^2}}{16384} - \frac{111315x^2\sqrt{}}{20}$$

Mathematica [A] time = 0.37, size = 95, normalized size = 0.57

$$\sqrt{2x^2-x+3}\left(\frac{625x^5}{24}+\frac{10075x^4}{96}+\frac{79425x^3}{512}-\frac{111315x^2}{2048}-\frac{14641(79x+101)}{1472\left(2x^2-x+3\right)}-\frac{8992487x}{16384}-\frac{31009685}{65536}\right)+\frac{310445587\sinh^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{131072\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)^4/(3 - x + 2*x^2)^(3/2),x]
```

```
[Out] Sqrt[3 - x + 2*x^2]*(-31009685/65536 - (8992487*x)/16384 - (111315*x^2)/204
8 + (79425*x^3)/512 + (10075*x^4)/96 + (625*x^5)/24 - (14641*(101 + 79*x))/
(1472*(3 - x + 2*x^2))) + (310445587*ArcSinh[(-1 + 4*x)/Sqrt[23]])/(131072*
Sqrt[2])
```

IntegrateAlgebraic [A] time = 1.32, size = 90, normalized size = 0.54

$$\frac{235520000x^7 + 831385600x^6 + 1281670400x^5 + 230669760x^4 - 2613624504x^3 - 2534760678x^2 - 8859305979x - 10961697147}{4521984\sqrt{2x^2 - x + 3}} - \frac{310445587 \log\left(2\sqrt{2}\sqrt{2x^2 - x + 3} - 4x + 1\right)}{131072\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 + 3*x + 5*x^2)^4/(3 - x + 2*x^2)^(3/2),x]
```

```
[Out] (-10961697147 - 8859305979*x - 2534760678*x^2 - 2613624504*x^3 + 230669760*
x^4 + 1281670400*x^5 + 831385600*x^6 + 235520000*x^7)/(4521984*Sqrt[3 - x +
 2*x^2]) - (310445587*Log[1 - 4*x + 2*Sqrt[2]*Sqrt[3 - x + 2*x^2]])/(131072
*Sqrt[2])
```

fricas [A] time = 0.41, size = 112, normalized size = 0.67

$$\frac{21420745503 \sqrt{2} \left(2 x^2 - x + 3\right) \log\left(-4 \sqrt{2} \sqrt{2 x^2 - x + 3} (4 x - 1) - 32 x^2 + 16 x - 25\right) + 8 \left(235520000 x^7 + 831385600 x^6 + 1281670400 x^5 + 230669760 x^4 - 2613624504 x^3 - 2534760678 x^2 - 8859305979 x - 10961697147\right)\sqrt{2 x^2 - x + 3}}{36175872 \left(2 x^2 - x + 3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/36175872*(21420745503*sqrt(2)*(2*x^2 - x + 3)*log(-4*sqrt(2)*sqrt(2*x^2 -
 x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 8*(235520000*x^7 + 831385600*x^6
+ 1281670400*x^5 + 230669760*x^4 - 2613624504*x^3 - 2534760678*x^2 - 885930
5979*x - 10961697147)*sqrt(2*x^2 - x + 3))/(2*x^2 - x + 3)
```

giac [A] time = 0.25, size = 82, normalized size = 0.49

$$-\frac{310445587}{262144} \sqrt{2} \log\left(-2 \sqrt{2}\left(\sqrt{2} x - \sqrt{2 x^2 - x + 3}\right) + 1\right) + \frac{(46 (4 (40 (20 (16 (100 x + 353) x + 8707) x + 31341) x - 14204481) x - 55103493) x - 8859305979) x - 10961697147}{4521984 \sqrt{2 x^2 - x + 3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^(3/2),x, algorithm="giac")
```

```
[Out] -310445587/262144*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))
+ 1) + 1/4521984*((46*(4*(40*(20*(16*(100*x + 353)*x + 8707)*x + 31341)*x -
 14204481)*x - 55103493)*x - 8859305979)*x - 10961697147)/sqrt(2*x^2 - x +
3)
```

maple [A] time = 0.03, size = 166, normalized size = 1.00

$$\frac{625x^7}{12\sqrt{2x^2 - x + 3}} + \frac{8825x^6}{48\sqrt{2x^2 - x + 3}} + \frac{217675x^5}{768\sqrt{2x^2 - x + 3}} + \frac{52235x^4}{1024\sqrt{2x^2 - x + 3}} - \frac{4734827x^3}{8192\sqrt{2x^2 - x + 3}} - \frac{18367831x^2}{32768\sqrt{2x^2 - x + 3}} - \frac{310445587x}{131072\sqrt{2x^2 - x + 3}} + \frac{310445587\sqrt{2}\ \operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{262144} - \frac{1217267299}{524288\sqrt{2x^2 - x + 3}} + \frac{\frac{1234044515x}{3014656} - \frac{1234044515}{12058624}}{\sqrt{2x^2 - x + 3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)^4/(2*x^2-x+3)^(3/2),x)
```

```
[Out] -1217267299/524288/(2*x^2-x+3)^(1/2)+1234044515/12058624*(4*x-1)/(2*x^2-x+3
)^(1/2)+310445587/262144*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))+217675/768*
x^5/(2*x^2-x+3)^(1/2)+52235/1024*x^4/(2*x^2-x+3)^(1/2)-4734827/8192*x^3/(2*
x^2-x+3)^(1/2)+625/12*x^7/(2*x^2-x+3)^(1/2)+8825/48*x^6/(2*x^2-x+3)^(1/2)-1
8367831/32768*x^2/(2*x^2-x+3)^(1/2)-310445587/131072*x/(2*x^2-x+3)^(1/2)
```

maxima [A] time = 0.98, size = 148, normalized size = 0.89

$$\frac{625\,x^{7}}{12\,\sqrt{2\,x^{2}-x+3}}+\frac{8825\,x^{6}}{48\,\sqrt{2\,x^{2}-x+3}}+\frac{217675\,x^{5}}{768\,\sqrt{2\,x^{2}-x+3}}+\frac{52235\,x^{4}}{1024\,\sqrt{2\,x^{2}-x+3}}-\frac{4734827\,x^{3}}{8192\,\sqrt{2\,x^{2}-x+3}}-\frac{18367831\,x^{2}}{32768\,\sqrt{2\,x^{2}-x+3}}+\frac{310445587}{262144}\,\sqrt{2}\,\operatorname{arsinh}\left(\frac{1}{23}\,\sqrt{23}\,(4\,x-1)\right)-\frac{2953101993\,x}{1507328\,\sqrt{2\,x^{2}-x+3}}-\frac{3653899049}{1507328\,\sqrt{2\,x^{2}-x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^(3/2),x, algorithm="maxima")
```

```
[Out] 625/12*x^7/sqrt(2*x^2 - x + 3) + 8825/48*x^6/sqrt(2*x^2 - x + 3) + 217675/7
68*x^5/sqrt(2*x^2 - x + 3) + 52235/1024*x^4/sqrt(2*x^2 - x + 3) - 4734827/8
192*x^3/sqrt(2*x^2 - x + 3) - 18367831/32768*x^2/sqrt(2*x^2 - x + 3) + 3104
45587/262144*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 2953101993/1507328*
x/sqrt(2*x^2 - x + 3) - 3653899049/1507328/sqrt(2*x^2 - x + 3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(5\,x^2+3\,x+2\right)}^4}{{\left(2\,x^2-x+3\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 5*x^2 + 2)^4/(2*x^2 - x + 3)^(3/2),x)
```

```
[Out] int((3*x + 5*x^2 + 2)^4/(2*x^2 - x + 3)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(5x^{2} + 3x + 2\right)^{4}}{\left(2x^{2} - x + 3\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)**4/(2*x**2-x+3)**(3/2),x)
```

```
[Out] Integral((5*x**2 + 3*x + 2)**4/(2*x**2 - x + 3)**(3/2), x)
```

3.84 $\int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^{3/2}}\,dx$

Optimal. Leaf size=124

$$\frac{1825}{64}\sqrt{2x^2-x+3}\,x^2+\frac{15565}{512}\sqrt{2x^2-x+3}\,x-\frac{181561\sqrt{2x^2-x+3}}{2048}-\frac{1331(17-45x)}{368\sqrt{2x^2-x+3}}+\frac{125}{16}\sqrt{2x^2-x+3}\,x^3+-$$

Rubi [A] time = 0.13, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1660, 1661, 640, 619, 215}

$$\frac{125}{16}\sqrt{2x^2-x+3}\,x^3+\frac{1825}{64}\sqrt{2x^2-x+3}\,x^2+\frac{15565}{512}\sqrt{2x^2-x+3}\,x-\frac{181561\sqrt{2x^2-x+3}}{2048}-\frac{1331(17-45x)}{368\sqrt{2x^2-x+3}}+\frac{1168881\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{4096\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + 5*x^2)^3/(3 - x + 2*x^2)^(3/2),x]
```

```
[Out] (-1331*(17 - 45*x))/(368*Sqrt[3 - x + 2*x^2]) - (181561*Sqrt[3 - x + 2*x^2]
)/2048 + (15565*x*Sqrt[3 - x + 2*x^2])/512 + (1825*x^2*Sqrt[3 - x + 2*x^2])
/64 + (125*x^3*Sqrt[3 - x + 2*x^2])/16 + (1168881*ArcSinh[(1 - 4*x)/Sqrt[23
]])/(4096*Sqrt[2])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(2+3x+5x^2\right)^3}{\left(3-x+2x^2\right)^{3/2}}\,dx = -\frac{1331(17-45x)}{368\sqrt{3-x+2x^2}} + \frac{2}{23}\int \frac{-\frac{110285}{64} - \frac{19067x}{32} + \frac{22195x^2}{16} + \frac{13225x^3}{8} + \frac{2875x^4}{4}}{\sqrt{3-x+2x^2}}\,dx$$

$$= -\frac{1331(17-45x)}{368\sqrt{3-x+2x^2}} + \frac{125}{16}x^3\sqrt{3-x+2x^2} + \frac{1}{92}\int \frac{-\frac{110285}{8} - \frac{19067x}{4} + \frac{18515x^2}{4} + \frac{125925x}{8}}{\sqrt{3-x+2x^2}}$$

$$= -\frac{1331(17-45x)}{368\sqrt{3-x+2x^2}} + \frac{1825}{64}x^2\sqrt{3-x+2x^2} + \frac{125}{16}x^3\sqrt{3-x+2x^2} + \frac{1}{552}\int \frac{-\frac{330855}{4} -}{\sqrt{}}$$

$$= -\frac{1331(17-45x)}{368\sqrt{3-x+2x^2}} + \frac{15565}{512}x\sqrt{3-x+2x^2} + \frac{1825}{64}x^2\sqrt{3-x+2x^2} + \frac{125}{16}x^3\sqrt{3-x+}$$

$$= -\frac{1331(17-45x)}{368\sqrt{3-x+2x^2}} - \frac{181561\sqrt{3-x+2x^2}}{2048} + \frac{15565}{512}x\sqrt{3-x+2x^2} + \frac{1825}{64}x^2\sqrt{3-x}$$

$$= -\frac{1331(17-45x)}{368\sqrt{3-x+2x^2}} - \frac{181561\sqrt{3-x+2x^2}}{2048} + \frac{15565}{512}x\sqrt{3-x+2x^2} + \frac{1825}{64}x^2\sqrt{3-x}$$

$$= -\frac{1331(17-45x)}{368\sqrt{3-x+2x^2}} - \frac{181561\sqrt{3-x+2x^2}}{2048} + \frac{15565}{512}x\sqrt{3-x+2x^2} + \frac{1825}{64}x^2\sqrt{3-x}$$

Mathematica [A] time = 0.23, size = 65, normalized size = 0.52

$$\frac{\frac{4\left(736000x^5+2318400x^4+2624760x^3-5754186x^2+16138403x-15423965\right)}{\sqrt{2x^2-x+3}} - 26884263\sqrt{2}\sinh^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{188416}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)^3/(3 - x + 2*x^2)^(3/2),x]
```

```
[Out] ((4*(-15423965 + 16138403*x - 5754186*x^2 + 2624760*x^3 + 2318400*x^4 + 736
000*x^5))/Sqrt[3 - x + 2*x^2] - 26884263*Sqrt[2]*ArcSinh[(-1 + 4*x)/Sqrt[23
]])/188416
```

IntegrateAlgebraic [A] time = 0.84, size = 80, normalized size = 0.65

$$\frac{1168881\log\left(2\sqrt{2}\sqrt{2x^2-x+3}-4x+1\right)}{4096\sqrt{2}} + \frac{736000x^5+2318400x^4+2624760x^3-5754186x^2+16138403x-15423965}{47104\sqrt{2x^2-x+3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 + 3*x + 5*x^2)^3/(3 - x + 2*x^2)^(3/2),x]
```

```
[Out] (-15423965 + 16138403*x - 5754186*x^2 + 2624760*x^3 + 2318400*x^4 + 736000*
x^5)/(47104*Sqrt[3 - x + 2*x^2]) + (1168881*Log[1 - 4*x + 2*Sqrt[2]*Sqrt[3
- x + 2*x^2]])/(4096*Sqrt[2])
```

fricas [A] time = 0.42, size = 102, normalized size = 0.82

$$\frac{26884263 \sqrt{2} \left(2 x^{2}-x+3\right) \log \left(4 \sqrt{2} \sqrt{2 x^{2}-x+3}(4 x-1)-32 x^{2}+16 x-25\right)+8 \left(736000 x^{5}+2318400 x^{4}+2624760 x^{3}-5754186 x^{2}+16138403 x-15423965\right) \sqrt{2 x^{2}-x+3}}{376832 \left(2 x^{2}-x+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/376832*(26884263*sqrt(2)*(2*x^2 - x + 3)*log(4*sqrt(2)*sqrt(2*x^2 - x + 3
)*(4*x - 1) - 32*x^2 + 16*x - 25) + 8*(736000*x^5 + 2318400*x^4 + 2624760*x
^3 - 5754186*x^2 + 16138403*x - 15423965)*sqrt(2*x^2 - x + 3))/(2*x^2 - x +
 3)
```

giac [A] time = 0.28, size = 72, normalized size = 0.58

$$\frac{1168881}{8192} \sqrt{2} \log \left(-2 \sqrt{2} \left(\sqrt{2} x-\sqrt{2 x^{2}-x+3}\right)+1\right)+\frac{(46 (20 (40 (20 x+63) x+2853) x-125091) x+16138403) x-15423965}{47104 \sqrt{2 x^{2}-x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^(3/2),x, algorithm="giac")
```

```
[Out] 1168881/8192*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1)
+ 1/47104*((46*(20*(40*(20*x + 63)*x + 2853)*x - 125091)*x + 16138403)*x -
15423965)/sqrt(2*x^2 - x + 3)
```

maple [A] time = 0.01, size = 132, normalized size = 1.06

$$\frac{125 x^{5}}{8 \sqrt{2 x^{2}-x+3}}+\frac{1575 x^{4}}{32 \sqrt{2 x^{2}-x+3}}+\frac{14265 x^{3}}{256 \sqrt{2 x^{2}-x+3}}-\frac{125091 x^{2}}{1024 \sqrt{2 x^{2}-x+3}}+\frac{1168881 x}{4096 \sqrt{2 x^{2}-x+3}}-\frac{1168881 \sqrt{2}\, \operatorname{arcsinh}\left(\frac{4 \sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{8192}-\frac{5130399}{16384 \sqrt{2 x^{2}-x+3}}+\frac{\frac{5392543 x}{94208}-\frac{5392543}{376832}}{\sqrt{2 x^{2}-x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)^3/(2*x^2-x+3)^(3/2),x)
```

```
[Out] -5130399/16384/(2*x^2-x+3)^(1/2)+5392543/376832*(4*x-1)/(2*x^2-x+3)^(1/2)-1
168881/8192*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))+125/8/(2*x^2-x+3)^(1/2)*
x^5+1575/32/(2*x^2-x+3)^(1/2)*x^4+14265/256/(2*x^2-x+3)^(1/2)*x^3-125091/10
24/(2*x^2-x+3)^(1/2)*x^2+1168881/4096/(2*x^2-x+3)^(1/2)*x
```

maxima [A] time = 0.97, size = 114, normalized size = 0.92

$$\frac{125\,x^5}{8\,\sqrt{2\,x^2-x+3}}+\frac{1575\,x^4}{32\,\sqrt{2\,x^2-x+3}}+\frac{14265\,x^3}{256\,\sqrt{2\,x^2-x+3}}-\frac{125091\,x^2}{1024\,\sqrt{2\,x^2-x+3}}-\frac{1168881}{8192}\sqrt{2}\,\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}\,(4\,x-1)\right)+\frac{16138403\,x}{47104\,\sqrt{2\,x^2-x+3}}-\frac{15423965}{47104\,\sqrt{2\,x^2-x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^(3/2),x, algorithm="maxima")
```

```
[Out] 125/8*x^5/sqrt(2*x^2 - x + 3) + 1575/32*x^4/sqrt(2*x^2 - x + 3) + 14265/256
*x^3/sqrt(2*x^2 - x + 3) - 125091/1024*x^2/sqrt(2*x^2 - x + 3) - 1168881/81
92*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) + 16138403/47104*x/sqrt(2*x^2 -
 x + 3) - 15423965/47104/sqrt(2*x^2 - x + 3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(5\,x^2+3\,x+2\right)^3}{\left(2\,x^2-x+3\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 5*x^2 + 2)^3/(2*x^2 - x + 3)^(3/2),x)
```

```
[Out] int((3*x + 5*x^2 + 2)^3/(2*x^2 - x + 3)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(5x^2+3x+2\right)^3}{\left(2x^2-x+3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)**3/(2*x**2-x+3)**(3/2),x)
```

```
[Out] Integral((5*x**2 + 3*x + 2)**3/(2*x**2 - x + 3)**(3/2), x)
```

3.85 $\int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^{3/2}}\,dx$

Optimal. Leaf size=82

$$\frac{121(19-7x)}{92\sqrt{2x^2-x+3}}+\frac{25}{8}x\sqrt{2x^2-x+3}+\frac{415}{32}\sqrt{2x^2-x+3}-\frac{223\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{64\sqrt{2}}$$

Rubi [A] time = 0.07, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1660, 1661, 640, 619, 215}

$$\frac{121(19-7x)}{92\sqrt{2x^2-x+3}}+\frac{25}{8}x\sqrt{2x^2-x+3}+\frac{415}{32}\sqrt{2x^2-x+3}-\frac{223\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{64\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + 5*x^2)^2/(3 - x + 2*x^2)^(3/2),x]
```

```
[Out] (121*(19 - 7*x))/(92*Sqrt[3 - x + 2*x^2]) + (415*Sqrt[3 - x + 2*x^2])/32 +
(25*x*Sqrt[3 - x + 2*x^2])/8 - (223*ArcSinh[(1 - 4*x)/Sqrt[23]])/(64*Sqrt[2
])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(2+3x+5x^2\right)^2}{\left(3-x+2x^2\right)^{3/2}}\,dx = \frac{121(19-7x)}{92\sqrt{3-x+2x^2}} + \frac{2}{23}\int \frac{\frac{1173}{16}+\frac{1955x}{8}+\frac{575x^2}{4}}{\sqrt{3-x+2x^2}}\,dx$$

$$= \frac{121(19-7x)}{92\sqrt{3-x+2x^2}} + \frac{25}{8}x\sqrt{3-x+2x^2} + \frac{1}{46}\int \frac{-138+\frac{9545x}{8}}{\sqrt{3-x+2x^2}}\,dx$$

$$= \frac{121(19-7x)}{92\sqrt{3-x+2x^2}} + \frac{415}{32}\sqrt{3-x+2x^2} + \frac{25}{8}x\sqrt{3-x+2x^2} + \frac{223}{64}\int \frac{1}{\sqrt{3-x+2x^2}}\,dx$$

$$= \frac{121(19-7x)}{92\sqrt{3-x+2x^2}} + \frac{415}{32}\sqrt{3-x+2x^2} + \frac{25}{8}x\sqrt{3-x+2x^2} + \frac{223\,\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{23}}}\,dx,\right.}{64\sqrt{46}}$$

$$= \frac{121(19-7x)}{92\sqrt{3-x+2x^2}} + \frac{415}{32}\sqrt{3-x+2x^2} + \frac{25}{8}x\sqrt{3-x+2x^2} - \frac{223\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{64\sqrt{2}}$$

Mathematica [A] time = 0.14, size = 55, normalized size = 0.67

$$\frac{4600x^3+16790x^2-9421x+47027}{736\sqrt{2x^2-x+3}} + \frac{223\sinh^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{64\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)^2/(3 - x + 2*x^2)^(3/2),x]
```

```
[Out] (47027 - 9421*x + 16790*x^2 + 4600*x^3)/(736*Sqrt[3 - x + 2*x^2]) + (223*Ar
cSinh[(-1 + 4*x)/Sqrt[23]])/(64*Sqrt[2])
```

IntegrateAlgebraic [A] time = 0.73, size = 70, normalized size = 0.85

$$\frac{4600x^3+16790x^2-9421x+47027}{736\sqrt{2x^2-x+3}}-\frac{223\log\left(2\sqrt{2}\sqrt{2x^2-x+3}-4x+1\right)}{64\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 + 3*x + 5*x^2)^2/(3 - x + 2*x^2)^(3/2),x]
```

```
[Out] (47027 - 9421*x + 16790*x^2 + 4600*x^3)/(736*Sqrt[3 - x + 2*x^2]) - (223*Lo
g[1 - 4*x + 2*Sqrt[2]*Sqrt[3 - x + 2*x^2]])/(64*Sqrt[2])
```

fricas [A] time = 0.41, size = 92, normalized size = 1.12

$$\frac{5129\sqrt{2}\left(2x^2-x+3\right)\log\left(-4\sqrt{2}\sqrt{2x^2-x+3}(4x-1)-32x^2+16x-25\right)+8\left(4600x^3+16790x^2-9421x+47027\right)\sqrt{2x^2-x+3}}{5888\left(2x^2-x+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/5888*(5129*sqrt(2)*(2*x^2 - x + 3)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*
x - 1) - 32*x^2 + 16*x - 25) + 8*(4600*x^3 + 16790*x^2 - 9421*x + 47027)*sq
rt(2*x^2 - x + 3))/(2*x^2 - x + 3)
```

giac [A] time = 0.24, size = 62, normalized size = 0.76

$$-\frac{223}{128}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+1\right)+\frac{(230(20x+73)x-9421)x+47027}{736\sqrt{2x^2-x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^(3/2),x, algorithm="giac")
```

```
[Out] -223/128*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) + 1/
736*((230*(20*x + 73)*x - 9421)*x + 47027)/sqrt(2*x^2 - x + 3)
```

maple [A] time = 0.01, size = 98, normalized size = 1.20

$$\frac{25x^3}{4\sqrt{2x^2-x+3}}+\frac{365x^2}{16\sqrt{2x^2-x+3}}-\frac{223x}{64\sqrt{2x^2-x+3}}+\frac{223\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{128}+\frac{15761}{256\sqrt{2x^2-x+3}}-\frac{13713(4x-1)}{5888\sqrt{2x^2-x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)^2/(2*x^2-x+3)^(3/2),x)
```

```
[Out] 25/4/(2*x^2-x+3)^(1/2)*x^3+365/16/(2*x^2-x+3)^(1/2)*x^2-223/64/(2*x^2-x+3)^
(1/2)*x+15761/256/(2*x^2-x+3)^(1/2)-13713/5888*(4*x-1)/(2*x^2-x+3)^(1/2)+22
3/128*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))
```

maxima [A] time = 0.96, size = 80, normalized size = 0.98

$$\frac{25\,x^3}{4\,\sqrt{2\,x^2-x+3}}+\frac{365\,x^2}{16\,\sqrt{2\,x^2-x+3}}+\frac{223}{128}\,\sqrt{2}\,\operatorname{arsinh}\left(\frac{1}{23}\,\sqrt{23}\,(4\,x-1)\right)-\frac{9421\,x}{736\,\sqrt{2\,x^2-x+3}}+\frac{47027}{736\,\sqrt{2\,x^2-x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^(3/2),x, algorithm="maxima")
```

```
[Out] 25/4*x^3/sqrt(2*x^2 - x + 3) + 365/16*x^2/sqrt(2*x^2 - x + 3) + 223/128*sqr
t(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 9421/736*x/sqrt(2*x^2 - x + 3) + 47
027/736/sqrt(2*x^2 - x + 3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(5\,x^2+3\,x+2\right)^2}{\left(2\,x^2-x+3\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 5*x^2 + 2)^2/(2*x^2 - x + 3)^(3/2),x)
```

```
[Out] int((3*x + 5*x^2 + 2)^2/(2*x^2 - x + 3)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(5x^2+3x+2\right)^2}{\left(2x^2-x+3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)**2/(2*x**2-x+3)**(3/2),x)
```

```
[Out] Integral((5*x**2 + 3*x + 2)**2/(2*x**2 - x + 3)**(3/2), x)
```

3.86 $\int \frac{2+3x+5x^2}{\left(3-x+2x^2\right)^{3/2}} \, dx$

Optimal. Leaf size=45

$$-\frac{11(3x+5)}{23\sqrt{2x^2-x+3}}-\frac{5\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{2\sqrt{2}}$$

Rubi [A] time = 0.03, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1660, 12, 619, 215}

$$-\frac{11(3x+5)}{23\sqrt{2x^2-x+3}}-\frac{5\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + 5*x^2)/(3 - x + 2*x^2)^(3/2),x]
```

```
[Out] (-11*(5 + 3*x))/(23*Sqrt[3 - x + 2*x^2]) - (5*ArcSinh[(1 - 4*x)/Sqrt[23]])/
(2*Sqrt[2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
```

```
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{2+3x+5x^2}{\left(3-x+2x^2\right)^{3/2}}\,dx &= -\frac{11(5+3x)}{23\sqrt{3-x+2x^2}}+\frac{2}{23}\int \frac{115}{4\sqrt{3-x+2x^2}}\,dx\\ &= -\frac{11(5+3x)}{23\sqrt{3-x+2x^2}}+\frac{5}{2}\int \frac{1}{\sqrt{3-x+2x^2}}\,dx\\ &= -\frac{11(5+3x)}{23\sqrt{3-x+2x^2}}+\frac{5\,\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{23}}}\,dx,x,-1+4x\right)}{2\sqrt{46}}\\ &= -\frac{11(5+3x)}{23\sqrt{3-x+2x^2}}-\frac{5\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{2\sqrt{2}}\end{aligned}$$

Mathematica [A] time = 0.08, size = 45, normalized size = 1.00

$$\frac{5\sinh^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{2\sqrt{2}}-\frac{11(3x+5)}{23\sqrt{2x^2-x+3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)/(3 - x + 2*x^2)^(3/2),x]
```

```
[Out] (-11*(5 + 3*x))/(23*Sqrt[3 - x + 2*x^2]) + (5*ArcSinh[(-1 + 4*x)/Sqrt[23]])
/(2*Sqrt[2])
```

IntegrateAlgebraic [A] time = 0.42, size = 60, normalized size = 1.33

$$-\frac{11(3x+5)}{23\sqrt{2x^2-x+3}}-\frac{5\log\left(2\sqrt{2}\sqrt{2x^2-x+3}-4x+1\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 + 3*x + 5*x^2)/(3 - x + 2*x^2)^(3/2),x]
```

[Out] (-11*(5 + 3*x))/(23*Sqrt[3 - x + 2*x^2]) - (5*Log[1 - 4*x + 2*Sqrt[2]*Sqrt[
3 - x + 2*x^2]])/(2*Sqrt[2])

fricas [B] time = 0.41, size = 82, normalized size = 1.82

$$\frac{115\sqrt{2}\left(2x^2-x+3\right)\log\left(-4\sqrt{2}\sqrt{2x^2-x+3}\,(4x-1)-32x^2+16x-25\right)-88\sqrt{2x^2-x+3}\,(3x+5)}{184\left(2x^2-x+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)/(2*x^2-x+3)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/184*(115*sqrt(2)*(2*x^2 - x + 3)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x
- 1) - 32*x^2 + 16*x - 25) - 88*sqrt(2*x^2 - x + 3)*(3*x + 5))/(2*x^2 - x +
 3)
```

giac [A] time = 0.23, size = 53, normalized size = 1.18

$$-\frac{5}{4}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+1\right)-\frac{11\,(3x+5)}{23\sqrt{2x^2-x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)/(2*x^2-x+3)^(3/2),x, algorithm="giac")
```

```
[Out] -5/4*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) - 11/23*
(3*x + 5)/sqrt(2*x^2 - x + 3)
```

maple [A] time = 0.01, size = 64, normalized size = 1.42

$$-\frac{5x}{2\sqrt{2x^2-x+3}}+\frac{5\sqrt{2}\ \operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{4}-\frac{17}{8\sqrt{2x^2-x+3}}+\frac{\frac{49x}{46}-\frac{49}{184}}{\sqrt{2x^2-x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)/(2*x^2-x+3)^(3/2),x)
```

```
[Out] -5/2/(2*x^2-x+3)^(1/2)*x-17/8/(2*x^2-x+3)^(1/2)+49/184*(4*x-1)/(2*x^2-x+3)^
(1/2)+5/4*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))
```

maxima [A] time = 0.95, size = 46, normalized size = 1.02

$$\frac{5}{4}\sqrt{2}\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}\,(4x-1)\right)-\frac{33x}{23\sqrt{2x^2-x+3}}-\frac{55}{23\sqrt{2x^2-x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)/(2*x^2-x+3)^(3/2),x, algorithm="maxima")
```

```
[Out] 5/4*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 33/23*x/sqrt(2*x^2 - x + 3)
- 55/23/sqrt(2*x^2 - x + 3)
```

mupad [B] time = 0.23, size = 87, normalized size = 1.93

$$\frac{5\sqrt{2}\ \ln\left(\sqrt{2\,x^2-x+3}+\frac{\sqrt{2}\,\left(2\,x-\frac{1}{2}\right)}{2}\right)}{4}+\frac{3\,\left(2\,x-12\right)}{23\,\sqrt{2\,x^2-x+3}}-\frac{10\,\left(\frac{11\,x}{2}+\frac{3}{2}\right)}{23\,\sqrt{2\,x^2-x+3}}+\frac{16\,x-4}{23\,\sqrt{2\,x^2-x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 5*x^2 + 2)/(2*x^2 - x + 3)^(3/2),x)
```

```
[Out] (5*2^(1/2)*log((2*x^2 - x + 3)^(1/2) + (2^(1/2)*(2*x - 1/2))/2))/4 + (3*(2*
x - 12))/(23*(2*x^2 - x + 3)^(1/2)) - (10*((11*x)/2 + 3/2))/(23*(2*x^2 - x
+ 3)^(1/2)) + (16*x - 4)/(23*(2*x^2 - x + 3)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{5x^2+3x+2}{\left(2x^2-x+3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)/(2*x**2-x+3)**(3/2),x)
```

```
[Out] Integral((5*x**2 + 3*x + 2)/(2*x**2 - x + 3)**(3/2), x)
```

3.87 $\int \frac{1}{(3-x+2x^2)^{3/2}(2+3x+5x^2)}\,dx$

Optimal. Leaf size=176

$$\frac{13-6x}{253\sqrt{2x^2-x+3}}+\frac{1}{22}\sqrt{\frac{1}{682}\left(247+500\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{11}{31\left(247+500\sqrt{2}\right)}}\left(\left(69+65\sqrt{2}\right)x+4\sqrt{2}+61\right)}{\sqrt{2x^2-x+3}}\right)-\frac{1}{22}\sqrt{\frac{1}{682}}$$

Rubi [A] time = 0.41, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {974, 1035, 1029, 206, 204}

$$\frac{13-6x}{253\sqrt{2x^2-x+3}}+\frac{1}{22}\sqrt{\frac{1}{682}\left(247+500\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{11}{31\left(247+500\sqrt{2}\right)}}\left(\left(69+65\sqrt{2}\right)x+4\sqrt{2}+61\right)}{\sqrt{2x^2-x+3}}\right)-\frac{1}{22}\sqrt{\frac{1}{682}\left(500\sqrt{2}-247\right)}\tanh^{-1}\left(\frac{\sqrt{\frac{11}{31\left(500\sqrt{2}-247\right)}}\left(\left(69-65\sqrt{2}\right)x-4\sqrt{2}+61\right)}{\sqrt{2x^2-x+3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)),x]
```

```
[Out] (13 - 6*x)/(253*Sqrt[3 - x + 2*x^2]) + (Sqrt[(247 + 500*Sqrt[2])/682]*ArcTa
n[(Sqrt[11/(31*(247 + 500*Sqrt[2]))]*(61 + 4*Sqrt[2] + (69 + 65*Sqrt[2])*x)
)/Sqrt[3 - x + 2*x^2]])/22 - (Sqrt[(-247 + 500*Sqrt[2])/682]*ArcTanh[(Sqrt[
11/(31*(-247 + 500*Sqrt[2]))]*(61 - 4*Sqrt[2] + (69 - 65*Sqrt[2])*x))/Sqrt[
3 - x + 2*x^2]])/22
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 974

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*
a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*(
d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
```

```
- b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q,
 0]
```

Rule 1029

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
 c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 1035

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

Rubi steps

$$\int \frac{1}{\left(3-x+2x^2\right)^{3/2}\left(2+3x+5x^2\right)}\,dx = \frac{13-6x}{253\sqrt{3-x+2x^2}} - \frac{\int \frac{-1012-\frac{1265x}{2}}{\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)}\,dx}{2783}$$

$$= \frac{13-6x}{253\sqrt{3-x+2x^2}} + \frac{\int \frac{\frac{2783}{2}\left(3+8\sqrt{2}\right)-\frac{2783}{2}\left(13-5\sqrt{2}\right)x}{\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)}\,dx}{61226\sqrt{2}} - \frac{\int \frac{\frac{2783}{2}\left(3-8\sqrt{2}\right)-\frac{2783}{2}(\ldots}{\sqrt{3-x+2x^2}\left(2+3x\ldots\right.}}{61226\sqrt{2}\ldots}$$

$$= \frac{13-6x}{253\sqrt{3-x+2x^2}} - \frac{1}{8}\left(253\left(1000-247\sqrt{2}\right)\right)\text{Subst}\left(\int \frac{\ldots}{-\frac{240097759}{4}\left(247\ldots\right.}\right.$$

$$= \frac{13-6x}{253\sqrt{3-x+2x^2}} + \frac{1}{22}\sqrt{\frac{1}{682}\left(247+500\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{11}{31\left(247+500\sqrt{2}\right)}}\left(\ldots\right.}{\sqrt{\ldots}}\right.$$

Mathematica [C] time = 1.29, size = 202, normalized size = 1.15

$$\frac{-\frac{27280(6x-13)}{\sqrt{2x^2-x+3}}-23\sqrt{682\left(13+i\sqrt{31}\right)}\left(13\sqrt{31}+69i\right)\tanh^{-1}\left(\frac{\left(-22-4i\sqrt{31}\right)x+i\sqrt{31}+63}{2\sqrt{286+22i\sqrt{31}}\sqrt{2x^2-x+3}}\right)-23\sqrt{682\left(13-i\sqrt{31}\right)}\left(13\sqrt{31}-69i\right)\tanh^{-1}\left(\frac{\left(-22+4i\sqrt{31}\right)x-i\sqrt{31}+63}{2\sqrt{286-22i\sqrt{31}}\sqrt{2x^2-x+3}}\right)}{6901840}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)),x]
```

```
[Out] ((-27280*(-13 + 6*x))/Sqrt[3 - x + 2*x^2] - 23*Sqrt[682*(13 + I*Sqrt[31])]*
(69*I + 13*Sqrt[31])*ArcTanh[(63 + I*Sqrt[31] + (-22 - (4*I)*Sqrt[31])*x)/(
2*Sqrt[286 + (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])] - 23*Sqrt[682*(13 - I*S
qrt[31])]*(-69*I + 13*Sqrt[31])*ArcTanh[(63 - I*Sqrt[31] + (-22 + (4*I)*Sqr
t[31])*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])])/6901840
```

IntegrateAlgebraic [C] time = 0.49, size = 199, normalized size = 1.13

$$\frac{1}{22}\text{RootSum}\left[-5\#1^4+6\sqrt{2}\#1^3+17\#1^2-26\sqrt{2}\#1-56\&,\frac{-5\#1^2\log\left(-\#1+\sqrt{2x^2-x+3}-\sqrt{2}x\right)+16\sqrt{2}\#1\log\left(-\#1+\sqrt{2x^2-x+3}-\sqrt{2}x\right)+23\log\left(-\#1+\sqrt{2x^2-x+3}-\sqrt{2}x\right)}{-10\#1^3+9\sqrt{2}\#1^2+17\#1-13\sqrt{2}}\&\right]+\frac{13-6x}{253\sqrt{2x^2-x+3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)),x]
```

```
[Out] (13 - 6*x)/(253*Sqrt[3 - x + 2*x^2]) + RootSum[-56 - 26*Sqrt[2]*#1 + 17*#1^
2 + 6*Sqrt[2]*#1^3 - 5*#1^4 & , (23*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2]
- #1] + 16*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1 - 5*Log[
```

```
-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1^2)/(-13*Sqrt[2] + 17*#1 + 9*Sqr
t[2]*#1^2 - 10*#1^3) & ]/22
```

fricas [B] time = 1.17, size = 2083, normalized size = 11.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^(3/2)/(5*x^2+3*x+2),x, algorithm="fricas")
```

```
[Out] -1/50921775520*(339388*sqrt(341)*50^(1/4)*sqrt(10)*sqrt(2)*(2*x^2 - x + 3)*
sqrt(247*sqrt(2) + 1000)*arctan(1/328782125*(14260*sqrt(341)*sqrt(10)*sqrt(
2*x^2 - x + 3)*(22*50^(3/4)*(57708*x^7 - 181278*x^6 + 400374*x^5 - 525676*x
^4 + 235088*x^3 - 46944*x^2 - sqrt(2)*(20846*x^7 - 109153*x^6 + 215386*x^5
- 427391*x^4 + 234360*x^3 - 156600*x^2 - 172800*x + 186624) - 373248*x + 17
2800) + 5*50^(1/4)*(125839*x^7 - 1864281*x^6 + 9323336*x^5 - 19725020*x^4 +
 24624288*x^3 - 10862496*x^2 - sqrt(2)*(56119*x^7 - 908994*x^6 + 5175980*x^
5 - 12895624*x^4 + 17261280*x^3 - 14184000*x^2 - 10533888*x + 9994752) - 19
989504*x + 10533888))*sqrt(247*sqrt(2) + 1000) + 933317000*sqrt(31)*sqrt(2)
*(28180*x^8 - 254666*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*
x^3 - 98496*x^2 - sqrt(2)*(8746*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5
+ 1320710*x^4 - 752088*x^3 + 396144*x^2 + 546048*x - 539136) + 1154304*x -
456192) - 2*sqrt(310/119)*(sqrt(341)*sqrt(10)*sqrt(2*x^2 - x + 3)*(22*50^(3
/4)*(246848*x^7 - 348192*x^6 + 1080672*x^5 - 178432*x^4 - 18432*x^3 + 10298
88*x^2 - sqrt(2)*(46522*x^7 - 71117*x^6 + 257247*x^5 - 273360*x^4 + 484920*
x^3 - 269568*x^2 + 269568*x) - 1029888*x) + 5*50^(1/4)*(516957*x^7 - 667694
8*x^6 + 25569820*x^5 - 31522752*x^4 + 34450848*x^3 + 46199808*x^2 - 4*sqrt(
2)*(38689*x^7 - 502244*x^6 + 1967660*x^5 - 2828160*x^4 + 4711680*x^3 - 1689
984*x^2 + 1689984*x) - 46199808*x))*sqrt(247*sqrt(2) + 1000) + 65450*sqrt(3
1)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 + 396480*x^
4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 118051*x^7 + 244047*x^6
 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*x) + 3276
288*x) + 2975*sqrt(31)*(254591*x^8 - 4815126*x^7 + 32303580*x^6 - 90866808*
x^5 + 108781920*x^4 - 74219328*x^3 - 168956928*x^2 - 15488*sqrt(2)*(4*x^8 -
 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3618*x^3 + 2268*x^2 - 1944*x) + 1
44820224*x))*sqrt(-(sqrt(341)*50^(1/4)*sqrt(31)*sqrt(10)*sqrt(2*x^2 - x + 3
)*(sqrt(2)*(37*x - 38) + x - 75)*sqrt(247*sqrt(2) + 1000) - 903805*x^2 - 81
1580*sqrt(2)*(2*x^2 - x + 3) + 2785195*x - 3689000)/x^2) + 10605875*sqrt(31
)*(2828123*x^8 - 9696916*x^7 + 53385560*x^6 - 142835344*x^5 + 254146592*x^4
 - 249300096*x^3 + 37981440*x^2 - 7744*sqrt(2)*(1348*x^8 - 2692*x^7 + 9789*
x^6 - 10070*x^5 + 15569*x^4 - 5568*x^3 + 1080*x^2 + 4320*x - 5184) + 223064
064*x - 94887936))/(2585191*x^8 - 4661200*x^7 + 14191920*x^6 + 490880*x^5 -
 13562944*x^4 + 44249088*x^3 - 34615296*x^2 - 24772608*x + 18579456)) + 339
388*sqrt(341)*50^(1/4)*sqrt(10)*sqrt(2)*(2*x^2 - x + 3)*sqrt(247*sqrt(2) +
1000)*arctan(1/328782125*(14260*sqrt(341)*sqrt(10)*sqrt(2*x^2 - x + 3)*(22*
50^(3/4)*(57708*x^7 - 181278*x^6 + 400374*x^5 - 525676*x^4 + 235088*x^3 - 4
```

```
6944*x^2 - sqrt(2)*(20846*x^7 - 109153*x^6 + 215386*x^5 - 427391*x^4 + 2343
60*x^3 - 156600*x^2 - 172800*x + 186624) - 373248*x + 172800) + 5*50^(1/4)*
(125839*x^7 - 1864281*x^6 + 9323336*x^5 - 19725020*x^4 + 24624288*x^3 - 108
62496*x^2 - sqrt(2)*(56119*x^7 - 908994*x^6 + 5175980*x^5 - 12895624*x^4 +
17261280*x^3 - 14184000*x^2 - 10533888*x + 9994752) - 19989504*x + 10533888
))*sqrt(247*sqrt(2) + 1000) - 933317000*sqrt(31)*sqrt(2)*(28180*x^8 - 25466
6*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - s
qrt(2)*(8746*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^4 - 752
088*x^3 + 396144*x^2 + 546048*x - 539136) + 1154304*x - 456192) - 2*sqrt(31
0/119)*(sqrt(341)*sqrt(10)*sqrt(2*x^2 - x + 3)*(22*50^(3/4)*(246848*x^7 - 3
48192*x^6 + 1080672*x^5 - 178432*x^4 - 18432*x^3 + 1029888*x^2 - sqrt(2)*(4
6522*x^7 - 71117*x^6 + 257247*x^5 - 273360*x^4 + 484920*x^3 - 269568*x^2 +
269568*x) - 1029888*x) + 5*50^(1/4)*(516957*x^7 - 6676948*x^6 + 25569820*x^
5 - 31522752*x^4 + 34450848*x^3 + 46199808*x^2 - 4*sqrt(2)*(38689*x^7 - 502
244*x^6 + 1967660*x^5 - 2828160*x^4 + 4711680*x^3 - 1689984*x^2 + 1689984*x
) - 46199808*x))*sqrt(247*sqrt(2) + 1000) - 65450*sqrt(31)*sqrt(2)*(123408*
x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 + 396480*x^4 + 798336*x^3 - 38
22336*x^2 - sqrt(2)*(15550*x^8 - 118051*x^7 + 244047*x^6 - 707374*x^5 + 105
3960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*x) + 3276288*x) - 2975*sqrt(
31)*(254591*x^8 - 4815126*x^7 + 32303580*x^6 - 90866808*x^5 + 108781920*x^4
 - 74219328*x^3 - 168956928*x^2 - 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6 -
 1536*x^5 + 2385*x^4 - 3618*x^3 + 2268*x^2 - 1944*x) + 144820224*x))*sqrt((
sqrt(341)*50^(1/4)*sqrt(31)*sqrt(10)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(37*x - 3
8) + x - 75)*sqrt(247*sqrt(2) + 1000) + 903805*x^2 + 811580*sqrt(2)*(2*x^2
- x + 3) - 2785195*x + 3689000)/x^2) - 10605875*sqrt(31)*(2828123*x^8 - 969
6916*x^7 + 53385560*x^6 - 142835344*x^5 + 254146592*x^4 - 249300096*x^3 + 3
7981440*x^2 - 7744*sqrt(2)*(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15
569*x^4 - 5568*x^3 + 1080*x^2 + 4320*x - 5184) + 223064064*x - 94887936))/(
2585191*x^8 - 4661200*x^7 + 14191920*x^6 + 490880*x^5 - 13562944*x^4 + 4424
9088*x^3 - 34615296*x^2 - 24772608*x + 18579456)) - 23*sqrt(341)*50^(1/4)*s
qrt(31)*sqrt(10)*(2000*x^2 - 247*sqrt(2)*(2*x^2 - x + 3) - 1000*x + 3000)*s
qrt(247*sqrt(2) + 1000)*log(3100000/119*(sqrt(341)*50^(1/4)*sqrt(31)*sqrt(1
0)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(37*x - 38) + x - 75)*sqrt(247*sqrt(2) + 10
00) + 903805*x^2 + 811580*sqrt(2)*(2*x^2 - x + 3) - 2785195*x + 3689000)/x^
2) + 23*sqrt(341)*50^(1/4)*sqrt(31)*sqrt(10)*(2000*x^2 - 247*sqrt(2)*(2*x^2
 - x + 3) - 1000*x + 3000)*sqrt(247*sqrt(2) + 1000)*log(-3100000/119*(sqrt(
341)*50^(1/4)*sqrt(31)*sqrt(10)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(37*x - 38) +
x - 75)*sqrt(247*sqrt(2) + 1000) - 903805*x^2 - 811580*sqrt(2)*(2*x^2 - x +
 3) + 2785195*x - 3689000)/x^2) + 201271840*sqrt(2*x^2 - x + 3)*(6*x - 13))
/(2*x^2 - x + 3)
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^(3/2)/(5*x^2+3*x+2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Francis algorithm failure for[-1.0,infinity,
infinity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,infi
nity]Francis algorithm failure for[-1.0,infinity,infinity,infinity,infinity
]proot error [1.0,infinity,infinity,infinity,infinity]Francis algorithm fai
lure for[-1.0,infinity,infinity,infinity,infinity]proot error [1.0,infinity
,infinity,infinity,infinity]Francis algorithm failure for[-1.0,infinity,inf
inity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,infinit
y]Evaluation time: 9.49Done
```

maple [B] time = 0.03, size = 718, normalized size = 4.08

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^2-x+3)^(3/2)/(5*x^2+3*x+2),x)
```

```
[Out] 1/465124*(8*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+3*2^(1/2)*(x+2^(1/2)-1)^2/(-x+
2^(1/2)+1)^2+8-3*2^(1/2))^(1/2)*2^(1/2)*(2197*2^(1/2)*(-8866+6820*2^(1/2))^
(1/2)*(-775687+549362*2^(1/2))^(1/2)*arctan(1/11692487*(-775687+549362*2^(1
/2))^(1/2)*(-23*(8+3*2^(1/2))*(-23*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+24*2^(1
/2)-41))^(1/2)*(6485*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+10368*(x+2^(1
/2)-1)^2/(-x+2^(1/2)+1)^2+22379*2^(1/2)+32016)/(23*(x+2^(1/2)-1)^4/(-x+2^(1
/2)+1)^4+82*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+23)*(x+2^(1/2)-1)/(-x+2^(1/2)+
1)*(8+3*2^(1/2)))+3070*(-8866+6820*2^(1/2))^(1/2)*(-775687+549362*2^(1/2))^
(1/2)*arctan(1/11692487*(-775687+549362*2^(1/2))^(1/2)*(-23*(8+3*2^(1/2))*(
-23*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+24*2^(1/2)-41))^(1/2)*(6485*2^(1/2)*(x
+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+10368*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+22379
*2^(1/2)+32016)/(23*(x+2^(1/2)-1)^4/(-x+2^(1/2)+1)^4+82*(x+2^(1/2)-1)^2/(-x
+2^(1/2)+1)^2+23)*(x+2^(1/2)-1)/(-x+2^(1/2)+1)*(8+3*2^(1/2)))+1712502*arcta
nh(31/2*(8*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+3*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2
^(1/2)+1)^2+8-3*2^(1/2))^(1/2)/(-8866+6820*2^(1/2))^(1/2))*2^(1/2)-6617446*
arctanh(31/2*(8*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+3*2^(1/2)*(x+2^(1/2)-1)^2/
(-x+2^(1/2)+1)^2+8-3*2^(1/2))^(1/2)/(-8866+6820*2^(1/2))^(1/2)))/((8*(x+2^(
1/2)-1)^2/(-x+2^(1/2)+1)^2+3*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+8-3*2
^(1/2))/(1+(x+2^(1/2)-1)/(-x+2^(1/2)+1))^2)^(1/2)/(1+(x+2^(1/2)-1)/(-x+2^(1
/2)+1))/(8+3*2^(1/2))/(-8866+6820*2^(1/2))^(1/2)+1/22/(2*x^2-x+3)^(1/2)-3/5
06*(4*x-1)/(2*x^2-x+3)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(5\,x^2+3\,x+2\right)\left(2\,x^2-x+3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^(3/2)/(5*x^2+3*x+2),x, algorithm="maxima")
[Out] integrate(1/((5*x^2 + 3*x + 2)*(2*x^2 - x + 3)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(2\,x^2-x+3\right)^{3/2}\,\left(5\,x^2+3\,x+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*x^2 - x + 3)^(3/2)*(3*x + 5*x^2 + 2)),x)
[Out] int(1/((2*x^2 - x + 3)^(3/2)*(3*x + 5*x^2 + 2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(2x^2-x+3\right)^{\frac{3}{2}}\left(5x^2+3x+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**2-x+3)**(3/2)/(5*x**2+3*x+2),x)
[Out] Integral(1/((2*x**2 - x + 3)**(3/2)*(5*x**2 + 3*x + 2)), x)
```

3.88 $\int \frac{1}{\left(3-x+2x^2\right)^{3/2}\left(2+3x+5x^2\right)^2}\,dx$

Optimal. Leaf size=211

$$-\frac{6315-2306x}{345092\sqrt{2x^2-x+3}}+\frac{65x+4}{682\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}+\frac{\sqrt{\frac{1}{682}\left(129694447+103775000\sqrt{2}\right)}\tan^{-1}\left(\sqrt{\frac{\ldots}{31(1296\ldots}}\ldots\right)}{3000\ldots}$$

Rubi [A] time = 0.47, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {974, 1060, 1035, 1029, 206, 204}

$$-\frac{6315-2306x}{345092\sqrt{2x^2-x+3}}+\frac{65x+4}{682\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}+\frac{\sqrt{\frac{1}{682}\left(129694447+103775000\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{11}{31\left(129694447+103775000\sqrt{2}\right)}}\left(\left(45519+29065\sqrt{2}\right)x+16454\sqrt{2}+12611\right)}{\sqrt{2x^2-x+3}}\right)}{30008}-\frac{\sqrt{\frac{1}{682}\left(103775000\sqrt{2}-129694447\right)}\tanh^{-1}\left(\frac{\sqrt{\frac{11}{31\left(103775000\sqrt{2}-129694447\right)}}\left(\left(45519-29065\sqrt{2}\right)x-16454\sqrt{2}+12611\right)}{\sqrt{2x^2-x+3}}\right)}{30008}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)^2),x]
```

```
[Out] -(6315 - 2306*x)/(345092*Sqrt[3 - x + 2*x^2]) + (4 + 65*x)/(682*Sqrt[3 - x
+ 2*x^2]*(2 + 3*x + 5*x^2)) + (Sqrt[(129694447 + 103775000*Sqrt[2])/682]*Ar
cTan[(Sqrt[11/(31*(129694447 + 103775000*Sqrt[2]))]*(12611 + 16454*Sqrt[2]
+ (45519 + 29065*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]])/30008 - (Sqrt[(-1296944
47 + 103775000*Sqrt[2])/682]*ArcTanh[(Sqrt[11/(31*(-129694447 + 103775000*S
qrt[2]))]*(12611 - 16454*Sqrt[2] + (45519 - 29065*Sqrt[2])*x))/Sqrt[3 - x +
 2*x^2]])/30008
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 974

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*
```

```
a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*(
d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q,
 0]
```

Rule 1029

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
 c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 1035

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

Rule 1060

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x +
 c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
2*a*(c*d - a*f)))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
```

```
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] && !( !IntegerQ[p] && ILtQ[q, -1]) && !
IGtQ[q, 0]
```

Rubi steps

$$\int \frac{1}{\left(3-x+2x^2\right)^{3/2}\left(2+3x+5x^2\right)^2}\,dx=\frac{4+65x}{682\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)}-\frac{\int \frac{-1782+\frac{3333x}{2}-2860x^2}{\left(3-x+2x^2\right)^{3/2}\left(2+3x+5x^2\right)}\,dx}{7502}$$

$$=-\frac{6315-2306x}{345092\sqrt{3-x+2x^2}}+\frac{4+65x}{682\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)}-\frac{\int \frac{-\frac{3002\ldots}{\ldots}}{\sqrt{3-x+\ldots}}}{2\ldots}$$

$$=-\frac{6315-2306x}{345092\sqrt{3-x+2x^2}}+\frac{4+65x}{682\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)}+\frac{\int \frac{30613}{4}(\ldots}{\ldots}$$

$$=-\frac{6315-2306x}{345092\sqrt{3-x+2x^2}}+\frac{4+65x}{682\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)}-\frac{\left(253\left(20\ldots\right.\right.}{\ldots}$$

$$=-\frac{6315-2306x}{345092\sqrt{3-x+2x^2}}+\frac{4+65x}{682\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)}+\frac{\sqrt{\frac{1}{682}\left(1\ldots\right.}}{\ldots}$$

Mathematica [C] time = 1.53, size = 740, normalized size = 3.51

$$\frac{100\left(\frac{682\left(\left(22-4i\sqrt{31}\right)x+i\sqrt{31}+52\right)}{\left(\sqrt{31}+13i\right)\left(10ix+\sqrt{31}+3i\right)\sqrt{2x^2-x+3}}+\frac{682\left(\left(22+4i\sqrt{31}\right)x-i\sqrt{31}+52\right)}{\left(\sqrt{31}-13i\right)\left(-10ix+\sqrt{31}-3i\right)\sqrt{2x^2-x+3}}+\frac{22\left(2\left(11\sqrt{31}-62i\right)x+52\sqrt{31}+31i\right)}{\left(\sqrt{31}+13i\right)\sqrt{2x^2-x+3}}+\frac{22\left(2\left(11\sqrt{31}+62i\right)x+52\sqrt{31}-31i\right)}{\left(\sqrt{31}-13i\right)\sqrt{2x^2-x+3}}+\frac{575i\sqrt{682\left(13+i\sqrt{31}\right)}\tanh^{-1}\left(\frac{\left(-22-4i\sqrt{31}\right)x+i\sqrt{31}+63}{2\sqrt{286+22i\sqrt{31}}\sqrt{2x^2-x+3}}\right)}{\left(\sqrt{31}-13i\right)^2}+\frac{155\left(44\left(16353+581i\sqrt{31}\right)\sqrt{2x^2-x+3}+345\sqrt{286+22i\sqrt{31}}\left(10\left(11+2i\sqrt{31}\right)x+17i\sqrt{31}-29\right)\tanh^{-1}\left(\frac{\left(-22-4i\sqrt{31}\right)x+i\sqrt{31}+63}{2\sqrt{286+22i\sqrt{31}}\sqrt{2x^2-x+3}}\right)\right)}{22\left(\sqrt{31}-13i\right)^3\left(-10ix+\sqrt{31}-3i\right)}+\frac{155\left(44\left(16353-581i\sqrt{31}\right)\sqrt{2x^2-x+3}+345\sqrt{286-22i\sqrt{31}}\left(\left(-110+20i\sqrt{31}\right)x+17i\sqrt{31}+29\right)\tanh^{-1}\left(\frac{\left(22-4i\sqrt{31}\right)x+i\sqrt{31}-63}{2\sqrt{286-22i\sqrt{31}}\sqrt{2x^2-x+3}}\right)\right)}{22\left(\sqrt{31}+13i\right)^3\left(10ix+\sqrt{31}+3i\right)}-\frac{575i\sqrt{682\left(13-i\sqrt{31}\right)}\tanh^{-1}\left(\frac{\left(-22+4i\sqrt{31}\right)x-i\sqrt{31}+63}{2\sqrt{286-22i\sqrt{31}}\sqrt{2x^2-x+3}}\right)}{\left(\sqrt{31}+13i\right)^2}\right)}{2674463}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)^2),x]
```

```
[Out] (100*((682*(52 + I*Sqrt[31] + (22 - (4*I)*Sqrt[31])*x))/((13*I + Sqrt[31])*
(3*I + Sqrt[31] + (10*I)*x)*Sqrt[3 - x + 2*x^2]) + (682*(52 - I*Sqrt[31] +
(22 + (4*I)*Sqrt[31])*x))/((-13*I + Sqrt[31])*(-3*I + Sqrt[31] - (10*I)*x)*
Sqrt[3 - x + 2*x^2]) + (22*(31*I + 52*Sqrt[31] + 2*(-62*I + 11*Sqrt[31])*x)
)/((13*I + Sqrt[31])*Sqrt[3 - x + 2*x^2]) + (22*(-31*I + 52*Sqrt[31] + 2*(6
2*I + 11*Sqrt[31])*x))/((-13*I + Sqrt[31])*Sqrt[3 - x + 2*x^2]) + ((575*I)*
Sqrt[682*(13 + I*Sqrt[31])]*ArcTanh[(63 + I*Sqrt[31] + (-22 - (4*I)*Sqrt[31
])*x)/(2*Sqrt[286 + (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])]])/(-13*I + Sqrt[3
1])^2 + (155*(44*(16353 + (581*I)*Sqrt[31])*Sqrt[3 - x + 2*x^2] + 345*Sqrt[
286 + (22*I)*Sqrt[31]]*(-29 + (17*I)*Sqrt[31] + 10*(11 + (2*I)*Sqrt[31])*x)
*ArcTanh[(63 + I*Sqrt[31] + (-22 - (4*I)*Sqrt[31])*x)/(2*Sqrt[286 + (22*I)*
Sqrt[31]]*Sqrt[3 - x + 2*x^2])]))/(22*(-13*I + Sqrt[31])^3*(-3*I + Sqrt[31]
 - (10*I)*x)) + (155*(44*(16353 - (581*I)*Sqrt[31])*Sqrt[3 - x + 2*x^2] + 3
45*Sqrt[286 - (22*I)*Sqrt[31]]*(29 + (17*I)*Sqrt[31] + (-110 + (20*I)*Sqrt[
31])*x)*ArcTanh[(-63 + I*Sqrt[31] + (22 - (4*I)*Sqrt[31])*x)/(2*Sqrt[286 -
(22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])]))/(22*(13*I + Sqrt[31])^3*(3*I + Sqr
t[31] + (10*I)*x)) - ((575*I)*Sqrt[682*(13 - I*Sqrt[31])]*ArcTanh[(63 - I*S
qrt[31] + (-22 + (4*I)*Sqrt[31])*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 -
 x + 2*x^2])])/(13*I + Sqrt[31])^2))/2674463
```

IntegrateAlgebraic [C] time = 0.95, size = 406, normalized size = 1.92

$$-\frac{1}{484}\text{RootSum}\left[-5\#1^4+6\sqrt{2}\#1^3+17\#1^2-26\sqrt{2}\#1-56\&,\frac{-15\#1^2\log\left(-\#1+\sqrt{2x^2-x+3}-\sqrt{2}x\right)+8\sqrt{2}\#1\log\left(-\#1+\sqrt{2x^2-x+3}-\sqrt{2}x\right)+225\log\left(-\#1+\sqrt{2x^2-x+3}-\sqrt{2}x\right)}{-10\#1^3+9\sqrt{2}\#1^2+17\#1-13\sqrt{2}}\&\right]+\frac{\text{RootSum}\left[-5\#1^4+6\sqrt{2}\#1^3+17\#1^2-26\sqrt{2}\#1-56\&,\frac{1565\sqrt{2}\#1^2\log\left(-\#1+\sqrt{2x^2-x+3}-\sqrt{2}x\right)+9624\#1\log\left(-\#1+\sqrt{2x^2-x+3}-\sqrt{2}x\right)+8623\sqrt{2}\log\left(-\#1+\sqrt{2x^2-x+3}-\sqrt{2}x\right)}{-10\#1^3+9\sqrt{2}\#1^2+17\#1-13\sqrt{2}}\&\right]}{30008\sqrt{2}}+\frac{11530x^3-24657x^2+18557x-10606}{345092\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)^2),x]
```

```
[Out] (-10606 + 18557*x - 24657*x^2 + 11530*x^3)/(345092*Sqrt[3 - x + 2*x^2]*(2 +
 3*x + 5*x^2)) - RootSum[-56 - 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3 - 5
*#1^4 & , (225*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1] + 8*Sqrt[2]*Log
[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1 - 15*Log[-(Sqrt[2]*x) + Sqrt[3
 - x + 2*x^2] - #1]*#1^2)/(-13*Sqrt[2] + 17*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3)
& ]/484 + RootSum[-56 - 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3 - 5*#1^4 &
 , (8623*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1] + 9624*Log[-(
Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1 + 1565*Sqrt[2]*Log[-(Sqrt[2]*x) +
 Sqrt[3 - x + 2*x^2] - #1]*#1^2)/(-13*Sqrt[2] + 17*#1 + 9*Sqrt[2]*#1^2 - 10
*#1^3) & ]/(30008*Sqrt[2])
```

fricas [B] time = 1.26, size = 2173, normalized size = 10.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^2,x, algorithm="fricas")

[Out] 1/2988924703884110987072O*(35183643812*3446160200^(1/4)*sqrt(20755)*sqrt(34
1)*sqrt(2)*(10*x^4 + x^3 + 16*x^2 + 7*x + 6)*sqrt(129694447*sqrt(2) + 20755
0000)*arctan(1/243787105524753264O924125*(59193260*sqrt(20755)*(11*34461602
00^(3/4)*sqrt(341)*(20748108*x^7 - 87744678*x^6 + 180517074*x^5 - 311740976
*x^4 + 161753488*x^3 - 89046144*x^2 - sqrt(2)*(18515146*x^7 - 65709803*x^6
+ 140687186*x^5 - 209710441*x^4 + 101256360*x^3 - 39198600*x^2 - 126316800*
x + 76909824) - 153819648*x + 126316800) + 643405*3446160200^(1/4)*sqrt(341
)*(1637219*x^7 - 25548801*x^6 + 138274456*x^5 - 324967420*x^4 + 425065248*x
^3 - 297030816*x^2 - sqrt(2)*(1361849*x^7 - 20608224*x^6 + 106575580*x^5 -
236322704*x^4 + 301502880*x^3 - 169632000*x^2 - 225358848*x + 143534592) -
287069184*x + 225358848))*sqrt(2*x^2 - x + 3)*sqrt(129694447*sqrt(2) + 2075
50000) + 6920408156831705561333000*sqrt(31)*sqrt(2)*(28180*x^8 - 254666*x^7
 + 704270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - sqrt(2
)*(8746*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^4 - 752088*x
^3 + 396144*x^2 + 546048*x - 539136) + 1154304*x - 456192) - 2*sqrt(41510/3
97951)*(sqrt(20755)*(11*3446160200^(3/4)*sqrt(341)*(66710248*x^7 - 96938292
*x^6 + 319739772*x^5 - 172116032*x^4 + 247423968*x^3 + 38700288*x^2 - sqrt(
2)*(71827622*x^7 - 102266467*x^6 + 323714097*x^5 - 93357360*x^4 + 79054920*
x^3 + 219532032*x^2 - 219532032*x) - 38700288*x) + 643405*3446160200^(1/4)*
sqrt(341)*(5462397*x^7 - 70721108*x^6 + 273784220*x^5 - 364358592*x^4 + 506
287008*x^3 + 144903168*x^2 - 2*sqrt(2)*(2586013*x^7 - 33428948*x^6 + 128512
220*x^5 - 162918720*x^4 + 196126560*x^3 + 173705472*x^2 - 173705472*x) - 14
4903168*x))*sqrt(2*x^2 - x + 3)*sqrt(129694447*sqrt(2) + 207550000) + 11691
2097033204550*sqrt(31)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578888*x^6 - 329
3072*x^5 + 396480*x^4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 118
051*x^7 + 244047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^2
 - 1036800*x) + 3276288*x) + 5314186228782025*sqrt(31)*(254591*x^8 - 481512
6*x^7 + 32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 74219328*x^3 - 168956
928*x^2 - 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3
618*x^3 + 2268*x^2 - 1944*x) + 144820224*x))*sqrt(-(3446160200^(1/4)*sqrt(2
0755)*sqrt(341)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(6137*x + 12812) - 18
949*x + 6675)*sqrt(129694447*sqrt(2) + 207550000) - 388930324332445*x^2 - 3
49243556543420*sqrt(2)*(2*x^2 - x + 3) + 1198540387228555*x - 1587470711561
000)/x^2) + 78641001782178472287875*sqrt(31)*(2828123*x^8 - 9696916*x^7 + 5
3385560*x^6 - 142835344*x^5 + 254146592*x^4 - 249300096*x^3 + 37981440*x^2
- 7744*sqrt(2)*(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^4 - 55
68*x^3 + 1080*x^2 + 4320*x - 5184) + 223064064*x - 94887936))/(2585191*x^8
- 4661200*x^7 + 14191920*x^6 + 490880*x^5 - 13562944*x^4 + 44249088*x^3 - 3
4615296*x^2 - 24772608*x + 18579456)) + 35183643812*3446160200^(1/4)*sqrt(2
0755)*sqrt(341)*sqrt(2)*(10*x^4 + x^3 + 16*x^2 + 7*x + 6)*sqrt(129694447*sq
rt(2) + 207550000)*arctan(1/243787105524753264O924125*(59193260*sqrt(20755)
*(11*3446160200^(3/4)*sqrt(341)*(20748108*x^7 - 87744678*x^6 + 180517074*x^
5 - 311740976*x^4 + 161753488*x^3 - 89046144*x^2 - sqrt(2)*(18515146*x^7 -
```

```
65709803*x^6 + 140687186*x^5 - 209710441*x^4 + 101256360*x^3 - 39198600*x^2
 - 126316800*x + 76909824) - 153819648*x + 126316800) + 643405*3446160200^(
1/4)*sqrt(341)*(1637219*x^7 - 25548801*x^6 + 138274456*x^5 - 324967420*x^4
+ 425065248*x^3 - 297030816*x^2 - sqrt(2)*(1361849*x^7 - 20608224*x^6 + 106
575580*x^5 - 236322704*x^4 + 301502880*x^3 - 169632000*x^2 - 225358848*x +
143534592) - 287069184*x + 225358848))*sqrt(2*x^2 - x + 3)*sqrt(129694447*s
qrt(2) + 207550000) - 6920408156831705561333000*sqrt(31)*sqrt(2)*(28180*x^8
 - 254666*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496
*x^2 - sqrt(2)*(8746*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x
^4 - 752088*x^3 + 396144*x^2 + 546048*x - 539136) + 1154304*x - 456192) - 2
*sqrt(41510/397951)*(sqrt(20755)*(11*3446160200^(3/4)*sqrt(341)*(66710248*x
^7 - 96938292*x^6 + 319739772*x^5 - 172116032*x^4 + 247423968*x^3 + 3870028
8*x^2 - sqrt(2)*(71827622*x^7 - 102266467*x^6 + 323714097*x^5 - 93357360*x^
4 + 79054920*x^3 + 219532032*x^2 - 219532032*x) - 38700288*x) + 643405*3446
160200^(1/4)*sqrt(341)*(5462397*x^7 - 70721108*x^6 + 273784220*x^5 - 364358
592*x^4 + 506287008*x^3 + 144903168*x^2 - 2*sqrt(2)*(2586013*x^7 - 33428948
*x^6 + 128512220*x^5 - 162918720*x^4 + 196126560*x^3 + 173705472*x^2 - 1737
05472*x) - 144903168*x))*sqrt(2*x^2 - x + 3)*sqrt(129694447*sqrt(2) + 20755
0000) - 116912097033204550*sqrt(31)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578
888*x^6 - 3293072*x^5 + 396480*x^4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15
550*x^8 - 118051*x^7 + 244047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3
+ 1209600*x^2 - 1036800*x) + 3276288*x) - 5314186228782025*sqrt(31)*(254591
*x^8 - 4815126*x^7 + 32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 74219328
*x^3 - 168956928*x^2 - 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 +
 2385*x^4 - 3618*x^3 + 2268*x^2 - 1944*x) + 144820224*x))*sqrt((3446160200^
(1/4)*sqrt(20755)*sqrt(341)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(6137*x +
 12812) - 18949*x + 6675)*sqrt(129694447*sqrt(2) + 207550000) + 38893032433
2445*x^2 + 349243556543420*sqrt(2)*(2*x^2 - x + 3) - 1198540387228555*x + 1
587470711561000)/x^2) - 7864100178217847228787 5*sqrt(31)*(2828123*x^8 - 969
6916*x^7 + 53385560*x^6 - 142835344*x^5 + 254146592*x^4 - 249300096*x^3 + 3
7981440*x^2 - 7744*sqrt(2)*(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15
569*x^4 - 5568*x^3 + 1080*x^2 + 4320*x - 5184) + 223064064*x - 94887936))/(
2585191*x^8 - 4661200*x^7 + 14191920*x^6 + 490880*x^5 - 13562944*x^4 + 4424
9088*x^3 - 34615296*x^2 - 24772608*x + 18579456)) + 23*3446160200^(1/4)*sqr
t(20755)*sqrt(341)*sqrt(31)*(2075500000*x^4 + 207550000*x^3 + 3320800000*x^
2 - 129694447*sqrt(2)*(10*x^4 + x^3 + 16*x^2 + 7*x + 6) + 1452850000*x + 12
45300000)*sqrt(129694447*sqrt(2) + 207550000)*log(1037750000000/397951*(344
6160200^(1/4)*sqrt(20755)*sqrt(341)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(
6137*x + 12812) - 18949*x + 6675)*sqrt(129694447*sqrt(2) + 207550000) + 388
930324332445*x^2 + 349243556543420*sqrt(2)*(2*x^2 - x + 3) - 11985403872285
55*x + 1587470711561000)/x^2) - 23*3446160200^(1/4)*sqrt(20755)*sqrt(341)*s
qrt(31)*(2075500000*x^4 + 207550000*x^3 + 3320800000*x^2 - 129694447*sqrt(2
)*(10*x^4 + x^3 + 16*x^2 + 7*x + 6) + 1452850000*x + 1245300000)*sqrt(12969
4447*sqrt(2) + 207550000)*log(-1037750000000/397951*(3446160200^(1/4)*sqrt(
20755)*sqrt(341)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(6137*x + 12812) - 1
```

```
8949*x + 6675)*sqrt(129694447*sqrt(2) + 207550000) - 388930324332445*x^2 -
349243556543420*sqrt(2)*(2*x^2 - x + 3) + 1198540387228555*x - 158747071156
1000)/x^2) + 86612402022768160*(11530*x^3 - 24657*x^2 + 18557*x - 10606)*sq
rt(2*x^2 - x + 3))/(10*x^4 + x^3 + 16*x^2 + 7*x + 6)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Francis algorithm failure for[-1.0,infinity,
infinity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,infi
nity]Francis algorithm failure for[-1.0,infinity,infinity,infinity,infinity
]proot error [1.0,infinity,infinity,infinity,infinity]Francis algorithm fai
lure for[-1.0,infinity,infinity,infinity,infinity]proot error [1.0,infinity
,infinity,infinity,infinity]Francis algorithm failure for[-1.0,infinity,inf
inity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,infinit
y]Evaluation time: 36.15Done
```

maple [B] time = 0.10, size = 5942, normalized size = 28.16

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^2,x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(5x^2+3x+2\right)^2\left(2x^2-x+3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((5*x^2 + 3*x + 2)^2*(2*x^2 - x + 3)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(2x^2-x+3\right)^{3/2}\left(5x^2+3x+2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*x^2 - x + 3)^(3/2)*(3*x + 5*x^2 + 2)^2),x)
```

```
[Out] int(1/((2*x^2 - x + 3)^(3/2)*(3*x + 5*x^2 + 2)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(2x^2 - x + 3\right)^{\frac{3}{2}} \left(5x^2 + 3x + 2\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**2-x+3)**(3/2)/(5*x**2+3*x+2)**2,x)
```

```
[Out] Integral(1/((2*x**2 - x + 3)**(3/2)*(5*x**2 + 3*x + 2)**2), x)
```

3.89 $\displaystyle\int \frac{1}{\left(3-x+2x^2\right)^{3/2}\left(2+3x+5x^2\right)^3}\,dx$

Optimal. Leaf size=246

$$-\frac{4353943-6508666x}{941410976\sqrt{2x^2-x+3}}+\frac{5(17315x+7318)}{1860496\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}+\frac{65x+4}{1364\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)^2}+\frac{3\sqrt{\frac{1}{682}}}{}$$

Rubi [A] time = 0.53, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {974, 1060, 1035, 1029, 206, 204}

$$-\frac{4353943-6508666x}{941410976\sqrt{2x^2-x+3}}+\frac{5(17315x+7318)}{1860496\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}+\frac{65x+4}{1364\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)^2}+\frac{3\sqrt{\frac{1}{682}\left(13874275807943+9819738650000\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{11}{31\left(13874275807943+9819738650000\sqrt{2}\right)}}\left(\left(13785797+9662095\sqrt{2}\right)x+4123702\sqrt{2}+5538393\right)}{\sqrt{2x^2-x+3}}\right)}{81861824}-\frac{3\sqrt{\frac{1}{682}\left(9819738650000\sqrt{2}-13874275807943\right)}\tanh^{-1}\left(\frac{\sqrt{\frac{11}{31\left(9819738650000\sqrt{2}-13874275807943\right)}}\left(\left(13785797-9662095\sqrt{2}\right)x-4123702\sqrt{2}+5538393\right)}{\sqrt{2x^2-x+3}}\right)}{81861824}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)^3),x]
```

```
[Out] -(4353943 - 6508666*x)/(941410976*Sqrt[3 - x + 2*x^2]) + (4 + 65*x)/(1364*S
qrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)^2) + (5*(7318 + 17315*x))/(1860496*Sqr
t[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)) + (3*Sqrt[(13874275807943 + 98197386500
00*Sqrt[2])/682]*ArcTan[(Sqrt[11/(31*(13874275807943 + 9819738650000*Sqrt[2
]))]*(5538393 + 4123702*Sqrt[2] + (13785797 + 9662095*Sqrt[2])*x))/Sqrt[3 -
 x + 2*x^2]])/81861824 - (3*Sqrt[(-13874275807943 + 9819738650000*Sqrt[2])/
682]*ArcTanh[(Sqrt[11/(31*(-13874275807943 + 9819738650000*Sqrt[2]))]*(5538
393 - 4123702*Sqrt[2] + (13785797 - 9662095*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2
]])/81861824
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 974

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*
a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*(
d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q,
 0]
```

Rule 1029

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
 c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 1035

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

Rule 1060

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x +
 c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
```

```
2*a*(c*d - a*f)))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !
IGtQ[q, 0]
```

Rubi steps

$$\int \frac{1}{\left(3-x+2x^2\right)^{3/2}\left(2+3x+5x^2\right)^3}\,dx = \frac{4+65x}{1364\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^2} - \frac{\int \frac{-5731+\frac{7557x}{2}-5720x^2}{\left(3-x+2x^2\right)^{3/2}\left(2+3x+5x^2\right)^2}\,dx}{15004}$$

$$= \frac{4+65x}{1364\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^2} + \frac{5(7318+17315x)}{1860496\sqrt{3-x+2x^2}\left(2+3x+\right.}$$

$$= -\frac{4353943-6508666x}{941410976\sqrt{3-x+2x^2}} + \frac{4+65x}{1364\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^2} + \frac{}{186}$$

$$= -\frac{4353943-6508666x}{941410976\sqrt{3-x+2x^2}} + \frac{4+65x}{1364\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^2} + \frac{}{186}$$

$$= -\frac{4353943-6508666x}{941410976\sqrt{3-x+2x^2}} + \frac{4+65x}{1364\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^2} + \frac{}{186}$$

$$= -\frac{4353943-6508666x}{941410976\sqrt{3-x+2x^2}} + \frac{4+65x}{1364\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^2} + \frac{}{186}$$

Mathematica [C] time = 2.21, size = 231, normalized size = 0.94

$$\frac{69\sqrt{286-22i\sqrt{31}}\left(13785797\sqrt{31}+14026539i\right)\tan^{-1}\left(\frac{-2\left(2\sqrt{31}+11i\right)x+\sqrt{31}+63i}{2\sqrt{286-22i\sqrt{31}}\sqrt{2x^2-x+3}}\right)-69i\sqrt{286+22i\sqrt{31}}\left(13785797\sqrt{31}-14026539i\right)\tanh^{-1}\left(\frac{\left(-22-4i\sqrt{31}\right)x+i\sqrt{31}+63}{2\sqrt{286+22i\sqrt{31}}\sqrt{2x^2-x+3}}\right)+\frac{27280\left(162716650x^5+86411405x^4+277167774x^3+175833195x^2+161806828x+22374044\right)}{\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)^2}}{25681691425280}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)^3),x]
```

```
[Out] ((27280*(22374044 + 161806828*x + 175833195*x^2 + 277167774*x^3 + 86411405*
x^4 + 162716650*x^5))/(Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)^2) + 69*Sqrt[2
86 - (22*I)*Sqrt[31]]*(14026539*I + 13785797*Sqrt[31])*ArcTan[(63*I + Sqrt[
31] - 2*(11*I + 2*Sqrt[31])*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 - x +
2*x^2])] - (69*I)*Sqrt[286 + (22*I)*Sqrt[31]]*(-14026539*I + 13785797*Sqrt[
31])*ArcTanh[(63 + I*Sqrt[31] + (-22 - (4*I)*Sqrt[31])*x)/(2*Sqrt[286 + (22
*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])])/25681691425280
```

IntegrateAlgebraic [C] time = 1.51, size = 611, normalized size = 2.48

$$-\frac{\text{RootSum}\left[-5\#1^4+6\sqrt{2}\#1^3+17\#1^2-26\sqrt{2}\#1-56\&,\frac{5\sqrt{2}\#1^2\log\left(-\#1+\sqrt{2x^2-x+3}-\sqrt{2}x\right)+208\#1\log\left(-\#1+\sqrt{2x^2-x+3}-\sqrt{2}x\right)-491\sqrt{2}\log\left(-\#1+\sqrt{2x^2-x+3}-\sqrt{2}x\right)}{-10\#1^3+9\sqrt{2}\#1^2+17\#1-13\sqrt{2}}\&\right]}{10648\sqrt{2}}+\frac{\text{RootSum}\left[-5\#1^4+6\sqrt{2}\#1^3+17\#1^2-26\sqrt{2}\#1-56\&,\frac{575915\sqrt{2}\#1^2\log\left(-\#1+\sqrt{2x^2-x+3}-\sqrt{2}x\right)-3798456\#1\log\left(-\#1+\sqrt{2x^2-x+3}-\sqrt{2}x\right)+7194481\sqrt{2}\log\left(-\#1+\sqrt{2x^2-x+3}-\sqrt{2}x\right)}{-10\#1^3+9\sqrt{2}\#1^2+17\#1-13\sqrt{2}}\&\right]}{15184048\sqrt{2}}-\frac{7\text{RootSum}\left[-5\#1^4+6\sqrt{2}\#1^3+17\#1^2-26\sqrt{2}\#1-56\&,\frac{6180225\sqrt{2}\#1^2\log\left(-\#1+\sqrt{2x^2-x+3}-\sqrt{2}x\right)-105962920\#1\log\left(-\#1+\sqrt{2x^2-x+3}-\sqrt{2}x\right)+143178771\sqrt{2}\log\left(-\#1+\sqrt{2x^2-x+3}-\sqrt{2}x\right)}{-10\#1^3+9\sqrt{2}\#1^2+17\#1-13\sqrt{2}}\&\right]}{1882821952\sqrt{2}}+\frac{162716650x^5+86411405x^4+277167774x^3+175833195x^2+161806828x+22374044}{941410976\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)^3),x]
```

```
[Out] (22374044 + 161806828*x + 175833195*x^2 + 277167774*x^3 + 86411405*x^4 + 16
2716650*x^5)/(941410976*Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)^2) - RootSum[
-56 - 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3 - 5*#1^4 & , (-491*Sqrt[2]*L
og[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1] + 208*Log[-(Sqrt[2]*x) + Sqrt[3
 - x + 2*x^2] - #1]*#1 + 5*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] -
 #1]*#1^2)/(-13*Sqrt[2] + 17*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3) & ]/(10648*Sqrt
[2]) + RootSum[-56 - 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3 - 5*#1^4 & ,
(7194481*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1] - 3798456*Log
[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1 + 575915*Sqrt[2]*Log[-(Sqrt[2]
*x) + Sqrt[3 - x + 2*x^2] - #1]*#1^2)/(-13*Sqrt[2] + 17*#1 + 9*Sqrt[2]*#1^2
 - 10*#1^3) & ]/(15184048*Sqrt[2]) - (7*RootSum[-56 - 26*Sqrt[2]*#1 + 17*#1
^2 + 6*Sqrt[2]*#1^3 - 5*#1^4 & , (143178771*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt
[3 - x + 2*x^2] - #1] - 105962920*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] -
#1]*#1 + 6180225*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1^2)
/(-13*Sqrt[2] + 17*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3) & ])/(1882821952*Sqrt[2])
```

fricas [B] time = 1.39, size = 2263, normalized size = 9.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

```
[Out] 1/33652296632397026886019646994897920*(920746859815884*1928545343086076450^
(1/4)*sqrt(1963947730)*sqrt(341)*sqrt(2)*(50*x^6 + 35*x^5 + 103*x^4 + 85*x^
3 + 83*x^2 + 32*x + 12)*sqrt(13874275807943*sqrt(2) + 19639477300000)*arcta
n(1/2252270155289097943751876925347228391692375*(2800589462980*sqrt(1963947
730)*(22*1928545343086076450^(3/4)*sqrt(341)*(7361410004*x^7 - 28555361914*
x^6 + 59872788262*x^5 - 96593638888*x^4 + 48573560944*x^3 - 23355012672*x^2
 - sqrt(2)*(5311119598*x^7 - 20292577289*x^6 + 42695479118*x^5 - 6800681868
3*x^4 + 33985514680*x^3 - 15860251800*x^2 - 37489478400*x + 26167456512) -
52334913024*x + 37489478400) + 30441189815*1928545343086076450^(1/4)*sqrt(3
41)*(560592897*x^7 - 8616399363*x^6 + 45618625128*x^5 - 104316505460*x^4 +
134890825824*x^3 - 85859939808*x^2 - sqrt(2)*(402019087*x^7 - 6162703212*x^
6 + 32499503540*x^5 - 73942829952*x^4 + 95407993440*x^3 - 59600016000*x^2 -
 68177562624*x + 47773380096) - 95546760192*x + 68177562624))*sqrt(2*x^2 -
x + 3)*sqrt(13874275807943*sqrt(2) + 19639477300000) + 63935410859819554532
31134497759874144159000*sqrt(31)*sqrt(2)*(28180*x^8 - 254666*x^7 + 704270*x
```

```
^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - sqrt(2)*(8746*x^8
 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^4 - 752088*x^3 + 396144
*x^2 + 546048*x - 539136) + 1154304*x - 456192) - 2*sqrt(1963947730/3471424
919)*(sqrt(1963947730)*(22*192854534308607645O^(3/4)*sqrt(341)*(26184810824
*x^7 - 37618468196*x^6 + 121297463436*x^5 - 48741866816*x^4 + 58784153184*x
^3 + 51583129344*x^2 - sqrt(2)*(19194187986*x^7 - 27528525721*x^6 + 8845761
3411*x^5 - 33685377680*x^4 + 38926767960*x^3 + 41764674816*x^2 - 4176467481
6*x) - 51583129344*x) + 30441189815*1928545343086076450^(1/4)*sqrt(341)*(19
98926311*x^7 - 25858659004*x^6 + 99738083860*x^5 - 129415692096*x^4 + 16744
6420704*x^3 + 96037622784*x^2 - 22*sqrt(2)*(65886479*x^7 - 852213084*x^6 +
3285070260*x^5 - 4244909760*x^4 + 5424792480*x^3 + 3393259776*x^2 - 3393259
776*x) - 96037622784*x))*sqrt(2*x^2 - x + 3)*sqrt(13874275807943*sqrt(2) +
19639477300000) + 228292692324094986130994862455O*sqrt(31)*sqrt(2)*(123408*
x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 + 396480*x^4 + 798336*x^3 - 38
22336*x^2 - sqrt(2)*(15550*x^8 - 118051*x^7 + 244047*x^6 - 707374*x^5 + 105
3960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*x) + 3276288*x) + 1037694056
01861357332270392025*sqrt(31)*(254591*x^8 - 4815126*x^7 + 32303580*x^6 - 90
866808*x^5 + 108781920*x^4 - 74219328*x^3 - 168956928*x^2 - 15488*sqrt(2)*(
4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3618*x^3 + 2268*x^2 - 1944
*x) + 144820224*x))*sqrt(-(1928545343086076450^(1/4)*sqrt(1963947730)*sqrt(
341)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(2995431*x + 1523456) - 4518887*
x - 1471975)*sqrt(13874275807943*sqrt(2) + 19639477300000) - 16051926912456
8199977215*x^2 - 144139751866959199979540*sqrt(2)*(2*x^2 - x + 3) + 4946614
21179791799929785*x - 655180690304359999907000)/x^2) + 72653875977067675604
899255656362206183625*sqrt(31)*(2828123*x^8 - 9696916*x^7 + 53385560*x^6 -
142835344*x^5 + 254146592*x^4 - 249300096*x^3 + 37981440*x^2 - 7744*sqrt(2)
*(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^4 - 5568*x^3 + 1080*
x^2 + 4320*x - 5184) + 223064064*x - 94887936))/(2585191*x^8 - 4661200*x^7
+ 14191920*x^6 + 490880*x^5 - 13562944*x^4 + 44249088*x^3 - 34615296*x^2 -
24772608*x + 18579456)) + 920746859815884*1928545343086076450^(1/4)*sqrt(19
63947730)*sqrt(341)*sqrt(2)*(50*x^6 + 35*x^5 + 103*x^4 + 85*x^3 + 83*x^2 +
32*x + 12)*sqrt(13874275807943*sqrt(2) + 19639477300000)*arctan(1/225227015
5289097943751876925347228391692375*(2800589462980*sqrt(1963947730)*(22*1928
545343086076450^(3/4)*sqrt(341)*(7361410004*x^7 - 28555361914*x^6 + 5987278
8262*x^5 - 96593638888*x^4 + 48573560944*x^3 - 23355012672*x^2 - sqrt(2)*(5
311119598*x^7 - 20292577289*x^6 + 42695479118*x^5 - 68006818683*x^4 + 33985
514680*x^3 - 15860251800*x^2 - 37489478400*x + 26167456512) - 52334913024*x
 + 37489478400) + 30441189815*1928545343086076450^(1/4)*sqrt(341)*(56059289
7*x^7 - 8616399363*x^6 + 45618625128*x^5 - 104316505460*x^4 + 134890825824*
x^3 - 85859939808*x^2 - sqrt(2)*(402019087*x^7 - 6162703212*x^6 + 324995035
40*x^5 - 73942829952*x^4 + 95407993440*x^3 - 59600016000*x^2 - 68177562624*
x + 47773380096) - 95546760192*x + 68177562624))*sqrt(2*x^2 - x + 3)*sqrt(1
3874275807943*sqrt(2) + 19639477300000) - 639354108598195545323113449775987
4144159000*sqrt(31)*sqrt(2)*(28180*x^8 - 254666*x^7 + 704270*x^6 - 1385256*
x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - sqrt(2)*(8746*x^8 - 102335*x^7
```

 + 396104*x^6 - 783113*x^5 + 1320710*x^4 - 752088*x^3 + 396144*x^2 + 546048
*x - 539136) + 1154304*x - 456192) - 2*sqrt(1963947730/3471424919)*(sqrt(19
63947730)*(22*1928545343086076450^(3/4)*sqrt(341)*(26184810824*x^7 - 376184
68196*x^6 + 121297463436*x^5 - 48741866816*x^4 + 58784153184*x^3 + 51583129
344*x^2 - sqrt(2)*(19194187986*x^7 - 27528525721*x^6 + 88457613411*x^5 - 33
685377680*x^4 + 38926767960*x^3 + 41764674816*x^2 - 41764674816*x) - 515831
29344*x) + 30441189815*1928545343086076450^(1/4)*sqrt(341)*(1998926311*x^7
- 25858659004*x^6 + 99738083860*x^5 - 129415692096*x^4 + 167446420704*x^3 +
 96037622784*x^2 - 22*sqrt(2)*(65886479*x^7 - 852213084*x^6 + 3285070260*x^
5 - 4244909760*x^4 + 5424792480*x^3 + 3393259776*x^2 - 3393259776*x) - 9603
7622784*x))*sqrt(2*x^2 - x + 3)*sqrt(13874275807943*sqrt(2) + 1963947730000
0) - 228292692324094986130994862455 0*sqrt(31)*sqrt(2)*(123408*x^8 - 914152*
x^7 + 1578888*x^6 - 3293072*x^5 + 396480*x^4 + 798336*x^3 - 3822336*x^2 - s
qrt(2)*(15550*x^8 - 118051*x^7 + 244047*x^6 - 707374*x^5 + 1053960*x^4 - 16
67952*x^3 + 1209600*x^2 - 1036800*x) + 3276288*x) - 10376940560186135733227
0392025*sqrt(31)*(254591*x^8 - 4815126*x^7 + 32303580*x^6 - 90866808*x^5 +
108781920*x^4 - 74219328*x^3 - 168956928*x^2 - 15488*sqrt(2)*(4*x^8 - 76*x^
7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3618*x^3 + 2268*x^2 - 1944*x) + 1448202
24*x))*sqrt((1928545343086076450^(1/4)*sqrt(1963947730)*sqrt(341)*sqrt(31)*
sqrt(2*x^2 - x + 3)*(sqrt(2)*(2995431*x + 1523456) - 4518887*x - 1471975)*s
qrt(13874275807943*sqrt(2) + 19639477300000) + 160519269124568199977215*x^2
 + 144139751866959199979540*sqrt(2)*(2*x^2 - x + 3) - 494661421179791799929
785*x + 655180690304359999907000)/x^2) - 726538759770676756048992556563622 0
6183625*sqrt(31)*(2828123*x^8 - 9696916*x^7 + 53385560*x^6 - 142835344*x^5
+ 254146592*x^4 - 249300096*x^3 + 37981440*x^2 - 7744*sqrt(2)*(1348*x^8 - 2
692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^4 - 5568*x^3 + 1080*x^2 + 4320*x -
 5184) + 223064064*x - 94887936))/(2585191*x^8 - 4661200*x^7 + 14191920*x^6
 + 490880*x^5 - 13562944*x^4 + 44249088*x^3 - 34615296*x^2 - 24772608*x + 1
8579456)) + 69*1928545343086076450^(1/4)*sqrt(1963947730)*sqrt(341)*sqrt(31
)*(981973865000000*x^6 + 687381705500000*x^5 + 2022866161900000*x^4 + 16693
55570500000*x^3 + 1630076615900000*x^2 - 13874275807943*sqrt(2)*(50*x^6 + 3
5*x^5 + 103*x^4 + 85*x^3 + 83*x^2 + 32*x + 12) + 628463273600000*x + 235673
727600000)*sqrt(13874275807943*sqrt(2) + 19639477300000)*log(17675529570000
00000/3471424919*(1928545343086076450^(1/4)*sqrt(1963947730)*sqrt(341)*sqrt
(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(2995431*x + 1523456) - 4518887*x - 14719
75)*sqrt(13874275807943*sqrt(2) + 19639477300000) + 16051926912456819997721
5*x^2 + 144139751866959199979540*sqrt(2)*(2*x^2 - x + 3) - 4946614211797917
99929785*x + 655180690304359999907000)/x^2) - 69*1928545343086076450^(1/4)*
sqrt(1963947730)*sqrt(341)*sqrt(31)*(981973865000000*x^6 + 687381705500000*
x^5 + 2022866161900000*x^4 + 1669355570500000*x^3 + 1630076615900000*x^2 -
13874275807943*sqrt(2)*(50*x^6 + 35*x^5 + 103*x^4 + 85*x^3 + 83*x^2 + 32*x
+ 12) + 628463273600000*x + 235673727600000)*sqrt(13874275807943*sqrt(2) +
19639477300000)*log(-1767552957000000000/3471424919*(1928545343086076450^(1
/4)*sqrt(1963947730)*sqrt(341)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(29954
31*x + 1523456) - 4518887*x - 1471975)*sqrt(13874275807943*sqrt(2) + 196394

```
77300000) - 160519269124568199977215*x^2 - 144139751866959199979540*sqrt(2)
*(2*x^2 - x + 3) + 494661421179791799929785*x - 655180690304359999907000)/x
^2) + 35746658463005881594925920*(162716650*x^5 + 86411405*x^4 + 277167774*
x^3 + 175833195*x^2 + 161806828*x + 22374044)*sqrt(2*x^2 - x + 3))/(50*x^6
+ 35*x^5 + 103*x^4 + 85*x^3 + 83*x^2 + 32*x + 12)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Francis algorithm failure for[-1.0,infinity,
infinity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,infi
nity]Francis algorithm failure for[-1.0,infinity,infinity,infinity,infinity
]proot error [1.0,infinity,infinity,infinity,infinity]Francis algorithm fai
lure for[-1.0,infinity,infinity,infinity,infinity]proot error [1.0,infinity
,infinity,infinity,infinity]Francis algorithm failure for[-1.0,infinity,inf
inity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,infinit
y]Evaluation time: 69.9Done
```

maple [B] time = 0.21, size = 18981, normalized size = 77.16

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^3,x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(5\,x^2+3\,x+2\right)^3\left(2\,x^2-x+3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^3,x, algorithm="maxima")
```

```
[Out] integrate(1/((5*x^2 + 3*x + 2)^3*(2*x^2 - x + 3)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(2\,x^2-x+3\right)}^{3/2}\,{\left(5\,x^2+3\,x+2\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*x^2 - x + 3)^(3/2)*(3*x + 5*x^2 + 2)^3),x)
[Out] int(1/((2*x^2 - x + 3)^(3/2)*(3*x + 5*x^2 + 2)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(2x^{2} - x + 3\right)^{\frac{3}{2}} \left(5x^{2} + 3x + 2\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**2-x+3)**(3/2)/(5*x**2+3*x+2)**3,x)
[Out] Integral(1/((2*x**2 - x + 3)**(3/2)*(5*x**2 + 3*x + 2)**3), x)
```

3.90 $\int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^{5/2}}\,dx$

Optimal. Leaf size=147

$$\frac{38375}{384}\sqrt{2x^2-x+3}\,x^2+\frac{526075\sqrt{2x^2-x+3}\,x}{3072}-\frac{1308645\sqrt{2x^2-x+3}}{4096}+\frac{1331(116368x+7409)}{101568\sqrt{2x^2-x+3}}-\frac{14641(79x+}{4416\left(2x^2-x\right.}$$

Rubi [A] time = 0.17, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1660, 1661, 640, 619, 215}

$$\frac{625}{32}\sqrt{2x^2-x+3}\,x^3+\frac{38375}{384}\sqrt{2x^2-x+3}\,x^2+\frac{526075\sqrt{2x^2-x+3}\,x}{3072}-\frac{1308645\sqrt{2x^2-x+3}}{4096}+\frac{1331(116368x+7409)}{101568\sqrt{2x^2-x+3}}-\frac{14641(79x+101)}{4416\left(2x^2-x+3\right)^{3/2}}+\frac{16955197\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{8192\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + 5*x^2)^4/(3 - x + 2*x^2)^(5/2),x]
```

```
[Out] (-14641*(101 + 79*x))/(4416*(3 - x + 2*x^2)^(3/2)) + (1331*(7409 + 116368*x
))/(101568*Sqrt[3 - x + 2*x^2]) - (1308645*Sqrt[3 - x + 2*x^2])/4096 + (526
075*x*Sqrt[3 - x + 2*x^2])/3072 + (38375*x^2*Sqrt[3 - x + 2*x^2])/384 + (62
5*x^3*Sqrt[3 - x + 2*x^2])/32 + (16955197*ArcSinh[(1 - 4*x)/Sqrt[23]])/(819
2*Sqrt[2])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(2+3x+5x^2\right)^4}{\left(3-x+2x^2\right)^{5/2}}\,dx = -\frac{14641(101+79x)}{4416\left(3-x+2x^2\right)^{3/2}} + \frac{2}{69}\int \frac{\frac{3839123}{256} - \frac{1983543x}{128} - \frac{1464801x^2}{64} + \frac{430905x^3}{32} + \frac{639975x^4}{16} + 250\cdots}{\left(3-x+2x^2\right)^{3/2}}$$

$$= -\frac{14641(101+79x)}{4416\left(3-x+2x^2\right)^{3/2}} + \frac{1331(7409+116368x)}{101568\sqrt{3-x+2x^2}} + \frac{4\int \frac{-\frac{141812733}{256} - \frac{1880595x}{16} + \frac{15512925x^2}{64} + \frac{3372375\cdots}{16}\cdots}{\sqrt{3-x+2x^2}}}{1587}$$

$$= -\frac{14641(101+79x)}{4416\left(3-x+2x^2\right)^{3/2}} + \frac{1331(7409+116368x)}{101568\sqrt{3-x+2x^2}} + \frac{625}{32}x^3\sqrt{3-x+2x^2} + \frac{\int \frac{-\frac{141812733}{32} - 1\cdots}{\cdots}}{\cdots}$$

$$= -\frac{14641(101+79x)}{4416\left(3-x+2x^2\right)^{3/2}} + \frac{1331(7409+116368x)}{101568\sqrt{3-x+2x^2}} + \frac{38375}{384}x^2\sqrt{3-x+2x^2} + \frac{625}{32}x^3\sqrt{3\cdots}$$

$$= -\frac{14641(101+79x)}{4416\left(3-x+2x^2\right)^{3/2}} + \frac{1331(7409+116368x)}{101568\sqrt{3-x+2x^2}} + \frac{526075x\sqrt{3-x+2x^2}}{3072} + \frac{38375}{384}x^2\sqrt{\cdots}$$

$$= -\frac{14641(101+79x)}{4416\left(3-x+2x^2\right)^{3/2}} + \frac{1331(7409+116368x)}{101568\sqrt{3-x+2x^2}} - \frac{1308645\sqrt{3-x+2x^2}}{4096} + \frac{526075x\sqrt{\cdots}}{3\cdots}$$

$$= -\frac{14641(101+79x)}{4416\left(3-x+2x^2\right)^{3/2}} + \frac{1331(7409+116368x)}{101568\sqrt{3-x+2x^2}} - \frac{1308645\sqrt{3-x+2x^2}}{4096} + \frac{526075x\sqrt{\cdots}}{3\cdots}$$

$$= -\frac{14641(101+79x)}{4416\left(3-x+2x^2\right)^{3/2}} + \frac{1331(7409+116368x)}{101568\sqrt{3-x+2x^2}} - \frac{1308645\sqrt{3-x+2x^2}}{4096} + \frac{526075x\sqrt{\cdots}}{3\cdots}$$

Mathematica [A] time = 0.51, size = 75, normalized size = 0.51

$$\frac{507840000x^7 + 2090608000x^6 + 3504730800x^5 - 5076781260x^4 + 39848900984x^3 - 36481630395x^2 + 49883864262x - 18974698519}{6500352\left(2x^2-x+3\right)^{3/2}} - \frac{16955197\sinh^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{8192\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)^4/(3 - x + 2*x^2)^(5/2),x]
```

[Out] (-18974698519 + 49883864262*x - 36481630395*x^2 + 39848900984*x^3 - 5076781
260*x^4 + 3504730800*x^5 + 2090608000*x^6 + 507840000*x^7)/(6500352*(3 - x
+ 2*x^2)^(3/2)) - (16955197*ArcSinh[(-1 + 4*x)/Sqrt[23]])/(8192*Sqrt[2])

IntegrateAlgebraic [A] time = 1.45, size = 90, normalized size = 0.61

$$\frac{16955197\log\left(2\sqrt{2}\sqrt{2x^2-x+3}-4x+1\right)}{8192\sqrt{2}}+\frac{507840000x^7+2090608000x^6+3504730800x^5-5076781260x^4+39848900984x^3-36481630395x^2+49883864262x-18974698519}{6500352\left(2x^2-x+3\right)^{3/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(2 + 3*x + 5*x^2)^4/(3 - x + 2*x^2)^(5/2),x]

[Out] (-18974698519 + 49883864262*x - 36481630395*x^2 + 39848900984*x^3 - 5076781
260*x^4 + 3504730800*x^5 + 2090608000*x^6 + 507840000*x^7)/(6500352*(3 - x
+ 2*x^2)^(3/2)) + (16955197*Log[1 - 4*x + 2*Sqrt[2]*Sqrt[3 - x + 2*x^2]])/(
8192*Sqrt[2])

fricas [A] time = 0.42, size = 132, normalized size = 0.90

$$\frac{26907897639\sqrt{2}\left(4x^4-4x^3+13x^2-6x+9\right)\log\left(4\sqrt{2}\sqrt{2x^2-x+3}(4x-1)-32x^2+16x-25\right)+8\left(507840000x^7+2090608000x^6+3504730800x^5-5076781260x^4+39848900984x^3-36481630395x^2+49883864262x-18974698519\right)\sqrt{2x^2-x+3}}{52002816\left(4x^4-4x^3+13x^2-6x+9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^(5/2),x, algorithm="fricas")

[Out] 1/52002816*(26907897639*sqrt(2)*(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)*log(4*sq
rt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 8*(507840000*x^
7 + 2090608000*x^6 + 3504730800*x^5 - 5076781260*x^4 + 39848900984*x^3 - 36
481630395*x^2 + 49883864262*x - 18974698519)*sqrt(2*x^2 - x + 3))/(4*x^4 -
4*x^3 + 13*x^2 - 6*x + 9)

giac [A] time = 0.25, size = 81, normalized size = 0.55

$$\frac{16955197}{16384}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+1\right)+\frac{((4(2645(20(40(60x+247)x+16563)x-479847)x+9962225246)x-36481630395)x+49883864262)x-18974698519}{6500352\left(2x^2-x+3\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^(5/2),x, algorithm="giac")

[Out] 16955197/16384*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1
) + 1/6500352*(((4*(2645*(20*(40*(60*x + 247)*x + 16563)*x - 479847)*x + 99
62225246)*x - 36481630395)*x + 49883864262)*x - 18974698519)/(2*x^2 - x + 3
)^(3/2)

maple [A] time = 0.03, size = 214, normalized size = 1.46

$$\frac{625x^7}{8\left(2x^2-x+3\right)^{\frac{3}{2}}}+\frac{30875x^6}{96\left(2x^2-x+3\right)^{\frac{3}{2}}}+\frac{138025x^5}{256\left(2x^2-x+3\right)^{\frac{3}{2}}}-\frac{799745x^4}{1024\left(2x^2-x+3\right)^{\frac{3}{2}}}+\frac{16955197x^3}{12288\left(2x^2-x+3\right)^{\frac{3}{2}}}-\frac{67488035x^2}{16384\left(2x^2-x+3\right)^{\frac{3}{2}}}+\frac{55167267x}{131072\left(2x^2-x+3\right)^{\frac{3}{2}}}+\frac{16955197x}{8192\sqrt{2x^2-x+3}}-\frac{16955197\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{16384}+\frac{16955197}{32768\sqrt{2x^2-x+3}}-\frac{2149616639}{524288\left(2x^2-x+3\right)^{\frac{3}{2}}}+\frac{\frac{992926033x}{3250176}-\frac{992926033}{13000704}}{\sqrt{2x^2-x+3}}+\frac{\frac{5141612725x}{9043968}-\frac{5141612725}{36175872}}{\left(2x^2-x+3\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)^4/(2*x^2-x+3)^(5/2),x)
```

```
[Out] 16955197/32768/(2*x^2-x+3)^(1/2)-2149616639/524288/(2*x^2-x+3)^(3/2)+992926
033/13000704*(4*x-1)/(2*x^2-x+3)^(1/2)-16955197/16384*2^(1/2)*arcsinh(4/23*
23^(1/2)*(x-1/4))+5141612725/36175872*(4*x-1)/(2*x^2-x+3)^(3/2)+625/8*x^7/(
2*x^2-x+3)^(3/2)+30875/96*x^6/(2*x^2-x+3)^(3/2)+138025/256*x^5/(2*x^2-x+3)^
(3/2)-799745/1024*x^4/(2*x^2-x+3)^(3/2)+16955197/12288*x^3/(2*x^2-x+3)^(3/2
)-67488035/16384*x^2/(2*x^2-x+3)^(3/2)+55167267/131072*x/(2*x^2-x+3)^(3/2)+
16955197/8192/(2*x^2-x+3)^(1/2)*x
```

maxima [B] time = 1.00, size = 253, normalized size = 1.72

$$\frac{625\,x^7}{8\,\left(2\,x^2-x+3\right)^{\frac{3}{2}}}+\frac{30875\,x^6}{96\,\left(2\,x^2-x+3\right)^{\frac{3}{2}}}+\frac{138025\,x^5}{256\,\left(2\,x^2-x+3\right)^{\frac{3}{2}}}-\frac{799745\,x^4}{1024\,\left(2\,x^2-x+3\right)^{\frac{3}{2}}}-\frac{16955197}{13000704}\,x\left(\frac{284\,x}{\sqrt{2\,x^2-x+3}}-\frac{3174\,x^2}{\left(2\,x^2-x+3\right)^{\frac{3}{2}}}-\frac{71}{\sqrt{2\,x^2-x+3}}+\frac{805\,x}{\left(2\,x^2-x+3\right)^{\frac{3}{2}}}-\frac{3243}{\left(2\,x^2-x+3\right)^{\frac{3}{2}}}\right)-\frac{16955197}{16384}\,\sqrt{2}\,\operatorname{arsinh}\left(\frac{1}{23}\,\sqrt{23}\,(4\,x-1)\right)+\frac{1203818987}{6500352}\,\sqrt{2\,x^2-x+3}+\frac{3536205583\,x}{3250176\,\sqrt{2\,x^2-x+3}}-\frac{2638851\,x^2}{512\,\left(2\,x^2-x+3\right)^{\frac{3}{2}}}+\frac{257773037}{1083392\,\sqrt{2\,x^2-x+3}}+\frac{29484067\,x}{23552\,\left(2\,x^2-x+3\right)^{\frac{3}{2}}}-\frac{374445479}{70656\,\left(2\,x^2-x+3\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^(5/2),x, algorithm="maxima")
```

```
[Out] 625/8*x^7/(2*x^2 - x + 3)^(3/2) + 30875/96*x^6/(2*x^2 - x + 3)^(3/2) + 1380
25/256*x^5/(2*x^2 - x + 3)^(3/2) - 799745/1024*x^4/(2*x^2 - x + 3)^(3/2) -
16955197/13000704*x*(284*x/sqrt(2*x^2 - x + 3) - 3174*x^2/(2*x^2 - x + 3)^(
3/2) - 71/sqrt(2*x^2 - x + 3) + 805*x/(2*x^2 - x + 3)^(3/2) - 3243/(2*x^2 -
 x + 3)^(3/2)) - 16955197/16384*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) +
1203818987/6500352*sqrt(2*x^2 - x + 3) + 3536205583/3250176*x/sqrt(2*x^2 -
x + 3) - 2638851/512*x^2/(2*x^2 - x + 3)^(3/2) + 257773037/1083392/sqrt(2*x
^2 - x + 3) + 29484067/23552*x/(2*x^2 - x + 3)^(3/2) - 374445479/70656/(2*x
^2 - x + 3)^(3/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(5\,x^2+3\,x+2\right)}^4}{{\left(2\,x^2-x+3\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 5*x^2 + 2)^4/(2*x^2 - x + 3)^(5/2),x)
```

```
[Out] int((3*x + 5*x^2 + 2)^4/(2*x^2 - x + 3)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(5x^{2} + 3x + 2\right)^{4}}{\left(2x^{2} - x + 3\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)**4/(2*x**2-x+3)**(5/2),x)
```

```
[Out] Integral((5*x**2 + 3*x + 2)**4/(2*x**2 - x + 3)**(5/2), x)
```

3.91 $\int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^{5/2}}\,dx$

Optimal. Leaf size=105

$$\frac{121(10679-6744x)}{8464\sqrt{2x^2-x+3}}+\frac{125}{16}x\sqrt{2x^2-x+3}+\frac{3175}{64}\sqrt{2x^2-x+3}-\frac{1331(17-45x)}{1104\left(2x^2-x+3\right)^{3/2}}-\frac{7495\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{128\sqrt{2}}$$

Rubi [A] time = 0.11, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1660, 1661, 640, 619, 215}

$$\frac{121(10679-6744x)}{8464\sqrt{2x^2-x+3}}+\frac{125}{16}x\sqrt{2x^2-x+3}+\frac{3175}{64}\sqrt{2x^2-x+3}-\frac{1331(17-45x)}{1104\left(2x^2-x+3\right)^{3/2}}-\frac{7495\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{128\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + 5*x^2)^3/(3 - x + 2*x^2)^(5/2),x]
```

```
[Out] (-1331*(17 - 45*x))/(1104*(3 - x + 2*x^2)^(3/2)) + (121*(10679 - 6744*x))/(
8464*Sqrt[3 - x + 2*x^2]) + (3175*Sqrt[3 - x + 2*x^2])/64 + (125*x*Sqrt[3 -
 x + 2*x^2])/16 - (7495*ArcSinh[(1 - 4*x)/Sqrt[23]])/(128*Sqrt[2])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(2+3x+5x^2\right)^3}{\left(3-x+2x^2\right)^{5/2}}\,dx = -\frac{1331(17-45x)}{1104\left(3-x+2x^2\right)^{3/2}} + \frac{2}{69}\int \frac{-\frac{91275}{64}-\frac{57201x}{32}+\frac{66585x^2}{16}+\frac{39675x^3}{8}+\frac{8625x^4}{4}}{\left(3-x+2x^2\right)^{3/2}}\,dx$$

$$= -\frac{1331(17-45x)}{1104\left(3-x+2x^2\right)^{3/2}} + \frac{121(10679-6744x)}{8464\sqrt{3-x+2x^2}} + \frac{4\int \frac{\frac{1452105}{64}+\frac{277725x}{8}+\frac{198375x^2}{16}}{\sqrt{3-x+2x^2}}\,dx}{1587}$$

$$= -\frac{1331(17-45x)}{1104\left(3-x+2x^2\right)^{3/2}} + \frac{121(10679-6744x)}{8464\sqrt{3-x+2x^2}} + \frac{125}{16}x\sqrt{3-x+2x^2} + \frac{\int \frac{\frac{214245}{4}+\frac{5038725x}{32}}{\sqrt{3-x+2x^2}}\,d}{1587}$$

$$= -\frac{1331(17-45x)}{1104\left(3-x+2x^2\right)^{3/2}} + \frac{121(10679-6744x)}{8464\sqrt{3-x+2x^2}} + \frac{3175}{64}\sqrt{3-x+2x^2} + \frac{125}{16}x\sqrt{3-x+2}$$

$$= -\frac{1331(17-45x)}{1104\left(3-x+2x^2\right)^{3/2}} + \frac{121(10679-6744x)}{8464\sqrt{3-x+2x^2}} + \frac{3175}{64}\sqrt{3-x+2x^2} + \frac{125}{16}x\sqrt{3-x+2}$$

$$= -\frac{1331(17-45x)}{1104\left(3-x+2x^2\right)^{3/2}} + \frac{121(10679-6744x)}{8464\sqrt{3-x+2x^2}} + \frac{3175}{64}\sqrt{3-x+2x^2} + \frac{125}{16}x\sqrt{3-x+2}$$

Mathematica [A] time = 0.34, size = 65, normalized size = 0.62

$$\frac{3174000x^5+16980900x^4-29423976x^3+101546529x^2-62463282x+89784565}{101568\left(2x^2-x+3\right)^{3/2}} + \frac{7495\sinh^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{128\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)^3/(3 - x + 2*x^2)^(5/2),x]
```

```
[Out] (89784565 - 62463282*x + 101546529*x^2 - 29423976*x^3 + 16980900*x^4 + 3174
000*x^5)/(101568*(3 - x + 2*x^2)^(3/2)) + (7495*ArcSinh[(-1 + 4*x)/Sqrt[23]
])/(128*Sqrt[2])
```

IntegrateAlgebraic [A] time = 1.03, size = 80, normalized size = 0.76

$$\frac{3174000x^5+16980900x^4-29423976x^3+101546529x^2-62463282x+89784565}{101568\left(2x^2-x+3\right)^{3/2}} - \frac{7495\log\left(2\sqrt{2}\sqrt{2x^2-x+3}-4x+1\right)}{128\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 + 3*x + 5*x^2)^3/(3 - x + 2*x^2)^(5/2),x]
```

```
[Out] (89784565 - 62463282*x + 101546529*x^2 - 29423976*x^3 + 16980900*x^4 + 3174
000*x^5)/(101568*(3 - x + 2*x^2)^(3/2)) - (7495*Log[1 - 4*x + 2*Sqrt[2]*Sqr
t[3 - x + 2*x^2]])/(128*Sqrt[2])
```

fricas [A] time = 0.43, size = 122, normalized size = 1.16

$$\frac{11894565 \sqrt{2} \left(4 x^{4} - 4 x^{3} + 13 x^{2} - 6 x + 9\right) \log\left(-4 \sqrt{2} \sqrt{2 x^{2} - x + 3} (4 x - 1) - 32 x^{2} + 16 x - 25\right) + 8 \left(3174000 x^{5} + 16980900 x^{4} - 29423976 x^{3} + 101546529 x^{2} - 62463282 x + 89784565\right) \sqrt{2 x^{2} - x + 3}}{812544 \left(4 x^{4} - 4 x^{3} + 13 x^{2} - 6 x + 9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/812544*(11894565*sqrt(2)*(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)*log(-4*sqrt(2
)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 8*(3174000*x^5 + 16
980900*x^4 - 29423976*x^3 + 101546529*x^2 - 62463282*x + 89784565)*sqrt(2*x
^2 - x + 3))/(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)
```

giac [A] time = 0.27, size = 72, normalized size = 0.69

$$-\frac{7495}{256} \sqrt{2} \log\left(-2 \sqrt{2} \left(\sqrt{2} x - \sqrt{2 x^{2} - x + 3}\right) + 1\right) + \frac{3 ((4 (13225 (20 x + 107) x - 2451998) x + 33848843) x - 20821094) x + 89784565}{101568 \left(2 x^{2} - x + 3\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^(5/2),x, algorithm="giac")
```

```
[Out] -7495/256*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) + 1
/101568*(3*((4*(13225*(20*x + 107)*x - 2451998)*x + 33848843)*x - 20821094)
*x + 89784565)/(2*x^2 - x + 3)^(3/2)
```

maple [B] time = 0.01, size = 180, normalized size = 1.71

$$\frac{125 x^{5}}{4 \left(2 x^{2}-x+3\right)^{\frac{3}{2}}}+\frac{2675 x^{4}}{16 \left(2 x^{2}-x+3\right)^{\frac{3}{2}}}-\frac{7495 x^{3}}{192 \left(2 x^{2}-x+3\right)^{\frac{3}{2}}}+\frac{222809 x^{2}}{256 \left(2 x^{2}-x+3\right)^{\frac{3}{2}}}-\frac{281177 x}{2048 \left(2 x^{2}-x+3\right)^{\frac{3}{2}}}-\frac{7495 x}{128 \sqrt{2 x^{2}-x+3}}+\frac{7495 \sqrt{2}\, \operatorname{arcsinh}\left(\frac{4 \sqrt{23}\, \left(x-\frac{1}{4}\right)}{23}\right)}{256}-\frac{7495}{512 \sqrt{2 x^{2}-x+3}}+\frac{20961031}{24576 \left(2 x^{2}-x+3\right)^{\frac{3}{2}}}-\frac{3391139 \left(4 x-1\right)}{203136 \sqrt{2 x^{2}-x+3}}-\frac{14081711 \left(4 x-1\right)}{565248 \left(2 x^{2}-x+3\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)^3/(2*x^2-x+3)^(5/2),x)
```

```
[Out] -7495/512/(2*x^2-x+3)^(1/2)+20961031/24576/(2*x^2-x+3)^(3/2)-3391139/203136
*(4*x-1)/(2*x^2-x+3)^(1/2)+7495/256*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))-
14081711/565248*(4*x-1)/(2*x^2-x+3)^(3/2)+125/4/(2*x^2-x+3)^(3/2)*x^5+2675/
16/(2*x^2-x+3)^(3/2)*x^4-7495/192/(2*x^2-x+3)^(3/2)*x^3+222809/256/(2*x^2-x
+3)^(3/2)*x^2-281177/2048/(2*x^2-x+3)^(3/2)*x-7495/128/(2*x^2-x+3)^(1/2)*x
```

maxima [B] time = 1.02, size = 219, normalized size = 2.09

$$\frac{125\,x^{5}}{4\,{\left(2\,x^{2}-x+3\right)}^{\frac{3}{2}}}+\frac{2675\,x^{4}}{16\,{\left(2\,x^{2}-x+3\right)}^{\frac{3}{2}}}+\frac{7495}{203136}\,x\left(\frac{284\,x}{\sqrt{2\,x^{2}-x+3}}-\frac{3174\,x^{2}}{{\left(2\,x^{2}-x+3\right)}^{\frac{3}{2}}}-\frac{71}{\sqrt{2\,x^{2}-x+3}}+\frac{805\,x}{{\left(2\,x^{2}-x+3\right)}^{\frac{3}{2}}}-\frac{3243}{{\left(2\,x^{2}-x+3\right)}^{\frac{3}{2}}}\right)+\frac{7495}{256}\,\sqrt{2}\,\operatorname{arsinh}\left(\frac{1}{23}\,\sqrt{23}\,(4\,x-1)\right)-\frac{532145}{101568}\,\sqrt{2\,x^{2}-x+3}-\frac{4515389\,x}{50784\,\sqrt{2\,x^{2}-x+3}}+\frac{7197\,x^{2}}{8\,{\left(2\,x^{2}-x+3\right)}^{\frac{3}{2}}}+\frac{396211}{50784\,\sqrt{2\,x^{2}-x+3}}-\frac{269783\,x}{1104\,{\left(2\,x^{2}-x+3\right)}^{\frac{3}{2}}}+\frac{1002137}{1104\,{\left(2\,x^{2}-x+3\right)}^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^(5/2),x, algorithm="maxima")
```

```
[Out] 125/4*x^5/(2*x^2 - x + 3)^(3/2) + 2675/16*x^4/(2*x^2 - x + 3)^(3/2) + 7495/
203136*x*(284*x/sqrt(2*x^2 - x + 3) - 3174*x^2/(2*x^2 - x + 3)^(3/2) - 71/s
qrt(2*x^2 - x + 3) + 805*x/(2*x^2 - x + 3)^(3/2) - 3243/(2*x^2 - x + 3)^(3/
2)) + 7495/256*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 532145/101568*sqr
t(2*x^2 - x + 3) - 4515389/50784*x/sqrt(2*x^2 - x + 3) + 7197/8*x^2/(2*x^2
- x + 3)^(3/2) + 396211/50784/sqrt(2*x^2 - x + 3) - 269783/1104*x/(2*x^2 -
x + 3)^(3/2) + 1002137/1104/(2*x^2 - x + 3)^(3/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(5\,x^2+3\,x+2\right)}^3}{{\left(2\,x^2-x+3\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 5*x^2 + 2)^3/(2*x^2 - x + 3)^(5/2),x)
```

```
[Out] int((3*x + 5*x^2 + 2)^3/(2*x^2 - x + 3)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(5x^{2} + 3x + 2\right)^{3}}{\left(2x^{2} - x + 3\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)**3/(2*x**2-x+3)**(5/2),x)
```

```
[Out] Integral((5*x**2 + 3*x + 2)**3/(2*x**2 - x + 3)**(5/2), x)
```

3.92 $\int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^{5/2}}\,dx$

Optimal. Leaf size=68

$$\frac{121(19-7x)}{276(2x^2-x+3)^{3/2}}-\frac{11(2336x+7351)}{6348\sqrt{2x^2-x+3}}-\frac{25\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{4\sqrt{2}}$$

Rubi [A] time = 0.06, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1660, 12, 619, 215}

$$\frac{121(19-7x)}{276(2x^2-x+3)^{3/2}}-\frac{11(2336x+7351)}{6348\sqrt{2x^2-x+3}}-\frac{25\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + 5*x^2)^2/(3 - x + 2*x^2)^(5/2),x]
```

```
[Out] (121*(19 - 7*x))/(276*(3 - x + 2*x^2)^(3/2)) - (11*(7351 + 2336*x))/(6348*S
qrt[3 - x + 2*x^2]) - (25*ArcSinh[(1 - 4*x)/Sqrt[23]])/(4*Sqrt[2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
```

```
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(2+3x+5x^2\right)^2}{\left(3-x+2x^2\right)^{5/2}}\,dx &= \frac{121(19-7x)}{276\left(3-x+2x^2\right)^{3/2}}+\frac{2}{69}\int \frac{\frac{131}{16}+\frac{5865x}{8}+\frac{1725x^2}{4}}{\left(3-x+2x^2\right)^{3/2}}\,dx \\
&= \frac{121(19-7x)}{276\left(3-x+2x^2\right)^{3/2}}-\frac{11(7351+2336x)}{6348\sqrt{3-x+2x^2}}+\frac{4\int \frac{39675}{16\sqrt{3-x+2x^2}}\,dx}{1587} \\
&= \frac{121(19-7x)}{276\left(3-x+2x^2\right)^{3/2}}-\frac{11(7351+2336x)}{6348\sqrt{3-x+2x^2}}+\frac{25}{4}\int \frac{1}{\sqrt{3-x+2x^2}}\,dx \\
&= \frac{121(19-7x)}{276\left(3-x+2x^2\right)^{3/2}}-\frac{11(7351+2336x)}{6348\sqrt{3-x+2x^2}}+\frac{25\,\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{23}}}\,dx,x,-1+4x\right)}{4\sqrt{46}} \\
&= \frac{121(19-7x)}{276\left(3-x+2x^2\right)^{3/2}}-\frac{11(7351+2336x)}{6348\sqrt{3-x+2x^2}}-\frac{25\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{4\sqrt{2}}
\end{aligned}$$

Mathematica [A] time = 0.24, size = 55, normalized size = 0.81

$$\frac{25\sinh^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{4\sqrt{2}}-\frac{11\left(2336x^3+6183x^2+714x+8623\right)}{3174\left(2x^2-x+3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)^2/(3 - x + 2*x^2)^(5/2),x]
```

```
[Out] (-11*(8623 + 714*x + 6183*x^2 + 2336*x^3))/(3174*(3 - x + 2*x^2)^(3/2)) + (
25*ArcSinh[(-1 + 4*x)/Sqrt[23]])/(4*Sqrt[2])
```

IntegrateAlgebraic [A] time = 0.74, size = 70, normalized size = 1.03

$$-\frac{25\log\left(2\sqrt{2}\sqrt{2x^2-x+3}-4x+1\right)}{4\sqrt{2}}-\frac{11\left(2336x^3+6183x^2+714x+8623\right)}{3174\left(2x^2-x+3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 + 3*x + 5*x^2)^2/(3 - x + 2*x^2)^(5/2),x]
```

```
[Out] (-11*(8623 + 714*x + 6183*x^2 + 2336*x^3))/(3174*(3 - x + 2*x^2)^(3/2)) - (
25*Log[1 - 4*x + 2*Sqrt[2]*Sqrt[3 - x + 2*x^2]])/(4*Sqrt[2])
```

fricas [B] time = 0.41, size = 112, normalized size = 1.65

$$\frac{39675\sqrt{2}\left(4x^4-4x^3+13x^2-6x+9\right)\log\left(-4\sqrt{2}\sqrt{2x^2-x+3}(4x-1)-32x^2+16x-25\right)-88\left(2336x^3+6183x^2+714x+8623\right)\sqrt{2x^2-x+3}}{25392\left(4x^4-4x^3+13x^2-6x+9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/25392*(39675*sqrt(2)*(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)*log(-4*sqrt(2)*sq
rt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) - 88*(2336*x^3 + 6183*x^2
 + 714*x + 8623)*sqrt(2*x^2 - x + 3))/(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)
```

giac [A] time = 0.27, size = 61, normalized size = 0.90

$$-\frac{25}{8}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+1\right)-\frac{11\left(((2336x+6183)x+714)x+8623\right)}{3174\left(2x^2-x+3\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^(5/2),x, algorithm="giac")
```

```
[Out] -25/8*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) - 11/31
74*(((2336*x + 6183)*x + 714)*x + 8623)/(2*x^2 - x + 3)^(3/2)
```

maple [B] time = 0.01, size = 146, normalized size = 2.15

$$-\frac{25x^3}{6\left(2x^2-x+3\right)^{\frac{3}{2}}}-\frac{145x^2}{8\left(2x^2-x+3\right)^{\frac{3}{2}}}-\frac{319x}{64\left(2x^2-x+3\right)^{\frac{3}{2}}}-\frac{25x}{4\sqrt{2x^2-x+3}}+\frac{25\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{8}-\frac{15775}{768\left(2x^2-x+3\right)^{\frac{3}{2}}}+\frac{\frac{8493x}{1472}-\frac{8493}{5888}}{\left(2x^2-x+3\right)^{\frac{3}{2}}}+\frac{\frac{2267x}{529}-\frac{2267}{2116}}{\sqrt{2x^2-x+3}}-\frac{25}{16\sqrt{2x^2-x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)^2/(2*x^2-x+3)^(5/2),x)
```

```
[Out] -25/6/(2*x^2-x+3)^(3/2)*x^3-145/8/(2*x^2-x+3)^(3/2)*x^2-319/64/(2*x^2-x+3)^
(3/2)*x-15775/768/(2*x^2-x+3)^(3/2)+8493/5888*(4*x-1)/(2*x^2-x+3)^(3/2)+226
7/2116*(4*x-1)/(2*x^2-x+3)^(1/2)-25/4/(2*x^2-x+3)^(1/2)*x-25/16/(2*x^2-x+3)
^(1/2)+25/8*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))
```

maxima [B] time = 0.98, size = 185, normalized size = 2.72

$$\frac{25}{6348}\,x\left(\frac{284\,x}{\sqrt{2\,x^2-x+3}}-\frac{3174\,x^2}{\left(2\,x^2-x+3\right)^{\frac{3}{2}}}-\frac{71}{\sqrt{2\,x^2-x+3}}+\frac{805\,x}{\left(2\,x^2-x+3\right)^{\frac{3}{2}}}-\frac{3243}{\left(2\,x^2-x+3\right)^{\frac{3}{2}}}\right)+\frac{25}{8}\,\sqrt{2}\,\operatorname{arsinh}\left(\frac{1}{23}\,\sqrt{23}\,(4\,x-1)\right)-\frac{1775}{3174}\,\sqrt{2\,x^2-x+3}+\frac{1017\,x}{529\,\sqrt{2\,x^2-x+3}}-\frac{15\,x^2}{\left(2\,x^2-x+3\right)^{\frac{3}{2}}}-\frac{6413}{3174\,\sqrt{2\,x^2-x+3}}-\frac{x}{138\left(2\,x^2-x+3\right)^{\frac{3}{2}}}-\frac{2593}{138\left(2\,x^2-x+3\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^(5/2),x, algorithm="maxima")
```

```
[Out] 25/6348*x*(284*x/sqrt(2*x^2 - x + 3) - 3174*x^2/(2*x^2 - x + 3)^(3/2) - 71/
sqrt(2*x^2 - x + 3) + 805*x/(2*x^2 - x + 3)^(3/2) - 3243/(2*x^2 - x + 3)^(3
/2)) + 25/8*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 1775/3174*sqrt(2*x^2
 - x + 3) + 1017/529*x/sqrt(2*x^2 - x + 3) - 15*x^2/(2*x^2 - x + 3)^(3/2) -
 6413/3174/sqrt(2*x^2 - x + 3) - 1/138*x/(2*x^2 - x + 3)^(3/2) - 2593/138/(
2*x^2 - x + 3)^(3/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(5\,x^2+3\,x+2\right)^2}{\left(2\,x^2-x+3\right)^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 5*x^2 + 2)^2/(2*x^2 - x + 3)^(5/2),x)
```

```
[Out] int((3*x + 5*x^2 + 2)^2/(2*x^2 - x + 3)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(5x^{2} + 3x + 2\right)^{2}}{\left(2x^{2} - x + 3\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)**2/(2*x**2-x+3)**(5/2),x)
```

```
[Out] Integral((5*x**2 + 3*x + 2)**2/(2*x**2 - x + 3)**(5/2), x)
```

3.93 $\int \frac{2+3x+5x^2}{\left(3-x+2x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=47

$$-\frac{71(1-4x)}{529\sqrt{2x^2-x+3}}-\frac{11(3x+5)}{69\left(2x^2-x+3\right)^{3/2}}$$

Rubi [A] time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {1660, 12, 613}

$$-\frac{71(1-4x)}{529\sqrt{2x^2-x+3}}-\frac{11(3x+5)}{69\left(2x^2-x+3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + 5*x^2)/(3 - x + 2*x^2)^(5/2),x]
```

```
[Out] (-11*(5 + 3*x))/(69*(3 - x + 2*x^2)^(3/2)) - (71*(1 - 4*x))/(529*Sqrt[3 - x
 + 2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{2+3x+5x^2}{\left(3-x+2x^2\right)^{5/2}}\,dx = -\frac{11(5+3x)}{69\left(3-x+2x^2\right)^{3/2}}+\frac{2}{69}\int \frac{213}{4\left(3-x+2x^2\right)^{3/2}}\,dx$$
$$= -\frac{11(5+3x)}{69\left(3-x+2x^2\right)^{3/2}}+\frac{71}{46}\int \frac{1}{\left(3-x+2x^2\right)^{3/2}}\,dx$$
$$= -\frac{11(5+3x)}{69\left(3-x+2x^2\right)^{3/2}}-\frac{71(1-4x)}{529\sqrt{3-x+2x^2}}$$

Mathematica [A] time = 0.10, size = 33, normalized size = 0.70

$$\frac{2\left(852x^3-639x^2+1005x-952\right)}{1587\left(2x^2-x+3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)/(3 - x + 2*x^2)^(5/2),x]
[Out] (2*(-952 + 1005*x - 639*x^2 + 852*x^3))/(1587*(3 - x + 2*x^2)^(3/2))
```

IntegrateAlgebraic [A] time = 0.52, size = 33, normalized size = 0.70

$$\frac{2\left(852x^3-639x^2+1005x-952\right)}{1587\left(2x^2-x+3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 + 3*x + 5*x^2)/(3 - x + 2*x^2)^(5/2),x]
[Out] (2*(-952 + 1005*x - 639*x^2 + 852*x^3))/(1587*(3 - x + 2*x^2)^(3/2))
```

fricas [A] time = 0.40, size = 51, normalized size = 1.09

$$\frac{2\left(852\,x^3-639\,x^2+1005\,x-952\right)\sqrt{2\,x^2-x+3}}{1587\left(4\,x^4-4\,x^3+13\,x^2-6\,x+9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)/(2*x^2-x+3)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/1587*(852*x^3 - 639*x^2 + 1005*x - 952)*sqrt(2*x^2 - x + 3)/(4*x^4 - 4*x^
3 + 13*x^2 - 6*x + 9)
```

giac [A] time = 0.23, size = 29, normalized size = 0.62

$$\frac{2\,(3\,(71\,(4\,x-3)x+335)x-952)}{1587\,\left(2\,x^2-x+3\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)/(2*x^2-x+3)^(5/2),x, algorithm="giac")
```

```
[Out] 2/1587*(3*(71*(4*x - 3)*x + 335)*x - 952)/(2*x^2 - x + 3)^(3/2)
```

maple [A] time = 0.00, size = 30, normalized size = 0.64

$$\frac{\frac{568}{529}x^3-\frac{426}{529}x^2+\frac{670}{529}x-\frac{1904}{1587}}{\left(2x^2-x+3\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)/(2*x^2-x+3)^(5/2),x)
```

```
[Out] 2/1587/(2*x^2-x+3)^(3/2)*(852*x^3-639*x^2+1005*x-952)
```

maxima [A] time = 0.43, size = 59, normalized size = 1.26

$$\frac{284\,x}{529\,\sqrt{2\,x^2-x+3}}-\frac{71}{529\,\sqrt{2\,x^2-x+3}}-\frac{11\,x}{23\,\left(2\,x^2-x+3\right)^{\frac{3}{2}}}-\frac{55}{69\,\left(2\,x^2-x+3\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)/(2*x^2-x+3)^(5/2),x, algorithm="maxima")
```

```
[Out] 284/529*x/sqrt(2*x^2 - x + 3) - 71/529/sqrt(2*x^2 - x + 3) - 11/23*x/(2*x^2
 - x + 3)^(3/2) - 55/69/(2*x^2 - x + 3)^(3/2)
```

mupad [B] time = 0.09, size = 29, normalized size = 0.62

$$\frac{2\,\left(852\,x^3-639\,x^2+1005\,x-952\right)}{1587\,{\left(2\,x^2-x+3\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 5*x^2 + 2)/(2*x^2 - x + 3)^(5/2),x)
[Out] (2*(1005*x - 639*x^2 + 852*x^3 - 952))/(1587*(2*x^2 - x + 3)^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5x^2 + 3x + 2}{(2x^2 - x + 3)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)/(2*x**2-x+3)**(5/2),x)
[Out] Integral((5*x**2 + 3*x + 2)/(2*x**2 - x + 3)**(5/2), x)
```

3.94 $\int \frac{1}{(3-x+2x^2)^{5/2}(2+3x+5x^2)}\,dx$

Optimal. Leaf size=199

$$\frac{3603-658x}{128018\sqrt{2x^2-x+3}}+\frac{13-6x}{759\left(2x^2-x+3\right)^{3/2}}+\frac{1}{484}\sqrt{\frac{1}{682}\left(25000\sqrt{2}-15457\right)}\tan^{-1}\left(\frac{\sqrt{\frac{11}{31\left(25000\sqrt{2}-15457\right)}}\left((247\right.}{\sqrt{2x^2}}\right.$$

Rubi [A] time = 0.46, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {974, 1060, 1035, 1029, 206, 204}

$$\frac{3603-658x}{128018\sqrt{2x^2-x+3}}+\frac{13-6x}{759\left(2x^2-x+3\right)^{3/2}}+\frac{1}{484}\sqrt{\frac{1}{682}\left(25000\sqrt{2}-15457\right)}\tan^{-1}\left(\frac{\sqrt{\frac{11}{31\left(25000\sqrt{2}-15457\right)}}\left(\left(247+345\sqrt{2}\right)x-98\sqrt{2}+443\right)}{\sqrt{2x^2-x+3}}\right)-\frac{1}{484}\sqrt{\frac{1}{682}\left(15457+25000\sqrt{2}\right)}\tanh^{-1}\left(\frac{\sqrt{\frac{11}{31\left(15457+25000\sqrt{2}\right)}}\left(\left(247-345\sqrt{2}\right)x+98\sqrt{2}+443\right)}{\sqrt{2x^2-x+3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - x + 2*x^2)^(5/2)*(2 + 3*x + 5*x^2)),x]
```

```
[Out] (13 - 6*x)/(759*(3 - x + 2*x^2)^(3/2)) + (3603 - 658*x)/(128018*Sqrt[3 - x
+ 2*x^2]) + (Sqrt[(-15457 + 25000*Sqrt[2])/682]*ArcTan[(Sqrt[11/(31*(-15457
 + 25000*Sqrt[2]))]*(443 - 98*Sqrt[2] + (247 + 345*Sqrt[2])*x))/Sqrt[3 - x
+ 2*x^2]])/484 - (Sqrt[(15457 + 25000*Sqrt[2])/682]*ArcTanh[(Sqrt[11/(31*(1
5457 + 25000*Sqrt[2]))]*(443 + 98*Sqrt[2] + (247 - 345*Sqrt[2])*x))/Sqrt[3
- x + 2*x^2]])/484
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 974

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*
a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*(
```

```
d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q,
 0]
```

Rule 1029

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
 c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 1035

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

Rule 1060

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x +
 c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
2*a*(c*d - a*f)))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
```

```
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !
IGtQ[q, 0]
```

Rubi steps

$$\int \frac{1}{\left(3-x+2x^2\right)^{5/2}\left(2+3x+5x^2\right)}\,dx = \frac{13-6x}{759\left(3-x+2x^2\right)^{3/2}} - \frac{\int \frac{-2772-\frac{3003x}{2}+660x^2}{\left(3-x+2x^2\right)^{3/2}\left(2+3x+5x^2\right)}\,dx}{8349}$$

$$= \frac{13-6x}{759\left(3-x+2x^2\right)^{3/2}} + \frac{3603-658x}{128018\sqrt{3-x+2x^2}} - \frac{\int \frac{-\frac{5184729}{2}-\frac{12481755x}{4}}{\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)}\,dx}{23235267}$$

$$= \frac{13-6x}{759\left(3-x+2x^2\right)^{3/2}} + \frac{3603-658x}{128018\sqrt{3-x+2x^2}} + \frac{\int \frac{-\frac{2112297}{4}\left(11-54\sqrt{2}\right)-\frac{21122}{4}}{\sqrt{3-x+2x^2}\left(2+3x\right.}}{511175874}$$

$$= \frac{13-6x}{759\left(3-x+2x^2\right)^{3/2}} + \frac{3603-658x}{128018\sqrt{3-x+2x^2}} - \frac{1}{32}\left(17457\left(50000-15\right.\right.$$

$$= \frac{13-6x}{759\left(3-x+2x^2\right)^{3/2}} + \frac{3603-658x}{128018\sqrt{3-x+2x^2}} + \frac{1}{484}\sqrt{\frac{1}{682}\left(-15457+2\right.}$$

Mathematica [C] time = 0.87, size = 218, normalized size = 1.10

$$-\frac{\sqrt{\frac{1}{682}\left(13+i\sqrt{31}\right)}\left(119\sqrt{31}+247i\right)\tanh^{-1}\left(\frac{\left(-22-4i\sqrt{31}\right)x+i\sqrt{31}+63}{2\sqrt{286+22i\sqrt{31}}\sqrt{2x^2-x+3}}\right)}{9680}+\frac{\sqrt{\frac{1}{682}\left(13-i\sqrt{31}\right)}\left(119\sqrt{31}-247i\right)\tanh^{-1}\left(\frac{\left(22-4i\sqrt{31}\right)x+i\sqrt{31}-63}{2\sqrt{286-22i\sqrt{31}}\sqrt{2x^2-x+3}}\right)}{9680}+\frac{-3948x^3+23592x^2-19767x+39005}{384054\left(2x^2-x+3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - x + 2*x^2)^(5/2)*(2 + 3*x + 5*x^2)),x]
```

```
[Out] (39005 - 19767*x + 23592*x^2 - 3948*x^3)/(384054*(3 - x + 2*x^2)^(3/2)) - (
Sqrt[(13 + I*Sqrt[31])/682]*(247*I + 119*Sqrt[31])*ArcTanh[(63 + I*Sqrt[31]
 + (-22 - (4*I)*Sqrt[31])*x)/(2*Sqrt[286 + (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*
x^2])])/9680 + (Sqrt[(13 - I*Sqrt[31])/682]*(-247*I + 119*Sqrt[31])*ArcTanh
[(-63 + I*Sqrt[31] + (22 - (4*I)*Sqrt[31])*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]
]*Sqrt[3 - x + 2*x^2])])/9680
```

IntegrateAlgebraic [C] time = 0.83, size = 209, normalized size = 1.05

$$\frac{1}{484}\text{RootSum}\left[-5\#1^4+6\sqrt{2}\#1^3+17\#1^2-26\sqrt{2}\#1-56\&,\frac{-65\#1^2\log\left(-\#1+\sqrt{2x^2-x+3}-\sqrt{2}x\right)+108\sqrt{2}\#1\log\left(-\#1+\sqrt{2x^2-x+3}-\sqrt{2}x\right)+249\log\left(-\#1+\sqrt{2x^2-x+3}-\sqrt{2}x\right)}{-10\#1^3+9\sqrt{2}\#1^2+17\#1-13\sqrt{2}}\&\right]+\frac{-3948x^3+23592x^2-19767x+39005}{384054\left(2x^2-x+3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((3 - x + 2*x^2)^(5/2)*(2 + 3*x + 5*x^2)),x]
```

```
[Out] (39005 - 19767*x + 23592*x^2 - 3948*x^3)/(384054*(3 - x + 2*x^2)^(3/2)) + R
ootSum[-56 - 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3 - 5*#1^4 & , (249*Log
[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1] + 108*Sqrt[2]*Log[-(Sqrt[2]*x) +
Sqrt[3 - x + 2*x^2] - #1]*#1 - 65*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] -
#1]*#1^2)/(-13*Sqrt[2] + 17*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3) & ]/484
```

fricas [B] time = 1.24, size = 2133, normalized size = 10.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^(5/2)/(5*x^2+3*x+2),x, algorithm="fricas")
```

```
[Out] 1/37097146779158400O*(1123856268*sqrt(341)*200^(1/4)*sqrt(2)*(4*x^4 - 4*x^3
 + 13*x^2 - 6*x + 9)*sqrt(-772850000*sqrt(2) + 2500000000)*arctan(-1/788938
9562500*(71300*sqrt(341)*sqrt(2*x^2 - x + 3)*(11*200^(3/4)*(347404*x^7 - 90
7814*x^6 + 2112962*x^5 - 2166688*x^4 + 787344*x^3 + 304128*x^2 - sqrt(2)*(3
5898*x^7 - 441939*x^6 + 782418*x^5 - 2117233*x^4 + 1272680*x^3 - 1081800*x^
2 - 518400*x + 1043712) - 2087424*x + 518400) + 5*200^(1/4)*(712757*x^7 - 1
0233303*x^6 + 48529768*x^5 - 94500260*x^4 + 113086944*x^3 - 22282848*x^2 -
sqrt(2)*(158647*x^7 - 2935272*x^6 + 19428740*x^5 - 55765712*x^4 + 78380640*
x^3 - 84096000*x^2 - 37407744*x + 53208576) - 106417152*x + 37407744))*sqrt
(-772850000*sqrt(2) + 2500000000) + 22395686500000*sqrt(31)*sqrt(2)*(28180*
x^8 - 254666*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98
496*x^2 - sqrt(2)*(8746*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 132071
0*x^4 - 752088*x^3 + 396144*x^2 + 546048*x - 539136) + 1154304*x - 456192)
- sqrt(310/5711)*(sqrt(341)*sqrt(2*x^2 - x + 3)*(11*200^(3/4)*(1665224*x^7
- 2325796*x^6 + 7065036*x^5 - 196416*x^4 - 2176416*x^3 + 8895744*x^2 + sqrt
```

(2)*(167914*x^7 - 195429*x^6 + 331239*x^5 + 1685680*x^4 - 3693960*x^3 + 419
5584*x^2 - 4195584*x) - 8895744*x) + 5*200^(1/4)*(3246491*x^7 - 41888524*x^
6 + 159670660*x^5 - 190080576*x^4 + 180496224*x^3 + 376648704*x^2 - 2*sqrt(
2)*(40239*x^7 - 558044*x^6 + 2804660*x^5 - 9524160*x^4 + 34843680*x^3 - 740
06784*x^2 + 74006784*x) - 376648704*x))*sqrt(-772850000*sqrt(2) + 250000000
0) + 314105000*sqrt(31)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578888*x^6 - 32
93072*x^5 + 396480*x^4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 11
8051*x^7 + 244047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^
2 - 1036800*x) + 3276288*x) + 14277500*sqrt(31)*(254591*x^8 - 4815126*x^7 +
 32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 74219328*x^3 - 168956928*x^2
 - 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3618*x^3
 + 2268*x^2 - 1944*x) + 144820224*x))*sqrt(-(sqrt(341)*200^(1/4)*sqrt(31)*s
qrt(2*x^2 - x + 3)*(sqrt(2)*(281*x - 444) + 163*x - 725)*sqrt(-772850000*sq
rt(2) + 2500000000) - 4337504500*x^2 - 3894902000*sqrt(2)*(2*x^2 - x + 3) +
 13366595500*x - 17704100000)/x^2) + 254496437500*sqrt(31)*(2828123*x^8 - 9
696916*x^7 + 53385560*x^6 - 142835344*x^5 + 254146592*x^4 - 249300096*x^3 +
 37981440*x^2 - 7744*sqrt(2)*(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 +
15569*x^4 - 5568*x^3 + 1080*x^2 + 4320*x - 5184) + 223064064*x - 94887936))
/(2585191*x^8 - 4661200*x^7 + 14191920*x^6 + 490880*x^5 - 13562944*x^4 + 44
249088*x^3 - 34615296*x^2 - 24772608*x + 18579456)) + 1123856268*sqrt(341)*
200^(1/4)*sqrt(2)*(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)*sqrt(-772850000*sqrt(2
) + 2500000000)*arctan(-1/7889389562500*(71300*sqrt(341)*sqrt(2*x^2 - x + 3
)*(11*200^(3/4)*(347404*x^7 - 907814*x^6 + 2112962*x^5 - 2166688*x^4 + 7873
44*x^3 + 304128*x^2 - sqrt(2)*(35898*x^7 - 441939*x^6 + 782418*x^5 - 211723
3*x^4 + 1272680*x^3 - 1081800*x^2 - 518400*x + 1043712) - 2087424*x + 51840
0) + 5*200^(1/4)*(712757*x^7 - 10233303*x^6 + 48529768*x^5 - 94500260*x^4 +
 113086944*x^3 - 22282848*x^2 - sqrt(2)*(158647*x^7 - 2935272*x^6 + 1942874
0*x^5 - 55765712*x^4 + 78380640*x^3 - 84096000*x^2 - 37407744*x + 53208576)
 - 106417152*x + 37407744))*sqrt(-772850000*sqrt(2) + 2500000000) - 2239568
6500000*sqrt(31)*sqrt(2)*(28180*x^8 - 254666*x^7 + 704270*x^6 - 1385256*x^5
 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - sqrt(2)*(8746*x^8 - 102335*x^7 +
396104*x^6 - 783113*x^5 + 1320710*x^4 - 752088*x^3 + 396144*x^2 + 546048*x
 - 539136) + 1154304*x - 456192) - sqrt(310/5711)*(sqrt(341)*sqrt(2*x^2 - x
 + 3)*(11*200^(3/4)*(1665224*x^7 - 2325796*x^6 + 7065036*x^5 - 196416*x^4 -
2176416*x^3 + 8895744*x^2 + sqrt(2)*(167914*x^7 - 195429*x^6 + 331239*x^5 +
 1685680*x^4 - 3693960*x^3 + 4195584*x^2 - 4195584*x) - 8895744*x) + 5*200^
(1/4)*(3246491*x^7 - 41888524*x^6 + 159670660*x^5 - 190080576*x^4 + 1804962
24*x^3 + 376648704*x^2 - 2*sqrt(2)*(40239*x^7 - 558044*x^6 + 2804660*x^5 -
9524160*x^4 + 34843680*x^3 - 74006784*x^2 + 74006784*x) - 376648704*x))*sqr
t(-772850000*sqrt(2) + 2500000000) - 314105000*sqrt(31)*sqrt(2)*(123408*x^8
 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 + 396480*x^4 + 798336*x^3 - 38223
36*x^2 - sqrt(2)*(15550*x^8 - 118051*x^7 + 244047*x^6 - 707374*x^5 + 105396
0*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*x) + 3276288*x) - 14277500*sqrt
(31)*(254591*x^8 - 4815126*x^7 + 32303580*x^6 - 90866808*x^5 + 108781920*x^
4 - 74219328*x^3 - 168956928*x^2 - 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6

```
- 1536*x^5 + 2385*x^4 - 3618*x^3 + 2268*x^2 - 1944*x) + 144820224*x))*sqrt(
(sqrt(341)*200^(1/4)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(281*x - 444) +
163*x - 725)*sqrt(-772850000*sqrt(2) + 2500000000) + 4337504500*x^2 + 38949
02000*sqrt(2)*(2*x^2 - x + 3) - 13366595500*x + 17704100000)/x^2) - 2544964
37500*sqrt(31)*(2828123*x^8 - 9696916*x^7 + 53385560*x^6 - 142835344*x^5 +
254146592*x^4 - 249300096*x^3 + 37981440*x^2 - 7744*sqrt(2)*(1348*x^8 - 269
2*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^4 - 5568*x^3 + 1080*x^2 + 4320*x - 5
184) + 223064064*x - 94887936))/(2585191*x^8 - 4661200*x^7 + 14191920*x^6 +
 490880*x^5 - 13562944*x^4 + 44249088*x^3 - 34615296*x^2 - 24772608*x + 185
79456)) + 1587*sqrt(341)*200^(1/4)*sqrt(31)*(200000*x^4 - 200000*x^3 + 6500
00*x^2 + 15457*sqrt(2)*(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9) - 300000*x + 4500
00)*sqrt(-772850000*sqrt(2) + 2500000000)*log(77500000/5711*(sqrt(341)*200^
(1/4)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(281*x - 444) + 163*x - 725)*sq
rt(-772850000*sqrt(2) + 2500000000) + 4337504500*x^2 + 3894902000*sqrt(2)*(
2*x^2 - x + 3) - 13366595500*x + 17704100000)/x^2) - 1587*sqrt(341)*200^(1/
4)*sqrt(31)*(200000*x^4 - 200000*x^3 + 650000*x^2 + 15457*sqrt(2)*(4*x^4 -
4*x^3 + 13*x^2 - 6*x + 9) - 300000*x + 450000)*sqrt(-772850000*sqrt(2) + 25
00000000)*log(-77500000/5711*(sqrt(341)*200^(1/4)*sqrt(31)*sqrt(2*x^2 - x +
 3)*(sqrt(2)*(281*x - 444) + 163*x - 725)*sqrt(-772850000*sqrt(2) + 2500000
000) - 4337504500*x^2 - 3894902000*sqrt(2)*(2*x^2 - x + 3) + 13366595500*x
- 17704100000)/x^2) - 965935696000*(3948*x^3 - 23592*x^2 + 19767*x - 39005)
*sqrt(2*x^2 - x + 3))/(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^(5/2)/(5*x^2+3*x+2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Francis algorithm failure for[-1.0,infinity,
infinity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,infi
nity]Francis algorithm failure for[-1.0,infinity,infinity,infinity,infinity
]proot error [1.0,infinity,infinity,infinity,infinity]Francis algorithm fai
lure for[-1.0,infinity,infinity,infinity,infinity]proot error [1.0,infinity
,infinity,infinity,infinity]Francis algorithm failure for[-1.0,infinity,inf
inity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,infinit
y]Evaluation time: 16.02Done
```

maple [B] time = 0.04, size = 751, normalized size = 3.77

$$\frac{\sqrt{\frac{8(x+\sqrt{2}-1)^2}{(-x+\sqrt{2}+1)^2}+\frac{3\sqrt{2}(x+\sqrt{2}-1)^2}{(-x+\sqrt{2}+1)^2}+8-3\sqrt{2}}\,\sqrt{2}\left(-993674\sqrt{2}\,\operatorname{arctanh}\left(\frac{31\sqrt{\frac{8(x+\sqrt{2}-1)^2}{(-x+\sqrt{2}+1)^2}+\frac{3\sqrt{2}(x+\sqrt{2}-1)^2}{(-x+\sqrt{2}+1)^2}+8-3\sqrt{2}}}{2\sqrt{-8866+6820\sqrt{2}}}\right)-42685698\,\operatorname{arctanh}\left(\frac{31\sqrt{\frac{8(x+\sqrt{2}-1)^2}{(-x+\sqrt{2}+1)^2}+\frac{3\sqrt{2}(x+\sqrt{2}-1)^2}{(-x+\sqrt{2}+1)^2}+8-3\sqrt{2}}}{2\sqrt{-8866+6820\sqrt{2}}}\right)+10111\sqrt{2}\sqrt{-8866+6820\sqrt{2}}\sqrt{-775687+549362\sqrt{2}}\arctan\left(\frac{\sqrt{-775687+549362\sqrt{2}}\sqrt{-23(8+3\sqrt{2})\left(-\frac{23(x+\sqrt{2}-1)^2}{(-x+\sqrt{2}+1)^2}+24\sqrt{2}-41\right)}\left(\frac{6485\sqrt{2}(x+\sqrt{2}-1)^2}{(-x+\sqrt{2}+1)^2}+\frac{10368(x+\sqrt{2}-1)^2}{(-x+\sqrt{2}+1)^2}+22379\sqrt{2}+32016\right)(x+\sqrt{2}-1)(8+3\sqrt{2})}{11692487\left(\frac{23(x+\sqrt{2}-1)^4}{(-x+\sqrt{2}+1)^4}+\frac{82(x+\sqrt{2}-1)^2}{(-x+\sqrt{2}+1)^2}+23\right)(-x+\sqrt{2}+1)}\right)+13910\sqrt{-8866+6820\sqrt{2}}\sqrt{-775687+549362\sqrt{2}}\arctan\left(\frac{\sqrt{-775687+549362\sqrt{2}}\sqrt{-23(8+3\sqrt{2})\left(-\frac{23(x+\sqrt{2}-1)^2}{(-x+\sqrt{2}+1)^2}+24\sqrt{2}-41\right)}\left(\frac{6485\sqrt{2}(x+\sqrt{2}-1)^2}{(-x+\sqrt{2}+1)^2}+\frac{10368(x+\sqrt{2}-1)^2}{(-x+\sqrt{2}+1)^2}+22379\sqrt{2}+32016\right)(x+\sqrt{2}-1)(8+3\sqrt{2})}{11692487\left(\frac{23(x+\sqrt{2}-1)^4}{(-x+\sqrt{2}+1)^4}+\frac{82(x+\sqrt{2}-1)^2}{(-x+\sqrt{2}+1)^2}+23\right)(-x+\sqrt{2}+1)}\right)\right)}{10232728\sqrt{\frac{\frac{8(x+\sqrt{2}-1)^2}{(-x+\sqrt{2}+1)^2}+\frac{3\sqrt{2}(x+\sqrt{2}-1)^2}{(-x+\sqrt{2}+1)^2}+8-3\sqrt{2}}{\left(1+\frac{x+\sqrt{2}-1}{-x+\sqrt{2}+1}\right)^2}}\left(1+\frac{x+\sqrt{2}-1}{-x+\sqrt{2}+1}\right)(8+3\sqrt{2})\sqrt{-8866+6820\sqrt{2}}}+\frac{13}{484\sqrt{2x^2-x+3}}-\frac{329(4x-1)}{256036\sqrt{2x^2-x+3}}+\frac{1}{66\left(2x^2-x+3\right)^{\frac{3}{2}}}-\frac{4x-1}{506\left(2x^2-x+3\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^2-x+3)^(5/2)/(5*x^2+3*x+2),x)
```

```
[Out] 1/10232728*(8*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+3*2^(1/2)*(x+2^(1/2)-1)^2/(-
x+2^(1/2)+1)^2+8-3*2^(1/2))^(1/2)*2^(1/2)*(10111*2^(1/2)*(-8866+6820*2^(1/2
))^(1/2)*(-775687+549362*2^(1/2))^(1/2)*arctan(1/11692487*(-775687+549362*2
^(1/2))^(1/2)*(-23*(8+3*2^(1/2))*(-23*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+24*2
^(1/2)-41))^(1/2)*(6485*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+10368*(x+2
^(1/2)-1)^2/(-x+2^(1/2)+1)^2+22379*2^(1/2)+32016)/(23*(x+2^(1/2)-1)^4/(-x+2
^(1/2)+1)^4+82*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+23)*(x+2^(1/2)-1)/(-x+2^(1/
2)+1)*(8+3*2^(1/2)))+13910*(-8866+6820*2^(1/2))^(1/2)*(-775687+549362*2^(1/
2))^(1/2)*arctan(1/11692487*(-775687+549362*2^(1/2))^(1/2)*(-23*(8+3*2^(1/2
))*(-23*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+24*2^(1/2)-41))^(1/2)*(6485*2^(1/2
)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+10368*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+2
2379*2^(1/2)+32016)/(23*(x+2^(1/2)-1)^4/(-x+2^(1/2)+1)^4+82*(x+2^(1/2)-1)^2
/(-x+2^(1/2)+1)^2+23)*(x+2^(1/2)-1)/(-x+2^(1/2)+1)*(8+3*2^(1/2)))-993674*ar
ctanh(31/2*(8*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+3*2^(1/2)*(x+2^(1/2)-1)^2/(-
x+2^(1/2)+1)^2+8-3*2^(1/2))^(1/2)/(-8866+6820*2^(1/2))^(1/2))*2^(1/2)-42685
698*arctanh(31/2*(8*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+3*2^(1/2)*(x+2^(1/2)-1
)^2/(-x+2^(1/2)+1)^2+8-3*2^(1/2))^(1/2)/(-8866+6820*2^(1/2))^(1/2)))/((8*(x
+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+3*2^(1/2)*(x+2^(1/2)-1)^2/(-x+2^(1/2)+1)^2+8
-3*2^(1/2))/(1+(x+2^(1/2)-1)/(-x+2^(1/2)+1))^2)^(1/2)/(1+(x+2^(1/2)-1)/(-x+
2^(1/2)+1))/(8+3*2^(1/2))/(-8866+6820*2^(1/2))^(1/2)+13/484/(2*x^2-x+3)^(1/
2)-329/256036*(4*x-1)/(2*x^2-x+3)^(1/2)+1/66/(2*x^2-x+3)^(3/2)-1/506*(4*x-1
)/(2*x^2-x+3)^(3/2)
```

maxima **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(5\,x^2+3\,x+2\right)\left(2\,x^2-x+3\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^(5/2)/(5*x^2+3*x+2),x, algorithm="maxima")
```

```
[Out] integrate(1/((5*x^2 + 3*x + 2)*(2*x^2 - x + 3)^(5/2)), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(2\,x^2-x+3\right)}^{5/2}\,\left(5\,x^2+3\,x+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*x^2 - x + 3)^(5/2)*(3*x + 5*x^2 + 2)),x)
```

```
[Out] int(1/((2*x^2 - x + 3)^(5/2)*(3*x + 5*x^2 + 2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(2x^2 - x + 3\right)^{\frac{5}{2}} \left(5x^2 + 3x + 2\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**2-x+3)**(5/2)/(5*x**2+3*x+2),x)
[Out] Integral(1/((2*x**2 - x + 3)**(5/2)*(5*x**2 + 3*x + 2)), x)
```

3.95 $\int \frac{1}{\left(3-x+2x^2\right)^{5/2}\left(2+3x+5x^2\right)^2}\,dx$

Optimal. Leaf size=234

$$-\frac{15101-8654x}{1035276\left(2x^2-x+3\right)^{3/2}}-\frac{1352542x+3133427}{523849656\sqrt{2x^2-x+3}}+\frac{65x+4}{682\left(2x^2-x+3\right)^{3/2}\left(5x^2+3x+2\right)}+\frac{625\sqrt{\frac{1}{682}\left(30463+\right.}}{}$$

Rubi [A] time = 0.54, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {974, 1060, 1035, 1029, 206, 204}

$$-\frac{15101-8654x}{1035276\left(2x^2-x+3\right)^{3/2}}-\frac{1352542x+3133427}{523849656\sqrt{2x^2-x+3}}+\frac{65x+4}{682\left(2x^2-x+3\right)^{3/2}\left(5x^2+3x+2\right)}+\frac{625\sqrt{\frac{1}{682}\left(30463+23600\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{11}{31\left(30463+23600\sqrt{2}\right)}}\left(\left(687+445\sqrt{2}\right)x+242\sqrt{2}+203\right)}{\sqrt{2x^2-x+3}}\right)}{660176}-\frac{625\sqrt{\frac{1}{682}\left(23600\sqrt{2}-30463\right)}\tanh^{-1}\left(\frac{\sqrt{\frac{11}{31\left(23600\sqrt{2}-30463\right)}}\left(\left(687-445\sqrt{2}\right)x-242\sqrt{2}+203\right)}{\sqrt{2x^2-x+3}}\right)}{660176}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - x + 2*x^2)^(5/2)*(2 + 3*x + 5*x^2)^2),x]
```

```
[Out] -(15101 - 8654*x)/(1035276*(3 - x + 2*x^2)^(3/2)) - (3133427 + 1352542*x)/(
523849656*Sqrt[3 - x + 2*x^2]) + (4 + 65*x)/(682*(3 - x + 2*x^2)^(3/2)*(2 +
 3*x + 5*x^2)) + (625*Sqrt[(30463 + 23600*Sqrt[2])/682]*ArcTan[(Sqrt[11/(31
*(30463 + 23600*Sqrt[2]))]*(203 + 242*Sqrt[2] + (687 + 445*Sqrt[2])*x))/Sqr
t[3 - x + 2*x^2]])/660176 - (625*Sqrt[(-30463 + 23600*Sqrt[2])/682]*ArcTanh
[(Sqrt[11/(31*(-30463 + 23600*Sqrt[2]))]*(203 - 242*Sqrt[2] + (687 - 445*Sq
rt[2])*x))/Sqrt[3 - x + 2*x^2]])/660176
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 974

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*
```

```
a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*(
d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q,
 0]
```

Rule 1029

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
 c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 1035

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

Rule 1060

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x +
 c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
2*a*(c*d - a*f)))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
```

```
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !
IGtQ[q, 0]
```

Rubi steps

$$\int \frac{1}{\left(3-x+2x^2\right)^{5/2}\left(2+3x+5x^2\right)^2}\,dx = \frac{4+65x}{682\left(3-x+2x^2\right)^{3/2}\left(2+3x+5x^2\right)} - \frac{\int \frac{-1738+\frac{4411x}{2}-5720x^2}{\left(3-x+2x^2\right)^{5/2}\left(2+3x+5x^2\right)}\,dx}{7502}$$

$$= -\frac{15101-8654x}{1035276\left(3-x+2x^2\right)^{3/2}} + \frac{4+65x}{682\left(3-x+2x^2\right)^{3/2}\left(2+3x+5x^2\right)} - \cdots$$

$$= -\frac{15101-8654x}{1035276\left(3-x+2x^2\right)^{3/2}} - \frac{3133427+1352542x}{523849656\sqrt{3-x+2x^2}} + \frac{\cdots}{682\left(3-x+\cdots\right.}$$

$$= -\frac{15101-8654x}{1035276\left(3-x+2x^2\right)^{3/2}} - \frac{3133427+1352542x}{523849656\sqrt{3-x+2x^2}} + \frac{\cdots}{682\left(3-x+\cdots\right.}$$

$$= -\frac{15101-8654x}{1035276\left(3-x+2x^2\right)^{3/2}} - \frac{3133427+1352542x}{523849656\sqrt{3-x+2x^2}} + \frac{\cdots}{682\left(3-x+\cdots\right.}$$

$$= -\frac{15101-8654x}{1035276\left(3-x+2x^2\right)^{3/2}} - \frac{3133427+1352542x}{523849656\sqrt{3-x+2x^2}} + \frac{\cdots}{682\left(3-x+\cdots\right.}$$

Mathematica [C] time = 1.15, size = 296, normalized size = 1.26

$$-\frac{198375i\sqrt{286+22i\sqrt{31}}\left(687\sqrt{31}+31i\right)\sqrt{2x^2-x+3}\left(10x^4+x^3+16x^2+7x+6\right)\tanh^{-1}\left(\frac{\left(-22-4i\sqrt{31}\right)x+i\sqrt{31}+63}{2\sqrt{286+22i\sqrt{31}}\sqrt{2x^2-x+3}}\right)+198375\sqrt{286-22i\sqrt{31}}\left(31+687i\sqrt{31}\right)\sqrt{2x^2-x+3}\left(10x^4+x^3+16x^2+7x+6\right)\tanh^{-1}\left(\frac{\left(22-4i\sqrt{31}\right)x+i\sqrt{31}-63}{2\sqrt{286-22i\sqrt{31}}\sqrt{2x^2-x+3}}\right)+5456\left(13525420x^5+32686812x^4+2879479x^3+84671384x^2-5712309x+31010342\right)}{2858123723136\left(2x^2-x+3\right)^{3/2}\left(5x^2+3x+2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - x + 2*x^2)^(5/2)*(2 + 3*x + 5*x^2)^2),x]
```

```
[Out] -1/2858123723136*(5456*(31010342 - 5712309*x + 84671384*x^2 + 2879479*x^3 +
 32686812*x^4 + 13525420*x^5) + (198375*I)*Sqrt[286 + (22*I)*Sqrt[31]]*(31*
I + 687*Sqrt[31])*Sqrt[3 - x + 2*x^2]*(6 + 7*x + 16*x^2 + x^3 + 10*x^4)*Arc
Tanh[(63 + I*Sqrt[31] + (-22 - (4*I)*Sqrt[31])*x)/(2*Sqrt[286 + (22*I)*Sqrt
[31]]*Sqrt[3 - x + 2*x^2])] + 198375*Sqrt[286 - (22*I)*Sqrt[31]]*(31 + (687
*I)*Sqrt[31])*Sqrt[3 - x + 2*x^2]*(6 + 7*x + 16*x^2 + x^3 + 10*x^4)*ArcTanh
```

```
[(-63 + I*Sqrt[31] + (22 - (4*I)*Sqrt[31])*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]
]*Sqrt[3 - x + 2*x^2])]])/((3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2))
```

IntegrateAlgebraic [C] time = 1.42, size = 416, normalized size = 1.78

$$\frac{\text{RootSum}\left[-5\#1^4+6\sqrt{2}\#1^3+17\#1^2-26\sqrt{2}\#1-56\&,\frac{95\#1^2\log\left(-\#1+\sqrt{2x^2-x+3}-\sqrt{2}x\right)+106\sqrt{2}\#1\log\left(-\#1+\sqrt{2x^2-x+3}-\sqrt{2}x\right)-1376\log\left(-\#1+\sqrt{2x^2-x+3}-\sqrt{2}x\right)}{-10\#1^3+9\sqrt{2}\#1^2+17\#1-13\sqrt{2}}\&\right]}{5324}+\frac{\text{RootSum}\left[-5\#1^4+6\sqrt{2}\#1^3+17\#1^2-26\sqrt{2}\#1-56\&,\frac{10095\sqrt{2}\#1^2\log\left(-\#1+\sqrt{2x^2-x+3}-\sqrt{2}x\right)+58712\#1\log\left(-\#1+\sqrt{2x^2-x+3}-\sqrt{2}x\right)+126249\sqrt{2}\log\left(-\#1+\sqrt{2x^2-x+3}-\sqrt{2}x\right)}{-10\#1^3+9\sqrt{2}\#1^2+17\#1-13\sqrt{2}}\&\right]}{660176\sqrt{2}}+\frac{-13525420x^5-32686812x^4-2879479x^3-84671384x^2+5712309x-31010342}{523849656\left(2x^2-x+3\right)^{3/2}\left(5x^2+3x+2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((3 - x + 2*x^2)^(5/2)*(2 + 3*x + 5*x^2)^2),x]
```

```
[Out] (-31010342 + 5712309*x - 84671384*x^2 - 2879479*x^3 - 32686812*x^4 - 135254
20*x^5)/(523849656*(3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)) + RootSum[-56 -
 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3 - 5*#1^4 & , (-1376*Log[-(Sqrt[2]
*x) + Sqrt[3 - x + 2*x^2] - #1] + 106*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x
 + 2*x^2] - #1]*#1 + 95*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1^2)/
(-13*Sqrt[2] + 17*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3) & ]/5324 + RootSum[-56 - 2
6*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3 - 5*#1^4 & , (126249*Sqrt[2]*Log[-(
Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1] + 58712*Log[-(Sqrt[2]*x) + Sqrt[3 -
x + 2*x^2] - #1]*#1 + 10095*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2]
- #1]*#1^2)/(-13*Sqrt[2] + 17*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3) & ]/(660176*Sq
rt[2])
```

fricas [B] time = 1.19, size = 2253, normalized size = 9.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

```
[Out] 1/2560433560253791411 2*(301208632500*6962^(1/4)*sqrt(341)*sqrt(118)*sqrt(2)
*(20*x^6 - 8*x^5 + 61*x^4 + x^3 + 53*x^2 + 15*x + 18)*sqrt(30463*sqrt(2) +
47200)*arctan(1/11117215998613*(168268*sqrt(118)*(22*6962^(3/4)*sqrt(341)*(
321084*x^7 - 1338894*x^6 + 2762802*x^5 - 4721048*x^4 + 2438224*x^3 - 131731
2*x^2 - sqrt(2)*(277258*x^7 - 994619*x^6 + 2123978*x^5 - 3198193*x^4 + 1552
680*x^3 - 621000*x^2 - 1900800*x + 1181952) - 2363904*x + 1900800) + 1829*6
962^(1/4)*sqrt(341)*(25187*x^7 - 392073*x^6 + 2114488*x^5 - 4948060*x^4 + 6
460704*x^3 - 4452768*x^2 - sqrt(2)*(20477*x^7 - 310452*x^6 + 1610140*x^5 -
3584192*x^4 + 4580640*x^3 - 2620800*x^2 - 3400704*x + 2198016) - 4396032*x
+ 3400704))*sqrt(2*x^2 - x + 3)*sqrt(30463*sqrt(2) + 47200) + 3155854864122
4*sqrt(31)*sqrt(2)*(28180*x^8 - 254666*x^7 + 704270*x^6 - 1385256*x^5 + 154
9144*x^4 - 642048*x^3 - 98496*x^2 - sqrt(2)*(8746*x^8 - 102335*x^7 + 396104
*x^6 - 783113*x^5 + 1320710*x^4 - 752088*x^3 + 396144*x^2 + 546048*x - 5391
36) + 1154304*x - 456192) - 2*sqrt(118/79)*(sqrt(118)*(22*6962^(3/4)*sqrt(3
41)*(1050904*x^7 - 1523916*x^6 + 5005956*x^5 - 2572736*x^4 + 3615264*x^3 +
877824*x^2 - sqrt(2)*(1065206*x^7 - 1518091*x^6 + 4815081*x^5 - 1448880*x^4
```

+ 1303560*x^3 + 3131136*x^2 - 3131136*x) - 877824*x) + 1829*6962^(1/4)*sqr
t(341)*(84981*x^7 - 1100084*x^6 + 4256060*x^5 - 5639616*x^4 + 7745184*x^3 +
 2571264*x^2 - 242*sqrt(2)*(319*x^7 - 4124*x^6 + 15860*x^5 - 20160*x^4 + 24
480*x^3 + 20736*x^2 - 20736*x) - 2571264*x))*sqrt(2*x^2 - x + 3)*sqrt(30463
*sqrt(2) + 47200) + 187549318*sqrt(31)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1
578888*x^6 - 3293072*x^5 + 396480*x^4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*
(15550*x^8 - 118051*x^7 + 244047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x
^3 + 1209600*x^2 - 1036800*x) + 3276288*x) + 8524969*sqrt(31)*(254591*x^8 -
 4815126*x^7 + 32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 74219328*x^3 -
 168956928*x^2 - 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*
x^4 - 3618*x^3 + 2268*x^2 - 1944*x) + 144820224*x))*sqrt(-(6962^(1/4)*sqrt(
341)*sqrt(118)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(101*x + 176) - 277*x
+ 75)*sqrt(30463*sqrt(2) + 47200) - 219481829*x^2 - 197085724*sqrt(2)*(2*x^
2 - x + 3) + 676362371*x - 895844200)/x^2) + 358619870923*sqrt(31)*(2828123
*x^8 - 9696916*x^7 + 53385560*x^6 - 142835344*x^5 + 254146592*x^4 - 2493000
96*x^3 + 37981440*x^2 - 7744*sqrt(2)*(1348*x^8 - 2692*x^7 + 9789*x^6 - 1007
0*x^5 + 15569*x^4 - 5568*x^3 + 1080*x^2 + 4320*x - 5184) + 223064064*x - 94
887936))/(2585191*x^8 - 4661200*x^7 + 14191920*x^6 + 490880*x^5 - 13562944*
x^4 + 44249088*x^3 - 34615296*x^2 - 24772608*x + 18579456)) + 301208632500*
6962^(1/4)*sqrt(341)*sqrt(118)*sqrt(2)*(20*x^6 - 8*x^5 + 61*x^4 + x^3 + 53*
x^2 + 15*x + 18)*sqrt(30463*sqrt(2) + 47200)*arctan(1/11117215998613*(16826
8*sqrt(118)*(22*6962^(3/4)*sqrt(341)*(321084*x^7 - 1338894*x^6 + 2762802*x^
5 - 4721048*x^4 + 2438224*x^3 - 1317312*x^2 - sqrt(2)*(277258*x^7 - 994619*
x^6 + 2123978*x^5 - 3198193*x^4 + 1552680*x^3 - 621000*x^2 - 1900800*x + 11
81952) - 2363904*x + 1900800) + 1829*6962^(1/4)*sqrt(341)*(25187*x^7 - 3920
73*x^6 + 2114488*x^5 - 4948060*x^4 + 6460704*x^3 - 4452768*x^2 - sqrt(2)*(2
0477*x^7 - 310452*x^6 + 1610140*x^5 - 3584192*x^4 + 4580640*x^3 - 2620800*x
^2 - 3400704*x + 2198016) - 4396032*x + 3400704))*sqrt(2*x^2 - x + 3)*sqrt(
30463*sqrt(2) + 47200) - 31558548641224*sqrt(31)*sqrt(2)*(28180*x^8 - 25466
6*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - s
qrt(2)*(8746*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^4 - 752
088*x^3 + 396144*x^2 + 546048*x - 539136) + 1154304*x - 456192) - 2*sqrt(11
8/79)*(sqrt(118)*(22*6962^(3/4)*sqrt(341)*(1050904*x^7 - 1523916*x^6 + 5005
956*x^5 - 2572736*x^4 + 3615264*x^3 + 877824*x^2 - sqrt(2)*(1065206*x^7 - 1
518091*x^6 + 4815081*x^5 - 1448880*x^4 + 1303560*x^3 + 3131136*x^2 - 313113
6*x) - 877824*x) + 1829*6962^(1/4)*sqrt(341)*(84981*x^7 - 1100084*x^6 + 425
6060*x^5 - 5639616*x^4 + 7745184*x^3 + 2571264*x^2 - 242*sqrt(2)*(319*x^7 -
 4124*x^6 + 15860*x^5 - 20160*x^4 + 24480*x^3 + 20736*x^2 - 20736*x) - 2571
264*x))*sqrt(2*x^2 - x + 3)*sqrt(30463*sqrt(2) + 47200) - 187549318*sqrt(31
)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 + 396480*x^4
 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 118051*x^7 + 244047*x^6
- 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*x) + 32762
88*x) - 8524969*sqrt(31)*(254591*x^8 - 4815126*x^7 + 32303580*x^6 - 9086680
8*x^5 + 108781920*x^4 - 74219328*x^3 - 168956928*x^2 - 15488*sqrt(2)*(4*x^8
 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3618*x^3 + 2268*x^2 - 1944*x) +

```
 144820224*x))*sqrt((6962^(1/4)*sqrt(341)*sqrt(118)*sqrt(31)*sqrt(2*x^2 - x
 + 3)*(sqrt(2)*(101*x + 176) - 277*x + 75)*sqrt(30463*sqrt(2) + 47200) + 21
9481829*x^2 + 197085724*sqrt(2)*(2*x^2 - x + 3) - 676362371*x + 895844200)/
x^2) - 358619870923*sqrt(31)*(2828123*x^8 - 9696916*x^7 + 53385560*x^6 - 14
2835344*x^5 + 254146592*x^4 - 249300096*x^3 + 37981440*x^2 - 7744*sqrt(2)*(
1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^4 - 5568*x^3 + 1080*x^
2 + 4320*x - 5184) + 223064064*x - 94887936))/(2585191*x^8 - 4661200*x^7 +
14191920*x^6 + 490880*x^5 - 13562944*x^4 + 44249088*x^3 - 34615296*x^2 - 24
772608*x + 18579456)) + 991875*6962^(1/4)*sqrt(341)*sqrt(118)*sqrt(31)*(944
000*x^6 - 377600*x^5 + 2879200*x^4 + 47200*x^3 + 2501600*x^2 - 30463*sqrt(2
)*(20*x^6 - 8*x^5 + 61*x^4 + x^3 + 53*x^2 + 15*x + 18) + 708000*x + 849600)
*sqrt(30463*sqrt(2) + 47200)*log(7375000000000/79*(6962^(1/4)*sqrt(341)*sqr
t(118)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(101*x + 176) - 277*x + 75)*sq
rt(30463*sqrt(2) + 47200) + 219481829*x^2 + 197085724*sqrt(2)*(2*x^2 - x +
3) - 676362371*x + 895844200)/x^2) - 991875*6962^(1/4)*sqrt(341)*sqrt(118)*
sqrt(31)*(944000*x^6 - 377600*x^5 + 2879200*x^4 + 47200*x^3 + 2501600*x^2 -
 30463*sqrt(2)*(20*x^6 - 8*x^5 + 61*x^4 + x^3 + 53*x^2 + 15*x + 18) + 70800
0*x + 849600)*sqrt(30463*sqrt(2) + 47200)*log(-7375000000000/79*(6962^(1/4)
*sqrt(341)*sqrt(118)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(101*x + 176) -
277*x + 75)*sqrt(30463*sqrt(2) + 47200) - 219481829*x^2 - 197085724*sqrt(2)
*(2*x^2 - x + 3) + 676362371*x - 895844200)/x^2) - 48877259552*(13525420*x^
5 + 32686812*x^4 + 2879479*x^3 + 84671384*x^2 - 5712309*x + 31010342)*sqrt(
2*x^2 - x + 3))/(20*x^6 - 8*x^5 + 61*x^4 + x^3 + 53*x^2 + 15*x + 18)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Francis algorithm failure for[-1.0,infinity,
infinity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,infi
nity]Francis algorithm failure for[-1.0,infinity,infinity,infinity,infinity
]proot error [1.0,infinity,infinity,infinity,infinity]Francis algorithm fai
lure for[-1.0,infinity,infinity,infinity,infinity]proot error [1.0,infinity
,infinity,infinity,infinity]Francis algorithm failure for[-1.0,infinity,inf
inity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,infinit
y]Evaluation time: 18.73Done
```

maple [B] time = 0.10, size = 5975, normalized size = 25.53

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^2,x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(5\,x^2+3\,x+2\right)^2\left(2\,x^2-x+3\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((5*x^2 + 3*x + 2)^2*(2*x^2 - x + 3)^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(2\,x^2-x+3\right)}^{5/2}\,{\left(5\,x^2+3\,x+2\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*x^2 - x + 3)^(5/2)*(3*x + 5*x^2 + 2)^2),x)
```

```
[Out] int(1/((2*x^2 - x + 3)^(5/2)*(3*x + 5*x^2 + 2)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(2x^{2}-x+3\right)^{\frac{5}{2}}\left(5x^{2}+3x+2\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**2-x+3)**(5/2)/(5*x**2+3*x+2)**2,x)
```

```
[Out] Integral(1/((2*x**2 - x + 3)**(5/2)*(5*x**2 + 3*x + 2)**2), x)
```

3.96 $\int \frac{1}{\left(3-x+2x^2\right)^{5/2}\left(2+3x+5x^2\right)^3}\,dx$

Optimal. Leaf size=269

$$-\frac{1134826571-1504660754x}{476353953856\sqrt{2x^2-x+3}}+\frac{86885x+46386}{1860496\left(2x^2-x+3\right)^{3/2}\left(5x^2+3x+2\right)}-\frac{12280939-19536786x}{2824232928\left(2x^2-x+3\right)^{3/2}}+\frac{\cdots}{1364\left(\cdots\right.}$$

Rubi [A] time = 0.59, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {974, 1060, 1035, 1029, 206, 204}

$$-\frac{1134826571-1504660754x}{476353953856\sqrt{2x^2-x+3}}+\frac{86885x+46386}{1860496\left(2x^2-x+3\right)^{3/2}\left(5x^2+3x+2\right)}-\frac{12280939-19536786x}{2824232928\left(2x^2-x+3\right)^{3/2}}+\frac{65x+4}{1364\left(2x^2-x+3\right)^{3/2}\left(5x^2+3x+2\right)^2}+\frac{35\sqrt{\frac{1}{682}\left(2243059557247+2011748500000\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{11}{31\left(2243059557247+2011748500000\sqrt{2}\right)}}\left(\left(6290431+3861685\sqrt{2}\right)x+2428746\sqrt{2}+1432939\right)}{\sqrt{2x^2-x+3}}\right)}{1800960128}-\frac{35\sqrt{\frac{1}{682}\left(2011748500000\sqrt{2}-2243059557247\right)}\tanh^{-1}\left(\frac{\sqrt{\frac{11}{31\left(2011748500000\sqrt{2}-2243059557247\right)}}\left(\left(6290431-3861685\sqrt{2}\right)x-2428746\sqrt{2}+1432939\right)}{\sqrt{2x^2-x+3}}\right)}{1800960128}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - x + 2*x^2)^(5/2)*(2 + 3*x + 5*x^2)^3),x]
```

```
[Out] -(12280939 - 19536786*x)/(2824232928*(3 - x + 2*x^2)^(3/2)) - (1134826571 -
 1504660754*x)/(476353953856*Sqrt[3 - x + 2*x^2]) + (4 + 65*x)/(1364*(3 - x
 + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)^2) + (46386 + 86885*x)/(1860496*(3 - x +
2*x^2)^(3/2)*(2 + 3*x + 5*x^2)) + (35*Sqrt[(2243059557247 + 2011748500000*S
qrt[2])/682]*ArcTan[(Sqrt[11/(31*(2243059557247 + 2011748500000*Sqrt[2]))]*
(1432939 + 2428746*Sqrt[2] + (6290431 + 3861685*Sqrt[2])*x))/Sqrt[3 - x + 2
*x^2]])/1800960128 - (35*Sqrt[(-2243059557247 + 2011748500000*Sqrt[2])/682]
*ArcTanh[(Sqrt[11/(31*(-2243059557247 + 2011748500000*Sqrt[2]))]*(1432939 -
 2428746*Sqrt[2] + (6290431 - 3861685*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]])/18
00960128
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 974

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*
a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*(
d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q,
 0]
```

Rule 1029

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
 c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 1035

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

Rule 1060

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x +
 c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
```

```
2*a*(c*d - a*f)))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !
IGtQ[q, 0]
```

Rubi steps

$$\int \frac{1}{\left(3-x+2x^2\right)^{5/2}\left(2+3x+5x^2\right)^3}\,dx = \frac{4+65x}{1364\left(3-x+2x^2\right)^{3/2}\left(2+3x+5x^2\right)^2} - \frac{\int \frac{-5687+\frac{8635x}{2}-8580x^2}{\left(3-x+2x^2\right)^{5/2}\left(2+3x+5x^2\right)^2}\,dx}{15004}$$

$$= \frac{4+65x}{1364\left(3-x+2x^2\right)^{3/2}\left(2+3x+5x^2\right)^2} + \frac{46386+86885x}{1860496\left(3-x+2x^2\right)^{3/2}\left(2+\cdots\right.}$$

$$= -\frac{12280939-19536786x}{2824232928\left(3-x+2x^2\right)^{3/2}} + \frac{4+65x}{1364\left(3-x+2x^2\right)^{3/2}\left(2+3x+5x^2\right)^2} \cdots$$

$$= -\frac{12280939-19536786x}{2824232928\left(3-x+2x^2\right)^{3/2}} - \frac{1134826571-1504660754x}{476353953856\sqrt{3-x+2x^2}} + \frac{\cdots}{1364\left(\cdots\right.}$$

$$= -\frac{12280939-19536786x}{2824232928\left(3-x+2x^2\right)^{3/2}} - \frac{1134826571-1504660754x}{476353953856\sqrt{3-x+2x^2}} + \frac{\cdots}{1364\left(\cdots\right.}$$

$$= -\frac{12280939-19536786x}{2824232928\left(3-x+2x^2\right)^{3/2}} - \frac{1134826571-1504660754x}{476353953856\sqrt{3-x+2x^2}} + \frac{\cdots}{1364\left(\cdots\right.}$$

$$= -\frac{12280939-19536786x}{2824232928\left(3-x+2x^2\right)^{3/2}} - \frac{1134826571-1504660754x}{476353953856\sqrt{3-x+2x^2}} + \frac{\cdots}{1364\left(\cdots\right.}$$

Mathematica [C] time = 1.96, size = 242, normalized size = 0.90

$$\frac{11109\sqrt{286+22i\sqrt{31}}\left(4541903-6290431i\sqrt{31}\right)\tanh^{-1}\left(\frac{\left(-22-4i\sqrt{31}\right)x+i\sqrt{31}+63}{2\sqrt{286+22i\sqrt{31}}\sqrt{2x^2-x+3}}\right)-11109i\sqrt{286-22i\sqrt{31}}\left(6290431\sqrt{31}-4541903i\right)\tanh^{-1}\left(\frac{\left(22-4i\sqrt{31}\right)x+i\sqrt{31}-63}{2\sqrt{286-22i\sqrt{31}}\sqrt{2x^2-x+3}}\right)+\frac{5456\left(225699113100x^7-12234606480x^6+592923725931x^5+174241614961x^4+519223213785x^3+178650961091x^2+218659985088x+9739335532\right)}{\left(2x^2-x+3\right)^{3/2}\left(5x^2+3x+2\right)^2}}{7796961516715008}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - x + 2*x^2)^(5/2)*(2 + 3*x + 5*x^2)^3),x]
```

```
[Out] ((5456*(9739335532 + 218659985088*x + 178650961091*x^2 + 519223213785*x^3 +
 174241614961*x^4 + 592923725931*x^5 - 12234606480*x^6 + 225699113100*x^7))
```

```
/((3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)^2) + 11109*Sqrt[286 + (22*I)*Sqrt
[31]]*(4541903 - (6290431*I)*Sqrt[31])*ArcTanh[(63 + I*Sqrt[31] + (-22 - (4
*I)*Sqrt[31])*x)/(2*Sqrt[286 + (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])] - (11
109*I)*Sqrt[286 - (22*I)*Sqrt[31]]*(-4541903*I + 6290431*Sqrt[31])*ArcTanh[
(-63 + I*Sqrt[31] + (22 - (4*I)*Sqrt[31])*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]
*Sqrt[3 - x + 2*x^2])])/7796961516715008
```

IntegrateAlgebraic [C] time = 1.61, size = 601, normalized size = 2.23

$$-\frac{\text{RootSum}\left[-5\#1^4+6\sqrt{2}\#1^3+17\#1^2-26\sqrt{2}\#1-56\&,\frac{-1193705\#1^2\log\left(-\#1+\sqrt{2x^2-x+3}-\sqrt{2}x\right)+37230166\sqrt{2}\#1\log\left(-\#1+\sqrt{2x^2-x+3}-\sqrt{2}x\right)-26154346\log\left(-\#1+\sqrt{2x^2-x+3}-\sqrt{2}x\right)}{-10\#1^3+9\sqrt{2}\#1^2+17\#1-13\sqrt{2}}\&\right]}{1920782072}-\frac{\text{RootSum}\left[-5\#1^4+6\sqrt{2}\#1^3+17\#1^2-26\sqrt{2}\#1-56\&,\frac{-485\#1^2\log\left(-\#1+\sqrt{2x^2-x+3}-\sqrt{2}x\right)+3172\sqrt{2}\#1\log\left(-\#1+\sqrt{2x^2-x+3}-\sqrt{2}x\right)-3647\log\left(-\#1+\sqrt{2x^2-x+3}-\sqrt{2}x\right)}{-10\#1^3+9\sqrt{2}\#1^2+17\#1-13\sqrt{2}}\&\right]}{234256}+\frac{5\text{RootSum}\left[-5\#1^4+6\sqrt{2}\#1^3+17\#1^2-26\sqrt{2}\#1-56\&,\frac{1004412885\sqrt{2}\#1^2\log\left(-\#1+\sqrt{2x^2-x+3}-\sqrt{2}x\right)+16445754136\#1\log\left(-\#1+\sqrt{2x^2-x+3}-\sqrt{2}x\right)-9138129081\sqrt{2}\log\left(-\#1+\sqrt{2x^2-x+3}-\sqrt{2}x\right)}{-10\#1^3+9\sqrt{2}\#1^2+17\#1-13\sqrt{2}}\&\right]}{952707907712\sqrt{2}}+\frac{225699113100x^7-12234606480x^6+592923725931x^5+174241614961x^4+519223213785x^3+178650961091x^2+218659985088x+9739335532}{1429061861568\left(2x^2-x+3\right)^{3/2}\left(5x^2+3x+2\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((3 - x + 2*x^2)^(5/2)*(2 + 3*x + 5*x^2)^3),x]
```

```
[Out] (9739335532 + 218659985088*x + 178650961091*x^2 + 519223213785*x^3 + 174241
614961*x^4 + 592923725931*x^5 - 12234606480*x^6 + 225699113100*x^7)/(142906
1861568*(3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)^2) - RootSum[-56 - 26*Sqrt[
2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3 - 5*#1^4 & , (-26154346*Log[-(Sqrt[2]*x) +
 Sqrt[3 - x + 2*x^2] - #1] + 37230166*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x
 + 2*x^2] - #1]*#1 - 1193705*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#
1^2)/(-13*Sqrt[2] + 17*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3) & ]/1920782072 - Root
Sum[-56 - 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3 - 5*#1^4 & , (-3647*Log[
-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1] + 3172*Sqrt[2]*Log[-(Sqrt[2]*x) +
Sqrt[3 - x + 2*x^2] - #1]*#1 - 485*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] -
 #1]*#1^2)/(-13*Sqrt[2] + 17*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3) & ]/234256 + (5
*RootSum[-56 - 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3 - 5*#1^4 & , (-9138
129081*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1] + 16445754136*L
og[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1 + 1004412885*Sqrt[2]*Log[-(S
qrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1^2)/(-13*Sqrt[2] + 17*#1 + 9*Sqrt[2
]*#1^2 - 10*#1^3) & ])/(952707907712*Sqrt[2])
```

fricas [B] time = 1.33, size = 2343, normalized size = 8.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

```
[Out] 1/611377875290135815296770157063555072*(216498859339875798O*129508224872072
^(1/4)*sqrt(4023497)*sqrt(341)*sqrt(2)*(100*x^8 + 20*x^7 + 321*x^6 + 172*x^
5 + 390*x^4 + 236*x^3 + 241*x^2 + 84*x + 36)*sqrt(2243059557247*sqrt(2) + 4
023497000000)*arctan(1/45253401157462826192523703385785943 9*(11475013444*sq
rt(4023497)*(11*129508224872072^(3/4)*sqrt(341)*(2673027292*x^7 - 117686842
22*x^6 + 24008796626*x^5 - 42687622824*x^4 + 22428040912*x^3 - 12956821056*
x^2 - sqrt(2)*(2612082154*x^7 - 9010050347*x^6 + 19426337114*x^5 - 28170626
609*x^4 + 13394761640*x^3 - 4698131400*x^2 - 17594323200*x + 10110341376) -
```

```
 20220682752*x + 17594323200) + 124728407*129508224872072^(1/4)*sqrt(341)*(
214583731*x^7 - 3372306249*x^6 + 18434388344*x^5 - 43845503580*x^4 + 576317
17152*x^3 - 41786349984*x^2 - sqrt(2)*(190078101*x^7 - 2862100476*x^6 + 146
88003420*x^5 - 32231022496*x^4 + 40927641120*x^3 - 21959568000*x^2 - 311565
03552*x + 19060075008) - 38120150016*x + 31156503552))*sqrt(2*x^2 - x + 3)*
sqrt(2243059557247*sqrt(2) + 4023497000000) + 128461267801829958223938254774
25536472*sqrt(31)*sqrt(2)*(28180*x^8 - 254666*x^7 + 704270*x^6 - 1385256*x^
5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - sqrt(2)*(8746*x^8 - 102335*x^7 +
 396104*x^6 - 783113*x^5 + 1320710*x^4 - 752088*x^3 + 396144*x^2 + 546048*x
 - 539136) + 1154304*x - 456192) - 2*sqrt(8046994/10139750351)*(sqrt(402349
7)*(11*129508224872072^(3/4)*sqrt(341)*(8140972152*x^7 - 11907581308*x^6 +
39777303828*x^5 - 24395365568*x^4 + 37103094432*x^3 - 1836165888*x^2 - sqrt
(2)*(10387383478*x^7 - 14753211883*x^6 + 46462095753*x^5 - 11926110640*x^4
+ 8224291080*x^3 + 34793549568*x^2 - 34793549568*x) + 1836165888*x) + 12472
8407*129508224872072^(1/4)*sqrt(341)*(692762453*x^7 - 8972954292*x^6 + 3480
3726780*x^5 - 46915651008*x^4 + 67421983392*x^3 + 10625375232*x^2 - 2*sqrt(
2)*(367903387*x^7 - 4754813452*x^6 + 18261523780*x^5 - 22991417280*x^4 + 27
054001440*x^3 + 26759248128*x^2 - 26759248128*x) - 10625375232*x))*sqrt(2*x
^2 - x + 3)*sqrt(2243059557247*sqrt(2) + 4023497000000) + 11194868609848920
9076292438*sqrt(31)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578888*x^6 - 329307
2*x^5 + 396480*x^4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 118051
*x^7 + 244047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^2 -
1036800*x) + 3276288*x) + 5088576640840418594376929*sqrt(31)*(254591*x^8 -
4815126*x^7 + 32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 74219328*x^3 -
168956928*x^2 - 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x
^4 - 3618*x^3 + 2268*x^2 - 1944*x) + 144820224*x))*sqrt(-(129508224872072^(
1/4)*sqrt(4023497)*sqrt(341)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(643213*
x + 2195288) - 2838501*x + 1552075)*sqrt(2243059557247*sqrt(2) + 4023497000
000) - 19211019462513817817783*x^2 - 17250711354094040489 48*sqrt(2)*(2*x^2 -
 x + 3) + 59201304874277275316 17*x - 78412324336791093134 00)/x^2) + 1459787
13411170407072657107696083 69*sqrt(31)*(2828123*x^8 - 9696916*x^7 + 53385560
*x^6 - 142835344*x^5 + 254146592*x^4 - 249300096*x^3 + 37981440*x^2 - 7744*
sqrt(2)*(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^4 - 5568*x^3
+ 1080*x^2 + 4320*x - 5184) + 223064064*x - 94887936))/(2585191*x^8 - 46612
00*x^7 + 14191920*x^6 + 490880*x^5 - 13562944*x^4 + 44249088*x^3 - 34615296
*x^2 - 24772608*x + 18579456)) + 2164988593398757980*129508224872072^(1/4)*
sqrt(4023497)*sqrt(341)*sqrt(2)*(100*x^8 + 20*x^7 + 321*x^6 + 172*x^5 + 390
*x^4 + 236*x^3 + 241*x^2 + 84*x + 36)*sqrt(2243059557247*sqrt(2) + 40234970
00000)*arctan(1/45253401157462826192523703385785943 9*(11475013444*sqrt(4023
497)*(11*129508224872072^(3/4)*sqrt(341)*(2673027292*x^7 - 11768684222*x^6
+ 24008796626*x^5 - 42687622824*x^4 + 22428040912*x^3 - 12956821056*x^2 - s
qrt(2)*(2612082154*x^7 - 9010050347*x^6 + 19426337114*x^5 - 28170626609*x^4
 + 13394761640*x^3 - 4698131400*x^2 - 17594323200*x + 10110341376) - 202206
82752*x + 17594323200) + 124728407*129508224872072^(1/4)*sqrt(341)*(2145837
31*x^7 - 3372306249*x^6 + 18434388344*x^5 - 43845503580*x^4 + 57631717152*x
```

```
^3 - 41786349984*x^2 - sqrt(2)*(190078101*x^7 - 2862100476*x^6 + 1468800342
0*x^5 - 32231022496*x^4 + 40927641120*x^3 - 21959568000*x^2 - 31156503552*x
 + 19060075008) - 38120150016*x + 31156503552))*sqrt(2*x^2 - x + 3)*sqrt(22
43059557247*sqrt(2) + 4023497000000) - 12846126780182995822393825477255364 7
2*sqrt(31)*sqrt(2)*(28180*x^8 - 254666*x^7 + 704270*x^6 - 1385256*x^5 + 154
9144*x^4 - 642048*x^3 - 98496*x^2 - sqrt(2)*(8746*x^8 - 102335*x^7 + 396104
*x^6 - 783113*x^5 + 1320710*x^4 - 752088*x^3 + 396144*x^2 + 546048*x - 5391
36) + 1154304*x - 456192) - 2*sqrt(8046994/10139750351)*(sqrt(4023497)*(11*
129508224872072^(3/4)*sqrt(341)*(8140972152*x^7 - 11907581308*x^6 + 3977730
3828*x^5 - 24395365568*x^4 + 37103094432*x^3 - 1836165888*x^2 - sqrt(2)*(10
387383478*x^7 - 14753211883*x^6 + 46462095753*x^5 - 11926110640*x^4 + 82242
91080*x^3 + 34793549568*x^2 - 34793549568*x) + 1836165888*x) + 124728407*12
9508224872072^(1/4)*sqrt(341)*(692762453*x^7 - 8972954292*x^6 + 34803726780
*x^5 - 46915651008*x^4 + 67421983392*x^3 + 10625375232*x^2 - 2*sqrt(2)*(367
903387*x^7 - 4754813452*x^6 + 18261523780*x^5 - 22991417280*x^4 + 270540014
40*x^3 + 26759248128*x^2 - 26759248128*x) - 10625375232*x))*sqrt(2*x^2 - x
+ 3)*sqrt(2243059557247*sqrt(2) + 4023497000000) - 1119486860984892090762 92
438*sqrt(31)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 +
 396480*x^4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 118051*x^7 +
244047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800
*x) + 3276288*x) - 508857664084041859437692 9*sqrt(31)*(254591*x^8 - 4815126
*x^7 + 32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 74219328*x^3 - 1689569
28*x^2 - 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 36
18*x^3 + 2268*x^2 - 1944*x) + 144820224*x))*sqrt((129508224872072^(1/4)*sqr
t(4023497)*sqrt(341)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(643213*x + 2195
288) - 2838501*x + 1552075)*sqrt(2243059557247*sqrt(2) + 4023497000000) + 1
92110194625138178178 3*x^2 + 172507113540940404894 8*sqrt(2)*(2*x^2 - x + 3)
 - 592013048742772753161 7*x + 784123243367910931340 0)/x^2) - 145978713411170
407072657107696083 69*sqrt(31)*(2828123*x^8 - 9696916*x^7 + 53385560*x^6 - 1
42835344*x^5 + 254146592*x^4 - 249300096*x^3 + 37981440*x^2 - 7744*sqrt(2)*
(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^4 - 5568*x^3 + 1080*x
^2 + 4320*x - 5184) + 223064064*x - 94887936))/(2585191*x^8 - 4661200*x^7 +
 14191920*x^6 + 490880*x^5 - 13562944*x^4 + 44249088*x^3 - 34615296*x^2 - 2
4772608*x + 18579456)) + 55545*129508224872072^(1/4)*sqrt(4023497)*sqrt(341
)*sqrt(31)*(402349700000000*x^8 + 80469940000000*x^7 + 1291542537000000*x^6
 + 692041484000000*x^5 + 1569163830000000*x^4 + 949545292000000*x^3 + 96966
2777000000*x^2 - 2243059557247*sqrt(2)*(100*x^8 + 20*x^7 + 321*x^6 + 172*x^
5 + 390*x^4 + 236*x^3 + 241*x^2 + 84*x + 36) + 337973748000000*x + 14484589
2000000)*sqrt(2243059557247*sqrt(2) + 4023497000000)*log(246439191250000000
0000/10139750351*(129508224872072^(1/4)*sqrt(4023497)*sqrt(341)*sqrt(31)*sq
rt(2*x^2 - x + 3)*(sqrt(2)*(643213*x + 2195288) - 2838501*x + 1552075)*sqrt
(2243059557247*sqrt(2) + 4023497000000) + 192110194625138178178 3*x^2 + 1725
07113540940404894 8*sqrt(2)*(2*x^2 - x + 3) - 592013048742772753161 7*x + 784
123243367910931340 0)/x^2) - 55545*129508224872072^(1/4)*sqrt(4023497)*sqrt(
341)*sqrt(31)*(402349700000000*x^8 + 80469940000000*x^7 + 1291542537000000*
```

```
x^6 + 692041484000000*x^5 + 1569163830000000*x^4 + 949545292000000*x^3 + 96
9662777000000*x^2 - 2243059557247*sqrt(2)*(100*x^8 + 20*x^7 + 321*x^6 + 172
*x^5 + 390*x^4 + 236*x^3 + 241*x^2 + 84*x + 36) + 337973748000000*x + 14484
5892000000)*sqrt(2243059557247*sqrt(2) + 4023497000000)*log(-24643919125000
00000000/10139750351*(129508224872072^(1/4)*sqrt(4023497)*sqrt(341)*sqrt(31
)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(643213*x + 2195288) - 2838501*x + 1552075)*
sqrt(2243059557247*sqrt(2) + 4023497000000) - 19211019462513817817 83*x^2 -
17250711354094040489 48*sqrt(2)*(2*x^2 - x + 3) + 592013048742772753161 7*x -
 784123243367910931340 0)/x^2) + 427817641581532204139104*(225699113100*x^7
- 12234606480*x^6 + 592923725931*x^5 + 174241614961*x^4 + 519223213785*x^3
+ 178650961091*x^2 + 218659985088*x + 9739335532)*sqrt(2*x^2 - x + 3))/(100
*x^8 + 20*x^7 + 321*x^6 + 172*x^5 + 390*x^4 + 236*x^3 + 241*x^2 + 84*x + 36
)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Francis algorithm failure for[-1.0,infinity,
infinity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,infi
nity]Francis algorithm failure for[-1.0,infinity,infinity,infinity,infinity
]proot error [1.0,infinity,infinity,infinity,infinity]Francis algorithm fai
lure for[-1.0,infinity,infinity,infinity,infinity]proot error [1.0,infinity
,infinity,infinity,infinity]Francis algorithm failure for[-1.0,infinity,inf
inity,infinity,infinity]proot error [1.0,infinity,infinity,infinity,infinit
y]Evaluation time: 70.56Done
```

maple [B] time = 0.21, size = 19014, normalized size = 70.68

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^3,x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(5\,x^2+3\,x+2\right)^3\left(2\,x^2-x+3\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^3,x, algorithm="maxima")
```

```
[Out] integrate(1/((5*x^2 + 3*x + 2)^3*(2*x^2 - x + 3)^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(2\,x^2-x+3\right)^{5/2}\,\left(5\,x^2+3\,x+2\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*x^2 - x + 3)^(5/2)*(3*x + 5*x^2 + 2)^3),x)
```

```
[Out] int(1/((2*x^2 - x + 3)^(5/2)*(3*x + 5*x^2 + 2)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(2x^{2} - x + 3\right)^{\frac{5}{2}} \left(5x^{2} + 3x + 2\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**2-x+3)**(5/2)/(5*x**2+3*x+2)**3,x)
```

```
[Out] Integral(1/((2*x**2 - x + 3)**(5/2)*(5*x**2 + 3*x + 2)**3), x)
```

3.97 $\int \sqrt{a+bx+cx^2}\,\left(d+ex+fx^2\right)^2\,dx$

Optimal. Leaf size=436

$$-\frac{\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\,\sqrt{a+bx+cx^2}}\right)\left(8c^2\left(2a^2f^2+12abef+5b^2\left(2df+e^2\right)\right)-56b^2cf(af+be)-32c^3\left(a\left(2df+e\right.\right.\right.}{1024c^{11/2}}$$

Rubi [A]   time = 0.79, antiderivative size = 436, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1661, 640, 612, 621, 206}

$$\frac{(b+2cx)\sqrt{a+bx+cx^2}\left(8c^2\left(2a^2f^2+12abef+5b^2\left(2df+e^2\right)\right)-56b^2cf(af+be)-32c^3\left(a\left(2df+e^2\right)+4bde\right)+21b^4f^2+128c^4d^2\right)}{512c^5}-\frac{\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(8c^2\left(2a^2f^2+12abef+5b^2\left(2df+e^2\right)\right)-56b^2cf(af+be)-32c^3\left(a\left(2df+e^2\right)+4bde\right)+21b^4f^2+128c^4d^2\right)}{1024c^{11/2}}+\frac{x\left(a+bx+cx^2\right)^{3/2}\left(-4cf(5af+14be)+21b^2f^2+40c^2\left(2df+e^2\right)\right)}{160c^3}+\frac{\left(a+bx+cx^2\right)^{3/2}\left(-8c^2\left(32aef+25b\left(2df+e^2\right)\right)+28bcf(7af+10be)-105b^3f^2+640c^3de\right)}{960c^4}+\frac{fx^2\left(a+bx+cx^2\right)^{3/2}(8ce-3bf)}{20c^2}+\frac{f^2x^3\left(a+bx+cx^2\right)^{3/2}}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)^2,x]
```

```
[Out] ((128*c^4*d^2 + 21*b^4*f^2 - 56*b^2*c*f*(b*e + a*f) - 32*c^3*(4*b*d*e + a*(
e^2 + 2*d*f)) + 8*c^2*(12*a*b*e*f + 2*a^2*f^2 + 5*b^2*(e^2 + 2*d*f)))*(b +
2*c*x)*Sqrt[a + b*x + c*x^2])/(512*c^5) + ((640*c^3*d*e - 105*b^3*f^2 + 28*
b*c*f*(10*b*e + 7*a*f) - 8*c^2*(32*a*e*f + 25*b*(e^2 + 2*d*f)))*(a + b*x +
c*x^2)^(3/2))/(960*c^4) + ((21*b^2*f^2 - 4*c*f*(14*b*e + 5*a*f) + 40*c^2*(e
^2 + 2*d*f))*x*(a + b*x + c*x^2)^(3/2))/(160*c^3) + (f*(8*c*e - 3*b*f)*x^2*
(a + b*x + c*x^2)^(3/2))/(20*c^2) + (f^2*x^3*(a + b*x + c*x^2)^(3/2))/(6*c)
 - ((b^2 - 4*a*c)*(128*c^4*d^2 + 21*b^4*f^2 - 56*b^2*c*f*(b*e + a*f) - 32*c
^3*(4*b*d*e + a*(e^2 + 2*d*f)) + 8*c^2*(12*a*b*e*f + 2*a^2*f^2 + 5*b^2*(e^2
 + 2*d*f)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(1024*c
^(11/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)^2 dx = \frac{f^2x^3\left(a+bx+cx^2\right)^{3/2}}{6c}+\frac{\int \sqrt{a+bx+cx^2}\left(6cd^2+12cdex-3\left(af^2-2c\right.\right.}{6c}$$

$$= \frac{f(8ce-3bf)x^2\left(a+bx+cx^2\right)^{3/2}}{20c^2}+\frac{f^2x^3\left(a+bx+cx^2\right)^{3/2}}{6c}+\frac{\int \sqrt{a+bx}}{}$$

$$= \frac{\left(21b^2f^2-4cf(14be+5af)+40c^2\left(e^2+2df\right)\right)x\left(a+bx+cx^2\right)^{3/2}}{160c^3}+\frac{f(}{}$$

$$= \frac{\left(640c^3de-105b^3f^2+28bcf(10be+7af)-8c^2\left(32aef+25b\left(e^2+2df\right.\right.\right.}{960c^4}$$

$$= \frac{\left(128c^4d^2+21b^4f^2-56b^2cf(be+af)-32c^3\left(4bde+a\left(e^2+2df\right)\right)+8\right.}{512c^5}$$

$$= \frac{\left(128c^4d^2+21b^4f^2-56b^2cf(be+af)-32c^3\left(4bde+a\left(e^2+2df\right)\right)+8\right.}{512c^5}$$

$$= \frac{\left(128c^4d^2+21b^4f^2-56b^2cf(be+af)-32c^3\left(4bde+a\left(e^2+2df\right)\right)+8\right.}{512c^5}$$

Mathematica [A] time = 0.93, size = 657, normalized size = 1.51

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)^2,x]
```

```
[Out] (3840*c^(9/2)*d^2*(b + 2*c*x)*Sqrt[a + x*(b + c*x)] + 10240*c^(9/2)*d*e*(a
+ x*(b + c*x))^(3/2) + 3840*c^(9/2)*(e^2 + 2*d*f)*x*(a + x*(b + c*x))^(3/2)
 + 6144*c^(9/2)*e*f*x^2*(a + x*(b + c*x))^(3/2) + 2560*c^(9/2)*f^2*x^3*(a +
 x*(b + c*x))^(3/2) - 1920*c^4*(b^2 - 4*a*c)*d^2*ArcTanh[(b + 2*c*x)/(2*Sqr
t[c]*Sqrt[a + x*(b + c*x)])] - 1920*b*c^3*d*e*(2*Sqrt[c]*(b + 2*c*x)*Sqrt[a
 + x*(b + c*x)] - (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(
b + c*x)])]) + 8*c*e*f*(-16*c^(3/2)*(-35*b^2 + 32*a*c + 42*b*c*x)*(a + x*(b
 + c*x))^(3/2) - 15*b*(7*b^2 - 12*a*c)*(2*Sqrt[c]*(b + 2*c*x)*Sqrt[a + x*(b
 + c*x)] - (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x
)])])) - 40*c^2*(e^2 + 2*d*f)*(80*b*c^(3/2)*(a + x*(b + c*x))^(3/2) - 3*(5*
b^2 - 4*a*c)*(2*Sqrt[c]*(b + 2*c*x)*Sqrt[a + x*(b + c*x)] - (b^2 - 4*a*c)*A
rcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])) - f^2*(2304*b*c^(7/
2)*x^2*(a + x*(b + c*x))^(3/2) + 16*c^(3/2)*(105*b^3 - 196*a*b*c - 126*b^2*
c*x + 120*a*c^2*x)*(a + x*(b + c*x))^(3/2) - 15*(21*b^4 - 56*a*b^2*c + 16*a
^2*c^2)*(2*Sqrt[c]*(b + 2*c*x)*Sqrt[a + x*(b + c*x)] - (b^2 - 4*a*c)*ArcTan
h[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])))/(15360*c^(11/2))
```

IntegrateAlgebraic [A] time = 2.82, size = 655, normalized size = 1.50

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)^2,x]
```

```
[Out] (Sqrt[a + b*x + c*x^2]*(1920*b*c^4*d^2 - 1920*b^2*c^3*d*e + 5120*a*c^4*d*e
+ 600*b^3*c^2*e^2 - 2080*a*b*c^3*e^2 + 1200*b^3*c^2*d*f - 4160*a*b*c^3*d*f
- 840*b^4*c*e*f + 3680*a*b^2*c^2*e*f - 2048*a^2*c^3*e*f + 315*b^5*f^2 - 168
0*a*b^3*c*f^2 + 1808*a^2*b*c^2*f^2 + 3840*c^5*d^2*x + 1280*b*c^4*d*e*x - 40
0*b^2*c^3*e^2*x + 960*a*c^4*e^2*x - 800*b^2*c^3*d*f*x + 1920*a*c^4*d*f*x +
560*b^3*c^2*e*f*x - 1856*a*b*c^3*e*f*x - 210*b^4*c*f^2*x + 896*a*b^2*c^2*f^
2*x - 480*a^2*c^3*f^2*x + 5120*c^5*d*e*x^2 + 320*b*c^4*e^2*x^2 + 640*b*c^4*
d*f*x^2 - 448*b^2*c^3*e*f*x^2 + 1024*a*c^4*e*f*x^2 + 168*b^3*c^2*f^2*x^2 -
544*a*b*c^3*f^2*x^2 + 1920*c^5*e^2*x^3 + 3840*c^5*d*f*x^3 + 384*b*c^4*e*f*x
^3 - 144*b^2*c^3*f^2*x^3 + 320*a*c^4*f^2*x^3 + 3072*c^5*e*f*x^4 + 128*b*c^4
*f^2*x^4 + 1280*c^5*f^2*x^5))/(7680*c^5) + ((128*b^2*c^4*d^2 - 512*a*c^5*d^
2 - 128*b^3*c^3*d*e + 512*a*b*c^4*d*e + 40*b^4*c^2*e^2 - 192*a*b^2*c^3*e^2
+ 128*a^2*c^4*e^2 + 80*b^4*c^2*d*f - 384*a*b^2*c^3*d*f + 256*a^2*c^4*d*f -
```

```
56*b^5*c*e*f + 320*a*b^3*c^2*e*f - 384*a^2*b*c^3*e*f + 21*b^6*f^2 - 140*a*b
^4*c*f^2 + 240*a^2*b^2*c^2*f^2 - 64*a^3*c^3*f^2)*Log[b + 2*c*x - 2*Sqrt[c]*
Sqrt[a + b*x + c*x^2]])/(1024*c^(11/2))
```

fricas [A] time = 0.74, size = 1269, normalized size = 2.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d)^2,x, algorithm="fricas")
```

```
[Out] [-1/30720*(15*(128*(b^2*c^4 - 4*a*c^5)*d^2 - 128*(b^3*c^3 - 4*a*b*c^4)*d*e
+ 8*(5*b^4*c^2 - 24*a*b^2*c^3 + 16*a^2*c^4)*e^2 + (21*b^6 - 140*a*b^4*c + 2
40*a^2*b^2*c^2 - 64*a^3*c^3)*f^2 + 8*(2*(5*b^4*c^2 - 24*a*b^2*c^3 + 16*a^2*
c^4)*d - (7*b^5*c - 40*a*b^3*c^2 + 48*a^2*b*c^3)*e)*f)*sqrt(c)*log(-8*c^2*x
^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) -
 4*(1280*c^6*f^2*x^5 + 1920*b*c^5*d^2 + 128*(24*c^6*e*f + b*c^5*f^2)*x^4 +
16*(120*c^6*e^2 - (9*b^2*c^4 - 20*a*c^5)*f^2 + 24*(10*c^6*d + b*c^5*e)*f)*x
^3 - 640*(3*b^2*c^4 - 8*a*c^5)*d*e + 40*(15*b^3*c^3 - 52*a*b*c^4)*e^2 + (31
5*b^5*c - 1680*a*b^3*c^2 + 1808*a^2*b*c^3)*f^2 + 8*(640*c^6*d*e + 40*b*c^5*
e^2 + (21*b^3*c^3 - 68*a*b*c^4)*f^2 + 8*(10*b*c^5*d - (7*b^2*c^4 - 16*a*c^5
)*e)*f)*x^2 + 8*(10*(15*b^3*c^3 - 52*a*b*c^4)*d - (105*b^4*c^2 - 460*a*b^2*
c^3 + 256*a^2*c^4)*e)*f + 2*(1920*c^6*d^2 + 640*b*c^5*d*e - 40*(5*b^2*c^4 -
 12*a*c^5)*e^2 - (105*b^4*c^2 - 448*a*b^2*c^3 + 240*a^2*c^4)*f^2 - 8*(10*(5
*b^2*c^4 - 12*a*c^5)*d - (35*b^3*c^3 - 116*a*b*c^4)*e)*f)*x)*sqrt(c*x^2 + b
*x + a))/c^6, 1/15360*(15*(128*(b^2*c^4 - 4*a*c^5)*d^2 - 128*(b^3*c^3 - 4*a
*b*c^4)*d*e + 8*(5*b^4*c^2 - 24*a*b^2*c^3 + 16*a^2*c^4)*e^2 + (21*b^6 - 140
*a*b^4*c + 240*a^2*b^2*c^2 - 64*a^3*c^3)*f^2 + 8*(2*(5*b^4*c^2 - 24*a*b^2*c
^3 + 16*a^2*c^4)*d - (7*b^5*c - 40*a*b^3*c^2 + 48*a^2*b*c^3)*e)*f)*sqrt(-c)
*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a
*c)) + 2*(1280*c^6*f^2*x^5 + 1920*b*c^5*d^2 + 128*(24*c^6*e*f + b*c^5*f^2)*
x^4 + 16*(120*c^6*e^2 - (9*b^2*c^4 - 20*a*c^5)*f^2 + 24*(10*c^6*d + b*c^5*e
)*f)*x^3 - 640*(3*b^2*c^4 - 8*a*c^5)*d*e + 40*(15*b^3*c^3 - 52*a*b*c^4)*e^2
 + (315*b^5*c - 1680*a*b^3*c^2 + 1808*a^2*b*c^3)*f^2 + 8*(640*c^6*d*e + 40*
b*c^5*e^2 + (21*b^3*c^3 - 68*a*b*c^4)*f^2 + 8*(10*b*c^5*d - (7*b^2*c^4 - 16
*a*c^5)*e)*f)*x^2 + 8*(10*(15*b^3*c^3 - 52*a*b*c^4)*d - (105*b^4*c^2 - 460*
a*b^2*c^3 + 256*a^2*c^4)*e)*f + 2*(1920*c^6*d^2 + 640*b*c^5*d*e - 40*(5*b^2
*c^4 - 12*a*c^5)*e^2 - (105*b^4*c^2 - 448*a*b^2*c^3 + 240*a^2*c^4)*f^2 - 8*
(10*(5*b^2*c^4 - 12*a*c^5)*d - (35*b^3*c^3 - 116*a*b*c^4)*e)*f)*x)*sqrt(c*x
^2 + b*x + a))/c^6]
```

giac [A] time = 0.36, size = 638, normalized size = 1.46

$$\frac{1}{7680}\sqrt{cx^2+bx+a}\left(2\left(4\left(2\left(8\left(10f^2x+\frac{bc^4f^2+24c^5fe}{c^5}\right)x+\frac{240c^5df-9b^2c^3f^2+20ac^4f^2+24bc^4fe+120c^5e^2}{c^5}\right)x+\frac{80bc^4df+21b^3c^2f^2-68abc^3f^2+640c^5de-56b^2c^3fe+128ac^4fe+40bc^4e^2}{c^5}\right)x+\frac{1920c^5d^2-400b^2c^3df+960ac^4df-105b^4cf^2+448ab^2c^2f^2-240a^2c^3f^2+640bc^4de+280b^3c^2fe-928abc^3fe-200b^2c^3e^2+480ac^4e^2}{c^5}\right)x+\frac{1920bc^4d^2+1200b^3c^2df-4160abc^3df+315b^5f^2-1680ab^3cf^2+1808a^2bc^2f^2-1920b^2c^3de+5120ac^4de-840b^4cfe+3680ab^2c^2fe-2048a^2c^3fe+600b^3c^2e^2-2080abc^3e^2}{c^5}\right)+\frac{\left(128b^2c^4d^2-512ac^5d^2+80b^4c^2df-384ab^2c^3df+256a^2c^4df+21b^6f^2-140ab^4cf^2+240a^2b^2c^2f^2-64a^3c^3f^2-128b^3c^3de+512abc^4de-56b^5cfe+320ab^3c^2fe-384a^2bc^3fe+40b^4c^2e^2-192ab^2c^3e^2+128a^2c^4e^2\right)\log\left(\left|-2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}-b\right|\right)}{1024c^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d)^2,x, algorithm="giac")
```

```
[Out] 1/7680*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*f^2*x + (b*c^4*f^2 + 24*c^5*f*
e)/c^5)*x + (240*c^5*d*f - 9*b^2*c^3*f^2 + 20*a*c^4*f^2 + 24*b*c^4*f*e + 12
0*c^5*e^2)/c^5)*x + (80*b*c^4*d*f + 21*b^3*c^2*f^2 - 68*a*b*c^3*f^2 + 640*c
^5*d*e - 56*b^2*c^3*f*e + 128*a*c^4*f*e + 40*b*c^4*e^2)/c^5)*x + (1920*c^5*
d^2 - 400*b^2*c^3*d*f + 960*a*c^4*d*f - 105*b^4*c*f^2 + 448*a*b^2*c^2*f^2 -
 240*a^2*c^3*f^2 + 640*b*c^4*d*e + 280*b^3*c^2*f*e - 928*a*b*c^3*f*e - 200*
b^2*c^3*e^2 + 480*a*c^4*e^2)/c^5)*x + (1920*b*c^4*d^2 + 1200*b^3*c^2*d*f -
4160*a*b*c^3*d*f + 315*b^5*f^2 - 1680*a*b^3*c*f^2 + 1808*a^2*b*c^2*f^2 - 19
20*b^2*c^3*d*e + 5120*a*c^4*d*e - 840*b^4*c*f*e + 3680*a*b^2*c^2*f*e - 2048
*a^2*c^3*f*e + 600*b^3*c^2*e^2 - 2080*a*b*c^3*e^2)/c^5) + 1/1024*(128*b^2*c
^4*d^2 - 512*a*c^5*d^2 + 80*b^4*c^2*d*f - 384*a*b^2*c^3*d*f + 256*a^2*c^4*d
*f + 21*b^6*f^2 - 140*a*b^4*c*f^2 + 240*a^2*b^2*c^2*f^2 - 64*a^3*c^3*f^2 -
128*b^3*c^3*d*e + 512*a*b*c^4*d*e - 56*b^5*c*f*e + 320*a*b^3*c^2*f*e - 384*
a^2*b*c^3*f*e + 40*b^4*c^2*e^2 - 192*a*b^2*c^3*e^2 + 128*a^2*c^4*e^2)*log(a
bs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(11/2)
```

maple [B] time = 0.02, size = 1429, normalized size = 3.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d)^2,x)
```

```
[Out] 1/6*f^2*x^3*(c*x^2+b*x+a)^(3/2)/c-7/32*f^2*b^2/c^3*a*x*(c*x^2+b*x+a)^(1/2)-
7/20*e*f*b/c^2*x*(c*x^2+b*x+a)^(3/2)-7/32*e*f*b^3/c^3*x*(c*x^2+b*x+a)^(1/2)
+2/3*d*e*(c*x^2+b*x+a)^(3/2)/c+1/4*d^2/c*(c*x^2+b*x+a)^(1/2)*b+1/2*d^2/c^(1
/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a-1/8*d^2/c^(3/2)*ln((1/2*b
+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*b^2+1/4*x*(c*x^2+b*x+a)^(3/2)/c*e^2-5/24
*b/c^2*(c*x^2+b*x+a)^(3/2)*e^2+5/64*b^3/c^3*(c*x^2+b*x+a)^(1/2)*e^2-5/128*b
^4/c^(7/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*e^2-1/8*a^2/c^(3/2)*
ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*e^2+21/512*f^2*b^5/c^5*(c*x^2+b
*x+a)^(1/2)-21/1024*f^2*b^6/c^(11/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(
1/2))+1/16*f^2*a^3/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-7/64
*f^2*b^3/c^4*(c*x^2+b*x+a)^(3/2)+3/8*e*f*b/c^2*a*x*(c*x^2+b*x+a)^(1/2)-5/16
*e*f*b^3/c^(7/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a+3/16*e*f*b^2
/c^3*a*(c*x^2+b*x+a)^(1/2)+3/8*e*f*b/c^(5/2)*a^2*ln((1/2*b+c*x)/c^(1/2)+(c*
x^2+b*x+a)^(1/2))+5/16*b^2/c^2*x*(c*x^2+b*x+a)^(1/2)*d*f+3/8*b^2/c^(5/2)*ln
((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a*d*f-1/4*a/c*x*(c*x^2+b*x+a)^(1/
2)*d*f-1/8*a/c^2*(c*x^2+b*x+a)^(1/2)*b*d*f-1/2*d*e*b/c*x*(c*x^2+b*x+a)^(1/2
)-1/2*d*e*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a+1/2*d^2*x
*(c*x^2+b*x+a)^(1/2)+21/256*f^2*b^4/c^4*x*(c*x^2+b*x+a)^(1/2)-5/12*b/c^2*(c
*x^2+b*x+a)^(3/2)*d*f+5/32*b^2/c^2*x*(c*x^2+b*x+a)^(1/2)*e^2+5/32*b^3/c^3*(
c*x^2+b*x+a)^(1/2)*d*f+3/16*b^2/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a
)^(1/2))*a*e^2-5/64*b^4/c^(7/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))
```

```
*d*f-1/8*a/c*x*(c*x^2+b*x+a)^(1/2)*e^2-1/16*a/c^2*(c*x^2+b*x+a)^(1/2)*b*e^2
-1/4*a^2/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*d*f-1/4*d*e*b^
2/c^2*(c*x^2+b*x+a)^(1/2)+35/256*f^2*b^4/c^(9/2)*ln((1/2*b+c*x)/c^(1/2)+(c*
x^2+b*x+a)^(1/2))*a-7/64*f^2*b^3/c^4*a*(c*x^2+b*x+a)^(1/2)-15/64*f^2*b^2/c^
(7/2)*a^2*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+49/240*f^2*b/c^3*a*(c
*x^2+b*x+a)^(3/2)-1/8*f^2*a/c^2*x*(c*x^2+b*x+a)^(3/2)+1/16*f^2*a^2/c^2*x*(c
*x^2+b*x+a)^(1/2)+1/32*f^2*a^2/c^3*(c*x^2+b*x+a)^(1/2)*b+2/5*e*f*x^2*(c*x^2
+b*x+a)^(3/2)/c+7/24*e*f*b^2/c^3*(c*x^2+b*x+a)^(3/2)-7/64*e*f*b^4/c^4*(c*x^
2+b*x+a)^(1/2)+7/128*e*f*b^5/c^(9/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(
1/2))-4/15*e*f*a/c^2*(c*x^2+b*x+a)^(3/2)-3/20*f^2*b/c^2*x^2*(c*x^2+b*x+a)^(
3/2)+21/160*f^2*b^2/c^3*x*(c*x^2+b*x+a)^(3/2)+1/2*x*(c*x^2+b*x+a)^(3/2)/c*d
*f+1/8*d*e*b^3/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [B] time = 5.31, size = 1299, normalized size = 2.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)^2,x)
```

```
[Out] d^2*(x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) + (e^2*x*(a + b*x + c*x^2)^(3/2
))/(4*c) + (a*f^2*((5*b*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/
2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a
 + b*x + c*x^2)^(1/2))/(24*c^2)))/(8*c) - (x*(a + b*x + c*x^2)^(3/2))/(4*c)
 + (a*((x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) + (log((b/2 + c*x)/c^(1/2) +
 (a + b*x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(4*c)))/(2*c) - (3*b
*f^2*((7*b*((5*b*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^
3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x
+ c*x^2)^(1/2))/(24*c^2)))/(8*c) - (x*(a + b*x + c*x^2)^(3/2))/(4*c) + (a*(
(x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) + (log((b/2 + c*x)/c^(1/2) + (a + b
*x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(4*c)))/(10*c) - (2*a*((log
((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5
/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^
2)))/(5*c) + (x^2*(a + b*x + c*x^2)^(3/2))/(5*c)))/(4*c) + (f^2*x^3*(a + b*
```

```
x + c*x^2)^(3/2))/(6*c) - (a*e^2*((x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) +
 (log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3
/2))))/(4*c) + (d^2*log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2))*(a*c
 - b^2/4))/(2*c^(3/2)) - (5*b*e^2*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x +
c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2
*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(8*c) - (4*a*e*f*((log((b + 2*c
*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((
8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(5*c
) - (5*b*d*f*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 -
4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*
x^2)^(1/2))/(24*c^2)))/(4*c) + (d*e*(8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a
+ b*x + c*x^2)^(1/2))/(12*c^2) + (d*f*x*(a + b*x + c*x^2)^(3/2))/(2*c) + (7
*b*e*f*((5*b*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 -
4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*
x^2)^(1/2))/(24*c^2)))/(8*c) - (x*(a + b*x + c*x^2)^(3/2))/(4*c) + (a*((x/2
 + b/(4*c))*(a + b*x + c*x^2)^(1/2) + (log((b/2 + c*x)/c^(1/2) + (a + b*x +
 c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(4*c)))/(5*c) + (2*e*f*x^2*(a +
 b*x + c*x^2)^(3/2))/(5*c) - (a*d*f*((x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2
) + (log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*c
^(3/2))))/(2*c) + (d*e*log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))
*(b^3 - 4*a*b*c))/(8*c^(5/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + bx + cx^2} \left(d + ex + fx^2\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)*(f*x**2+e*x+d)**2,x)
```

```
[Out] Integral(sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2)**2, x)
```

3.98 $\int \sqrt{a+bx+cx^2} \left(d+ex+fx^2\right) dx$

Optimal. Leaf size=175

$$-\frac{\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-4c(af+2be)+5b^2f+16c^2d\right)}{128c^{7/2}}+\frac{(b+2cx)\sqrt{a+bx+cx^2}\left(-4acf+5b^2f-\right.}{64c^3}$$

Rubi [A] time = 0.16, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1661, 640, 612, 621, 206}

$$\frac{(b+2cx)\sqrt{a+bx+cx^2}\left(-4acf+5b^2f-8bce+16c^2d\right)}{64c^3}-\frac{\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-4c(af+2be)+5b^2f+16c^2d\right)}{128c^{7/2}}+\frac{\left(a+bx+cx^2\right)^{3/2}(8ce-5bf)}{24c^2}+\frac{fx\left(a+bx+cx^2\right)^{3/2}}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2),x]
```

```
[Out] ((16*c^2*d - 8*b*c*e + 5*b^2*f - 4*a*c*f)*(b + 2*c*x)*Sqrt[a + b*x + c*x^2]
)/(64*c^3) + ((8*c*e - 5*b*f)*(a + b*x + c*x^2)^(3/2))/(24*c^2) + (f*x*(a +
 b*x + c*x^2)^(3/2))/(4*c) - ((b^2 - 4*a*c)*(16*c^2*d + 5*b^2*f - 4*c*(2*b*
e + a*f))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(128*c^(7
/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)dx = \frac{fx\left(a+bx+cx^2\right)^{3/2}}{4c} + \frac{\int \left(4cd-af+\frac{1}{2}(8ce-5bf)x\right)\sqrt{a+bx+cx^2}\,dx}{4c}$$

$$= \frac{(8ce-5bf)\left(a+bx+cx^2\right)^{3/2}}{24c^2} + \frac{fx\left(a+bx+cx^2\right)^{3/2}}{4c} + \frac{\left(16c^2d-8bce+5b\right.}{}$$

$$= \frac{\left(16c^2d-8bce+5b^2f-4acf\right)(b+2cx)\sqrt{a+bx+cx^2}}{64c^3} + \frac{(8ce-5bf)\left(a+\right.}{24c^2}$$

$$= \frac{\left(16c^2d-8bce+5b^2f-4acf\right)(b+2cx)\sqrt{a+bx+cx^2}}{64c^3} + \frac{(8ce-5bf)\left(a+\right.}{24c^2}$$

$$= \frac{\left(16c^2d-8bce+5b^2f-4acf\right)(b+2cx)\sqrt{a+bx+cx^2}}{64c^3} + \frac{(8ce-5bf)\left(a+\right.}{24c^2}$$

Mathematica [A] time = 0.27, size = 173, normalized size = 0.99

$$\frac{2\sqrt{c}\sqrt{a+x(b+cx)}\left(4bc\left(2c\left(6d+2ex+fx^2\right)-13af\right)+8c^2\left(a(8e+3fx)+2cx\left(6d+4ex+3fx^2\right)\right)+15b^3f-2b^2c(12e+5fx)\right)-3\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)\left(-4c(af+2be)+5b^2f+16c^2d\right)}{384c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2),x]
```

```
[Out] (2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(15*b^3*f - 2*b^2*c*(12*e + 5*f*x) + 4*b*c
*(-13*a*f + 2*c*(6*d + 2*e*x + f*x^2)) + 8*c^2*(a*(8*e + 3*f*x) + 2*c*x*(6*
```

```
d + 4*e*x + 3*f*x^2))) - 3*(b^2 - 4*a*c)*(16*c^2*d + 5*b^2*f - 4*c*(2*b*e +
 a*f))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]])/(384*c^(7/2)
)
```

IntegrateAlgebraic [A] time = 0.77, size = 207, normalized size = 1.18

$$\frac{\log\left(-2\sqrt{c}\sqrt{a+bx+cx^2}+b+2cx\right)\left(16a^2c^2f-24ab^2cf+32abc^2e-64ac^3d+5b^4f-8b^3ce+16b^2c^2d\right)}{128c^{7/2}}+\frac{\sqrt{a+bx+cx^2}\left(-52abcf+64ac^2e+24ac^2fx+15b^3f-24b^2ce-10b^2cfx+48bc^2d+16bc^2ex+8bc^2fx^2+96c^3dx+64c^3ex^2+48c^3fx^3\right)}{192c^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2),x]
```

```
[Out] (Sqrt[a + b*x + c*x^2]*(48*b*c^2*d - 24*b^2*c*e + 64*a*c^2*e + 15*b^3*f - 5
2*a*b*c*f + 96*c^3*d*x + 16*b*c^2*e*x - 10*b^2*c*f*x + 24*a*c^2*f*x + 64*c^
3*e*x^2 + 8*b*c^2*f*x^2 + 48*c^3*f*x^3))/(192*c^3) + ((16*b^2*c^2*d - 64*a*
c^3*d - 8*b^3*c*e + 32*a*b*c^2*e + 5*b^4*f - 24*a*b^2*c*f + 16*a^2*c^2*f)*L
og[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + b*x + c*x^2]])/(128*c^(7/2))
```

fricas [A] time = 0.47, size = 465, normalized size = 2.66

$$\left[\frac{3\left(16\left(b^2c^2-4ac^3\right)d-8\left(b^3c-4abc^2\right)e+\left(5b^4-24ab^2c+16a^2c^2\right)f\right)\sqrt{c}\log\left(-8c^2x^2-8bcx-b^2+4\sqrt{cx^2+bx+a}(2cx+b)\sqrt{c}-4ac\right)+4\left(48c^4fx^3+48bc^3d+8\left(8c^4e+bc^3f\right)x^2-8\left(3b^2c^2-8ac^3\right)e+\left(15b^3c-52abc^2\right)f+2\left(48c^4d+8bc^3e-\left(5b^2c^2-12ac^3\right)f\right)x\right)\sqrt{cx^2+bx+a}}{768c^4}, \frac{3\left(16\left(b^2c^2-4ac^3\right)d-8\left(b^3c-4abc^2\right)e+\left(5b^4-24ab^2c+16a^2c^2\right)f\right)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^2+bx+a}(2cx+b)\sqrt{-c}}{2\left(c^2x^2+bcx+ac\right)}\right)+2\left(48c^4fx^3+48bc^3d+8\left(8c^4e+bc^3f\right)x^2-8\left(3b^2c^2-8ac^3\right)e+\left(15b^3c-52abc^2\right)f+2\left(48c^4d+8bc^3e-\left(5b^2c^2-12ac^3\right)f\right)x\right)\sqrt{cx^2+bx+a}}{384c^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] [1/768*(3*(16*(b^2*c^2 - 4*a*c^3)*d - 8*(b^3*c - 4*a*b*c^2)*e + (5*b^4 - 24
*a*b^2*c + 16*a^2*c^2)*f)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c
*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(48*c^4*f*x^3 + 48*b*c^3*d
 + 8*(8*c^4*e + b*c^3*f)*x^2 - 8*(3*b^2*c^2 - 8*a*c^3)*e + (15*b^3*c - 52*a
*b*c^2)*f + 2*(48*c^4*d + 8*b*c^3*e - (5*b^2*c^2 - 12*a*c^3)*f)*x)*sqrt(c*x
^2 + b*x + a))/c^4, 1/384*(3*(16*(b^2*c^2 - 4*a*c^3)*d - 8*(b^3*c - 4*a*b*c
^2)*e + (5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*f)*sqrt(-c)*arctan(1/2*sqrt(c*x^2
 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(48*c^4*f*x^3
 + 48*b*c^3*d + 8*(8*c^4*e + b*c^3*f)*x^2 - 8*(3*b^2*c^2 - 8*a*c^3)*e + (15
*b^3*c - 52*a*b*c^2)*f + 2*(48*c^4*d + 8*b*c^3*e - (5*b^2*c^2 - 12*a*c^3)*f
)*x)*sqrt(c*x^2 + b*x + a))/c^4]
```

giac [A] time = 0.30, size = 212, normalized size = 1.21

$$\frac{1}{192}\sqrt{cx^2+bx+a}\left(2\left(4\left(6fx+\frac{bc^2f+8c^3e}{c^3}\right)x+\frac{48c^3d-5b^2cf+12ac^2f+8bc^2e}{c^3}\right)x+\frac{48bc^2d+15b^3f-52abcf-24b^2ce+64ac^2e}{c^3}\right)+\frac{\left(16b^2c^2d-64ac^3d+5b^4f-24ab^2cf+16a^2c^2f-8b^3ce+32abc^2e\right)\log\left(\left|-2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}-b\right|\right)}{128c^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] 1/192*sqrt(c*x^2 + b*x + a)*(2*(4*(6*f*x + (b*c^2*f + 8*c^3*e)/c^3)*x + (48
*c^3*d - 5*b^2*c*f + 12*a*c^2*f + 8*b*c^2*e)/c^3)*x + (48*b*c^2*d + 15*b^3*
```

```
f - 52*a*b*c*f - 24*b^2*c*e + 64*a*c^2*e)/c^3) + 1/128*(16*b^2*c^2*d - 64*a
*c^3*d + 5*b^4*f - 24*a*b^2*c*f + 16*a^2*c^2*f - 8*b^3*c*e + 32*a*b*c^2*e)*
log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(7/2)
```

maple [B] time = 0.01, size = 453, normalized size = 2.59

$$-\frac{a^{2} f \ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\sqrt{c x^{2}+bx+a}\right)}{8 c^{\frac{3}{2}}}+\frac{3 a b^{2} f \ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\sqrt{c x^{2}+bx+a}\right)}{16 c^{\frac{5}{2}}}-\frac{a b e \ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\sqrt{c x^{2}+bx+a}\right)}{4 c^{\frac{3}{2}}}+\frac{a d \ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\sqrt{c x^{2}+bx+a}\right)}{2 \sqrt{c}}-\frac{5 b^{4} f \ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\sqrt{c x^{2}+bx+a}\right)}{128 c^{\frac{7}{2}}}+\frac{b^{3} e \ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\sqrt{c x^{2}+bx+a}\right)}{16 c^{\frac{5}{2}}}-\frac{b^{2} d \ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\sqrt{c x^{2}+bx+a}\right)}{8 c^{\frac{3}{2}}}-\frac{\sqrt{c x^{2}+bx+a}\, a f x}{8 c}+\frac{5 \sqrt{c x^{2}+bx+a}\, b^{2} f x}{32 c^{2}}-\frac{\sqrt{c x^{2}+bx+a}\, b e x}{4 c}+\frac{\sqrt{c x^{2}+bx+a}\, d x}{2}-\frac{\sqrt{c x^{2}+bx+a}\, a b f}{16 c^{2}}+\frac{5 \sqrt{c x^{2}+bx+a}\, b^{3} f}{64 c^{3}}-\frac{\sqrt{c x^{2}+bx+a}\, b^{2} e}{8 c^{2}}+\frac{\sqrt{c x^{2}+bx+a}\, b d}{4 c}+\frac{\left(c x^{2}+bx+a\right)^{\frac{3}{2}} f x}{4 c}-\frac{5 \left(c x^{2}+bx+a\right)^{\frac{3}{2}} b f}{24 c^{2}}+\frac{\left(c x^{2}+bx+a\right)^{\frac{3}{2}} e}{3 c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d),x)
```

```
[Out] 1/4*f*x*(c*x^2+b*x+a)^(3/2)/c-5/24*f*b/c^2*(c*x^2+b*x+a)^(3/2)+5/32*f*b^2/c
^2*(c*x^2+b*x+a)^(1/2)*x+5/64*f*b^3/c^3*(c*x^2+b*x+a)^(1/2)+3/16*f*b^2/c^(5
/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a-5/128*f*b^4/c^(7/2)*ln((c
*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-1/8*f*a/c*(c*x^2+b*x+a)^(1/2)*x-1/16
*f*a/c^2*(c*x^2+b*x+a)^(1/2)*b-1/8*f*a^2/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*
x^2+b*x+a)^(1/2))+1/3*e*(c*x^2+b*x+a)^(3/2)/c-1/4*e*b/c*(c*x^2+b*x+a)^(1/2)
*x-1/8*e*b^2/c^2*(c*x^2+b*x+a)^(1/2)-1/4*e*b/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)
+(c*x^2+b*x+a)^(1/2))*a+1/16*e*b^3/c^(5/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*
x+a)^(1/2))+1/2*d*(c*x^2+b*x+a)^(1/2)*x+1/4*d/c*(c*x^2+b*x+a)^(1/2)*b+1/2*d
/c^(1/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a-1/8*d/c^(3/2)*ln((c*
x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*b^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [B] time = 3.91, size = 320, normalized size = 1.83

$$d\left(\frac{x}{2}+\frac{b}{4c}\right)\sqrt{cx^2+bx+a}-\frac{af\left(\left(\frac{x}{2}+\frac{b}{4c}\right)\sqrt{cx^2+bx+a}+\frac{\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{cx^2+bx+a}\right)\left(ac-\frac{b^2}{4}\right)}{2c^{3/2}}\right)}{4c}+\frac{d\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{cx^2+bx+a}\right)\left(ac-\frac{b^2}{4}\right)}{2c^{3/2}}+\frac{e\ln\left(\frac{b+2cx}{\sqrt{c}}+2\sqrt{cx^2+bx+a}\right)\left(b^3-4abc\right)}{16c^{5/2}}-\frac{5bf\left(\frac{\ln\left(\frac{b+2cx}{\sqrt{c}}+2\sqrt{cx^2+bx+a}\right)\left(b^3-4abc\right)}{16c^{5/2}}+\frac{\left(-3b^2+2cxb+8c\left(cx^2+a\right)\right)\sqrt{cx^2+bx+a}}{24c^2}\right)}{8c}+\frac{e\left(-3b^2+2cxb+8c\left(cx^2+a\right)\right)\sqrt{cx^2+bx+a}}{24c^2}+\frac{fx\left(cx^2+bx+a\right)^{3/2}}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2),x)
```

```
[Out] d*(x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) - (a*f*((x/2 + b/(4*c))*(a + b*x
+ c*x^2)^(1/2) + (log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2))*(a*c -
```

```
 b^2/4))/(2*c^(3/2))))/(4*c) + (d*log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^
2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2)) + (e*log((b + 2*c*x)/c^(1/2) + 2*(a +
b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) - (5*b*f*((log((b + 2*c*x
)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*
c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(8*c)
+ (e*(8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)
+ (f*x*(a + b*x + c*x^2)^(3/2))/(4*c)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)*(f*x**2+e*x+d),x)
```

```
[Out] Integral(sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2), x)
```

3.99 $\int \frac{\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx$

Optimal. Leaf size=431

$$-\frac{\sqrt{f\left(2af-b\left(e-\sqrt{e^2-4df}\right)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}\,(ce-bf)-bef-2cdf}}\right.}{\sqrt{2}\,f\sqrt{e^2-4df}}$$

Rubi [A] time = 1.05, antiderivative size = 431, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {989, 621, 206, 1032, 724}

$$-\frac{\sqrt{f\left(2af-b\left(e-\sqrt{e^2-4df}\right)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}\,(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}\,f\sqrt{e^2-4df}}+\frac{\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}\,(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}\,f\sqrt{e^2-4df}}+\frac{\sqrt{c}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x + c*x^2]/(d + e*x + f*x^2),x]
```

```
[Out] (Sqrt[c]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/f - (Sqrt[
c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e - Sqrt[e^2 - 4*d*f]
))]*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 -
 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f
)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*f*Sqrt[e^2 - 4*d*f])
 + (Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2
 - 4*d*f]))]*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + S
qrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (
c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*f*Sqrt[e^2
- 4*d*f])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 989

```
Int[Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]/((d_) + (e_.)*(x_) + (f_.)*(x_)^
2), x_Symbol] :> Dist[c/f, Int[1/Sqrt[a + b*x + c*x^2], x], x] - Dist[1/f,
Int[(c*d - a*f + (c*e - b*f)*x)/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 -
4*d*f, 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx = -\frac{\int \frac{cd-af+(ce-bf)x}{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}\,dx}{f} + \frac{c\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{f}$$

$$= \frac{(2c)\,\mathrm{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{f} - \frac{\left(2f(cd-af)-(ce-bf)\left(e-\sqrt{e^2-4df}\right)\right)\int \frac{}{(e-}}{f\sqrt{e^2-4df}}$$

$$= \frac{\sqrt{c}\,\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\,\sqrt{a+bx+cx^2}}\right)}{f} + \frac{\left(2\left(2f(cd-af)-(ce-bf)\left(e-\sqrt{e^2-4df}\right)\right)\right)\mathrm{Subst}\left(\int \frac{}{1}\right.}{}$$

$$= \frac{\sqrt{c}\,\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\,\sqrt{a+bx+cx^2}}\right)}{f} - \frac{\sqrt{c\left(e^2-2df-e\sqrt{e^2-4df}\right)+f\left(2af-b\left(e-\sqrt{e^2-4df}\right.\right.}}{\sqrt{2}\,f\sqrt{e}}$$

Mathematica [A] time = 1.22, size = 417, normalized size = 0.97

$$\frac{\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}\tanh^{-1}\left(\frac{4af-b\left(\sqrt{e^2-4df}+e-2fx\right)-2cx\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+x(b+cx)}\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)-\sqrt{f\left(2af+b\sqrt{e^2-4df}+b(-e)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}\tanh^{-1}\left(\frac{4af+b\left(\sqrt{e^2-4df}-e+2fx\right)+2cx\left(\sqrt{e^2-4df}-e\right)}{2\sqrt{2}\sqrt{a+x(b+cx)}\sqrt{f\left(2af+b\sqrt{e^2-4df}+b(-e)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}\,f\sqrt{e^2-4df}}+\frac{\sqrt{c}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]/(d + e*x + f*x^2),x]
```

```
[Out] (Sqrt[c]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/f + (Sqrt[
c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]
))]*ArcTanh[(4*a*f - 2*c*(e + Sqrt[e^2 - 4*d*f])*x - b*(e + Sqrt[e^2 - 4*d*
f] - 2*f*x))/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a
*f - b*(e + Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c*x)])] - Sqrt[f*(-(b*e) +
 2*a*f + b*Sqrt[e^2 - 4*d*f]) + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*ArcT
anh[(4*a*f + 2*c*(-e + Sqrt[e^2 - 4*d*f])*x + b*(-e + Sqrt[e^2 - 4*d*f] + 2
*f*x))/(2*Sqrt[2]*Sqrt[f*(-(b*e) + 2*a*f + b*Sqrt[e^2 - 4*d*f]) + c*(e^2 -
2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + x*(b + c*x)])])/(Sqrt[2]*f*Sqrt[e^2
- 4*d*f])
```

IntegrateAlgebraic [C] time = 0.60, size = 402, normalized size = 0.93

$$\frac{\text{RootSum}\left[\#1^4 f-2\#1^3\sqrt{c}e-2\#1^2af+\#1^2be+4\#1^2cd+2\#1a\sqrt{c}e-4\#1b\sqrt{c}d+a^2f-abe+b^2d\&,\frac{\#1^2ce\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)-\#1^2bf\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)-2\#1c^{3/2}d\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)+bcd\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)-ace\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)+2\#1a\sqrt{c}f\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)}{-2\#1^3f+3\#1^2\sqrt{c}e+2\#1af-\#1be-4\#1cd-a\sqrt{c}e+2b\sqrt{c}d}\&\right]}{f}-\frac{\sqrt{c}\log\left(-2\sqrt{c}f\sqrt{a+bx+cx^2}+bf+2cfx\right)}{f}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b*x + c*x^2]/(d + e*x + f*x^2),x]
```

```
[Out] -((Sqrt[c]*Log[b*f + 2*c*f*x - 2*Sqrt[c]*f*Sqrt[a + b*x + c*x^2]])/f) + Roo
tSum[b^2*d - a*b*e + a^2*f - 4*b*Sqrt[c]*d*#1 + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1
^2 + b*e*#1^2 - 2*a*f*#1^2 - 2*Sqrt[c]*e*#1^3 + f*#1^4 & , (b*c*d*Log[-(Sqr
t[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - a*c*e*Log[-(Sqrt[c]*x) + Sqrt[a + b
*x + c*x^2] - #1] - 2*c^(3/2)*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] -
#1]*#1 + 2*a*Sqrt[c]*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 +
c*e*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2 - b*f*Log[-(Sqrt[c]
*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2)/(2*b*Sqrt[c]*d - a*Sqrt[c]*e - 4*c*
d*#1 - b*e*#1 + 2*a*f*#1 + 3*Sqrt[c]*e*#1^2 - 2*f*#1^3) & ]/f
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.05, size = 6019, normalized size = 13.97

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x)
```

```
[Out] result too large to display
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re details)Is 4*d*f-e^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^2+b\,x+a}}{f\,x^2+e\,x+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^(1/2)/(d + e*x + f*x^2),x)
```

```
[Out] int((a + b*x + c*x^2)^(1/2)/(d + e*x + f*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + bx + cx^2}}{d + ex + fx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d),x)
```

```
[Out] Integral(sqrt(a + b*x + c*x**2)/(d + e*x + f*x**2), x)
```

3.100 $\int \frac{\sqrt{a+bx+cx^2}}{\left(d+ex+fx^2\right)^2}\,dx$

Optimal. Leaf size=488

$$-\frac{\left(f(be-4af)-\left(e-\sqrt{e^2-4df}\right)(ce-bf)\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}\,(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}\left(e^2-4df\right)^{3/2}\sqrt{2af^2-\sqrt{e^2-4df}\,(ce-bf)-bef-2cdf+ce^2}}+\frac{\left(f(be-4af\right.}{}$$

Rubi [A] time = 2.93, antiderivative size = 488, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {971, 1032, 724, 206}

$$-\frac{\left(f(be-4af)-\left(e-\sqrt{e^2-4df}\right)(ce-bf)\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}\,(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}\left(e^2-4df\right)^{3/2}\sqrt{2af^2-\sqrt{e^2-4df}\,(ce-bf)-bef-2cdf+ce^2}}+\frac{\left(f(be-4af)-\left(\sqrt{e^2-4df}+e\right)(ce-bf)\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}\,(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}\left(e^2-4df\right)^{3/2}\sqrt{2af^2+\sqrt{e^2-4df}\,(ce-bf)-bef-2cdf+ce^2}}-\frac{(e+2fx)\sqrt{a+bx+cx^2}}{\left(e^2-4df\right)\left(d+ex+fx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x + c*x^2]/(d + e*x + f*x^2)^2,x]
```

```
[Out] -(((e + 2*f*x)*Sqrt[a + b*x + c*x^2])/((e^2 - 4*d*f)*(d + e*x + f*x^2))) -
((f*(b*e - 4*a*f) - (c*e - b*f)*(e - Sqrt[e^2 - 4*d*f]))*ArcTanh[(4*a*f - b
*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2
]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*S
qrt[a + b*x + c*x^2])])/(Sqrt[2]*(e^2 - 4*d*f)^(3/2)*Sqrt[c*e^2 - 2*c*d*f -
 b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]) + ((f*(b*e - 4*a*f) - (c
*e - b*f)*(e + Sqrt[e^2 - 4*d*f]))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f
]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f
 - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])]
)/(Sqrt[2]*(e^2 - 4*d*f)^(3/2)*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*
e - b*f)*Sqrt[e^2 - 4*d*f]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
```

```
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 971

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((b + 2*c*x)*(a + b*x + c*x^2)^(p + 1)*(d + e
*x + f*x^2)^q)/((b^2 - 4*a*c)*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*(p + 1))
, Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*Simp[2*c*d*(2*p +
 3) + b*e*q + (2*b*f*q + 2*c*e*(2*p + q + 3))*x + 2*c*f*(2*p + 2*q + 3)*x^2
, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
e^2 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0] &&  !IGtQ[q, 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}}{\left(d+ex+fx^2\right)^2}\,dx = -\frac{(e+2fx)\sqrt{a+bx+cx^2}}{\left(e^2-4df\right)\left(d+ex+fx^2\right)} - \frac{\int \frac{\frac{1}{2}(be-4af)+(ce-bf)x}{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}\,dx}{-e^2+4df}$$

$$= -\frac{(e+2fx)\sqrt{a+bx+cx^2}}{\left(e^2-4df\right)\left(d+ex+fx^2\right)} - \frac{\left(ce\left(e-\sqrt{e^2-4df}\right)+f\left(4af-b\left(2e-\sqrt{e^2-4df}\right)\right)\right)\int}{\left(e^2-4df\right)^{3/2}}$$

$$= -\frac{(e+2fx)\sqrt{a+bx+cx^2}}{\left(e^2-4df\right)\left(d+ex+fx^2\right)} + \frac{\left(2\left(ce\left(e-\sqrt{e^2-4df}\right)+f\left(4af-b\left(2e-\sqrt{e^2-4df}\right)\right)\right)\right.}{}$$

$$= -\frac{(e+2fx)\sqrt{a+bx+cx^2}}{\left(e^2-4df\right)\left(d+ex+fx^2\right)} + \frac{\left(ce\left(e-\sqrt{e^2-4df}\right)+f\left(4af-b\left(2e-\sqrt{e^2-4df}\right)\right)\right)\tan}{\sqrt{2}\left(e^2-4df\right)^{3/2}\sqrt{ce^2-2cdf-be}}$$

Mathematica [A] time = 5.09, size = 555, normalized size = 1.14

$$\frac{4f(e+2fx)\sqrt{a+x(b+cx)}}{\left(e^2-4df\right)\left(\sqrt{e^2-4df}-e-2fx\right)\left(\sqrt{e^2-4df}+e+2fx\right)}+\frac{\left(ce\left(\sqrt{e^2-4df}-e\right)-f\left(4af+b\left(\sqrt{e^2-4df}-2e\right)\right)\right)\tanh^{-1}\left(\frac{-4af+b\left(-\sqrt{e^2-4df}+e-2fx\right)+2cx\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+x(b+cx)}\sqrt{f\left(2af+b\left(\sqrt{e^2-4df}-e\right)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}\left(e^2-4df\right)^{3/2}\sqrt{f\left(2af+b\left(\sqrt{e^2-4df}-e\right)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{\left(f\left(4af-b\left(\sqrt{e^2-4df}+2e\right)\right)+ce\left(\sqrt{e^2-4df}+e\right)\right)\tanh^{-1}\left(\frac{4af-b\left(\sqrt{e^2-4df}+e-2fx\right)-2cx\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+x(b+cx)}\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}\left(e^2-4df\right)^{3/2}\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]/(d + e*x + f*x^2)^2,x]
```

```
[Out] (4*f*(e + 2*f*x)*Sqrt[a + x*(b + c*x)])/((e^2 - 4*d*f)*(-e + Sqrt[e^2 - 4*d
*f] - 2*f*x)*(e + Sqrt[e^2 - 4*d*f] + 2*f*x)) + ((c*e*(-e + Sqrt[e^2 - 4*d*
f]) - f*(4*a*f + b*(-2*e + Sqrt[e^2 - 4*d*f])))*ArcTanh[(-4*a*f + 2*c*(e -
Sqrt[e^2 - 4*d*f])*x + b*(e - Sqrt[e^2 - 4*d*f] - 2*f*x))/(2*Sqrt[2]*Sqrt[c
*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f + b*(-e + Sqrt[e^2 - 4*d*f]
))]*Sqrt[a + x*(b + c*x)])])/(Sqrt[2]*(e^2 - 4*d*f)^(3/2)*Sqrt[c*(e^2 - 2*d
*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f + b*(-e + Sqrt[e^2 - 4*d*f]))]) - ((c*
e*(e + Sqrt[e^2 - 4*d*f]) + f*(4*a*f - b*(2*e + Sqrt[e^2 - 4*d*f])))*ArcTan
h[(4*a*f - 2*c*(e + Sqrt[e^2 - 4*d*f])*x - b*(e + Sqrt[e^2 - 4*d*f] - 2*f*x
))/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e
+ Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c*x)])])/(Sqrt[2]*(e^2 - 4*d*f)^(3/2
)*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 -
 4*d*f]))])
```

IntegrateAlgebraic [C] time = 2.55, size = 946, normalized size = 1.94

$$\frac{\sqrt{cx^2+bx+a}\,(-e-2fx)}{\left(e^2-4df\right)\left(fx^2+ex+d\right)}-\frac{c\,\mathrm{RootSum}\left[f\#1^4-2\sqrt{c}e\#1^3+4cd\#1^2+be\#1^2-2af\#1^2-4b\sqrt{c}d\#1+2a\sqrt{c}e\#1+b^2d-abe+a^2f\&,\frac{4ce\log\left(-\sqrt{c}x-\#1+\sqrt{cx^2+bx+a}\right)-5bf\log\left(-\sqrt{c}x-\#1+\sqrt{cx^2+bx+a}\right)+2\sqrt{c}f\#1\log\left(-\sqrt{c}x-\#1+\sqrt{cx^2+bx+a}\right)}{2f\#1^3-3\sqrt{c}e\#1^2+4cd\#1+be\#1-2af\#1-2b\sqrt{c}d+a\sqrt{c}e}\&\right]}{f^2}-\frac{\mathrm{RootSum}\left[f\#1^4-2\sqrt{c}e\#1^3+4cd\#1^2+be\#1^2-2af\#1^2-4b\sqrt{c}d\#1+2a\sqrt{c}e\#1+b^2d-abe+a^2f\&,\frac{8c^2\log\left(-\sqrt{c}x-\#1+\sqrt{cx^2+bx+a}\right)e^3-10bcf\log\left(-\sqrt{c}x-\#1+\sqrt{cx^2+bx+a}\right)e^2+4c^{3/2}f\log\left(-\sqrt{c}x-\#1+\sqrt{cx^2+bx+a}\right)\#1e^2+2cf^2\log\left(-\sqrt{c}x-\#1+\sqrt{cx^2+bx+a}\right)\#1^2e+b^2f^2\log\left(-\sqrt{c}x-\#1+\sqrt{cx^2+bx+a}\right)e-2acf^2\log\left(-\sqrt{c}x-\#1+\sqrt{cx^2+bx+a}\right)e-32c^2df\log\left(-\sqrt{c}x-\#1+\sqrt{cx^2+bx+a}\right)e-2b\sqrt{c}f^2\log\left(-\sqrt{c}x-\#1+\sqrt{cx^2+bx+a}\right)\#1e-2bf^3\log\left(-\sqrt{c}x-\#1+\sqrt{cx^2+bx+a}\right)\#1^2-2abf^3\log\left(-\sqrt{c}x-\#1+\sqrt{cx^2+bx+a}\right)+40bcdf^2\log\left(-\sqrt{c}x-\#1+\sqrt{cx^2+bx+a}\right)+8a\sqrt{c}f^3\log\left(-\sqrt{c}x-\#1+\sqrt{cx^2+bx+a}\right)\#1-16c^{3/2}df^2\log\left(-\sqrt{c}x-\#1+\sqrt{cx^2+bx+a}\right)\#1}{2f\#1^3-3\sqrt{c}e\#1^2+4cd\#1+be\#1-2af\#1-2b\sqrt{c}d+a\sqrt{c}e}\&\right]}{2f^2\left(4df-e^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b*x + c*x^2]/(d + e*x + f*x^2)^2,x]
```

```
[Out] ((-e - 2*f*x)*Sqrt[a + b*x + c*x^2])/((e^2 - 4*d*f)*(d + e*x + f*x^2)) - (c
*RootSum[b^2*d - a*b*e + a^2*f - 4*b*Sqrt[c]*d*#1 + 2*a*Sqrt[c]*e*#1 + 4*c*
d*#1^2 + b*e*#1^2 - 2*a*f*#1^2 - 2*Sqrt[c]*e*#1^3 + f*#1^4 & , (4*c*e*Log[-
(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 5*b*f*Log[-(Sqrt[c]*x) + Sqrt[a
 + b*x + c*x^2] - #1] + 2*Sqrt[c]*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2
] - #1]*#1)/(-2*b*Sqrt[c]*d + a*Sqrt[c]*e + 4*c*d*#1 + b*e*#1 - 2*a*f*#1 -
3*Sqrt[c]*e*#1^2 + 2*f*#1^3) & ])/f^2 - RootSum[b^2*d - a*b*e + a^2*f - 4*b
*Sqrt[c]*d*#1 + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 + b*e*#1^2 - 2*a*f*#1^2 - 2*S
qrt[c]*e*#1^3 + f*#1^4 & , (8*c^2*e^3*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x
^2] - #1] - 32*c^2*d*e*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 1
0*b*c*e^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + 40*b*c*d*f^2*L
og[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + b^2*e*f^2*Log[-(Sqrt[c]*x)
+ Sqrt[a + b*x + c*x^2] - #1] - 2*a*c*e*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x
 + c*x^2] - #1] - 2*a*b*f^3*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]
+ 4*c^(3/2)*e^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - 16*c^
(3/2)*d*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - 2*b*Sqrt[c]
```

```
*e*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 + 8*a*Sqrt[c]*f^3*
Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 + 2*c*e*f^2*Log[-(Sqrt[c]
*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2 - 2*b*f^3*Log[-(Sqrt[c]*x) + Sqrt[a
+ b*x + c*x^2] - #1]*#1^2)/(-2*b*Sqrt[c]*d + a*Sqrt[c]*e + 4*c*d*#1 + b*e*#
1 - 2*a*f*#1 - 3*Sqrt[c]*e*#1^2 + 2*f*#1^3) & ]/(2*f^2*(-e^2 + 4*d*f))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d)^2,x, algorithm="fricas")
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d)^2,x, algorithm="giac")
[Out] Timed out
```

maple [B] time = 0.04, size = 22287, normalized size = 45.67

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d)^2,x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2 + bx + a}}{\left(fx^2 + ex + d\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d)^2,x, algorithm="maxima")
[Out] integrate(sqrt(c*x^2 + b*x + a)/(f*x^2 + e*x + d)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^2+b\,x+a}}{\left(f\,x^2+e\,x+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^(1/2)/(d + e*x + f*x^2)^2,x)
```

```
[Out] int((a + b*x + c*x^2)^(1/2)/(d + e*x + f*x^2)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d)**2,x)
```

```
[Out] Timed out
```

3.101 $\int \left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)^2\,dx$

Optimal. Leaf size=564

$$\frac{\left(b^2-4ac\right)^2\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(16c^2\left(3a^2f^2+24abef+14b^2\left(2df+e^2\right)\right)-72b^2cf(3af+4be)-128c^3\left(a\left(2\right.\right.\right.}{32768c^{13/2}}$$

Rubi [A] time = 0.93, antiderivative size = 564, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1661, 640, 612, 621, 206}

$$\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}\left(16c^2\left(3a^2f^2+24abef+14b^2\left(2df+e^2\right)\right)-72b^2cf(3af+4be)-128c^3\left(a\left(2df+e^2\right)+6bde\right)+99b^4f^2+768c^4d^2\right)}{6144c^5}-\frac{\left(b^2-4ac\right)(b+2cx)\sqrt{a+bx+cx^2}\left(16c^2\left(3a^2f^2+24abef+14b^2\left(2df+e^2\right)\right)-72b^2cf(3af+4be)-128c^3\left(a\left(2df+e^2\right)+6bde\right)+99b^4f^2+768c^4d^2\right)}{16384c^6}+\frac{\left(b^2-4ac\right)^2\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(16c^2\left(3a^2f^2+24abef+14b^2\left(2df+e^2\right)\right)-72b^2cf(3af+4be)-128c^3\left(a\left(2df+e^2\right)+6bde\right)+99b^4f^2+768c^4d^2\right)}{32768c^{13/2}}+\frac{x\left(a+bx+cx^2\right)^{5/2}\left(-12cf(7af+24be)+99b^2f^2+224c^2\left(2df+e^2\right)\right)}{1344c^3}+\frac{\left(a+bx+cx^2\right)^{5/2}\left(-32c^2\left(48aef+49b\left(2df+e^2\right)\right)+36bcf(31af+56be)-693b^3f^2+5376c^3de\right)}{13440c^4}+\frac{fx^2\left(a+bx+cx^2\right)^{5/2}(32ce-11bf)}{112c^2}+\frac{f^2x^3\left(a+bx+cx^2\right)^{5/2}}{8c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)^2,x]
```

```
[Out] -((b^2 - 4*a*c)*(768*c^4*d^2 + 99*b^4*f^2 - 72*b^2*c*f*(4*b*e + 3*a*f) - 12
8*c^3*(6*b*d*e + a*(e^2 + 2*d*f)) + 16*c^2*(24*a*b*e*f + 3*a^2*f^2 + 14*b^2
*(e^2 + 2*d*f)))*(b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(16384*c^6) + ((768*c^4
*d^2 + 99*b^4*f^2 - 72*b^2*c*f*(4*b*e + 3*a*f) - 128*c^3*(6*b*d*e + a*(e^2
+ 2*d*f)) + 16*c^2*(24*a*b*e*f + 3*a^2*f^2 + 14*b^2*(e^2 + 2*d*f)))*(b + 2*
c*x)*(a + b*x + c*x^2)^(3/2))/(6144*c^5) + ((5376*c^3*d*e - 693*b^3*f^2 + 3
6*b*c*f*(56*b*e + 31*a*f) - 32*c^2*(48*a*e*f + 49*b*(e^2 + 2*d*f)))*(a + b*
x + c*x^2)^(5/2))/(13440*c^4) + ((99*b^2*f^2 - 12*c*f*(24*b*e + 7*a*f) + 22
4*c^2*(e^2 + 2*d*f))*x*(a + b*x + c*x^2)^(5/2))/(1344*c^3) + (f*(32*c*e - 1
1*b*f)*x^2*(a + b*x + c*x^2)^(5/2))/(112*c^2) + (f^2*x^3*(a + b*x + c*x^2)^
(5/2))/(8*c) + ((b^2 - 4*a*c)^2*(768*c^4*d^2 + 99*b^4*f^2 - 72*b^2*c*f*(4*b
*e + 3*a*f) - 128*c^3*(6*b*d*e + a*(e^2 + 2*d*f)) + 16*c^2*(24*a*b*e*f + 3*
a^2*f^2 + 14*b^2*(e^2 + 2*d*f)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*
x + c*x^2])])/(32768*c^(13/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right)^2 \, dx = \frac{f^2x^3\left(a+bx+cx^2\right)^{5/2}}{8c} + \frac{\int \left(a+bx+cx^2\right)^{3/2}\left(8cd^2+16cdex-\left(3af^2\right.\right.}{8}$$

$$= \frac{f(32ce-11bf)x^2\left(a+bx+cx^2\right)^{5/2}}{112c^2} + \frac{f^2x^3\left(a+bx+cx^2\right)^{5/2}}{8c} + \frac{\int \left(a+\right.}{}$$

$$= \frac{\left(99b^2f^2-12cf(24be+7af)+224c^2\left(e^2+2df\right)\right)x\left(a+bx+cx^2\right)^{5/2}}{1344c^3}$$

$$= \frac{\left(5376c^3de-693b^3f^2+36bcf(56be+31af)-32c^2\left(48aef+49b\left(e^2\right.\right.\right.}{13440c^4}$$

$$= \frac{\left(768c^4d^2+99b^4f^2-72b^2cf(4be+3af)-128c^3\left(6bde+a\left(e^2+2df\right.\right.\right.}{6}$$

$$= -\frac{\left(b^2-4ac\right)\left(768c^4d^2+99b^4f^2-72b^2cf(4be+3af)-128c^3\left(6bde+\right.\right.}{}$$

$$= -\frac{\left(b^2-4ac\right)\left(768c^4d^2+99b^4f^2-72b^2cf(4be+3af)-128c^3\left(6bde+\right.\right.}{}$$

$$= -\frac{\left(b^2-4ac\right)\left(768c^4d^2+99b^4f^2-72b^2cf(4be+3af)-128c^3\left(6bde+\right.\right.}{}$$

Mathematica [A] time = 1.76, size = 829, normalized size = 1.47

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)^2,x]
```

```
[Out] (430080*d^2*(b + 2*c*x)*(a + x*(b + c*x))^(3/2) + 1376256*d*e*(a + x*(b + c
*x))^(5/2) + 573440*(e^2 + 2*d*f)*x*(a + x*(b + c*x))^(5/2) + 983040*e*f*x^
2*(a + x*(b + c*x))^(5/2) + 430080*f^2*x^3*(a + x*(b + c*x))^(5/2) + (80640
*(b^2 - 4*a*c)*d^2*(-2*Sqrt[c]*(b + 2*c*x)*Sqrt[a + x*(b + c*x)] + (b^2 - 4
*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]))/c^(3/2) - (2
6880*b*d*e*(16*c^(3/2)*(b + 2*c*x)*(a + x*(b + c*x))^(3/2) - 3*(b^2 - 4*a*c
)*(2*Sqrt[c]*(b + 2*c*x)*Sqrt[a + x*(b + c*x)] - (b^2 - 4*a*c)*ArcTanh[(b +
 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])))/c^(5/2) + (96*e*f*(-256*c^(5/
2)*(-21*b^2 + 16*a*c + 30*b*c*x)*(a + x*(b + c*x))^(5/2) - 35*b*(3*b^2 - 4*
a*c)*(16*c^(3/2)*(b + 2*c*x)*(a + x*(b + c*x))^(3/2) - 3*(b^2 - 4*a*c)*(2*S
```

```
qrt[c]*(b + 2*c*x)*Sqrt[a + x*(b + c*x)] - (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x
)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]))))/c^(9/2) - (224*(e^2 + 2*d*f)*(1792
*b*c^(5/2)*(a + x*(b + c*x))^(5/2) - 5*(7*b^2 - 4*a*c)*(16*c^(3/2)*(b + 2*c
*x)*(a + x*(b + c*x))^(3/2) - 3*(b^2 - 4*a*c)*(2*Sqrt[c]*(b + 2*c*x)*Sqrt[a
 + x*(b + c*x)] - (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(
b + c*x)])]))))/c^(7/2) - (3*f^2*(112640*b*c^(9/2)*x^2*(a + x*(b + c*x))^(5
/2) + 256*c^(5/2)*(231*b^3 - 372*a*b*c - 330*b^2*c*x + 280*a*c^2*x)*(a + x*
(b + c*x))^(5/2) - 35*(33*b^4 - 72*a*b^2*c + 16*a^2*c^2)*(16*c^(3/2)*(b + 2
*c*x)*(a + x*(b + c*x))^(3/2) - 3*(b^2 - 4*a*c)*(2*Sqrt[c]*(b + 2*c*x)*Sqrt
[a + x*(b + c*x)] - (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x
*(b + c*x)])]))))/c^(11/2))/(3440640*c)
```

IntegrateAlgebraic [B] time = 7.79, size = 1210, normalized size = 2.15

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)^2,x]
```

```
[Out] (Sqrt[a + b*x + c*x^2]*(-80640*b^3*c^4*d^2 + 537600*a*b*c^5*d^2 + 80640*b^4
*c^3*d*e - 537600*a*b^2*c^4*d*e + 688128*a^2*c^5*d*e - 23520*b^5*c^2*e^2 +
170240*a*b^3*c^3*e^2 - 290304*a^2*b*c^4*e^2 - 47040*b^5*c^2*d*f + 340480*a*
b^3*c^3*d*f - 580608*a^2*b*c^4*d*f + 30240*b^6*c*e*f - 241920*a*b^4*c^2*e*f
 + 526848*a^2*b^2*c^3*e*f - 196608*a^3*c^4*e*f - 10395*b^7*f^2 + 91980*a*b^
5*c*f^2 - 244944*a^2*b^3*c^2*f^2 + 176448*a^3*b*c^3*f^2 + 53760*b^2*c^5*d^2
*x + 1075200*a*c^6*d^2*x - 53760*b^3*c^4*d*e*x + 301056*a*b*c^5*d*e*x + 156
80*b^4*c^3*e^2*x - 96768*a*b^2*c^4*e^2*x + 107520*a^2*c^5*e^2*x + 31360*b^4
*c^3*d*f*x - 193536*a*b^2*c^4*d*f*x + 215040*a^2*c^5*d*f*x - 20160*b^5*c^2*
e*f*x + 139776*a*b^3*c^3*e*f*x - 224256*a^2*b*c^4*e*f*x + 6930*b^6*c*f^2*x
- 53928*a*b^4*c^2*f^2*x + 113376*a^2*b^2*c^3*f^2*x - 40320*a^3*c^4*f^2*x +
645120*b*c^6*d^2*x^2 + 43008*b^2*c^5*d*e*x^2 + 1376256*a*c^6*d*e*x^2 - 1254
4*b^3*c^4*e^2*x^2 + 64512*a*b*c^5*e^2*x^2 - 25088*b^3*c^4*d*f*x^2 + 129024*
a*b*c^5*d*f*x^2 + 16128*b^4*c^3*e*f*x^2 - 95232*a*b^2*c^4*e*f*x^2 + 98304*a
^2*c^5*e*f*x^2 - 5544*b^5*c^2*f^2*x^2 + 37440*a*b^3*c^3*f^2*x^2 - 57984*a^2
*b*c^4*f^2*x^2 + 430080*c^7*d^2*x^3 + 946176*b*c^6*d*e*x^3 + 10752*b^2*c^5*
e^2*x^3 + 501760*a*c^6*e^2*x^3 + 21504*b^2*c^5*d*f*x^3 + 1003520*a*c^6*d*f*
x^3 - 13824*b^3*c^4*e*f*x^3 + 67584*a*b*c^5*e*f*x^3 + 4752*b^4*c^3*f^2*x^3
- 27264*a*b^2*c^4*f^2*x^3 + 26880*a^2*c^5*f^2*x^3 + 688128*c^7*d*e*x^4 + 37
2736*b*c^6*e^2*x^4 + 745472*b*c^6*d*f*x^4 + 12288*b^2*c^5*e*f*x^4 + 786432*
a*c^6*e*f*x^4 - 4224*b^3*c^4*f^2*x^4 + 19968*a*b*c^5*f^2*x^4 + 286720*c^7*e
^2*x^5 + 573440*c^7*d*f*x^5 + 614400*b*c^6*e*f*x^5 + 3840*b^2*c^5*f^2*x^5 +
 322560*a*c^6*f^2*x^5 + 491520*c^7*e*f*x^6 + 261120*b*c^6*f^2*x^6 + 215040*
c^7*f^2*x^7))/(1720320*c^6) + ((-768*b^4*c^4*d^2 + 6144*a*b^2*c^5*d^2 - 122
88*a^2*c^6*d^2 + 768*b^5*c^3*d*e - 6144*a*b^3*c^4*d*e + 12288*a^2*b*c^5*d*e
 - 224*b^6*c^2*e^2 + 1920*a*b^4*c^3*e^2 - 4608*a^2*b^2*c^4*e^2 + 2048*a^3*c
```

```
^5*e^2 - 448*b^6*c^2*d*f + 3840*a*b^4*c^3*d*f - 9216*a^2*b^2*c^4*d*f + 4096
*a^3*c^5*d*f + 288*b^7*c*e*f - 2688*a*b^5*c^2*e*f + 7680*a^2*b^3*c^3*e*f -
6144*a^3*b*c^4*e*f - 99*b^8*f^2 + 1008*a*b^6*c*f^2 - 3360*a^2*b^4*c^2*f^2 +
 3840*a^3*b^2*c^3*f^2 - 768*a^4*c^4*f^2)*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a +
 b*x + c*x^2]])/(32768*c^(13/2))
```

fricas [B] time = 1.94, size = 2179, normalized size = 3.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)^2,x, algorithm="fricas")
```

```
[Out] [1/6881280*(105*(768*(b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*d^2 - 768*(b^5*c^
3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*d*e + 32*(7*b^6*c^2 - 60*a*b^4*c^3 + 144*a^
2*b^2*c^4 - 64*a^3*c^5)*e^2 + 3*(33*b^8 - 336*a*b^6*c + 1120*a^2*b^4*c^2 -
1280*a^3*b^2*c^3 + 256*a^4*c^4)*f^2 + 32*(2*(7*b^6*c^2 - 60*a*b^4*c^3 + 144
*a^2*b^2*c^4 - 64*a^3*c^5)*d - 3*(3*b^7*c - 28*a*b^5*c^2 + 80*a^2*b^3*c^3 -
 64*a^3*b*c^4)*e)*f)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2
+ b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(215040*c^8*f^2*x^7 + 15360*(32
*c^8*e*f + 17*b*c^7*f^2)*x^6 + 1280*(224*c^8*e^2 + 3*(b^2*c^6 + 84*a*c^7)*f
^2 + 32*(14*c^8*d + 15*b*c^7*e)*f)*x^5 + 128*(5376*c^8*d*e + 2912*b*c^7*e^2
 - 3*(11*b^3*c^5 - 52*a*b*c^6)*f^2 + 32*(182*b*c^7*d + 3*(b^2*c^6 + 64*a*c^
7)*e)*f)*x^4 + 16*(26880*c^8*d^2 + 59136*b*c^7*d*e + 224*(3*b^2*c^6 + 140*a
*c^7)*e^2 + 3*(99*b^4*c^4 - 568*a*b^2*c^5 + 560*a^2*c^6)*f^2 + 32*(14*(3*b^
2*c^6 + 140*a*c^7)*d - 3*(9*b^3*c^5 - 44*a*b*c^6)*e)*f)*x^3 - 26880*(3*b^3*
c^5 - 20*a*b*c^6)*d^2 + 5376*(15*b^4*c^4 - 100*a*b^2*c^5 + 128*a^2*c^6)*d*e
 - 224*(105*b^5*c^3 - 760*a*b^3*c^4 + 1296*a^2*b*c^5)*e^2 - 3*(3465*b^7*c -
 30660*a*b^5*c^2 + 81648*a^2*b^3*c^3 - 58816*a^3*b*c^4)*f^2 + 8*(80640*b*c^
7*d^2 + 5376*(b^2*c^6 + 32*a*c^7)*d*e - 224*(7*b^3*c^5 - 36*a*b*c^6)*e^2 -
3*(231*b^5*c^3 - 1560*a*b^3*c^4 + 2416*a^2*b*c^5)*f^2 - 32*(14*(7*b^3*c^5 -
 36*a*b*c^6)*d - 3*(21*b^4*c^4 - 124*a*b^2*c^5 + 128*a^2*c^6)*e)*f)*x^2 - 3
2*(14*(105*b^5*c^3 - 760*a*b^3*c^4 + 1296*a^2*b*c^5)*d - 3*(315*b^6*c^2 - 2
520*a*b^4*c^3 + 5488*a^2*b^2*c^4 - 2048*a^3*c^5)*e)*f + 2*(26880*(b^2*c^6 +
 20*a*c^7)*d^2 - 5376*(5*b^3*c^5 - 28*a*b*c^6)*d*e + 224*(35*b^4*c^4 - 216*
a*b^2*c^5 + 240*a^2*c^6)*e^2 + 3*(1155*b^6*c^2 - 8988*a*b^4*c^3 + 18896*a^2
*b^2*c^4 - 6720*a^3*c^5)*f^2 + 32*(14*(35*b^4*c^4 - 216*a*b^2*c^5 + 240*a^2
*c^6)*d - 3*(105*b^5*c^3 - 728*a*b^3*c^4 + 1168*a^2*b*c^5)*e)*f)*x)*sqrt(c*
x^2 + b*x + a))/c^7, -1/3440640*(105*(768*(b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c
^6)*d^2 - 768*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*d*e + 32*(7*b^6*c^2 -
60*a*b^4*c^3 + 144*a^2*b^2*c^4 - 64*a^3*c^5)*e^2 + 3*(33*b^8 - 336*a*b^6*c
+ 1120*a^2*b^4*c^2 - 1280*a^3*b^2*c^3 + 256*a^4*c^4)*f^2 + 32*(2*(7*b^6*c^2
 - 60*a*b^4*c^3 + 144*a^2*b^2*c^4 - 64*a^3*c^5)*d - 3*(3*b^7*c - 28*a*b^5*c
^2 + 80*a^2*b^3*c^3 - 64*a^3*b*c^4)*e)*f)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 +
b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(215040*c^8*f^2*
x^7 + 15360*(32*c^8*e*f + 17*b*c^7*f^2)*x^6 + 1280*(224*c^8*e^2 + 3*(b^2*c^
```

```
6 + 84*a*c^7)*f^2 + 32*(14*c^8*d + 15*b*c^7*e)*f)*x^5 + 128*(5376*c^8*d*e +
 2912*b*c^7*e^2 - 3*(11*b^3*c^5 - 52*a*b*c^6)*f^2 + 32*(182*b*c^7*d + 3*(b^
2*c^6 + 64*a*c^7)*e)*f)*x^4 + 16*(26880*c^8*d^2 + 59136*b*c^7*d*e + 224*(3*
b^2*c^6 + 140*a*c^7)*e^2 + 3*(99*b^4*c^4 - 568*a*b^2*c^5 + 560*a^2*c^6)*f^2
 + 32*(14*(3*b^2*c^6 + 140*a*c^7)*d - 3*(9*b^3*c^5 - 44*a*b*c^6)*e)*f)*x^3
- 26880*(3*b^3*c^5 - 20*a*b*c^6)*d^2 + 5376*(15*b^4*c^4 - 100*a*b^2*c^5 + 1
28*a^2*c^6)*d*e - 224*(105*b^5*c^3 - 760*a*b^3*c^4 + 1296*a^2*b*c^5)*e^2 -
3*(3465*b^7*c - 30660*a*b^5*c^2 + 81648*a^2*b^3*c^3 - 58816*a^3*b*c^4)*f^2
+ 8*(80640*b*c^7*d^2 + 5376*(b^2*c^6 + 32*a*c^7)*d*e - 224*(7*b^3*c^5 - 36*
a*b*c^6)*e^2 - 3*(231*b^5*c^3 - 1560*a*b^3*c^4 + 2416*a^2*b*c^5)*f^2 - 32*(
14*(7*b^3*c^5 - 36*a*b*c^6)*d - 3*(21*b^4*c^4 - 124*a*b^2*c^5 + 128*a^2*c^6
)*e)*f)*x^2 - 32*(14*(105*b^5*c^3 - 760*a*b^3*c^4 + 1296*a^2*b*c^5)*d - 3*(
315*b^6*c^2 - 2520*a*b^4*c^3 + 5488*a^2*b^2*c^4 - 2048*a^3*c^5)*e)*f + 2*(2
6880*(b^2*c^6 + 20*a*c^7)*d^2 - 5376*(5*b^3*c^5 - 28*a*b*c^6)*d*e + 224*(35
*b^4*c^4 - 216*a*b^2*c^5 + 240*a^2*c^6)*e^2 + 3*(1155*b^6*c^2 - 8988*a*b^4*
c^3 + 18896*a^2*b^2*c^4 - 6720*a^3*c^5)*f^2 + 32*(14*(35*b^4*c^4 - 216*a*b^
2*c^5 + 240*a^2*c^6)*d - 3*(105*b^5*c^3 - 728*a*b^3*c^4 + 1168*a^2*b*c^5)*e
)*f)*x)*sqrt(c*x^2 + b*x + a))/c^7]
```

giac [B] time = 0.60, size = 1150, normalized size = 2.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)^2,x, algorithm="giac")
```

```
[Out] 1/1720320*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*(12*(14*c*f^2*x + (17*b*c^7
*f^2 + 32*c^8*f*e)/c^7)*x + (448*c^8*d*f + 3*b^2*c^6*f^2 + 252*a*c^7*f^2 +
480*b*c^7*f*e + 224*c^8*e^2)/c^7)*x + (5824*b*c^7*d*f - 33*b^3*c^5*f^2 + 15
6*a*b*c^6*f^2 + 5376*c^8*d*e + 96*b^2*c^6*f*e + 6144*a*c^7*f*e + 2912*b*c^7
*e^2)/c^7)*x + (26880*c^8*d^2 + 1344*b^2*c^6*d*f + 62720*a*c^7*d*f + 297*b^
4*c^4*f^2 - 1704*a*b^2*c^5*f^2 + 1680*a^2*c^6*f^2 + 59136*b*c^7*d*e - 864*b
^3*c^5*f*e + 4224*a*b*c^6*f*e + 672*b^2*c^6*e^2 + 31360*a*c^7*e^2)/c^7)*x +
 (80640*b*c^7*d^2 - 3136*b^3*c^5*d*f + 16128*a*b*c^6*d*f - 693*b^5*c^3*f^2
+ 4680*a*b^3*c^4*f^2 - 7248*a^2*b*c^5*f^2 + 5376*b^2*c^6*d*e + 172032*a*c^7
*d*e + 2016*b^4*c^4*f*e - 11904*a*b^2*c^5*f*e + 12288*a^2*c^6*f*e - 1568*b^
3*c^5*e^2 + 8064*a*b*c^6*e^2)/c^7)*x + (26880*b^2*c^6*d^2 + 537600*a*c^7*d^
2 + 15680*b^4*c^4*d*f - 96768*a*b^2*c^5*d*f + 107520*a^2*c^6*d*f + 3465*b^6
*c^2*f^2 - 26964*a*b^4*c^3*f^2 + 56688*a^2*b^2*c^4*f^2 - 20160*a^3*c^5*f^2
- 26880*b^3*c^5*d*e + 150528*a*b*c^6*d*e - 10080*b^5*c^3*f*e + 69888*a*b^3*
c^4*f*e - 112128*a^2*b*c^5*f*e + 7840*b^4*c^4*e^2 - 48384*a*b^2*c^5*e^2 + 5
3760*a^2*c^6*e^2)/c^7)*x - (80640*b^3*c^5*d^2 - 537600*a*b*c^6*d^2 + 47040*
b^5*c^3*d*f - 340480*a*b^3*c^4*d*f + 580608*a^2*b*c^5*d*f + 10395*b^7*c*f^2
 - 91980*a*b^5*c^2*f^2 + 244944*a^2*b^3*c^3*f^2 - 176448*a^3*b*c^4*f^2 - 80
640*b^4*c^4*d*e + 537600*a*b^2*c^5*d*e - 688128*a^2*c^6*d*e - 30240*b^6*c^2
```

```
*f*e + 241920*a*b^4*c^3*f*e - 526848*a^2*b^2*c^4*f*e + 196608*a^3*c^5*f*e +
 23520*b^5*c^3*e^2 - 170240*a*b^3*c^4*e^2 + 290304*a^2*b*c^5*e^2)/c^7) - 1/
32768*(768*b^4*c^4*d^2 - 6144*a*b^2*c^5*d^2 + 12288*a^2*c^6*d^2 + 448*b^6*c
^2*d*f - 3840*a*b^4*c^3*d*f + 9216*a^2*b^2*c^4*d*f - 4096*a^3*c^5*d*f + 99*
b^8*f^2 - 1008*a*b^6*c*f^2 + 3360*a^2*b^4*c^2*f^2 - 3840*a^3*b^2*c^3*f^2 +
768*a^4*c^4*f^2 - 768*b^5*c^3*d*e + 6144*a*b^3*c^4*d*e - 12288*a^2*b*c^5*d*
e - 288*b^7*c*f*e + 2688*a*b^5*c^2*f*e - 7680*a^2*b^3*c^3*f*e + 6144*a^3*b*
c^4*f*e + 224*b^6*c^2*e^2 - 1920*a*b^4*c^3*e^2 + 4608*a^2*b^2*c^4*e^2 - 204
8*a^3*c^5*e^2)*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))
/c^(13/2)
```

maple [B] time = 0.02, size = 2458, normalized size = 4.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)^2,x)
```

```
[Out] 1/8*f^2*x^3*(c*x^2+b*x+a)^(5/2)/c+1/8*d^2/c*(c*x^2+b*x+a)^(3/2)*b+3/8*d^2*(
c*x^2+b*x+a)^(1/2)*x*a-3/64*d^2/c^2*(c*x^2+b*x+a)^(1/2)*b^3+3/8*d^2/c^(1/2)
*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^2+3/128*d^2/c^(5/2)*ln((c*x+
1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*b^4+2/5*d*e*(c*x^2+b*x+a)^(5/2)/c+1/6*x
*(c*x^2+b*x+a)^(5/2)/c*e^2-7/60*b/c^2*(c*x^2+b*x+a)^(5/2)*e^2+7/192*b^3/c^3
*(c*x^2+b*x+a)^(3/2)*e^2-7/512*b^5/c^4*(c*x^2+b*x+a)^(1/2)*e^2+7/1024*b^6/c
^(9/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*e^2-1/16*a^3/c^(3/2)*ln(
(c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*e^2+3/128*f^2*a^4/c^(5/2)*ln((c*x+
1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+99/32768*f^2*b^8/c^(13/2)*ln((c*x+1/2*b
)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-33/640*f^2*b^3/c^4*(c*x^2+b*x+a)^(5/2)+33/20
48*f^2*b^5/c^5*(c*x^2+b*x+a)^(3/2)-99/16384*f^2*b^7/c^6*(c*x^2+b*x+a)^(1/2)
-3/16*e*f*b^3/c^3*(c*x^2+b*x+a)^(1/2)*x*a+1/8*e*f*b/c^2*a*x*(c*x^2+b*x+a)^(
3/2)+3/16*e*f*b/c^2*a^2*(c*x^2+b*x+a)^(1/2)*x+1/4*b^2/c^2*(c*x^2+b*x+a)^(1/
2)*x*a*d*f-3/8*d*e*b/c*(c*x^2+b*x+a)^(1/2)*x*a+3/16*d*e*b^3/c^(5/2)*ln((c*x
+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a+1/16*e*f*b^2/c^3*a*(c*x^2+b*x+a)^(3/
2)+3/32*e*f*b^2/c^3*a^2*(c*x^2+b*x+a)^(1/2)-3/14*e*f*b/c^2*x*(c*x^2+b*x+a)^
(5/2)-3/32*e*f*b^3/c^3*x*(c*x^2+b*x+a)^(3/2)+9/256*e*f*b^5/c^4*(c*x^2+b*x+a
)^(1/2)*x-3/32*e*f*b^4/c^4*(c*x^2+b*x+a)^(1/2)*a-15/64*e*f*b^3/c^(7/2)*ln((
c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^2+21/256*e*f*b^5/c^(9/2)*ln((c*x+
1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a+3/16*e*f*b/c^(5/2)*a^3*ln((c*x+1/2*b)
/c^(1/2)+(c*x^2+b*x+a)^(1/2))+7/48*b^2/c^2*x*(c*x^2+b*x+a)^(3/2)*d*f+1/8*b^
2/c^2*(c*x^2+b*x+a)^(1/2)*x*a*e^2-7/128*b^4/c^3*(c*x^2+b*x+a)^(1/2)*x*d*f-5
7/512*f^2*b^2/c^3*a^2*(c*x^2+b*x+a)^(1/2)*x+153/2048*f^2*b^4/c^4*(c*x^2+b*x
+a)^(1/2)*x*a-9/128*f^2*b^2/c^3*a*x*(c*x^2+b*x+a)^(3/2)+1/8*b^3/c^3*(c*x^2+
b*x+a)^(1/2)*a*d*f+9/32*b^2/c^(5/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1
/2))*a^2*d*f-15/128*b^4/c^(7/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))
*a*d*f-1/12*a/c*x*(c*x^2+b*x+a)^(3/2)*d*f-1/24*a/c^2*(c*x^2+b*x+a)^(3/2)*b*
```

```
d*f-1/8*a^2/c*(c*x^2+b*x+a)^(1/2)*x*d*f-1/16*a^2/c^2*(c*x^2+b*x+a)^(1/2)*b*
d*f-1/4*d*e*b/c*x*(c*x^2+b*x+a)^(3/2)+3/32*d*e*b^3/c^2*(c*x^2+b*x+a)^(1/2)*
x-3/16*d*e*b^2/c^2*(c*x^2+b*x+a)^(1/2)*a-3/8*d*e*b/c^(3/2)*ln((c*x+1/2*b)/c
^(1/2)+(c*x^2+b*x+a)^(1/2))*a^2+1/4*d^2*x*(c*x^2+b*x+a)^(3/2)+153/4096*f^2*
b^5/c^5*(c*x^2+b*x+a)^(1/2)*a+93/1120*f^2*b/c^3*a*(c*x^2+b*x+a)^(5/2)-11/11
2*f^2*b/c^2*x^2*(c*x^2+b*x+a)^(5/2)+105/1024*f^2*b^4/c^(9/2)*ln((c*x+1/2*b)
/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^2-1/8*a^3/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c
*x^2+b*x+a)^(1/2))*d*f-7/30*b/c^2*(c*x^2+b*x+a)^(5/2)*d*f+7/96*b^2/c^2*x*(c
*x^2+b*x+a)^(3/2)*e^2+7/96*b^3/c^3*(c*x^2+b*x+a)^(3/2)*d*f-7/256*b^4/c^3*(c
*x^2+b*x+a)^(1/2)*x*e^2+1/16*b^3/c^3*(c*x^2+b*x+a)^(1/2)*a*e^2-9/1024*e*f*b
^7/c^(11/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-4/35*e*f*a/c^2*(c*x
^2+b*x+a)^(5/2)+3/20*e*f*b^2/c^3*(c*x^2+b*x+a)^(5/2)-3/64*e*f*b^4/c^4*(c*x^
2+b*x+a)^(3/2)+9/512*e*f*b^6/c^5*(c*x^2+b*x+a)^(1/2)+2/7*e*f*x^2*(c*x^2+b*x
+a)^(5/2)/c-9/256*f^2*b^3/c^4*a*(c*x^2+b*x+a)^(3/2)-57/1024*f^2*b^3/c^4*a^2
*(c*x^2+b*x+a)^(1/2)+33/448*f^2*b^2/c^3*x*(c*x^2+b*x+a)^(5/2)-63/2048*f^2*b
^6/c^(11/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a-15/128*f^2*b^2/c^
(7/2)*a^3*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+1/64*f^2*a^2/c^2*x*(c
*x^2+b*x+a)^(3/2)+1/128*f^2*a^2/c^3*(c*x^2+b*x+a)^(3/2)*b+3/128*f^2*a^3/c^2
*(c*x^2+b*x+a)^(1/2)*x+3/256*f^2*a^3/c^3*(c*x^2+b*x+a)^(1/2)*b-1/16*f^2*a/c
^2*x*(c*x^2+b*x+a)^(5/2)+33/1024*f^2*b^4/c^4*x*(c*x^2+b*x+a)^(3/2)-99/8192*
f^2*b^6/c^5*(c*x^2+b*x+a)^(1/2)*x-15/256*b^4/c^(7/2)*ln((c*x+1/2*b)/c^(1/2)
+(c*x^2+b*x+a)^(1/2))*a*e^2+7/512*b^6/c^(9/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2
+b*x+a)^(1/2))*d*f-1/24*a/c*x*(c*x^2+b*x+a)^(3/2)*e^2-1/48*a/c^2*(c*x^2+b*x
+a)^(3/2)*b*e^2-1/16*a^2/c*(c*x^2+b*x+a)^(1/2)*x*e^2+1/3*x*(c*x^2+b*x+a)^(5
/2)/c*d*f-1/8*d*e*b^2/c^2*(c*x^2+b*x+a)^(3/2)+3/64*d*e*b^4/c^3*(c*x^2+b*x+a
)^(1/2)-3/128*d*e*b^5/c^(7/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-3
/32*d^2/c*(c*x^2+b*x+a)^(1/2)*x*b^2+3/16*d^2/c*(c*x^2+b*x+a)^(1/2)*b*a-3/16
*d^2/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*b^2*a-1/32*a^2/c^2
*(c*x^2+b*x+a)^(1/2)*b*e^2-7/256*b^5/c^4*(c*x^2+b*x+a)^(1/2)*d*f+9/64*b^2/c
^(5/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^2*e^2
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(c\,x^2+b\,x+a\right)^{3/2}\,\left(f\,x^2+e\,x+d\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)^2,x)
[Out] int((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+bx+cx^2\right)^{\frac{3}{2}}\left(d+ex+fx^2\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d)**2,x)
[Out] Integral((a + b*x + c*x**2)**(3/2)*(d + e*x + f*x**2)**2, x)
```

3.102 $\int \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right) dx$

Optimal. Leaf size=236

$$\frac{\left(b^2-4ac\right)^2 \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-4c(af+3be)+7b^2f+24c^2d\right)}{1024c^{9/2}}-\frac{\left(b^2-4ac\right)(b+2cx)\sqrt{a+bx+cx^2}\left(-4c(af\right.}{512c^4}$$

Rubi [A] time = 0.23, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1661, 640, 612, 621, 206}

$$\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}\left(-4acf+7b^2f-12bce+24c^2d\right)}{192c^3}-\frac{\left(b^2-4ac\right)(b+2cx)\sqrt{a+bx+cx^2}\left(-4c(af+3be)+7b^2f+24c^2d\right)}{512c^4}+\frac{\left(b^2-4ac\right)^2\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-4c(af+3be)+7b^2f+24c^2d\right)}{1024c^{9/2}}+\frac{\left(a+bx+cx^2\right)^{5/2}(12ce-7bf)}{60c^2}+\frac{fx\left(a+bx+cx^2\right)^{5/2}}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

```
[Out] -((b^2 - 4*a*c)*(24*c^2*d + 7*b^2*f - 4*c*(3*b*e + a*f))*(b + 2*c*x)*Sqrt[a
 + b*x + c*x^2])/(512*c^4) + ((24*c^2*d - 12*b*c*e + 7*b^2*f - 4*a*c*f)*(b
+ 2*c*x)*(a + b*x + c*x^2)^(3/2))/(192*c^3) + ((12*c*e - 7*b*f)*(a + b*x +
c*x^2)^(5/2))/(60*c^2) + (f*x*(a + b*x + c*x^2)^(5/2))/(6*c) + ((b^2 - 4*a*
c)^2*(24*c^2*d + 7*b^2*f - 4*c*(3*b*e + a*f))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c
]*Sqrt[a + b*x + c*x^2])])/(1024*c^(9/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)dx = \frac{fx\left(a+bx+cx^2\right)^{5/2}}{6c} + \frac{\int \left(6cd-af+\frac{1}{2}(12ce-7bf)x\right)\left(a+bx+cx^2\right)^{3}}{6c}$$

$$= \frac{(12ce-7bf)\left(a+bx+cx^2\right)^{5/2}}{60c^2} + \frac{fx\left(a+bx+cx^2\right)^{5/2}}{6c} + \frac{\left(2c(6cd-af)\right.}{}$$

$$= \frac{\left(24c^2d-12bce+7b^2f-4acf\right)(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{192c^3} + \frac{(12ce-7}{}$$

$$= -\frac{\left(b^2-4ac\right)\left(24c^2d+7b^2f-4c(3be+af)\right)(b+2cx)\sqrt{a+bx+cx^2}}{512c^4} +$$

$$= -\frac{\left(b^2-4ac\right)\left(24c^2d+7b^2f-4c(3be+af)\right)(b+2cx)\sqrt{a+bx+cx^2}}{512c^4} +$$

$$= -\frac{\left(b^2-4ac\right)\left(24c^2d+7b^2f-4c(3be+af)\right)(b+2cx)\sqrt{a+bx+cx^2}}{512c^4} +$$

Mathematica [A] time = 0.61, size = 392, normalized size = 1.66

$$\frac{\frac{360d(b^2-4ac)\left((b^2-4ac)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)-2\sqrt{c}(b+2cx)\sqrt{a+x(b+cx)}\right)}{c^{3/2}} - 60be\left(\frac{3(b^2-4ac)\left((b^2-4ac)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)-2\sqrt{c}(b+2cx)\sqrt{a+x(b+cx)}\right)}{c^{5/2}}+\frac{16(b+2cx)(a+x(b+cx))^{3/2}}{c}\right) + \frac{f\left(5(7b^2-4ac)\left(\frac{3(b^2-4ac)\left((b^2-4ac)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)-2\sqrt{c}(b+2cx)\sqrt{a+x(b+cx)}\right)}{c^{5/2}}+\frac{16(b+2cx)(a+x(b+cx))^{3/2}}{c}\right)-1792b(a+x(b+cx))^{5/2}\right)}{c} + 1920d(b+2cx)(a+x(b+cx))^{3/2} + 3072e(a+x(b+cx))^{5/2} + 2560fx(a+x(b+cx))^{5/2}}{15360c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

```
[Out] (1920*d*(b + 2*c*x)*(a + x*(b + c*x))^(3/2) + 3072*e*(a + x*(b + c*x))^(5/2
) + 2560*f*x*(a + x*(b + c*x))^(5/2) + (360*(b^2 - 4*a*c)*d*(-2*Sqrt[c]*(b
+ 2*c*x)*Sqrt[a + x*(b + c*x)] + (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[
c]*Sqrt[a + x*(b + c*x)])]))/c^(3/2) - 60*b*e*((16*(b + 2*c*x)*(a + x*(b +
c*x))^(3/2))/c + (3*(b^2 - 4*a*c)*(-2*Sqrt[c]*(b + 2*c*x)*Sqrt[a + x*(b + c
*x)] + (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]
))/c^(5/2)) + (f*(-1792*b*(a + x*(b + c*x))^(5/2) + 5*(7*b^2 - 4*a*c)*((16*
(b + 2*c*x)*(a + x*(b + c*x))^(3/2))/c + (3*(b^2 - 4*a*c)*(-2*Sqrt[c]*(b +
2*c*x)*Sqrt[a + x*(b + c*x)] + (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]
*Sqrt[a + x*(b + c*x)])]))/c^(5/2))))/c)/(15360*c)
```

IntegrateAlgebraic [A] time = 1.70, size = 408, normalized size = 1.73

$$\frac{\sqrt{a+bx+cx^2}\left(-1296a^2bc^2f+1536a^2c^3e+480a^2c^3fx+760ab^3cf-1200ab^2c^2e-432ab^2c^2fx+2400abc^3d+672abc^3ex+288abc^3fx^2+4800ac^4dx+3072ac^4ex^2+2240ac^4fx^3-105b^5f+180b^4ce+70b^4cfx-360b^3c^2d-120b^3c^2ex-56b^3c^2fx^2+240b^2c^3dx+96b^2c^3ex^2+48b^2c^3fx^3+2880bc^4dx^2+2112bc^4ex^3+1664bc^4fx^4+1920c^5dx^3+1536c^5ex^4+1280c^5fx^5\right)}{7680c^4}+\frac{\log\left(-2\sqrt{c}\sqrt{a+bx+cx^2}+b+2cx\right)\left(64a^3c^3f-144a^2b^2c^2f+192a^2bc^3e-384a^2c^4d+60ab^4cf-96ab^3c^2e+192ab^2c^3d-7b^6f+12b^5ce-24b^4c^2d\right)}{1024c^{9/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

```
[Out] (Sqrt[a + b*x + c*x^2]*(-360*b^3*c^2*d + 2400*a*b*c^3*d + 180*b^4*c*e - 120
0*a*b^2*c^2*e + 1536*a^2*c^3*e - 105*b^5*f + 760*a*b^3*c*f - 1296*a^2*b*c^2
*f + 240*b^2*c^3*d*x + 4800*a*c^4*d*x - 120*b^3*c^2*e*x + 672*a*b*c^3*e*x +
 70*b^4*c*f*x - 432*a*b^2*c^2*f*x + 480*a^2*c^3*f*x + 2880*b*c^4*d*x^2 + 96
*b^2*c^3*e*x^2 + 3072*a*c^4*e*x^2 - 56*b^3*c^2*f*x^2 + 288*a*b*c^3*f*x^2 +
1920*c^5*d*x^3 + 2112*b*c^4*e*x^3 + 48*b^2*c^3*f*x^3 + 2240*a*c^4*f*x^3 + 1
536*c^5*e*x^4 + 1664*b*c^4*f*x^4 + 1280*c^5*f*x^5))/(7680*c^4) + ((-24*b^4*
c^2*d + 192*a*b^2*c^3*d - 384*a^2*c^4*d + 12*b^5*c*e - 96*a*b^3*c^2*e + 192
*a^2*b*c^3*e - 7*b^6*f + 60*a*b^4*c*f - 144*a^2*b^2*c^2*f + 64*a^3*c^3*f)*L
og[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + b*x + c*x^2]])/(1024*c^(9/2))
```

fricas [A] time = 0.84, size = 839, normalized size = 3.56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] [-1/30720*(15*(24*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d - 12*(b^5*c - 8*a*
b^3*c^2 + 16*a^2*b*c^3)*e + (7*b^6 - 60*a*b^4*c + 144*a^2*b^2*c^2 - 64*a^3*
c^3)*f)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)*(2
*c*x + b)*sqrt(c) - 4*a*c) - 4*(1280*c^6*f*x^5 + 128*(12*c^6*e + 13*b*c^5*f
)*x^4 + 16*(120*c^6*d + 132*b*c^5*e + (3*b^2*c^4 + 140*a*c^5)*f)*x^3 + 8*(3
60*b*c^5*d + 12*(b^2*c^4 + 32*a*c^5)*e - (7*b^3*c^3 - 36*a*b*c^4)*f)*x^2 -
120*(3*b^3*c^3 - 20*a*b*c^4)*d + 12*(15*b^4*c^2 - 100*a*b^2*c^3 + 128*a^2*c
^4)*e - (105*b^5*c - 760*a*b^3*c^2 + 1296*a^2*b*c^3)*f + 2*(120*(b^2*c^4 +
20*a*c^5)*d - 12*(5*b^3*c^3 - 28*a*b*c^4)*e + (35*b^4*c^2 - 216*a*b^2*c^3 +
```

```
 240*a^2*c^4)*f)*x)*sqrt(c*x^2 + b*x + a))/c^5, -1/15360*(15*(24*(b^4*c^2 -
 8*a*b^2*c^3 + 16*a^2*c^4)*d - 12*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*e +
(7*b^6 - 60*a*b^4*c + 144*a^2*b^2*c^2 - 64*a^3*c^3)*f)*sqrt(-c)*arctan(1/2*
sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(12
80*c^6*f*x^5 + 128*(12*c^6*e + 13*b*c^5*f)*x^4 + 16*(120*c^6*d + 132*b*c^5*
e + (3*b^2*c^4 + 140*a*c^5)*f)*x^3 + 8*(360*b*c^5*d + 12*(b^2*c^4 + 32*a*c^
5)*e - (7*b^3*c^3 - 36*a*b*c^4)*f)*x^2 - 120*(3*b^3*c^3 - 20*a*b*c^4)*d + 1
2*(15*b^4*c^2 - 100*a*b^2*c^3 + 128*a^2*c^4)*e - (105*b^5*c - 760*a*b^3*c^2
 + 1296*a^2*b*c^3)*f + 2*(120*(b^2*c^4 + 20*a*c^5)*d - 12*(5*b^3*c^3 - 28*a
*b*c^4)*e + (35*b^4*c^2 - 216*a*b^2*c^3 + 240*a^2*c^4)*f)*x)*sqrt(c*x^2 + b
*x + a))/c^5]
```

giac [A] time = 0.61, size = 417, normalized size = 1.77

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] 1/7680*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*c*f*x + (13*b*c^5*f + 12*c^6*e
)/c^5)*x + (120*c^6*d + 3*b^2*c^4*f + 140*a*c^5*f + 132*b*c^5*e)/c^5)*x + (
360*b*c^5*d - 7*b^3*c^3*f + 36*a*b*c^4*f + 12*b^2*c^4*e + 384*a*c^5*e)/c^5)
*x + (120*b^2*c^4*d + 2400*a*c^5*d + 35*b^4*c^2*f - 216*a*b^2*c^3*f + 240*a
^2*c^4*f - 60*b^3*c^3*e + 336*a*b*c^4*e)/c^5)*x - (360*b^3*c^3*d - 2400*a*b
*c^4*d + 105*b^5*c*f - 760*a*b^3*c^2*f + 1296*a^2*b*c^3*f - 180*b^4*c^2*e +
 1200*a*b^2*c^3*e - 1536*a^2*c^4*e)/c^5) - 1/1024*(24*b^4*c^2*d - 192*a*b^2
*c^3*d + 384*a^2*c^4*d + 7*b^6*f - 60*a*b^4*c*f + 144*a^2*b^2*c^2*f - 64*a^
3*c^3*f - 12*b^5*c*e + 96*a*b^3*c^2*e - 192*a^2*b*c^3*e)*log(abs(-2*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(9/2)
```

maple [B] time = 0.01, size = 862, normalized size = 3.65

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x)
```

```
[Out] 1/8*f*b^2/c^2*(c*x^2+b*x+a)^(1/2)*x*a-3/16*e*b/c*(c*x^2+b*x+a)^(1/2)*x*a+1/
4*d*(c*x^2+b*x+a)^(3/2)*x+1/5*e*(c*x^2+b*x+a)^(5/2)/c+1/6*f*x*(c*x^2+b*x+a)
^(5/2)/c-3/32*d/c*(c*x^2+b*x+a)^(1/2)*x*b^2-3/16*e*b/c^(3/2)*ln((c*x+1/2*b)
/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^2+3/128*d/c^(5/2)*ln((c*x+1/2*b)/c^(1/2)+(c
*x^2+b*x+a)^(1/2))*b^4+1/8*d/c*(c*x^2+b*x+a)^(3/2)*b-1/16*e*b^2/c^2*(c*x^2+
b*x+a)^(3/2)+3/128*e*b^4/c^3*(c*x^2+b*x+a)^(1/2)-3/256*e*b^5/c^(7/2)*ln((c*
x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-7/60*f*b/c^2*(c*x^2+b*x+a)^(5/2)+7/19
```

```
2*f*b^3/c^3*(c*x^2+b*x+a)^(3/2)-7/512*f*b^5/c^4*(c*x^2+b*x+a)^(1/2)+7/1024*
f*b^6/c^(9/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-1/16*f*a^3/c^(3/2
)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+3/8*d/c^(1/2)*ln((c*x+1/2*b)/
c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^2+3/8*d*(c*x^2+b*x+a)^(1/2)*x*a-3/64*d/c^2*(
c*x^2+b*x+a)^(1/2)*b^3-1/24*f*a/c*(c*x^2+b*x+a)^(3/2)*x-1/48*f*a/c^2*(c*x^2
+b*x+a)^(3/2)*b-1/16*f*a^2/c*(c*x^2+b*x+a)^(1/2)*x+7/96*f*b^2/c^2*(c*x^2+b*
x+a)^(3/2)*x-7/256*f*b^4/c^3*(c*x^2+b*x+a)^(1/2)*x+1/16*f*b^3/c^3*(c*x^2+b*
x+a)^(1/2)*a+9/64*f*b^2/c^(5/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))
*a^2-15/256*f*b^4/c^(7/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a-3/1
6*d/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*b^2*a+3/16*d/c*(c*x
^2+b*x+a)^(1/2)*b*a+3/64*e*b^3/c^2*(c*x^2+b*x+a)^(1/2)*x-3/32*e*b^2/c^2*(c*
x^2+b*x+a)^(1/2)*a-1/8*e*b/c*(c*x^2+b*x+a)^(3/2)*x-1/32*f*a^2/c^2*(c*x^2+b*
x+a)^(1/2)*b+3/32*e*b^3/c^(5/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))
*a
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(c\,x^2+b\,x+a\right)^{3/2}\,\left(f\,x^2+e\,x+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2),x)
```

```
[Out] int((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + bx + cx^{2}\right)^{\frac{3}{2}} \left(d + ex + fx^{2}\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d),x)
```

```
[Out] Integral((a + b*x + c*x**2)**(3/2)*(d + e*x + f*x**2), x)
```

3.103 $\int \frac{(a+bx+cx^2)^{3/2}}{d+ex+fx^2}\,dx$

Optimal. Leaf size=679

$$\frac{\left(\left(e-\sqrt{e^2-4df}\right)(ce-bf)\left(f(be-2af)-c\left(e^2-2df\right)\right)-2f\left(-f^2\left(b^2d-a^2f\right)+2cdf(be-af)+c^2(-d)\left(e^2-d\right.\right.\right.}{\sqrt{2}\,f^3\sqrt{e^2-4df}\,\sqrt{2af^2-\sqrt{e^2-4df}\,(ce-bf)-bef-2cd}}$$

Rubi [A] time = 11.03, antiderivative size = 678, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {977, 1076, 621, 206, 1032, 724}

$$\frac{\left(\left(e-\sqrt{e^2-4df}\right)(ce-bf)\left(f(be-2af)-c\left(e^2-2df\right)\right)-2f\left(-f^2\left(b^2d-a^2f\right)+2cdf(be-af)+c^2(-d)\left(e^2-df\right)\right)\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}\,f^3\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}\,(ce-bf)-bef-2cdf+ce^2}}+\frac{\left(-2f^3\left(b^2d-a^2f\right)-\left(\sqrt{e^2-4df}+e\right)(ce-bf)\left(f(be-2af)-c\left(e^2-2df\right)\right)+4cdf^2(be-af)-2c^2df\left(e^2-df\right)\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}\,f^3\sqrt{e^2-4df}\sqrt{2af^2+\sqrt{e^2-4df}\,(ce-bf)-bef-2cdf+ce^2}}+\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-12cf(be-af)+3b^2f^2+8c^2\left(e^2-df\right)\right)}{8\sqrt{c}\,f^3}-\frac{\sqrt{a+bx+cx^2}\,(-5bf+4ce-2cfx)}{4f^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(3/2)/(d + e*x + f*x^2),x]
```

```
[Out] -((4*c*e - 5*b*f - 2*c*f*x)*Sqrt[a + b*x + c*x^2])/(4*f^2) + ((3*b^2*f^2 -
12*c*f*(b*e - a*f) + 8*c^2*(e^2 - d*f))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt
[a + b*x + c*x^2])])/(8*Sqrt[c]*f^3) + (((c*e - b*f)*(e - Sqrt[e^2 - 4*d*f]
)*(f*(b*e - 2*a*f) - c*(e^2 - 2*d*f)) - 2*f*(2*c*d*f*(b*e - a*f) - f^2*(b^2
*d - a^2*f) - c^2*d*(e^2 - d*f)))*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]
) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f
- b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])
/(Sqrt[2]*f^3*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c
*e - b*f)*Sqrt[e^2 - 4*d*f]]) + ((4*c*d*f^2*(b*e - a*f) - 2*f^3*(b^2*d - a^
2*f) - 2*c^2*d*f*(e^2 - d*f) - (c*e - b*f)*(e + Sqrt[e^2 - 4*d*f])*(f*(b*e
- 2*a*f) - c*(e^2 - 2*d*f)))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2
*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e
*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqr
t[2]*f^3*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e -
b*f)*Sqrt[e^2 - 4*d*f]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
```

```
b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 977

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((b*f*(3*p + 2*q) - c*e*(2*p + q) + 2*c*f*(p
+ q)*x)*(a + b*x + c*x^2)^(p - 1)*(d + e*x + f*x^2)^(q + 1))/(2*f^2*(p + q)
*(2*p + 2*q + 1)), x] - Dist[1/(2*f^2*(p + q)*(2*p + 2*q + 1)), Int[(a + b*
x + c*x^2)^(p - 2)*(d + e*x + f*x^2)^q*Simp[(b*d - a*e)*(c*e - b*f)*(1 - p)
*(2*p + q) - (p + q)*(b^2*d*f*(1 - p) - a*(f*(b*e - 2*a*f)*(2*p + 2*q + 1)
+ c*(2*d*f - e^2*(2*p + q)))) + (2*(c*d - a*f)*(c*e - b*f)*(1 - p)*(2*p + q
) - (p + q)*((b^2 - 4*a*c)*e*f*(1 - p) + b*(c*(e^2 - 4*d*f)*(2*p + q) + f*(
2*c*d - b*e + 2*a*f)*(2*p + 2*q + 1))))*x + ((c*e - b*f)^2*(1 - p)*p + c*(p
 + q)*(f*(b*e - 2*a*f)*(4*p + 2*q - 1) - c*(2*d*f*(1 - 2*p) + e^2*(3*p + q
- 1))))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a
*c, 0] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 1] && NeQ[p + q, 0] && NeQ[2*p + 2*
q + 1, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1076

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqr
t[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}}{d+ex+fx^2}\,dx = -\frac{(4ce-5bf-2cfx)\sqrt{a+bx+cx^2}}{4f^2} - \frac{\int \frac{\frac{1}{4}\left(-4bcde+5b^2df+4af(cd-2af)\right)-\frac{1}{4}\left(8c^2de-4acef-bf(5be-\right.}{\sqrt{a+bx+cx}}}{}$$

$$= -\frac{(4ce-5bf-2cfx)\sqrt{a+bx+cx^2}}{4f^2} - \frac{\int \frac{1}{4}f\left(-4bcde+5b^2df+4af(cd-2af)\right)-\frac{1}{4}d\left(-3b^2f^2+12cf(be-a\right.}{}$$

$$= -\frac{(4ce-5bf-2cfx)\sqrt{a+bx+cx^2}}{4f^2} + \frac{\left(3b^2f^2-12cf(be-af)+8c^2\left(e^2-df\right)\right)\text{Subst}}{4f^3}$$

$$= -\frac{(4ce-5bf-2cfx)\sqrt{a+bx+cx^2}}{4f^2} + \frac{\left(3b^2f^2-12cf(be-af)+8c^2\left(e^2-df\right)\right)\tanh^{-}}{8\sqrt{c}\,f^3}$$

$$= -\frac{(4ce-5bf-2cfx)\sqrt{a+bx+cx^2}}{4f^2} + \frac{\left(3b^2f^2-12cf(be-af)+8c^2\left(e^2-df\right)\right)\tanh^{-}}{8\sqrt{c}\,f^3}$$

Mathematica [A] time = 4.58, size = 1232, normalized size = 1.81

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x + c*x^2)^(3/2)/(d + e*x + f*x^2),x]
```

```
[Out] ((-2*b^2*f^2 - 16*a*c*f^2 + 10*b*c*f*(e + Sqrt[e^2 - 4*d*f]) - 4*c^2*(e + S
qrt[e^2 - 4*d*f])^2 - 4*b*c*f^2*x + 4*c^2*f*(e + Sqrt[e^2 - 4*d*f])*x)*Sqrt
[a + x*(b + c*x)] + 2*Sqrt[a + x*(b + c*x)]*(b^2*f^2 - 2*c^2*(-2*e^2 + 4*d*
f + 2*e*Sqrt[e^2 - 4*d*f] + e*f*x - f*Sqrt[e^2 - 4*d*f]*x) + c*f*(8*a*f + b
*(-5*e + 5*Sqrt[e^2 - 4*d*f] + 2*f*x))) - ((b*f + c*(-e + Sqrt[e^2 - 4*d*f]
))*(b^2*f^2 + 4*c^2*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f]) - 4*c*f*(3*a*f + b
*(-e + Sqrt[e^2 - 4*d*f])))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b +
c*x)])])/(Sqrt[c]*f) + ((-(b*f) + c*(e + Sqrt[e^2 - 4*d*f]))*(-(b^2*f^2) +
4*c^2*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) - 4*c*f*(-3*a*f + b*(e + Sqrt[e^2
 - 4*d*f])))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/(Sqrt[
c]*f) + (8*Sqrt[2]*c*(c^2*(e^4 - 4*d*e^2*f + 2*d^2*f^2 + e^3*Sqrt[e^2 - 4*d
*f] - 2*d*e*f*Sqrt[e^2 - 4*d*f]) + f^2*(2*a^2*f^2 - 2*a*b*f*(e + Sqrt[e^2 -
 4*d*f]) + b^2*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])) + 2*c*f*(a*f*(e^2 - 2*d
*f + e*Sqrt[e^2 - 4*d*f]) - b*(e^3 - 3*d*e*f + e^2*Sqrt[e^2 - 4*d*f] - d*f*
```

```
Sqrt[e^2 - 4*d*f])))*ArcTanh[(4*a*f - 2*c*(e + Sqrt[e^2 - 4*d*f])*x - b*(e
+ Sqrt[e^2 - 4*d*f] - 2*f*x))/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 -
 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c*x)])])/
(f*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2
- 4*d*f]))]) + (8*Sqrt[2]*c*(c^2*(-e^4 + 4*d*e^2*f - 2*d^2*f^2 + e^3*Sqrt[e
^2 - 4*d*f] - 2*d*e*f*Sqrt[e^2 - 4*d*f]) + f^2*(-2*a^2*f^2 + 2*a*b*f*(e - S
qrt[e^2 - 4*d*f]) + b^2*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f])) + 2*c*f*(a*f*
(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f]) + b*(e^3 - 3*d*e*f - e^2*Sqrt[e^2 - 4*
d*f] + d*f*Sqrt[e^2 - 4*d*f])))*ArcTanh[(4*a*f + 2*c*(-e + Sqrt[e^2 - 4*d*f
])*x + b*(-e + Sqrt[e^2 - 4*d*f] + 2*f*x))/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f -
 e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f + b*(-e + Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*
(b + c*x)])])/(f*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f + b*
(-e + Sqrt[e^2 - 4*d*f]))]))/(16*c*f^2*Sqrt[e^2 - 4*d*f])
```

IntegrateAlgebraic [C] time = 1.67, size = 1160, normalized size = 1.71

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x + c*x^2)^(3/2)/(d + e*x + f*x^2),x]
```

```
[Out] ((-4*c*e + 5*b*f + 2*c*f*x)*Sqrt[a + b*x + c*x^2])/(4*f^2) + ((-8*c^2*e^2 +
 8*c^2*d*f + 12*b*c*e*f - 3*b^2*f^2 - 12*a*c*f^2)*Log[b + 2*c*x - 2*Sqrt[c]
*Sqrt[a + b*x + c*x^2]])/(8*Sqrt[c]*f^3) + RootSum[b^2*d - a*b*e + a^2*f -
4*b*Sqrt[c]*d*#1 + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 + b*e*#1^2 - 2*a*f*#1^2 -
2*Sqrt[c]*e*#1^3 + f*#1^4 & , (b*c^2*d*e^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x
+ c*x^2] - #1] - a*c^2*e^3*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] -
 b*c^2*d^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*b^2*c*d*e*f
*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + 2*a*c^2*d*e*f*Log[-(Sqrt[
c]*x) + Sqrt[a + b*x + c*x^2] - #1] + 2*a*b*c*e^2*f*Log[-(Sqrt[c]*x) + Sqrt
[a + b*x + c*x^2] - #1] + b^3*d*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2
] - #1] - a*b^2*e*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*a^
2*c*e*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + a^2*b*f^3*Log[-(
Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*c^(5/2)*d*e^2*Log[-(Sqrt[c]*x)
 + Sqrt[a + b*x + c*x^2] - #1]*#1 + 2*c^(5/2)*d^2*f*Log[-(Sqrt[c]*x) + Sqrt
[a + b*x + c*x^2] - #1]*#1 + 4*b*c^(3/2)*d*e*f*Log[-(Sqrt[c]*x) + Sqrt[a +
b*x + c*x^2] - #1]*#1 - 2*b^2*Sqrt[c]*d*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x
 + c*x^2] - #1]*#1 - 4*a*c^(3/2)*d*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*
x^2] - #1]*#1 + 2*a^2*Sqrt[c]*f^3*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2]
- #1]*#1 + c^2*e^3*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2 - 2*
c^2*d*e*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2 - 2*b*c*e^2*f
*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2 + 2*b*c*d*f^2*Log[-(Sq
rt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2 + b^2*e*f^2*Log[-(Sqrt[c]*x) +
Sqrt[a + b*x + c*x^2] - #1]*#1^2 + 2*a*c*e*f^2*Log[-(Sqrt[c]*x) + Sqrt[a +
b*x + c*x^2] - #1]*#1^2 - 2*a*b*f^3*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2
```

```
] - #1]*#1^2)/(2*b*Sqrt[c]*d - a*Sqrt[c]*e - 4*c*d*#1 - b*e*#1 + 2*a*f*#1 +
 3*Sqrt[c]*e*#1^2 - 2*f*#1^3) & ]/f^3
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.03, size = 22523, normalized size = 33.17

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x)
```

```
[Out] result too large to display
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re details)Is 4*d*f-e^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^2+b\,x+a\right)^{3/2}}{f\,x^2+e\,x+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^(3/2)/(d + e*x + f*x^2),x)
[Out] int((a + b*x + c*x^2)^(3/2)/(d + e*x + f*x^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)/(f*x**2+e*x+d),x)
[Out] Timed out
```

3.104 $\int \frac{\left(a+bx+cx^2\right)^{3/2}}{\left(d+ex+fx^2\right)^2} \, dx$

Optimal. Leaf size=704

$$-\frac{\left(\left(e-\sqrt{e^2-4df}\right)(ce-bf)\left(f(be-2af)+2c\left(e^2-5df\right)\right)-2f\left(f\left(-be(3af+cd)+4af(af+cd)+2b^2df\right)+2\right.}{2\sqrt{2}f^2\left(e^2-4df\right)^{3/2}\sqrt{2af^2-\sqrt{e^2-4df}\,(ce-bf)-b}}$$

Rubi [A] time = 11.95, antiderivative size = 704, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {971, 1066, 1076, 621, 206, 1032, 724}

$$-\frac{\left(\left(e-\sqrt{e^2-4df}\right)(ce-bf)\left(f(be-2af)+2c\left(e^2-5df\right)\right)-2f\left(f\left(-be(3af+cd)+4af(af+cd)+2b^2df\right)+2c^2d\left(e^2-4df\right)\right)\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{2\sqrt{2}f^2\left(e^2-4df\right)^{3/2}\sqrt{2af^2-\sqrt{e^2-4df}\,(ce-bf)-bef-2cdf+ce^2}}+\frac{\left(\left(\sqrt{e^2-4df}+e\right)(ce-bf)\left(f(be-2af)+2c\left(e^2-5df\right)\right)-2f\left(f\left(-be(3af+cd)+4af(af+cd)+2b^2df\right)+2c^2d\left(e^2-4df\right)\right)\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{2\sqrt{2}f^2\left(e^2-4df\right)^{3/2}\sqrt{2af^2+\sqrt{e^2-4df}\,(ce-bf)-bef-2cdf+ce^2}}+\frac{c^{3/2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{f^2}-\frac{\sqrt{a+bx+cx^2}\,(-2bf+ce-2cfx)}{f\left(e^2-4df\right)}-\frac{(e+2fx)\left(a+bx+cx^2\right)^{3/2}}{\left(e^2-4df\right)\left(d+ex+fx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(3/2)/(d + e*x + f*x^2)^2,x]
```

```
[Out] -(((c*e - 2*b*f - 2*c*f*x)*Sqrt[a + b*x + c*x^2])/(f*(e^2 - 4*d*f))) - ((e
+ 2*f*x)*(a + b*x + c*x^2)^(3/2))/((e^2 - 4*d*f)*(d + e*x + f*x^2)) + (c^(3
/2)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/f^2 - (((c*e -
b*f)*(f*(b*e - 2*a*f) + 2*c*(e^2 - 5*d*f))*(e - Sqrt[e^2 - 4*d*f]) - 2*f*(2
*c^2*d*(e^2 - 4*d*f) + f*(2*b^2*d*f + 4*a*f*(c*d + a*f) - b*e*(c*d + 3*a*f)
)))*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 -
 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f
)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[2]*f^2*(e^2 - 4*d*f)^
(3/2)*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f
]]) + (((c*e - b*f)*(f*(b*e - 2*a*f) + 2*c*(e^2 - 5*d*f))*(e + Sqrt[e^2 - 4
*d*f]) - 2*f*(2*c^2*d*(e^2 - 4*d*f) + f*(2*b^2*d*f + 4*a*f*(c*d + a*f) - b*
e*(c*d + 3*a*f))))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*
(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f
^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[2]*f^2
*(e^2 - 4*d*f)^(3/2)*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*S
qrt[e^2 - 4*d*f]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 971

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((b + 2*c*x)*(a + b*x + c*x^2)^(p + 1)*(d + e
*x + f*x^2)^q)/((b^2 - 4*a*c)*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*(p + 1))
, Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*Simp[2*c*d*(2*p +
 3) + b*e*q + (2*b*f*q + 2*c*e*(2*p + q + 3))*x + 2*c*f*(2*p + 2*q + 3)*x^2
, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
e^2 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0] &&  !IGtQ[q, 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1066

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((B*c*f*(2*
p + 2*q + 3) + C*(b*f*p - c*e*(2*p + q + 2)) + 2*c*C*f*(p + q + 1)*x)*(a +
b*x + c*x^2)^p*(d + e*x + f*x^2)^(q + 1))/(2*c*f^2*(p + q + 1)*(2*p + 2*q +
 3)), x] - Dist[1/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), Int[(a + b*x + c*x
^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[p*(b*d - a*e)*(C*(c*e - b*f)*(q + 1) -
 c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(b^2*C*d*f*p + a*c*(C*(2*d*f
- e^2*(2*p + q + 2)) + f*(B*e - 2*A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*f)
*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*e
*f*p*(b^2 - 4*a*c) - b*c*(C*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e +
2*A*f)*(2*p + 2*q + 3))))*x + (p*(c*e - b*f)*(C*(c*e - b*f)*(q + 1) - c*(C*
e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(b^2 - 4*a*c) - c^2*(C*(e^
2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3))))*x^2,
```

```
x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*
q + 3, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

Rule 1076

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqr
t[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}}{\left(d+ex+fx^2\right)^2}\,dx = -\frac{(e+2fx)\left(a+bx+cx^2\right)^{3/2}}{\left(e^2-4df\right)\left(d+ex+fx^2\right)} - \frac{\int \frac{\sqrt{a+bx+cx^2}\left(\frac{1}{2}(3be-4af)+(3ce+bf)x+4cfx^2\right)}{d+ex+fx^2}\,dx}{-e^2+4df}$$

$$= -\frac{(ce-2bf-2cfx)\sqrt{a+bx+cx^2}}{f\left(e^2-4df\right)} - \frac{(e+2fx)\left(a+bx+cx^2\right)^{3/2}}{\left(e^2-4df\right)\left(d+ex+fx^2\right)} - \frac{\int \frac{cf\left(2b^2df+4af(cd+a\ldots\right.}{\ldots}}{\ldots}$$

$$= -\frac{(ce-2bf-2cfx)\sqrt{a+bx+cx^2}}{f\left(e^2-4df\right)} - \frac{(e+2fx)\left(a+bx+cx^2\right)^{3/2}}{\left(e^2-4df\right)\left(d+ex+fx^2\right)} + \frac{c^2\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{f^2}\ldots$$

$$= -\frac{(ce-2bf-2cfx)\sqrt{a+bx+cx^2}}{f\left(e^2-4df\right)} - \frac{(e+2fx)\left(a+bx+cx^2\right)^{3/2}}{\left(e^2-4df\right)\left(d+ex+fx^2\right)} + \frac{\left(2c^2\right)\text{Subst}\left(\int \frac{1}{4c-\ldots}\ldots\right.}{\ldots}$$

$$= -\frac{(ce-2bf-2cfx)\sqrt{a+bx+cx^2}}{f\left(e^2-4df\right)} - \frac{(e+2fx)\left(a+bx+cx^2\right)^{3/2}}{\left(e^2-4df\right)\left(d+ex+fx^2\right)} + \frac{c^{3/2}\tanh^{-1}\left(\frac{b\ldots}{2\sqrt{c}\sqrt{\ldots}}\right.}{f^2}\ldots$$

$$= -\frac{(ce-2bf-2cfx)\sqrt{a+bx+cx^2}}{f\left(e^2-4df\right)} - \frac{(e+2fx)\left(a+bx+cx^2\right)^{3/2}}{\left(e^2-4df\right)\left(d+ex+fx^2\right)} + \frac{c^{3/2}\tanh^{-1}\left(\frac{b\ldots}{2\sqrt{c}\sqrt{\ldots}}\right.}{f^2}\ldots$$

Mathematica [B] time = 6.83, size = 2843, normalized size = 4.04

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x + c*x^2)^(3/2)/(d + e*x + f*x^2)^2,x]
```

```
[Out] (-2*f*(a + x*(b + c*x))^(3/2))/((e^2 - 4*d*f)*(e - Sqrt[e^2 - 4*d*f] + 2*f*
x)) - (2*f*(a + x*(b + c*x))^(3/2))/((e^2 - 4*d*f)*(e + Sqrt[e^2 - 4*d*f] +
 2*f*x)) - (3*f*(a + x*(b + c*x))^(3/2)*(((-4*b*c*f - 2*c*(b*f + 2*c*(-e +
Sqrt[e^2 - 4*d*f])) - 4*c^2*f*x)*Sqrt[a + b*x + c*x^2])/(8*c*f^2) - ((2*Sqr
t[c]*(b^2*f^2 + 4*c^2*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + 4*c*f*(a*f - b*
(e - Sqrt[e^2 - 4*d*f])))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x
^2])])/f + (2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - c*e*Sqrt[e^2
 - 4*d*f] + b*f*Sqrt[e^2 - 4*d*f]]*(4*c*f*(8*a*b*f^2 - 3*b^2*f*(e - Sqrt[e^
2 - 4*d*f]) - 4*a*c*f*(e - Sqrt[e^2 - 4*d*f]) + 4*b*c*(e^2 - 2*d*f - e*Sqrt
[e^2 - 4*d*f])) + 4*c*(-e + Sqrt[e^2 - 4*d*f])*(b^2*f^2 + 4*c^2*(e^2 - 2*d*
f - e*Sqrt[e^2 - 4*d*f]) + 4*c*f*(a*f - b*(e - Sqrt[e^2 - 4*d*f]))))*ArcTan
h[(-4*a*f - b*(-e + Sqrt[e^2 - 4*d*f]) - (2*b*f + 2*c*(-e + Sqrt[e^2 - 4*d*
f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - c*e*Sqrt[e^2 -
4*d*f] + b*f*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(f*(16*a*f^2 + 8*b
*f*(-e + Sqrt[e^2 - 4*d*f]) + 4*c*(-e + Sqrt[e^2 - 4*d*f])^2)))/(16*c*f^2))
)/((e^2 - 4*d*f)*(a + b*x + c*x^2)^(3/2)) + (f*(a + x*(b + c*x))^(3/2)*(((-
4*c*f*(4*a*f - b*(e - Sqrt[e^2 - 4*d*f])) - 2*(b*f - c*(e - Sqrt[e^2 - 4*d*
f]))*(b*f + 2*c*(-e + Sqrt[e^2 - 4*d*f])) - 4*c*f*(b*f - c*(e - Sqrt[e^2 -
4*d*f]))*x)*Sqrt[a + b*x + c*x^2])/(8*c*f^2) - ((-2*(b*f - c*(e - Sqrt[e^2
- 4*d*f]))*(b^2*f^2 - 4*c^2*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) - 4*c*f*(3*
a*f - b*(e - Sqrt[e^2 - 4*d*f])))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b
*x + c*x^2])])/(Sqrt[c]*f) + (2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*
f^2 - c*e*Sqrt[e^2 - 4*d*f] + b*f*Sqrt[e^2 - 4*d*f]]*(-4*(-e + Sqrt[e^2 - 4
*d*f])*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*(b^2*f^2 - 4*c^2*(e^2 - 2*d*f - e*
Sqrt[e^2 - 4*d*f]) - 4*c*f*(3*a*f - b*(e - Sqrt[e^2 - 4*d*f]))) + 4*f*(2*c*
f*(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]))^2 - (e - Sqrt[e^2 - 4*d*f])*(b*f - c*
(e - Sqrt[e^2 - 4*d*f]))*(b^2*f + 4*a*c*f - 2*b*c*(e - Sqrt[e^2 - 4*d*f])))
)*ArcTanh[(-4*a*f - b*(-e + Sqrt[e^2 - 4*d*f]) - (2*b*f + 2*c*(-e + Sqrt[e^
2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - c*e*Sqr
t[e^2 - 4*d*f] + b*f*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(f*(16*a*f
^2 + 8*b*f*(-e + Sqrt[e^2 - 4*d*f]) + 4*c*(-e + Sqrt[e^2 - 4*d*f])^2)))/(16
*c*f^2)))/((e^2 - 4*d*f)^(3/2)*(a + b*x + c*x^2)^(3/2)) - (f*(a + x*(b + c*
x))^(3/2)*(((4*c*f*(-4*a*f + b*(e + Sqrt[e^2 - 4*d*f])) + 2*(b*f - c*(e + S
qrt[e^2 - 4*d*f]))*(-(b*f) + 2*c*(e + Sqrt[e^2 - 4*d*f])) - 4*c*f*(b*f - c*
(e + Sqrt[e^2 - 4*d*f]))*x)*Sqrt[a + b*x + c*x^2])/(8*c*f^2) - ((-2*(b*f -
c*(e + Sqrt[e^2 - 4*d*f]))*(b^2*f^2 - 4*c^2*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d
*f]) - 4*c*f*(3*a*f - b*(e + Sqrt[e^2 - 4*d*f])))*ArcTanh[(b + 2*c*x)/(2*Sq
rt[c]*Sqrt[a + b*x + c*x^2])])/(Sqrt[c]*f) - (2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*
f - b*e*f + 2*a*f^2 + c*e*Sqrt[e^2 - 4*d*f] - b*f*Sqrt[e^2 - 4*d*f]]*(4*(e
+ Sqrt[e^2 - 4*d*f])*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*(b^2*f^2 - 4*c^2*(e^
2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) - 4*c*f*(3*a*f - b*(e + Sqrt[e^2 - 4*d*f])
```

```
)) + 4*f*(2*c*f*(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]))^2 - (e + Sqrt[e^2 - 4*d
*f])*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*(b^2*f + 4*a*c*f - 2*b*c*(e + Sqrt[e
^2 - 4*d*f]))))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) - (-2*b*f + 2*c*
(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f
^2 + c*e*Sqrt[e^2 - 4*d*f] - b*f*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])]
)/(f*(16*a*f^2 - 8*b*f*(e + Sqrt[e^2 - 4*d*f]) + 4*c*(e + Sqrt[e^2 - 4*d*f]
)^2)))/(16*c*f^2)))/((e^2 - 4*d*f)^(3/2)*(a + b*x + c*x^2)^(3/2)) + (3*f*(a
 + x*(b + c*x))^(3/2)*(((4*b*c*f - 2*c*(-(b*f) + 2*c*(e + Sqrt[e^2 - 4*d*f]
)) + 4*c^2*f*x)*Sqrt[a + b*x + c*x^2])/(8*c*f^2) - ((-2*Sqrt[c]*(b^2*f^2 +
4*c^2*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + 4*c*f*(a*f - b*(e + Sqrt[e^2 -
4*d*f])))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/f - (2*Sq
rt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + c*e*Sqrt[e^2 - 4*d*f] - b*f*
Sqrt[e^2 - 4*d*f]]*(4*c*(e + Sqrt[e^2 - 4*d*f])*(b^2*f^2 + 4*c^2*(e^2 - 2*d
*f + e*Sqrt[e^2 - 4*d*f]) + 4*c*f*(a*f - b*(e + Sqrt[e^2 - 4*d*f]))) + 4*c*
f*(3*b^2*f*(e + Sqrt[e^2 - 4*d*f]) + 4*a*c*f*(e + Sqrt[e^2 - 4*d*f]) - 4*b*
(2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]))))*ArcTanh[(4*a*f - b*(e +
 Sqrt[e^2 - 4*d*f]) - (-2*b*f + 2*c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*
Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + c*e*Sqrt[e^2 - 4*d*f] - b*f*Sqrt[e
^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(f*(16*a*f^2 - 8*b*f*(e + Sqrt[e^2 -
4*d*f]) + 4*c*(e + Sqrt[e^2 - 4*d*f])^2)))/(16*c*f^2)))/((e^2 - 4*d*f)*(a +
 b*x + c*x^2)^(3/2))
```

IntegrateAlgebraic [C] time = 5.18, size = 2427, normalized size = 3.45

Result too large to show

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x + c*x^2)^(3/2)/(d + e*x + f*x^2)^2,x]
```

```
[Out] ((c*d*e - 2*b*d*f + a*e*f + c*e^2*x - 2*c*d*f*x - b*e*f*x + 2*a*f^2*x)*Sqrt
[a + b*x + c*x^2])/(f*(-e^2 + 4*d*f)*(d + e*x + f*x^2)) - (c^(3/2)*Log[b +
2*c*x - 2*Sqrt[c]*Sqrt[a + b*x + c*x^2]])/f^2 - RootSum[b^2*d - a*b*e + a^2
*f - 4*b*Sqrt[c]*d*#1 + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 + b*e*#1^2 - 2*a*f*#1
^2 - 2*Sqrt[c]*e*#1^3 + f*#1^4 & , (4*c^3*e^3*Log[-(Sqrt[c]*x) + Sqrt[a + b
*x + c*x^2] - #1] - 8*c^3*d*e*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] -
#1] - 9*b*c^2*e^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + 10*b*c
^2*d*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + 6*b^2*c*e*f^2*Log
[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + 6*a*c^2*e*f^2*Log[-(Sqrt[c]*x
) + Sqrt[a + b*x + c*x^2] - #1] - b^3*f^3*Log[-(Sqrt[c]*x) + Sqrt[a + b*x +
 c*x^2] - #1] - 8*a*b*c*f^3*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]
+ 2*c^(5/2)*e^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - 4*c^(
5/2)*d*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - 4*b*c^(3/2)*
e*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 + 2*b^2*Sqrt[c]*f^3
*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 + 4*a*c^(3/2)*f^3*Log[-(
Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 + 2*c^2*e*f^2*Log[-(Sqrt[c]*x)
```

```
+ Sqrt[a + b*x + c*x^2] - #1]*#1^2 - 2*b*c*f^3*Log[-(Sqrt[c]*x) + Sqrt[a +
b*x + c*x^2] - #1]*#1^2)/(-2*b*Sqrt[c]*d + a*Sqrt[c]*e + 4*c*d*#1 + b*e*#1
- 2*a*f*#1 - 3*Sqrt[c]*e*#1^2 + 2*f*#1^3) & ]/f^4 - RootSum[b^2*d - a*b*e +
 a^2*f - 4*b*Sqrt[c]*d*#1 + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 + b*e*#1^2 - 2*a*
f*#1^2 - 2*Sqrt[c]*e*#1^3 + f*#1^4 & , (8*c^3*e^5*Log[-(Sqrt[c]*x) + Sqrt[a
 + b*x + c*x^2] - #1] - 48*c^3*d*e^3*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*
x^2] - #1] - 18*b*c^2*e^4*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]
+ 64*c^3*d^2*e*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + 90*b*c^
2*d*e^2*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + 12*b^2*c*e^3*f
^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + 14*a*c^2*e^3*f^2*Log[-(
Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 72*b*c^2*d^2*f^3*Log[-(Sqrt[c]*x
) + Sqrt[a + b*x + c*x^2] - #1] - 47*b^2*c*d*e*f^3*Log[-(Sqrt[c]*x) + Sqrt[
a + b*x + c*x^2] - #1] - 58*a*c^2*d*e*f^3*Log[-(Sqrt[c]*x) + Sqrt[a + b*x +
 c*x^2] - #1] - 2*b^3*e^2*f^3*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1
] - 17*a*b*c*e^2*f^3*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + 6*b^3
*d*f^4*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + 70*a*b*c*d*f^4*Log[
-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + 2*a*b^2*e*f^4*Log[-(Sqrt[c]*x)
 + Sqrt[a + b*x + c*x^2] - #1] - 2*a^2*c*e*f^4*Log[-(Sqrt[c]*x) + Sqrt[a +
b*x + c*x^2] - #1] - 2*a^2*b*f^5*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] -
 #1] + 4*c^(5/2)*e^4*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 -
20*c^(5/2)*d*e^2*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - 8*
b*c^(3/2)*e^3*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 + 16*c^
(5/2)*d^2*f^3*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 + 30*b*c^(3
/2)*d*e*f^3*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 + 4*b^2*Sqrt[
c]*e^2*f^3*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 + 8*a*c^(3/2)*
e^2*f^3*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - 12*b^2*Sqrt[c]*
d*f^4*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - 24*a*c^(3/2)*d*f^
4*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - 6*a*b*Sqrt[c]*e*f^4*L
og[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 + 8*a^2*Sqrt[c]*f^5*Log[-(
Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 + 2*c^2*e^3*f^2*Log[-(Sqrt[c]*x
) + Sqrt[a + b*x + c*x^2] - #1]*#1^2 - 6*c^2*d*e*f^3*Log[-(Sqrt[c]*x) + Sqr
t[a + b*x + c*x^2] - #1]*#1^2 - 3*b*c*e^2*f^3*Log[-(Sqrt[c]*x) + Sqrt[a + b
*x + c*x^2] - #1]*#1^2 + 6*b*c*d*f^4*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^
2] - #1]*#1^2 + b^2*e*f^4*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1
^2 + 2*a*c*e*f^4*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2 - 2*a*
b*f^5*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2)/(-2*b*Sqrt[c]*d
+ a*Sqrt[c]*e + 4*c*d*#1 + b*e*#1 - 2*a*f*#1 - 3*Sqrt[c]*e*#1^2 + 2*f*#1^3)
 & ]/(2*f^4*(-e^2 + 4*d*f))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{%%{[-1,0]:[1,0,%%%{-1,[1]%%%}]%%},[8,4,8,0,0,0]%%%}+%%%{%%{[%%%{16,[
1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[8,4,6,0,1,0]%%%}+%%%{%%{[%%%{-96,[2]%%%}
,0]:[1,0,%%%{-1,[1]%%%}]%%},[8,4,4,0,2,0]%%%}+%%%{%%{[%%%{256,[3]%%%},0]:[1
,0,%%%{-1,[1]%%%}]%%},[8,4,2,0,3,0]%%%}+%%%{%%{[%%%{-256,[4]%%%},0]:[1,0,%%
%{-1,[1]%%%}]%%},[8,4,0,0,4,0]%%%}+%%%{%%%{4,[1]%%%},[7,3,8,1,0,0]%%%}+%%%{
%%%{-64,[2]%%%},[7,3,6,1,1,0]%%%}+%%%{%%%{384,[3]%%%},[7,3,4,1,2,0]%%%}+%%%
{%%%{-1024,[4]%%%},[7,3,2,1,3,0]%%%}+%%%{%%%{1024,[5]%%%},[7,3,0,1,4,0]%%%}
+%%%{%%{[4,0]:[1,0,%%%{-1,[1]%%%}]%%},[6,4,8,0,1,0]%%%}+%%%{%%{[%%%{-64,[1]
%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[6,4,6,0,2,0]%%%}+%%%{%%{[%%%{384,[2]%%%},0
]:[1,0,%%%{-1,[1]%%%}]%%},[6,4,4,0,3,0]%%%}+%%%{%%{[%%%{-1024,[3]%%%},0]:[1
,0,%%%{-1,[1]%%%}]%%},[6,4,2,0,4,0]%%%}+%%%{%%{[%%%{1024,[4]%%%},0]:[1,0,%%
%{-1,[1]%%%}]%%},[6,4,0,0,5,0]%%%}+%%%{%%{[-2,0]:[1,0,%%%{-1,[1]%%%}]%%},[6
,3,9,1,0,0]%%%}+%%%{%%{[%%%{-8,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[6,3,8,0,
0,1]%%%}+%%%{%%{[%%%{32,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[6,3,7,1,1,0]%%%
}+%%%{%%{[%%%{128,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[6,3,6,0,1,1]%%%}+%%%{
%%{[%%%{-192,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[6,3,5,1,2,0]%%%}+%%%{%%{[%
%%{-768,[3]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[6,3,4,0,2,1]%%%}+%%%{%%{[%%%{51
2,[3]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[6,3,3,1,3,0]%%%}+%%%{%%{[%%%{2048,[4]
%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[6,3,2,0,3,1]%%%}+%%%{%%{[%%%{-512,[4]%%%},
0]:[1,0,%%%{-1,[1]%%%}]%%},[6,3,1,1,4,0]%%%}+%%%{%%{[%%%{-2048,[5]%%%},0]:[
1,0,%%%{-1,[1]%%%}]%%},[6,3,0,0,4,1]%%%}+%%%{%%{[%%%{-4,[1]%%%},0]:[1,0,%%%
{-1,[1]%%%}]%%},[6,2,8,2,0,0]%%%}+%%%{%%{[%%%{64,[2]%%%},0]:[1,0,%%%{-1,[1]
%%%}]%%},[6,2,6,2,1,0]%%%}+%%%{%%{[%%%{-384,[3]%%%},0]:[1,0,%%%{-1,[1]%%%}]
%%},[6,2,4,2,2,0]%%%}+%%%{%%{[%%%{1024,[4]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[
6,2,2,2,3,0]%%%}+%%%{%%{[%%%{-1024,[5]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[6,2,
0,2,4,0]%%%}+%%%{%%%{8,[1]%%%},[5,3,9,0,0,1]%%%}+%%%{%%%{-12,[1]%%%},[5,3,8
,1,1,0]%%%}+%%%{%%%{-128,[2]%%%},[5,3,7,0,1,1]%%%}+%%%{%%%{192,[2]%%%},[5,3
,6,1,2,0]%%%}+%%%{%%%{768,[3]%%%},[5,3,5,0,2,1]%%%}+%%%{%%%{-1152,[3]%%%},[
5,3,4,1,3,0]%%%}+%%%{%%%{-2048,[4]%%%},[5,3,3,0,3,1]%%%}+%%%{%%%{3072,[4]%%
%},[5,3,2,1,4,0]%%%}+%%%{%%%{2048,[5]%%%},[5,3,1,0,4,1]%%%}+%%%{%%%{-3072,[
5]%%%},[5,3,0,1,5,0]%%%}+%%%{%%%{4,[1]%%%},[5,2,9,2,0,0]%%%}+%%%{%%%{16,[2]
%%%},[5,2,8,1,0,1]%%%}+%%%{%%%{-64,[2]%%%},[5,2,7,2,1,0]%%%}+%%%{%%%{-256,[
3]%%%},[5,2,6,1,1,1]%%%}+%%%{%%%{384,[3]%%%},[5,2,5,2,2,0]%%%}+%%%{%%%{1536
,[4]%%%},[5,2,4,1,2,1]%%%}+%%%{%%%{-1024,[4]%%%},[5,2,3,2,3,0]%%%}+%%%{%%%{
```

-4096,[5]%%%},[5,2,2,1,3,1]%%%}+%%%{%%%{1024,[5]%%%},[5,2,1,2,4,0]%%%}+%%%{
%%%{4096,[6]%%%},[5,2,0,1,4,1]%%%}+%%%{%%{[-6,0]:[1,0,%%%{-1,[1]%%%}]%%},[4
,4,8,0,2,0]%%%}+%%%{%%{[%%%{96,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[4,4,6,0,
3,0]%%%}+%%%{%%{[%%%{-576,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[4,4,4,0,4,0]%
%%}+%%%{%%{[%%%{1536,[3]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[4,4,2,0,5,0]%%%}+%
%%{%%{[%%%{-1536,[4]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[4,4,0,0,6,0]%%%}+%%%{%
%{[-2,0]:[1,0,%%%{-1,[1]%%%}]%%},[4,3,10,0,0,1]%%%}+%%%{%%{[6,0]:[1,0,%%%{-
1,[1]%%%}]%%},[4,3,9,1,1,0]%%%}+%%%{%%{[%%%{48,[1]%%%},0]:[1,0,%%%{-1,[1]%%
%}]%%},[4,3,8,0,1,1]%%%}+%%%{%%{[%%%{-96,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%}
,[4,3,7,1,2,0]%%%}+%%%{%%{[%%%{-448,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[4,3
,6,0,2,1]%%%}+%%%{%%{[%%%{576,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[4,3,5,1,3
,0]%%%}+%%%{%%{[%%%{2048,[3]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[4,3,4,0,3,1]%%
%}+%%%{%%{[%%%{-1536,[3]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[4,3,3,1,4,0]%%%}+%
%%{%%{[%%%{-4608,[4]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[4,3,2,0,4,1]%%%}+%%%{%
%{[%%%{1536,[4]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[4,3,1,1,5,0]%%%}+%%%{%%{[%%
%{4096,[5]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[4,3,0,0,5,1]%%%}+%%%{%%{[-1,0]:[
1,0,%%%{-1,[1]%%%}]%%},[4,2,10,2,0,0]%%%}+%%%{%%{[%%%{-24,[1]%%%},0]:[1,0,%
%%{-1,[1]%%%}]%%},[4,2,9,1,0,1]%%%}+%%%{%%{[%%%{24,[1]%%%},0]:[1,0,%%%{-1,[
1]%%%}]%%},[4,2,8,2,1,0]%%%}+%%%{%%{[%%%{-16,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}
]%%},[4,2,8,0,0,2]%%%}+%%%{%%{[%%%{384,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[
4,2,7,1,1,1]%%%}+%%%{%%{[%%%{-224,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[4,2,6
,2,2,0]%%%}+%%%{%%{[%%%{256,[3]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[4,2,6,0,1,2
]%%%}+%%%{%%{[%%%{-2304,[3]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[4,2,5,1,2,1]%%%
}+%%%{%%{[%%%{1024,[3]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[4,2,4,2,3,0]%%%}+%%%
{%%{[%%%{-1536,[4]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[4,2,4,0,2,2]%%%}+%%%{%%{
[%%%{6144,[4]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[4,2,3,1,3,1]%%%}+%%%{%%{[%%%{
-2304,[4]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[4,2,2,2,4,0]%%%}+%%%{%%{[%%%{4096
,[5]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[4,2,2,0,3,2]%%%}+%%%{%%{[%%%{-6144,[5]
%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[4,2,1,1,4,1]%%%}+%%%{%%{[%%%{2048,[5]%%%},
0]:[1,0,%%%{-1,[1]%%%}]%%},[4,2,0,2,5,0]%%%}+%%%{%%{[%%%{-4096,[6]%%%},0]:[
1,0,%%%{-1,[1]%%%}]%%},[4,2,0,0,4,2]%%%}+%%%{%%%{-16,[1]%%%},[3,3,9,0,1,1]%
%%}+%%%{%%%{12,[1]%%%},[3,3,8,1,2,0]%%%}+%%%{%%%{256,[2]%%%},[3,3,7,0,2,1]%
%%}+%%%{%%%{-192,[2]%%%},[3,3,6,1,3,0]%%%}+%%%{%%%{-1536,[3]%%%},[3,3,5,0,3
,1]%%%}+%%%{%%%{1152,[3]%%%},[3,3,4,1,4,0]%%%}+%%%{%%%{4096,[4]%%%},[3,3,3,
0,4,1]%%%}+%%%{%%%{-3072,[4]%%%},[3,3,2,1,5,0]%%%}+%%%{%%%{-4096,[5]%%%},[3
,3,1,0,5,1]%%%}+%%%{%%%{3072,[5]%%%},[3,3,0,1,6,0]%%%}+%%%{%%%{12,[1]%%%},[
3,2,10,1,0,1]%%%}+%%%{%%%{-8,[1]%%%},[3,2,9,2,1,0]%%%}+%%%{%%%{32,[2]%%%},[
3,2,9,0,0,2]%%%}+%%%{%%%{-208,[2]%%%},[3,2,8,1,1,1]%%%}+%%%{%%%{128,[2]%%%}
,[3,2,7,2,2,0]%%%}+%%%{%%%{-512,[3]%%%},[3,2,7,0,1,2]%%%}+%%%{%%%{1408,[3]%
%%},[3,2,6,1,2,1]%%%}+%%%{%%%{-768,[3]%%%},[3,2,5,2,3,0]%%%}+%%%{%%%{3072,[
4]%%%},[3,2,5,0,2,2]%%%}+%%%{%%%{-4608,[4]%%%},[3,2,4,1,3,1]%%%}+%%%{%%%{20
48,[4]%%%},[3,2,3,2,4,0]%%%}+%%%{%%%{-8192,[5]%%%},[3,2,3,0,3,2]%%%}+%%%{%%
%{7168,[5]%%%},[3,2,2,1,4,1]%%%}+%%%{%%%{-2048,[5]%%%},[3,2,1,2,5,0]%%%}+%%
%{%%%{8192,[6]%%%},[3,2,1,0,4,2]%%%}+%%%{%%%{-4096,[6]%%%},[3,2,0,1,5,1]%%%
}+%%%{%%{[4,0]:[1,0,%%%{-1,[1]%%%}]%%},[2,4,8,0,3,0]%%%}+%%%{%%{[%%%{-64,[1

]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[2,4,6,0,4,0]%%%}+%%%{%%{[%%%{384,[2]%%%},
0]:[1,0,%%%{-1,[1]%%%}]%%},[2,4,4,0,5,0]%%%}+%%%{%%{[%%%{-1024,[3]%%%},0]:[
1,0,%%%{-1,[1]%%%}]%%},[2,4,2,0,6,0]%%%}+%%%{%%{[%%%{1024,[4]%%%},0]:[1,0,%
%%{-1,[1]%%%}]%%},[2,4,0,0,7,0]%%%}+%%%{%%{[4,0]:[1,0,%%%{-1,[1]%%%}]%%},[2
,3,10,0,1,1]%%%}+%%%{%%{[-6,0]:[1,0,%%%{-1,[1]%%%}]%%},[2,3,9,1,2,0]%%%}+%%
%{%%{[%%%{-72,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[2,3,8,0,2,1]%%%}+%%%{%%{[
%%%{96,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[2,3,7,1,3,0]%%%}+%%%{%%{[%%%{512
,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[2,3,6,0,3,1]%%%}+%%%{%%{[%%%{-576,[2]%
%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[2,3,5,1,4,0]%%%}+%%%{%%{[%%%{-1792,[3]%%%},
0]:[1,0,%%%{-1,[1]%%%}]%%},[2,3,4,0,4,1]%%%}+%%%{%%{[%%%{1536,[3]%%%},0]:[1
,0,%%%{-1,[1]%%%}]%%},[2,3,3,1,5,0]%%%}+%%%{%%{[%%%{3072,[4]%%%},0]:[1,0,%%
%{-1,[1]%%%}]%%},[2,3,2,0,5,1]%%%}+%%%{%%{[%%%{-1536,[4]%%%},0]:[1,0,%%%{-1
,[1]%%%}]%%},[2,3,1,1,6,0]%%%}+%%%{%%{[%%%{-2048,[5]%%%},0]:[1,0,%%%{-1,[1]
%%%}]%%},[2,3,0,0,6,1]%%%}+%%%{%%{[-2,0]:[1,0,%%%{-1,[1]%%%}]%%},[2,2,11,1,
0,1]%%%}+%%%{%%{[2,0]:[1,0,%%%{-1,[1]%%%}]%%},[2,2,10,2,1,0]%%%}+%%%{%%{[%%
%{-24,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[2,2,10,0,0,2]%%%}+%%%{%%{[%%%{56,
[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[2,2,9,1,1,1]%%%}+%%%{%%{[%%%{-36,[1]%%%
},0]:[1,0,%%%{-1,[1]%%%}]%%},[2,2,8,2,2,0]%%%}+%%%{%%{[%%%{384,[2]%%%},0]:[
1,0,%%%{-1,[1]%%%}]%%},[2,2,8,0,1,2]%%%}+%%%{%%{[%%%{-576,[2]%%%},0]:[1,0,%
%%{-1,[1]%%%}]%%},[2,2,7,1,2,1]%%%}+%%%{%%{[%%%{256,[2]%%%},0]:[1,0,%%%{-1,
[1]%%%}]%%},[2,2,6,2,3,0]%%%}+%%%{%%{[%%%{-2304,[3]%%%},0]:[1,0,%%%{-1,[1]%
%%}]%%},[2,2,6,0,2,2]%%%}+%%%{%%{[%%%{2816,[3]%%%},0]:[1,0,%%%{-1,[1]%%%}]%
%},[2,2,5,1,3,1]%%%}+%%%{%%{[%%%{-896,[3]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[2
,2,4,2,4,0]%%%}+%%%{%%{[%%%{6144,[4]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[2,2,4,
0,3,2]%%%}+%%%{%%{[%%%{-6656,[4]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[2,2,3,1,4,
1]%%%}+%%%{%%{[%%%{1536,[4]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[2,2,2,2,5,0]%%%
}+%%%{%%{[%%%{-6144,[5]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[2,2,2,0,4,2]%%%}+%%
%{%%{[%%%{6144,[5]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[2,2,1,1,5,1]%%%}+%%%{%%{
[%%%{-1024,[5]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[2,2,0,2,6,0]%%%}+%%%{%%%{8,[
1]%%%},[1,3,9,0,2,1]%%%}+%%%{%%%{-4,[1]%%%},[1,3,8,1,3,0]%%%}+%%%{%%%{-128,
[2]%%%},[1,3,7,0,3,1]%%%}+%%%{%%%{64,[2]%%%},[1,3,6,1,4,0]%%%}+%%%{%%%{768,
[3]%%%},[1,3,5,0,4,1]%%%}+%%%{%%%{-384,[3]%%%},[1,3,4,1,5,0]%%%}+%%%{%%%{-2
048,[4]%%%},[1,3,3,0,5,1]%%%}+%%%{%%%{1024,[4]%%%},[1,3,2,1,6,0]%%%}+%%%{%%
%{2048,[5]%%%},[1,3,1,0,6,1]%%%}+%%%{%%%{-1024,[5]%%%},[1,3,0,1,7,0]%%%}+%%
%{%%%{8,[1]%%%},[1,2,11,0,0,2]%%%}+%%%{%%%{-12,[1]%%%},[1,2,10,1,1,1]%%%}+%
%%{%%%{4,[1]%%%},[1,2,9,2,2,0]%%%}+%%%{%%%{-128,[2]%%%},[1,2,9,0,1,2]%%%}+%
%%{%%%{192,[2]%%%},[1,2,8,1,2,1]%%%}+%%%{%%%{-64,[2]%%%},[1,2,7,2,3,0]%%%}+
%%%{%%%{768,[3]%%%},[1,2,7,0,2,2]%%%}+%%%{%%%{-1152,[3]%%%},[1,2,6,1,3,1]%%
%}+%%%{%%%{384,[3]%%%},[1,2,5,2,4,0]%%%}+%%%{%%%{-2048,[4]%%%},[1,2,5,0,3,2
]%%%}+%%%{%%%{3072,[4]%%%},[1,2,4,1,4,1]%%%}+%%%{%%%{-1024,[4]%%%},[1,2,3,2
,5,0]%%%}+%%%{%%%{2048,[5]%%%},[1,2,3,0,4,2]%%%}+%%%{%%%{-3072,[5]%%%},[1,2
,2,1,5,1]%%%}+%%%{%%%{1024,[5]%%%},[1,2,1,2,6,0]%%%}+%%%{%%{[-1,0]:[1,0,%%%
{-1,[1]%%%}]%%},[0,4,8,0,4,0]%%%}+%%%{%%{[%%%{16,[1]%%%},0]:[1,0,%%%{-1,[1]
%%%}]%%},[0,4,6,0,5,0]%%%}+%%%{%%{[%%%{-96,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%
%},[0,4,4,0,6,0]%%%}+%%%{%%{[%%%{256,[3]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[0,

4,2,0,7,0]%%%}+%%%{%%{[%%%{-256,[4]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[0,4,0,0
,8,0]%%%}+%%%{%%{[-2,0]:[1,0,%%%{-1,[1]%%%}]%%},[0,3,10,0,2,1]%%%}+%%%{%%{[
2,0]:[1,0,%%%{-1,[1]%%%}]%%},[0,3,9,1,3,0]%%%}+%%%{%%{[%%%{32,[1]%%%},0]:[1
,0,%%%{-1,[1]%%%}]%%},[0,3,8,0,3,1]%%%}+%%%{%%{[%%%{-32,[1]%%%},0]:[1,0,%%%
{-1,[1]%%%}]%%},[0,3,7,1,4,0]%%%}+%%%{%%{[%%%{-192,[2]%%%},0]:[1,0,%%%{-1,[
1]%%%}]%%},[0,3,6,0,4,1]%%%}+%%%{%%{[%%%{192,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}
]%%},[0,3,5,1,5,0]%%%}+%%%{%%{[%%%{512,[3]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[
0,3,4,0,5,1]%%%}+%%%{%%{[%%%{-512,[3]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[0,3,3
,1,6,0]%%%}+%%%{%%{[%%%{-512,[4]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[0,3,2,0,6,
1]%%%}+%%%{%%{[%%%{512,[4]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[0,3,1,1,7,0]%%%}
+%%%{%%{[-1,0]:[1,0,%%%{-1,[1]%%%}]%%},[0,2,12,0,0,2]%%%}+%%%{%%{[2,0]:[1,0
,%%%{-1,[1]%%%}]%%},[0,2,11,1,1,1]%%%}+%%%{%%{[-1,0]:[1,0,%%%{-1,[1]%%%}]%%
},[0,2,10,2,2,0]%%%}+%%%{%%{[%%%{16,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[0,2
,10,0,1,2]%%%}+%%%{%%{[%%%{-32,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[0,2,9,1,
2,1]%%%}+%%%{%%{[%%%{16,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[0,2,8,2,3,0]%%%
}+%%%{%%{[%%%{-96,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[0,2,8,0,2,2]%%%}+%%%{
%%{[%%%{192,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[0,2,7,1,3,1]%%%}+%%%{%%{[%%
%{-96,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[0,2,6,2,4,0]%%%}+%%%{%%{[%%%{256,
[3]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[0,2,6,0,3,2]%%%}+%%%{%%{[%%%{-512,[3]%%
%},0]:[1,0,%%%{-1,[1]%%%}]%%},[0,2,5,1,4,1]%%%}+%%%{%%{[%%%{256,[3]%%%},0]:
[1,0,%%%{-1,[1]%%%}]%%},[0,2,4,2,5,0]%%%}+%%%{%%{[%%%{-256,[4]%%%},0]:[1,0,
%%%{-1,[1]%%%}]%%},[0,2,4,0,4,2]%%%}+%%%{%%{[%%%{512,[4]%%%},0]:[1,0,%%%{-1
,[1]%%%}]%%},[0,2,3,1,5,1]%%%}+%%%{%%{[%%%{-256,[4]%%%},0]:[1,0,%%%{-1,[1]%
%%}]%%},[0,2,2,2,6,0]%%%} / %%%{%%%{1,[1]%%%},[8,2,0,0,0,0]%%%}+%%%{%%{poly
1[%%%{-4,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[7,1,0,1,0,0]%%%}+%%%{%%%{-4,[1
]%%%},[6,2,0,0,1,0]%%%}+%%%{%%%{2,[1]%%%},[6,1,1,1,0,0]%%%}+%%%{%%%{8,[2]%%
%},[6,1,0,0,0,1]%%%}+%%%{%%%{4,[2]%%%},[6,0,0,2,0,0]%%%}+%%%{%%{poly1[%%%{-
8,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[5,1,1,0,0,1]%%%}+%%%{%%{poly1[%%%{12,
[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[5,1,0,1,1,0]%%%}+%%%{%%{poly1[%%%{-4,[1
]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[5,0,1,2,0,0]%%%}+%%%{%%{poly1[%%%{-16,[2]
%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[5,0,0,1,0,1]%%%}+%%%{%%%{6,[1]%%%},[4,2,0,
0,2,0]%%%}+%%%{%%%{2,[1]%%%},[4,1,2,0,0,1]%%%}+%%%{%%%{-6,[1]%%%},[4,1,1,1,
1,0]%%%}+%%%{%%%{-16,[2]%%%},[4,1,0,0,1,1]%%%}+%%%{%%%{1,[1]%%%},[4,0,2,2,0
,0]%%%}+%%%{%%%{24,[2]%%%},[4,0,1,1,0,1]%%%}+%%%{%%%{-8,[2]%%%},[4,0,0,2,1,
0]%%%}+%%%{%%%{16,[3]%%%},[4,0,0,0,0,2]%%%}+%%%{%%{poly1[%%%{16,[1]%%%},0]:
[1,0,%%%{-1,[1]%%%}]%%},[3,1,1,0,1,1]%%%}+%%%{%%{poly1[%%%{-12,[1]%%%},0]:[
1,0,%%%{-1,[1]%%%}]%%},[3,1,0,1,2,0]%%%}+%%%{%%{poly1[%%%{-12,[1]%%%},0]:[1
,0,%%%{-1,[1]%%%}]%%},[3,0,2,1,0,1]%%%}+%%%{%%{poly1[%%%{8,[1]%%%},0]:[1,0,
%%%{-1,[1]%%%}]%%},[3,0,1,2,1,0]%%%}+%%%{%%{poly1[%%%{-32,[2]%%%},0]:[1,0,%
%%{-1,[1]%%%}]%%},[3,0,1,0,0,2]%%%}+%%%{%%{poly1[%%%{16,[2]%%%},0]:[1,0,%%%
{-1,[1]%%%}]%%},[3,0,0,1,1,1]%%%}+%%%{%%%{-4,[1]%%%},[2,2,0,0,3,0]%%%}+%%%{
%%%{-4,[1]%%%},[2,1,2,0,1,1]%%%}+%%%{%%%{6,[1]%%%},[2,1,1,1,2,0]%%%}+%%%{%%
%{8,[2]%%%},[2,1,0,0,2,1]%%%}+%%%{%%%{2,[1]%%%},[2,0,3,1,0,1]%%%}+%%%{%%%{-
2,[1]%%%},[2,0,2,2,1,0]%%%}+%%%{%%%{24,[2]%%%},[2,0,2,0,0,2]%%%}+%%%{%%%{-2
4,[2]%%%},[2,0,1,1,1,1]%%%}+%%%{%%%{4,[2]%%%},[2,0,0,2,2,0]%%%}+%%%{%%{poly

1[%%%{-8,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[1,1,1,0,2,1]%%%}+%%%{%%{poly1[
%%%{4,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[1,1,0,1,3,0]%%%}+%%%{%%{poly1[%%%
{-8,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[1,0,3,0,0,2]%%%}+%%%{%%{poly1[%%%{1
2,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[1,0,2,1,1,1]%%%}+%%%{%%{poly1[%%%{-4,
[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[1,0,1,2,2,0]%%%}+%%%{%%%{1,[1]%%%},[0,2
,0,0,4,0]%%%}+%%%{%%%{2,[1]%%%},[0,1,2,0,2,1]%%%}+%%%{%%%{-2,[1]%%%},[0,1,1
,1,3,0]%%%}+%%%{%%%{1,[1]%%%},[0,0,4,0,0,2]%%%}+%%%{%%%{-2,[1]%%%},[0,0,3,1
,1,1]%%%}+%%%{%%%{1,[1]%%%},[0,0,2,2,2,0]%%%} Error: Bad Argument Value

maple [B] time = 0.04, size = 72576, normalized size = 103.09

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d)^2,x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^2 + bx + a\right)^{\frac{3}{2}}}{\left(fx^2 + ex + d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d)^2,x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^(3/2)/(f*x^2 + e*x + d)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c\,x^2+b\,x+a\right)}^{3/2}}{{\left(f\,x^2+e\,x+d\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^(3/2)/(d + e*x + f*x^2)^2,x)
```

```
[Out] int((a + b*x + c*x^2)^(3/2)/(d + e*x + f*x^2)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)/(f*x**2+e*x+d)**2,x)
```

```
[Out] Timed out
```

3.105 $\int \frac{\left(a+bx+cx^2\right)^{3/2}}{\left(d+ex+fx^2\right)^3}\,dx$

Optimal. Leaf size=671

$$-\frac{3\left(2\left(e-\sqrt{e^2-4df}\right)(ce-bf)(2af-be+2cd)-f\left(4be(3af+cd)-4a\left(4af^2+ce^2\right)-\left(b^2\left(4df+e^2\right)\right)\right)\right)\tanh}{4\sqrt{2}\left(e^2-4df\right)^{5/2}\sqrt{2af^2-\sqrt{e^2-4df}\,(ce-bf)-bef-2cdf+}}$$

Rubi [A] time = 11.60, antiderivative size = 669, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {971, 1013, 1032, 724, 206}

$$\frac{3\left(-2\left(e-\sqrt{e^2-4df}\right)(ce-bf)(2af-be+2cd)+4bef(3af+cd)-4af\left(4af^2+ce^2\right)+b^2(-f)\left(4df+e^2\right)\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}\,(ce-bf)-bef-2cdf+ce^2}}\right)}{4\sqrt{2}\left(e^2-4df\right)^{5/2}\sqrt{2af^2-\sqrt{e^2-4df}\,(ce-bf)-bef-2cdf+ce^2}}-\frac{3\left(-2\left(\sqrt{e^2-4df}+e\right)(ce-bf)(2af-be+2cd)+4bef(3af+cd)-4af\left(4af^2+ce^2\right)+b^2(-f)\left(4df+e^2\right)\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}\,(ce-bf)-bef-2cdf+ce^2}}\right)}{4\sqrt{2}\left(e^2-4df\right)^{5/2}\sqrt{2af^2+\sqrt{e^2-4df}\,(ce-bf)-bef-2cdf+ce^2}}+\frac{3\sqrt{a+bx+cx^2}\left(2x\left(4af^2-2bef+ce^2\right)+4aef-b\left(4df+e^2\right)+4cde\right)}{4\left(e^2-4df\right)^2\left(d+ex+fx^2\right)}-\frac{(e+2fx)\left(a+bx+cx^2\right)^{3/2}}{2\left(e^2-4df\right)\left(d+ex+fx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(3/2)/(d + e*x + f*x^2)^3,x]
```

```
[Out] -((e + 2*f*x)*(a + b*x + c*x^2)^(3/2))/(2*(e^2 - 4*d*f)*(d + e*x + f*x^2)^2
) + (3*(4*c*d*e + 4*a*e*f - b*(e^2 + 4*d*f) + 2*(c*e^2 - 2*b*e*f + 4*a*f^2)
*x)*Sqrt[a + b*x + c*x^2])/(4*(e^2 - 4*d*f)^2*(d + e*x + f*x^2)) + (3*(4*b*
e*f*(c*d + 3*a*f) - b^2*f*(e^2 + 4*d*f) - 4*a*f*(c*e^2 + 4*a*f^2) - 2*(2*c*
d - b*e + 2*a*f)*(c*e - b*f)*(e - Sqrt[e^2 - 4*d*f]))*ArcTanh[(4*a*f - b*(e
 - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*S
qrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt
[a + b*x + c*x^2])])/(4*Sqrt[2]*(e^2 - 4*d*f)^(5/2)*Sqrt[c*e^2 - 2*c*d*f -
b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]) - (3*(4*b*e*f*(c*d + 3*a*
f) - b^2*f*(e^2 + 4*d*f) - 4*a*f*(c*e^2 + 4*a*f^2) - 2*(2*c*d - b*e + 2*a*f
)*(c*e - b*f)*(e + Sqrt[e^2 - 4*d*f]))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4
*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c
*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^
2])])/(4*Sqrt[2]*(e^2 - 4*d*f)^(5/2)*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2
 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 971

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((b + 2*c*x)*(a + b*x + c*x^2)^(p + 1)*(d + e
*x + f*x^2)^q)/((b^2 - 4*a*c)*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*(p + 1))
, Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*Simp[2*c*d*(2*p +
 3) + b*e*q + (2*b*f*q + 2*c*e*(2*p + q + 3))*x + 2*c*f*(2*p + 2*q + 3)*x^2
, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
e^2 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0] &&  !IGtQ[q, 0]
```

Rule 1013

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e
_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((g*b - 2*a*h - (b*h - 2*g
*c)*x)*(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q)/((b^2 - 4*a*c)*(p + 1
)), x] - Dist[1/((b^2 - 4*a*c)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d +
 e*x + f*x^2)^(q - 1)*Simp[e*q*(g*b - 2*a*h) - d*(b*h - 2*g*c)*(2*p + 3) +
(2*f*q*(g*b - 2*a*h) - e*(b*h - 2*g*c)*(2*p + q + 3))*x - f*(b*h - 2*g*c)*(
2*p + 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}}{\left(d+ex+fx^2\right)^3}\,dx = -\frac{(e+2fx)\left(a+bx+cx^2\right)^{3/2}}{2\left(e^2-4df\right)\left(d+ex+fx^2\right)^2} + \frac{\int \frac{\left(\frac{3}{2}(be-4af)+3(ce-bf)x\right)\sqrt{a+bx+cx^2}}{\left(d+ex+fx^2\right)^2}\,dx}{2\left(e^2-4df\right)}$$

$$= -\frac{(e+2fx)\left(a+bx+cx^2\right)^{3/2}}{2\left(e^2-4df\right)\left(d+ex+fx^2\right)^2} + \frac{3\left(4cde+4aef-b\left(e^2+4df\right)+2\left(ce^2-2bef+4af\right.\right.}{4\left(e^2-4df\right)^2\left(d+ex+fx^2\right)}$$

$$= -\frac{(e+2fx)\left(a+bx+cx^2\right)^{3/2}}{2\left(e^2-4df\right)\left(d+ex+fx^2\right)^2} + \frac{3\left(4cde+4aef-b\left(e^2+4df\right)+2\left(ce^2-2bef+4af\right.\right.}{4\left(e^2-4df\right)^2\left(d+ex+fx^2\right)}$$

$$= -\frac{(e+2fx)\left(a+bx+cx^2\right)^{3/2}}{2\left(e^2-4df\right)\left(d+ex+fx^2\right)^2} + \frac{3\left(4cde+4aef-b\left(e^2+4df\right)+2\left(ce^2-2bef+4af\right.\right.}{4\left(e^2-4df\right)^2\left(d+ex+fx^2\right)}$$

$$= -\frac{(e+2fx)\left(a+bx+cx^2\right)^{3/2}}{2\left(e^2-4df\right)\left(d+ex+fx^2\right)^2} + \frac{3\left(4cde+4aef-b\left(e^2+4df\right)+2\left(ce^2-2bef+4af\right.\right.}{4\left(e^2-4df\right)^2\left(d+ex+fx^2\right)}$$

Mathematica [B] time = 7.23, size = 4727, normalized size = 7.04

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x + c*x^2)^(3/2)/(d + e*x + f*x^2)^3,x]
```

```
[Out] (-2*f^2*(a + x*(b + c*x))^(3/2))/((e^2 - 4*d*f)^(3/2)*(e - Sqrt[e^2 - 4*d*f
] + 2*f*x)^2) + (6*f^2*(a + x*(b + c*x))^(3/2))/((e^2 - 4*d*f)^2*(e - Sqrt[
e^2 - 4*d*f] + 2*f*x)) + (2*f^2*(a + x*(b + c*x))^(3/2))/((e^2 - 4*d*f)^(3/
2)*(e + Sqrt[e^2 - 4*d*f] + 2*f*x)^2) + (6*f^2*(a + x*(b + c*x))^(3/2))/((e
^2 - 4*d*f)^2*(e + Sqrt[e^2 - 4*d*f] + 2*f*x)) + (9*f^2*(a + x*(b + c*x))^(
3/2)*(((-4*b*c*f - 2*c*(b*f + 2*c*(-e + Sqrt[e^2 - 4*d*f])) - 4*c^2*f*x)*Sq
rt[a + b*x + c*x^2])/(8*c*f^2) - ((2*Sqrt[c]*(b^2*f^2 + 4*c^2*(e^2 - 2*d*f
- e*Sqrt[e^2 - 4*d*f]) + 4*c*f*(a*f - b*(e - Sqrt[e^2 - 4*d*f])))*ArcTanh[(
b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/f + (2*Sqrt[2]*Sqrt[c*e^2 -
2*c*d*f - b*e*f + 2*a*f^2 - c*e*Sqrt[e^2 - 4*d*f] + b*f*Sqrt[e^2 - 4*d*f]]*
(4*c*f*(8*a*b*f^2 - 3*b^2*f*(e - Sqrt[e^2 - 4*d*f]) - 4*a*c*f*(e - Sqrt[e^2
 - 4*d*f]) + 4*b*c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])) + 4*c*(-e + Sqrt[e^
```

2 - 4*d*f])*(b^2*f^2 + 4*c^2*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + 4*c*f*(a
f - b(e - Sqrt[e^2 - 4*d*f]))))*ArcTanh[(-4*a*f - b*(-e + Sqrt[e^2 - 4*d*
f]) - (2*b*f + 2*c*(-e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c
*d*f - b*e*f + 2*a*f^2 - c*e*Sqrt[e^2 - 4*d*f] + b*f*Sqrt[e^2 - 4*d*f]]*Sqr
t[a + b*x + c*x^2])])/(f*(16*a*f^2 + 8*b*f*(-e + Sqrt[e^2 - 4*d*f]) + 4*c*(
-e + Sqrt[e^2 - 4*d*f])^2)))/(16*c*f^2)))/((e^2 - 4*d*f)^2*(a + b*x + c*x^2
)^(3/2)) - (3*f^2*(a + x*(b + c*x))^(3/2)*(((-4*c*f*(4*a*f - b*(e - Sqrt[e^
2 - 4*d*f])) - 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*(b*f + 2*c*(-e + Sqrt[e^
2 - 4*d*f])) - 4*c*f*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)*Sqrt[a + b*x + c*
x^2])/(8*c*f^2) - ((-2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*(b^2*f^2 - 4*c^2*(
e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) - 4*c*f*(3*a*f - b*(e - Sqrt[e^2 - 4*d*f
]))))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(Sqrt[c]*f) +
(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - c*e*Sqrt[e^2 - 4*d*f] +
 b*f*Sqrt[e^2 - 4*d*f]]*(-4*(-e + Sqrt[e^2 - 4*d*f])*(b*f - c*(e - Sqrt[e^2
 - 4*d*f]))*(b^2*f^2 - 4*c^2*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) - 4*c*f*(3
*a*f - b*(e - Sqrt[e^2 - 4*d*f]))) + 4*f*(2*c*f*(4*a*f - b*(e - Sqrt[e^2 -
4*d*f]))^2 - (e - Sqrt[e^2 - 4*d*f])*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*(b^2
*f + 4*a*c*f - 2*b*c*(e - Sqrt[e^2 - 4*d*f]))))*ArcTanh[(-4*a*f - b*(-e + S
qrt[e^2 - 4*d*f]) - (2*b*f + 2*c*(-e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sq
rt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - c*e*Sqrt[e^2 - 4*d*f] + b*f*Sqrt[e^2
 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(f*(16*a*f^2 + 8*b*f*(-e + Sqrt[e^2 - 4
*d*f]) + 4*c*(-e + Sqrt[e^2 - 4*d*f])^2)))/(16*c*f^2)))/((e^2 - 4*d*f)^(5/2
)*(a + b*x + c*x^2)^(3/2)) + (3*f^2*(a + x*(b + c*x))^(3/2)*(((-2*b*f - 2*c
*(-e + Sqrt[e^2 - 4*d*f]))*(a + b*x + c*x^2)^(3/2))/((-4*a*f^2 - 2*b*f*(-e
+ Sqrt[e^2 - 4*d*f]) - c*(-e + Sqrt[e^2 - 4*d*f])^2)*(-e + Sqrt[e^2 - 4*d*f
] - 2*f*x)) + (((-4*c*f*(b^2*f + 4*a*c*f - 2*b*c*(e - Sqrt[e^2 - 4*d*f])) -
 4*c*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*(b*f + 2*c*(-e + Sqrt[e^2 - 4*d*f]))
 - 8*c^2*f*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)*Sqrt[a + b*x + c*x^2])/(8*c
*f^2) - ((16*c^(3/2)*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*(c*(e^2 - 2*d*f - e*
Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e - Sqrt[e^2 - 4*d*f])))*ArcTanh[(b + 2*
c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/f + (2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*
f - b*e*f + 2*a*f^2 - c*e*Sqrt[e^2 - 4*d*f] + b*f*Sqrt[e^2 - 4*d*f]]*(32*c^
2*(-e + Sqrt[e^2 - 4*d*f])*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*(c*(e^2 - 2*d*
f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e - Sqrt[e^2 - 4*d*f]))) + 16*c*f*
(b^2*f + 4*a*c*f - 2*b*c*(e - Sqrt[e^2 - 4*d*f]))*(c*(e^2 - 2*d*f - e*Sqrt[
e^2 - 4*d*f]) + f*(2*a*f - b*(e - Sqrt[e^2 - 4*d*f]))))*ArcTanh[(-4*a*f - b
*(-e + Sqrt[e^2 - 4*d*f]) - (2*b*f + 2*c*(-e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sq
rt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - c*e*Sqrt[e^2 - 4*d*f] + b*f*
Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(f*(16*a*f^2 + 8*b*f*(-e + Sqrt
[e^2 - 4*d*f]) + 4*c*(-e + Sqrt[e^2 - 4*d*f])^2)))/(16*c*f^2))/(-4*a*f^2 -
2*b*f*(-e + Sqrt[e^2 - 4*d*f]) - c*(-e + Sqrt[e^2 - 4*d*f])^2)))/((e^2 - 4*
d*f)^(3/2)*(a + b*x + c*x^2)^(3/2)) + (3*f^2*(a + x*(b + c*x))^(3/2)*(((4*c
f(-4*a*f + b*(e + Sqrt[e^2 - 4*d*f])) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]
))*(-(b*f) + 2*c*(e + Sqrt[e^2 - 4*d*f])) - 4*c*f*(b*f - c*(e + Sqrt[e^2 -
4*d*f]))*x)*Sqrt[a + b*x + c*x^2])/(8*c*f^2) - ((-2*(b*f - c*(e + Sqrt[e^2

- 4*d*f]))*(b^2*f^2 - 4*c^2*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) - 4*c*f*(3*a*f - b*(e + Sqrt[e^2 - 4*d*f])))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(Sqrt[c]*f) - (2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + c*e*Sqrt[e^2 - 4*d*f] - b*f*Sqrt[e^2 - 4*d*f]]*(4*(e + Sqrt[e^2 - 4*d*f])*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*(b^2*f^2 - 4*c^2*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) - 4*c*f*(3*a*f - b*(e + Sqrt[e^2 - 4*d*f]))) + 4*f*(2*c*f*(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]))^2 - (e + Sqrt[e^2 - 4*d*f])*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*(b^2*f + 4*a*c*f - 2*b*c*(e + Sqrt[e^2 - 4*d*f]))))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) - (-2*b*f + 2*c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + c*e*Sqrt[e^2 - 4*d*f] - b*f*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(f*(16*a*f^2 - 8*b*f*(e + Sqrt[e^2 - 4*d*f]) + 4*c*(e + Sqrt[e^2 - 4*d*f])^2)))/(16*c*f^2)))/((e^2 - 4*d*f)^(5/2)*(a + b*x + c*x^2)^(3/2)) - (9*f^2*(a + x*(b + c*x))^(3/2)*(((4*b*c*f - 2*c*(-(b*f) + 2*c*(e + Sqrt[e^2 - 4*d*f])) + 4*c^2*f*x)*Sqrt[a + b*x + c*x^2])/(8*c*f^2) - ((-2*Sqrt[c]*(b^2*f^2 + 4*c^2*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + 4*c*f*(a*f - b*(e + Sqrt[e^2 - 4*d*f])))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/f - (2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + c*e*Sqrt[e^2 - 4*d*f] - b*f*Sqrt[e^2 - 4*d*f]]*(4*c*(e + Sqrt[e^2 - 4*d*f])*(b^2*f^2 + 4*c^2*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + 4*c*f*(a*f - b*(e + Sqrt[e^2 - 4*d*f]))) + 4*c*f*(3*b^2*f*(e + Sqrt[e^2 - 4*d*f]) + 4*a*c*f*(e + Sqrt[e^2 - 4*d*f]) - 4*b*(2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]))))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) - (-2*b*f + 2*c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + c*e*Sqrt[e^2 - 4*d*f] - b*f*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(f*(16*a*f^2 - 8*b*f*(e + Sqrt[e^2 - 4*d*f]) + 4*c*(e + Sqrt[e^2 - 4*d*f])^2)))/(16*c*f^2)))/((e^2 - 4*d*f)^2*(a + b*x + c*x^2)^(3/2)) - (3*f^2*(a + x*(b + c*x))^(3/2)*(((2*b*f - 2*c*(e + Sqrt[e^2 - 4*d*f]))*(a + b*x + c*x^2)^(3/2))/((-4*a*f^2 + 2*b*f*(e + Sqrt[e^2 - 4*d*f]) - c*(e + Sqrt[e^2 - 4*d*f])^2)*(e + Sqrt[e^2 - 4*d*f] + 2*f*x)) + (((4*c*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*(-(b*f) + 2*c*(e + Sqrt[e^2 - 4*d*f])) + 4*c*f*(-(b^2*f) - 4*a*c*f + 2*b*c*(e + Sqrt[e^2 - 4*d*f])) - 8*c^2*f*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)*Sqrt[a + b*x + c*x^2])/(8*c*f^2) - ((16*c^(3/2)*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*(c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f])))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/f - (2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + c*e*Sqrt[e^2 - 4*d*f] - b*f*Sqrt[e^2 - 4*d*f]]*(-32*c^2*(e + Sqrt[e^2 - 4*d*f])*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*(c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))) + 16*c*f*(b^2*f + 4*a*c*f - 2*b*c*(e + Sqrt[e^2 - 4*d*f]))*(c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) - (-2*b*f + 2*c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + c*e*Sqrt[e^2 - 4*d*f] - b*f*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(f*(16*a*f^2 - 8*b*f*(e + Sqrt[e^2 - 4*d*f]) + 4*c*(e + Sqrt[e^2 - 4*d*f])^2)))/(16*c*f^2))/(-4*a*f^2 + 2*b*f*(e + Sqrt[e^2 - 4*d*f]) - c*(e + Sqrt[e^2 - 4*d*f])^2)))/((e^2 - 4*d*f)^(3/2)*(a + b*x + c*x

```
^2)^(3/2))
```

IntegrateAlgebraic [F] time = 180.01, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x + c*x^2)^(3/2)/(d + e*x + f*x^2)^3,x]
[Out] $Aborted
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d)^3,x, algorithm="fricas")
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d)^3,x, algorithm="giac")
[Out] Timed out
```

maple [B] time = 0.07, size = 178044, normalized size = 265.34

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d)^3,x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^2 + bx + a\right)^{\frac{3}{2}}}{\left(fx^2 + ex + d\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d)^3,x, algorithm="maxima")
[Out] integrate((c*x^2 + b*x + a)^(3/2)/(f*x^2 + e*x + d)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^2+b\,x+a\right)^{3/2}}{\left(f\,x^2+e\,x+d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^(3/2)/(d + e*x + f*x^2)^3,x)
[Out] int((a + b*x + c*x^2)^(3/2)/(d + e*x + f*x^2)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)/(f*x**2+e*x+d)**3,x)
[Out] Timed out
```

3.106 $\int \frac{(d+ex+fx^2)^3}{\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=717

$$\frac{x\sqrt{a+bx+cx^2}\left(24c^2f\left(50a^2f^2+322abef+175b^2\left(df+e^2\right)\right)-252b^2cf^2(14af+15be)-160c^3\left(27af\left(df+e^2\right)\right.\right.}{3840c^5}$$

Rubi [A] time = 2.71, antiderivative size = 717, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1661, 640, 621, 206}

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)^3/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] ((23040*c^5*d^2*e - 3465*b^5*f^3 + 420*b^3*c*f^2*(27*b*e + 34*a*f) - 504*b*
c^2*f*(70*a*b*e*f + 22*a^2*f^2 + 25*b^2*(e^2 + d*f)) - 640*c^4*(27*b*d*(e^2
 + d*f) + 8*a*e*(e^2 + 6*d*f)) + 96*c^3*(128*a^2*e*f^2 + 275*a*b*f*(e^2 + d
*f) + 50*b^2*(e^3 + 6*d*e*f)))*Sqrt[a + b*x + c*x^2])/(7680*c^6) + ((1155*b
^4*f^3 - 252*b^2*c*f^2*(15*b*e + 14*a*f) + 5760*c^4*d*(e^2 + d*f) + 24*c^2*
f*(322*a*b*e*f + 50*a^2*f^2 + 175*b^2*(e^2 + d*f)) - 160*c^3*(27*a*f*(e^2 +
 d*f) + 10*b*(e^3 + 6*d*e*f)))*x*Sqrt[a + b*x + c*x^2])/(3840*c^5) - ((231*
b^3*f^3 - 36*b*c*f^2*(21*b*e + 13*a*f) - 320*c^3*(e^3 + 6*d*e*f) + 24*c^2*f
*(32*a*e*f + 35*b*(e^2 + d*f)))*x^2*Sqrt[a + b*x + c*x^2])/(960*c^4) + (f*(
99*b^2*f^2 - 4*c*f*(81*b*e + 25*a*f) + 360*c^2*(e^2 + d*f))*x^3*Sqrt[a + b*
x + c*x^2])/(480*c^3) + (f^2*(36*c*e - 11*b*f)*x^4*Sqrt[a + b*x + c*x^2])/(
60*c^2) + (f^3*x^5*Sqrt[a + b*x + c*x^2])/(6*c) + ((1024*c^6*d^3 + 231*b^6*
f^3 - 252*b^4*c*f^2*(3*b*e + 5*a*f) - 1536*c^5*d*(b*d*e + a*(e^2 + d*f)) +
840*b^2*c^2*f*(4*a*b*e*f + 2*a^2*f^2 + b^2*(e^2 + d*f)) + 384*c^4*(3*b^2*d*
(e^2 + d*f) + 3*a^2*f*(e^2 + d*f) + 2*a*b*e*(e^2 + 6*d*f)) - 320*c^3*(9*a^2
*b*e*f^2 + a^3*f^3 + 9*a*b^2*f*(e^2 + d*f) + b^3*(e^3 + 6*d*e*f)))*ArcTanh[
(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(1024*c^(13/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(d+ex+fx^2\right)^3}{\sqrt{a+bx+cx^2}}\,dx = \frac{f^3x^5\sqrt{a+bx+cx^2}}{6c} + \frac{\int \frac{6cd^3+18cd^2ex+18cd\left(e^2+df\right)x^2+6ce\left(e^2+6df\right)x^3-f\left(5af^2-18c\left(e^2+df\right)\right)x^4+\frac{1}{2}f^2(36ce}{\sqrt{a+bx+cx^2}}}{6c}$$

$$= \frac{f^2(36ce-11bf)x^4\sqrt{a+bx+cx^2}}{60c^2} + \frac{f^3x^5\sqrt{a+bx+cx^2}}{6c} + \int \frac{30c^2d^3+90c^2d^2ex+90c^2d\left(e^2+df\right)x^2}{}$$

$$= \frac{f\left(99b^2f^2-4cf(81be+25af)+360c^2\left(e^2+df\right)\right)x^3\sqrt{a+bx+cx^2}}{480c^3} + \frac{f^2(36ce-11bf)x}{60c}$$

$$= -\frac{\left(231b^3f^3-36bcf^2(21be+13af)-320c^3\left(e^3+6def\right)+24c^2f\left(32aef+35b\left(e^2+df\right)\right)\right.}{960c^4}$$

$$= \frac{\left(1155b^4f^3-252b^2cf^2(15be+14af)+5760c^4d\left(e^2+df\right)+24c^2f\left(322abef+50a^2f^2+\right.\right.}{3840c^5}$$

$$= \frac{\left(23040c^5d^2e-3465b^5f^3+420b^3cf^2(27be+34af)-504bc^2f\left(70abef+22a^2f^2+25b^2\right.\right.}{}$$

$$= \frac{\left(23040c^5d^2e-3465b^5f^3+420b^3cf^2(27be+34af)-504bc^2f\left(70abef+22a^2f^2+25b^2\right.\right.}{}$$

$$= \frac{\left(23040c^5d^2e-3465b^5f^3+420b^3cf^2(27be+34af)-504bc^2f\left(70abef+22a^2f^2+25b^2\right.\right.}{}$$

Mathematica [A] time = 1.35, size = 615, normalized size = 0.86

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)^3/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (Sqrt[a + x*(b + c*x)]*(-3465*b^5*f^3 + 210*b^3*c*f^2*(54*b*e + 68*a*f + 11
*b*f*x) - 168*b*c^2*f*(66*a^2*f^2 + 42*a*b*f*(5*e + f*x) + b^2*(75*e^2 + 75
*d*f + 45*e*f*x + 11*f^2*x^2)) + 128*c^5*(90*d^2*(2*e + f*x) + 15*d*x*(6*e^
2 + 8*e*f*x + 3*f^2*x^2) + x^2*(20*e^3 + 45*e^2*f*x + 36*e*f^2*x^2 + 10*f^3
*x^3)) + 48*c^3*(2*a^2*f^2*(128*e + 25*f*x) + b^2*(100*e^3 + 175*e^2*f*x +
6*e*f*(100*d + 21*f*x^2) + f^2*x*(175*d + 33*f*x^2)) + 2*a*b*f*(275*e^2 + 1
61*e*f*x + f*(275*d + 39*f*x^2))) - 64*c^4*(a*(80*e^3 + 135*e^2*f*x + 96*e*
f*(5*d + f*x^2) + 5*f^2*x*(27*d + 5*f*x^2)) + b*(270*d^2*f + 15*d*(18*e^2 +
```

```
 20*e*f*x + 7*f^2*x^2) + x*(50*e^3 + 105*e^2*f*x + 81*e*f^2*x^2 + 22*f^3*x^
3))))))/(7680*c^6) + ((1024*c^6*d^3 + 231*b^6*f^3 - 252*b^4*c*f^2*(3*b*e + 5
*a*f) - 1536*c^5*d*(b*d*e + a*(e^2 + d*f)) + 840*b^2*c^2*f*(4*a*b*e*f + 2*a
^2*f^2 + b^2*(e^2 + d*f)) + 384*c^4*(3*b^2*d*(e^2 + d*f) + 3*a^2*f*(e^2 + d
*f) + 2*a*b*e*(e^2 + 6*d*f)) - 320*c^3*(9*a^2*b*e*f^2 + a^3*f^3 + 9*a*b^2*f
*(e^2 + d*f) + b^3*(e^3 + 6*d*e*f)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a
+ x*(b + c*x)])])/(1024*c^(13/2))
```

IntegrateAlgebraic [A] time = 4.22, size = 847, normalized size = 1.18

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d + e*x + f*x^2)^3/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (Sqrt[a + b*x + c*x^2]*(23040*c^5*d^2*e - 17280*b*c^4*d*e^2 + 4800*b^2*c^3*
e^3 - 5120*a*c^4*e^3 - 17280*b*c^4*d^2*f + 28800*b^2*c^3*d*e*f - 30720*a*c^
4*d*e*f - 12600*b^3*c^2*e^2*f + 26400*a*b*c^3*e^2*f - 12600*b^3*c^2*d*f^2 +
 26400*a*b*c^3*d*f^2 + 11340*b^4*c*e*f^2 - 35280*a*b^2*c^2*e*f^2 + 12288*a^
2*c^3*e*f^2 - 3465*b^5*f^3 + 14280*a*b^3*c*f^3 - 11088*a^2*b*c^2*f^3 + 1152
0*c^5*d*e^2*x - 3200*b*c^4*e^3*x + 11520*c^5*d^2*f*x - 19200*b*c^4*d*e*f*x
+ 8400*b^2*c^3*e^2*f*x - 8640*a*c^4*e^2*f*x + 8400*b^2*c^3*d*f^2*x - 8640*a
*c^4*d*f^2*x - 7560*b^3*c^2*e*f^2*x + 15456*a*b*c^3*e*f^2*x + 2310*b^4*c*f^
3*x - 7056*a*b^2*c^2*f^3*x + 2400*a^2*c^3*f^3*x + 2560*c^5*e^3*x^2 + 15360*
c^5*d*e*f*x^2 - 6720*b*c^4*e^2*f*x^2 - 6720*b*c^4*d*f^2*x^2 + 6048*b^2*c^3*
e*f^2*x^2 - 6144*a*c^4*e*f^2*x^2 - 1848*b^3*c^2*f^3*x^2 + 3744*a*b*c^3*f^3*
x^2 + 5760*c^5*e^2*f*x^3 + 5760*c^5*d*f^2*x^3 - 5184*b*c^4*e*f^2*x^3 + 1584
*b^2*c^3*f^3*x^3 - 1600*a*c^4*f^3*x^3 + 4608*c^5*e*f^2*x^4 - 1408*b*c^4*f^3
*x^4 + 1280*c^5*f^3*x^5))/(7680*c^6) + ((-1024*c^6*d^3 + 1536*b*c^5*d^2*e -
 1152*b^2*c^4*d*e^2 + 1536*a*c^5*d*e^2 + 320*b^3*c^3*e^3 - 768*a*b*c^4*e^3
- 1152*b^2*c^4*d^2*f + 1536*a*c^5*d^2*f + 1920*b^3*c^3*d*e*f - 4608*a*b*c^4
*d*e*f - 840*b^4*c^2*e^2*f + 2880*a*b^2*c^3*e^2*f - 1152*a^2*c^4*e^2*f - 84
0*b^4*c^2*d*f^2 + 2880*a*b^2*c^3*d*f^2 - 1152*a^2*c^4*d*f^2 + 756*b^5*c*e*f
^2 - 3360*a*b^3*c^2*e*f^2 + 2880*a^2*b*c^3*e*f^2 - 231*b^6*f^3 + 1260*a*b^4
*c*f^3 - 1680*a^2*b^2*c^2*f^3 + 320*a^3*c^3*f^3)*Log[b + 2*c*x - 2*Sqrt[c]*
Sqrt[a + b*x + c*x^2]])/(1024*c^(13/2))
```

fricas [A] time = 1.41, size = 1583, normalized size = 2.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/30720*(15*(1024*c^6*d^3 - 1536*b*c^5*d^2*e + 384*(3*b^2*c^4 - 4*a*c^5)*
d*e^2 - 64*(5*b^3*c^3 - 12*a*b*c^4)*e^3 + (231*b^6 - 1260*a*b^4*c + 1680*a^
```

```
2*b^2*c^2 - 320*a^3*c^3)*f^3 + 12*(2*(35*b^4*c^2 - 120*a*b^2*c^3 + 48*a^2*c
^4)*d - (63*b^5*c - 280*a*b^3*c^2 + 240*a^2*b*c^3)*e)*f^2 + 24*(16*(3*b^2*c
^4 - 4*a*c^5)*d^2 - 16*(5*b^3*c^3 - 12*a*b*c^4)*d*e + (35*b^4*c^2 - 120*a*b
^2*c^3 + 48*a^2*c^4)*e^2)*f)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqr
t(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(1280*c^6*f^3*x^5 + 230
40*c^6*d^2*e - 17280*b*c^5*d*e^2 + 128*(36*c^6*e*f^2 - 11*b*c^5*f^3)*x^4 +
320*(15*b^2*c^4 - 16*a*c^5)*e^3 - 21*(165*b^5*c - 680*a*b^3*c^2 + 528*a^2*b
*c^3)*f^3 + 16*(360*c^6*e^2*f + (99*b^2*c^4 - 100*a*c^5)*f^3 + 36*(10*c^6*d
 - 9*b*c^5*e)*f^2)*x^3 - 12*(50*(21*b^3*c^3 - 44*a*b*c^4)*d - (945*b^4*c^2
- 2940*a*b^2*c^3 + 1024*a^2*c^4)*e)*f^2 + 8*(320*c^6*e^3 - 3*(77*b^3*c^3 -
156*a*b*c^4)*f^3 - 12*(70*b*c^5*d - (63*b^2*c^4 - 64*a*c^5)*e)*f^2 + 120*(1
6*c^6*d*e - 7*b*c^5*e^2)*f)*x^2 - 120*(144*b*c^5*d^2 - 16*(15*b^2*c^4 - 16*
a*c^5)*d*e + 5*(21*b^3*c^3 - 44*a*b*c^4)*e^2)*f + 2*(5760*c^6*d*e^2 - 1600*
b*c^5*e^3 + 3*(385*b^4*c^2 - 1176*a*b^2*c^3 + 400*a^2*c^4)*f^3 + 12*(10*(35
*b^2*c^4 - 36*a*c^5)*d - 7*(45*b^3*c^3 - 92*a*b*c^4)*e)*f^2 + 120*(48*c^6*d
^2 - 80*b*c^5*d*e + (35*b^2*c^4 - 36*a*c^5)*e^2)*f)*x)*sqrt(c*x^2 + b*x + a
))/c^7, -1/15360*(15*(1024*c^6*d^3 - 1536*b*c^5*d^2*e + 384*(3*b^2*c^4 - 4*
a*c^5)*d*e^2 - 64*(5*b^3*c^3 - 12*a*b*c^4)*e^3 + (231*b^6 - 1260*a*b^4*c +
1680*a^2*b^2*c^2 - 320*a^3*c^3)*f^3 + 12*(2*(35*b^4*c^2 - 120*a*b^2*c^3 + 4
8*a^2*c^4)*d - (63*b^5*c - 280*a*b^3*c^2 + 240*a^2*b*c^3)*e)*f^2 + 24*(16*(
3*b^2*c^4 - 4*a*c^5)*d^2 - 16*(5*b^3*c^3 - 12*a*b*c^4)*d*e + (35*b^4*c^2 -
120*a*b^2*c^3 + 48*a^2*c^4)*e^2)*f)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x +
a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(1280*c^6*f^3*x^5 + 23
040*c^6*d^2*e - 17280*b*c^5*d*e^2 + 128*(36*c^6*e*f^2 - 11*b*c^5*f^3)*x^4 +
 320*(15*b^2*c^4 - 16*a*c^5)*e^3 - 21*(165*b^5*c - 680*a*b^3*c^2 + 528*a^2*
b*c^3)*f^3 + 16*(360*c^6*e^2*f + (99*b^2*c^4 - 100*a*c^5)*f^3 + 36*(10*c^6*
d - 9*b*c^5*e)*f^2)*x^3 - 12*(50*(21*b^3*c^3 - 44*a*b*c^4)*d - (945*b^4*c^2
 - 2940*a*b^2*c^3 + 1024*a^2*c^4)*e)*f^2 + 8*(320*c^6*e^3 - 3*(77*b^3*c^3 -
 156*a*b*c^4)*f^3 - 12*(70*b*c^5*d - (63*b^2*c^4 - 64*a*c^5)*e)*f^2 + 120*(
16*c^6*d*e - 7*b*c^5*e^2)*f)*x^2 - 120*(144*b*c^5*d^2 - 16*(15*b^2*c^4 - 16
*a*c^5)*d*e + 5*(21*b^3*c^3 - 44*a*b*c^4)*e^2)*f + 2*(5760*c^6*d*e^2 - 1600
*b*c^5*e^3 + 3*(385*b^4*c^2 - 1176*a*b^2*c^3 + 400*a^2*c^4)*f^3 + 12*(10*(3
5*b^2*c^4 - 36*a*c^5)*d - 7*(45*b^3*c^3 - 92*a*b*c^4)*e)*f^2 + 120*(48*c^6*
d^2 - 80*b*c^5*d*e + (35*b^2*c^4 - 36*a*c^5)*e^2)*f)*x)*sqrt(c*x^2 + b*x +
a))/c^7]
```

giac [A] time = 0.53, size = 824, normalized size = 1.15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/7680*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*f^3*x/c - (11*b*c^4*f^3 - 36*c
^5*f^2*e)/c^6)*x + (360*c^5*d*f^2 + 99*b^2*c^3*f^3 - 100*a*c^4*f^3 - 324*b*
```

```
c^4*f^2*e + 360*c^5*f*e^2)/c^6)*x - (840*b*c^4*d*f^2 + 231*b^3*c^2*f^3 - 46
8*a*b*c^3*f^3 - 1920*c^5*d*f*e - 756*b^2*c^3*f^2*e + 768*a*c^4*f^2*e + 840*
b*c^4*f*e^2 - 320*c^5*e^3)/c^6)*x + (5760*c^5*d^2*f + 4200*b^2*c^3*d*f^2 -
4320*a*c^4*d*f^2 + 1155*b^4*c*f^3 - 3528*a*b^2*c^2*f^3 + 1200*a^2*c^3*f^3 -
 9600*b*c^4*d*f*e - 3780*b^3*c^2*f^2*e + 7728*a*b*c^3*f^2*e + 5760*c^5*d*e^
2 + 4200*b^2*c^3*f*e^2 - 4320*a*c^4*f*e^2 - 1600*b*c^4*e^3)/c^6)*x - (17280
*b*c^4*d^2*f + 12600*b^3*c^2*d*f^2 - 26400*a*b*c^3*d*f^2 + 3465*b^5*f^3 - 1
4280*a*b^3*c*f^3 + 11088*a^2*b*c^2*f^3 - 23040*c^5*d^2*e - 28800*b^2*c^3*d*
f*e + 30720*a*c^4*d*f*e - 11340*b^4*c*f^2*e + 35280*a*b^2*c^2*f^2*e - 12288
*a^2*c^3*f^2*e + 17280*b*c^4*d*e^2 + 12600*b^3*c^2*f*e^2 - 26400*a*b*c^3*f*
e^2 - 4800*b^2*c^3*e^3 + 5120*a*c^4*e^3)/c^6) - 1/1024*(1024*c^6*d^3 + 1152
*b^2*c^4*d^2*f - 1536*a*c^5*d^2*f + 840*b^4*c^2*d*f^2 - 2880*a*b^2*c^3*d*f^
2 + 1152*a^2*c^4*d*f^2 + 231*b^6*f^3 - 1260*a*b^4*c*f^3 + 1680*a^2*b^2*c^2*
f^3 - 320*a^3*c^3*f^3 - 1536*b*c^5*d^2*e - 1920*b^3*c^3*d*f*e + 4608*a*b*c^
4*d*f*e - 756*b^5*c*f^2*e + 3360*a*b^3*c^2*f^2*e - 2880*a^2*b*c^3*f^2*e + 1
152*b^2*c^4*d*e^2 - 1536*a*c^5*d*e^2 + 840*b^4*c^2*f*e^2 - 2880*a*b^2*c^3*f
*e^2 + 1152*a^2*c^4*f*e^2 - 320*b^3*c^3*e^3 + 768*a*b*c^4*e^3)*log(abs(-2*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(13/2)
```

maple [B] time = 0.03, size = 1930, normalized size = 2.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)^3/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] 3*d^2*e/c*(c*x^2+b*x+a)^(1/2)+1/3*x^2/c*(c*x^2+b*x+a)^(1/2)*e^3+5/8*b^2/c^3
*(c*x^2+b*x+a)^(1/2)*e^3-5/16*b^3/c^(7/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x
+a)^(1/2))*e^3-2/3*a/c^2*(c*x^2+b*x+a)^(1/2)*e^3-231/512*f^3*b^5/c^6*(c*x^2
+b*x+a)^(1/2)+231/1024*f^3*b^6/c^(13/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a
)^(1/2))-5/16*f^3*a^3/c^(7/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+d
^3*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)+161/80*e*f^2*b/c^3*a
*x*(c*x^2+b*x+a)^(1/2)-5/2*b/c^2*x*(c*x^2+b*x+a)^(1/2)*d*e*f+9/2*b/c^(5/2)*
a*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*d*e*f-45/16*b^2/c^(7/2)*a*ln(
(c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*e^2*f+55/16*b/c^3*a*(c*x^2+b*x+a)^
(1/2)*d*f^2+55/16*b/c^3*a*(c*x^2+b*x+a)^(1/2)*e^2*f-9/8*a/c^2*x*(c*x^2+b*x+
a)^(1/2)*d*f^2-147/160*f^3*b^2/c^4*a*x*(c*x^2+b*x+a)^(1/2)+39/80*f^3*b/c^3*
a*x^2*(c*x^2+b*x+a)^(1/2)-9/8*a/c^2*x*(c*x^2+b*x+a)^(1/2)*e^2*f+2*x^2/c*(c*
x^2+b*x+a)^(1/2)*d*e*f+15/4*b^2/c^3*(c*x^2+b*x+a)^(1/2)*d*e*f-15/8*b^3/c^(7
/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*d*e*f-4*a/c^2*(c*x^2+b*x+a)
^(1/2)*d*e*f-27/40*e*f^2*b/c^2*x^3*(c*x^2+b*x+a)^(1/2)+63/80*e*f^2*b^2/c^3*
x^2*(c*x^2+b*x+a)^(1/2)-63/64*e*f^2*b^3/c^4*x*(c*x^2+b*x+a)^(1/2)+105/32*e*
f^2*b^3/c^(9/2)*a*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-147/32*e*f^2*
b^2/c^4*a*(c*x^2+b*x+a)^(1/2)-45/16*e*f^2*b/c^(7/2)*a^2*ln((c*x+1/2*b)/c^(1
/2)+(c*x^2+b*x+a)^(1/2))-4/5*e*f^2*a/c^2*x^2*(c*x^2+b*x+a)^(1/2)-7/8*b/c^2*
```

```
x^2*(c*x^2+b*x+a)^(1/2)*d*f^2-7/8*b/c^2*x^2*(c*x^2+b*x+a)^(1/2)*e^2*f+35/32
*b^2/c^3*x*(c*x^2+b*x+a)^(1/2)*d*f^2+35/32*b^2/c^3*x*(c*x^2+b*x+a)^(1/2)*e^
2*f-45/16*b^2/c^(7/2)*a*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*d*f^2+1
/6*f^3*x^5*(c*x^2+b*x+a)^(1/2)/c+9/8*a^2/c^(5/2)*ln((c*x+1/2*b)/c^(1/2)+(c*
x^2+b*x+a)^(1/2))*e^2*f-5/12*b/c^2*x*(c*x^2+b*x+a)^(1/2)*e^3+3/4*b/c^(5/2)*
a*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*e^3-105/64*b^3/c^4*(c*x^2+b*x
+a)^(1/2)*e^2*f+105/128*b^4/c^(9/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1
/2))*d*f^2+105/128*b^4/c^(9/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*
e^2*f+9/8*a^2/c^(5/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*d*f^2+3/4
*x^3/c*(c*x^2+b*x+a)^(1/2)*d*f^2+3/4*x^3/c*(c*x^2+b*x+a)^(1/2)*e^2*f-105/64
*b^3/c^4*(c*x^2+b*x+a)^(1/2)*d*f^2+3/5*e*f^2*x^4/c*(c*x^2+b*x+a)^(1/2)+189/
128*e*f^2*b^4/c^5*(c*x^2+b*x+a)^(1/2)-189/256*e*f^2*b^5/c^(11/2)*ln((c*x+1/
2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+8/5*e*f^2*a^2/c^3*(c*x^2+b*x+a)^(1/2)-315
/256*f^3*b^4/c^(11/2)*a*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+119/64*
f^3*b^3/c^5*a*(c*x^2+b*x+a)^(1/2)+105/64*f^3*b^2/c^(9/2)*a^2*ln((c*x+1/2*b)
/c^(1/2)+(c*x^2+b*x+a)^(1/2))-231/160*f^3*b/c^4*a^2*(c*x^2+b*x+a)^(1/2)-5/2
4*f^3*a/c^2*x^3*(c*x^2+b*x+a)^(1/2)+5/16*f^3*a^2/c^3*x*(c*x^2+b*x+a)^(1/2)-
11/60*f^3*b/c^2*x^4*(c*x^2+b*x+a)^(1/2)+33/160*f^3*b^2/c^3*x^3*(c*x^2+b*x+a
)^(1/2)-77/320*f^3*b^3/c^4*x^2*(c*x^2+b*x+a)^(1/2)+77/256*f^3*b^4/c^5*x*(c*
x^2+b*x+a)^(1/2)-9/4*b/c^2*(c*x^2+b*x+a)^(1/2)*f*d^2-9/4*b/c^2*(c*x^2+b*x+a
)^(1/2)*e^2*d+9/8*b^2/c^(5/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*f
*d^2+9/8*b^2/c^(5/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*e^2*d-3/2*
a/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*f*d^2-3/2*a/c^(3/2)*l
n((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*e^2*d-3/2*d^2*e*b/c^(3/2)*ln((c*
x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+3/2*x/c*(c*x^2+b*x+a)^(1/2)*f*d^2+3/2
*x/c*(c*x^2+b*x+a)^(1/2)*e^2*d
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(f\,x^2+e\,x+d\right)^3}{\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2)^3/(a + b*x + c*x^2)^(1/2),x)
```

```
[Out] int((d + e*x + f*x^2)^3/(a + b*x + c*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d + ex + fx^2\right)^3}{\sqrt{a + bx + cx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)**3/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x + f*x**2)**3/sqrt(a + b*x + c*x**2), x)
```

3.107 $\int \frac{(d+ex+fx^2)^2}{\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=316

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\,\sqrt{a+bx+cx^2}}\right)\left(48c^2\left(a^2f^2+4abef+b^2\left(2df+e^2\right)\right)-40b^2cf(3af+2be)-64c^3\left(a\left(2df+e^2\right)+2bde\right)\right.}{128c^{9/2}}$$

Rubi [A] time = 0.63, antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1661, 640, 621, 206}

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\,\sqrt{a+bx+cx^2}}\right)\left(48c^2\left(a^2f^2+4abef+b^2\left(2df+e^2\right)\right)-40b^2cf(3af+2be)-64c^3\left(a\left(2df+e^2\right)+2bde\right)+35b^4f^2+128c^4d^2\right)}{128c^{9/2}}+\frac{x\sqrt{a+bx+cx^2}\left(-4cf(9af+20be)+35b^2f^2+48c^2\left(2df+e^2\right)\right)}{96c^3}+\frac{\sqrt{a+bx+cx^2}\left(-16c^2\left(16aef+9b\left(2df+e^2\right)\right)+20bcf(11af+12be)-105b^3f^2+384c^3de\right)}{192c^4}+\frac{fx^2\sqrt{a+bx+cx^2}\,(16ce-7bf)}{24c^2}+\frac{f^2x^3\sqrt{a+bx+cx^2}}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)^2/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] ((384*c^3*d*e - 105*b^3*f^2 + 20*b*c*f*(12*b*e + 11*a*f) - 16*c^2*(16*a*e*f
 + 9*b*(e^2 + 2*d*f)))*Sqrt[a + b*x + c*x^2])/(192*c^4) + ((35*b^2*f^2 - 4*
c*f*(20*b*e + 9*a*f) + 48*c^2*(e^2 + 2*d*f))*x*Sqrt[a + b*x + c*x^2])/(96*c
^3) + (f*(16*c*e - 7*b*f)*x^2*Sqrt[a + b*x + c*x^2])/(24*c^2) + (f^2*x^3*Sq
rt[a + b*x + c*x^2])/(4*c) + ((128*c^4*d^2 + 35*b^4*f^2 - 40*b^2*c*f*(2*b*e
 + 3*a*f) - 64*c^3*(2*b*d*e + a*(e^2 + 2*d*f)) + 48*c^2*(4*a*b*e*f + a^2*f^
2 + b^2*(e^2 + 2*d*f)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2
])])/(128*c^(9/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
```

```
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(d+ex+fx^2\right)^2}{\sqrt{a+bx+cx^2}}\,dx = \frac{f^2x^3\sqrt{a+bx+cx^2}}{4c} + \frac{\int \frac{4cd^2+8cdex-\left(3af^2-4c\left(e^2+2df\right)\right)x^2+\frac{1}{2}f(16ce-7bf)x^3}{\sqrt{a+bx+cx^2}}\,dx}{4c}$$

$$= \frac{f(16ce-7bf)x^2\sqrt{a+bx+cx^2}}{24c^2} + \frac{f^2x^3\sqrt{a+bx+cx^2}}{4c} + \frac{\int \frac{12c^2d^2+\left(24c^2de-16acef+7abf^2\right)x+}{}}{}$$

$$= \frac{\left(35b^2f^2-4cf(20be+9af)+48c^2\left(e^2+2df\right)\right)x\sqrt{a+bx+cx^2}}{96c^3} + \frac{f(16ce-7bf)x^2\sqrt{a}}{24c^2}$$

$$= \frac{\left(384c^3de-105b^3f^2+20bcf(12be+11af)-16c^2\left(16aef+9b\left(e^2+2df\right)\right)\right)\sqrt{a+bx+}}{192c^4}$$

$$= \frac{\left(384c^3de-105b^3f^2+20bcf(12be+11af)-16c^2\left(16aef+9b\left(e^2+2df\right)\right)\right)\sqrt{a+bx+}}{192c^4}$$

$$= \frac{\left(384c^3de-105b^3f^2+20bcf(12be+11af)-16c^2\left(16aef+9b\left(e^2+2df\right)\right)\right)\sqrt{a+bx+}}{192c^4}$$

Mathematica [A] time = 0.50, size = 251, normalized size = 0.79

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)\left(48c^2\left(a^2f^2+4abef+b^2\left(2df+e^2\right)\right)-40b^2cf(3af+2be)-64c^3\left(a\left(2df+e^2\right)+2bde\right)+35b^4f^2+128c^4d^2\right)}{128c^{9/2}}+\frac{\sqrt{a+x(b+cx)}\left(-8c^2\left(af(32e+9fx)+b\left(36df+18e^2+20efx+7f^2x^2\right)\right)+10bcf(22af+24be+7bfx)-105b^3f^2+16c^3\left(12d(2e+fx)+x\left(6e^2+8efx+3f^2x^2\right)\right)\right)}{192c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)^2/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (Sqrt[a + x*(b + c*x)]*(-105*b^3*f^2 + 10*b*c*f*(24*b*e + 22*a*f + 7*b*f*x)
 + 16*c^3*(12*d*(2*e + f*x) + x*(6*e^2 + 8*e*f*x + 3*f^2*x^2)) - 8*c^2*(a*f
```

```
*(32*e + 9*f*x) + b*(18*e^2 + 36*d*f + 20*e*f*x + 7*f^2*x^2))))/(192*c^4) +
 ((128*c^4*d^2 + 35*b^4*f^2 - 40*b^2*c*f*(2*b*e + 3*a*f) - 64*c^3*(2*b*d*e
+ a*(e^2 + 2*d*f)) + 48*c^2*(4*a*b*e*f + a^2*f^2 + b^2*(e^2 + 2*d*f)))*ArcT
anh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/(128*c^(9/2))
```

IntegrateAlgebraic [A] time = 1.09, size = 296, normalized size = 0.94

$$\frac{\log\left(-2\sqrt{c}\sqrt{a+bx+cx^2}+b+2cx\right)\left(-48a^2c^2f^2+120ab^2cf^2-192abc^2ef+128ac^3df+64ac^3e^2-35b^4f^2+80b^3cef-96b^2c^2df-48b^2c^2e^2+128bc^3de-128c^4d^2\right)}{128c^{9/2}}+\frac{\sqrt{a+bx+cx^2}\left(220abcf^2-256ac^2ef-72ac^2f^2x-105b^3f^2+240b^2cef+70b^2cf^2x-288bc^2df-144bc^2e^2-160bc^2efx-56bc^2f^2x^2+384c^3de+192c^3dfx+96c^3e^2x+128c^3efx^2+48c^3f^2x^3\right)}{192c^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d + e*x + f*x^2)^2/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (Sqrt[a + b*x + c*x^2]*(384*c^3*d*e - 144*b*c^2*e^2 - 288*b*c^2*d*f + 240*b
^2*c*e*f - 256*a*c^2*e*f - 105*b^3*f^2 + 220*a*b*c*f^2 + 96*c^3*e^2*x + 192
*c^3*d*f*x - 160*b*c^2*e*f*x + 70*b^2*c*f^2*x - 72*a*c^2*f^2*x + 128*c^3*e*
f*x^2 - 56*b*c^2*f^2*x^2 + 48*c^3*f^2*x^3))/(192*c^4) + ((-128*c^4*d^2 + 12
8*b*c^3*d*e - 48*b^2*c^2*e^2 + 64*a*c^3*e^2 - 96*b^2*c^2*d*f + 128*a*c^3*d*
f + 80*b^3*c*e*f - 192*a*b*c^2*e*f - 35*b^4*f^2 + 120*a*b^2*c*f^2 - 48*a^2*
c^2*f^2)*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + b*x + c*x^2]])/(128*c^(9/2))
```

fricas [A] time = 0.80, size = 637, normalized size = 2.02

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/768*(3*(128*c^4*d^2 - 128*b*c^3*d*e + 16*(3*b^2*c^2 - 4*a*c^3)*e^2 + (35
*b^4 - 120*a*b^2*c + 48*a^2*c^2)*f^2 + 16*(2*(3*b^2*c^2 - 4*a*c^3)*d - (5*b
^3*c - 12*a*b*c^2)*e)*f)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*
x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(48*c^4*f^2*x^3 + 384*c^4*d
*e - 144*b*c^3*e^2 - 5*(21*b^3*c - 44*a*b*c^2)*f^2 + 8*(16*c^4*e*f - 7*b*c^
3*f^2)*x^2 - 16*(18*b*c^3*d - (15*b^2*c^2 - 16*a*c^3)*e)*f + 2*(48*c^4*e^2
+ (35*b^2*c^2 - 36*a*c^3)*f^2 + 16*(6*c^4*d - 5*b*c^3*e)*f)*x)*sqrt(c*x^2 +
 b*x + a))/c^5, -1/384*(3*(128*c^4*d^2 - 128*b*c^3*d*e + 16*(3*b^2*c^2 - 4*
a*c^3)*e^2 + (35*b^4 - 120*a*b^2*c + 48*a^2*c^2)*f^2 + 16*(2*(3*b^2*c^2 - 4
*a*c^3)*d - (5*b^3*c - 12*a*b*c^2)*e)*f)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b
*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(48*c^4*f^2*x^3 +
 384*c^4*d*e - 144*b*c^3*e^2 - 5*(21*b^3*c - 44*a*b*c^2)*f^2 + 8*(16*c^4*e*
f - 7*b*c^3*f^2)*x^2 - 16*(18*b*c^3*d - (15*b^2*c^2 - 16*a*c^3)*e)*f + 2*(4
8*c^4*e^2 + (35*b^2*c^2 - 36*a*c^3)*f^2 + 16*(6*c^4*d - 5*b*c^3*e)*f)*x)*sq
rt(c*x^2 + b*x + a))/c^5]
```

giac [A] time = 0.63, size = 304, normalized size = 0.96

$$\frac{1}{192}\sqrt{cx^2+bx+a}\left(2\left(4\left(\frac{6f^2x}{c}-\frac{7bc^2f^2-16c^3fe}{c^4}\right)x+\frac{96c^3df+35b^2cf^2-36ac^2f^2-80bc^2fe+48c^3e^2}{c^4}\right)x-\frac{288bc^2df+105b^3f^2-220abcf^2-384c^3de-240b^2cfe+256ac^2fe+144bc^2e^2}{c^4}\right)-\frac{\left(128c^4d^2+96b^2c^2df-128ac^3df+35b^4f^2-120ab^2cf^2+48a^2c^2f^2-128bc^3de-80b^3cfe+192abc^2fe+48b^2c^2e^2-64ac^3e^2\right)\log\left(\left|-2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}-b\right|\right)}{128c^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/192*sqrt(c*x^2 + b*x + a)*(2*(4*(6*f^2*x/c - (7*b*c^2*f^2 - 16*c^3*f*e)/c
^4)*x + (96*c^3*d*f + 35*b^2*c*f^2 - 36*a*c^2*f^2 - 80*b*c^2*f*e + 48*c^3*e
^2)/c^4)*x - (288*b*c^2*d*f + 105*b^3*f^2 - 220*a*b*c*f^2 - 384*c^3*d*e - 2
40*b^2*c*f*e + 256*a*c^2*f*e + 144*b*c^2*e^2)/c^4) - 1/128*(128*c^4*d^2 + 9
6*b^2*c^2*d*f - 128*a*c^3*d*f + 35*b^4*f^2 - 120*a*b^2*c*f^2 + 48*a^2*c^2*f
^2 - 128*b*c^3*d*e - 80*b^3*c*f*e + 192*a*b*c^2*f*e + 48*b^2*c^2*e^2 - 64*a
*c^3*e^2)*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(9
/2)
```

maple [B] time = 0.01, size = 706, normalized size = 2.23

$$\frac{\sqrt{cx^2+bx+a}\,f^2x^3}{4c}-\frac{7\sqrt{cx^2+bx+a}\,bf^2x^2}{24c^2}+\frac{2\sqrt{cx^2+bx+a}\,efx^2}{3c}+\frac{3a^2f^2\ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\sqrt{cx^2+bx+a}\right)}{8c^{\frac{5}{2}}}-\frac{15ab^2f^2\ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\sqrt{cx^2+bx+a}\right)}{16c^{\frac{7}{2}}}+\frac{3abef\ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\sqrt{cx^2+bx+a}\right)}{2c^{\frac{5}{2}}}-\frac{adf\ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\sqrt{cx^2+bx+a}\right)}{c^{\frac{3}{2}}}-\frac{ae^2\ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\sqrt{cx^2+bx+a}\right)}{2c^{\frac{3}{2}}}+\frac{35b^4f^2\ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\sqrt{cx^2+bx+a}\right)}{128c^{\frac{9}{2}}}-\frac{5b^3ef\ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\sqrt{cx^2+bx+a}\right)}{8c^{\frac{7}{2}}}+\frac{3b^2df\ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\sqrt{cx^2+bx+a}\right)}{4c^{\frac{5}{2}}}+\frac{3b^2e^2\ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\sqrt{cx^2+bx+a}\right)}{8c^{\frac{5}{2}}}-\frac{bde\ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\sqrt{cx^2+bx+a}\right)}{c^{\frac{3}{2}}}+\frac{d^2\ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\sqrt{cx^2+bx+a}\right)}{\sqrt{c}}-\frac{3\sqrt{cx^2+bx+a}\,af^2x}{8c^2}+\frac{35\sqrt{cx^2+bx+a}\,b^2f^2x}{96c^3}-\frac{5\sqrt{cx^2+bx+a}\,befx}{6c^2}+\frac{\sqrt{cx^2+bx+a}\,dfx}{c}+\frac{\sqrt{cx^2+bx+a}\,e^2x}{2c}+\frac{55\sqrt{cx^2+bx+a}\,abf^2}{48c^3}-\frac{4\sqrt{cx^2+bx+a}\,aef}{3c^2}-\frac{35\sqrt{cx^2+bx+a}\,b^3f^2}{64c^4}+\frac{5\sqrt{cx^2+bx+a}\,b^2ef}{4c^3}-\frac{3\sqrt{cx^2+bx+a}\,bdf}{2c^2}-\frac{3\sqrt{cx^2+bx+a}\,be^2}{4c^2}+\frac{2\sqrt{cx^2+bx+a}\,de}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)^2/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] 2*d*e/c*(c*x^2+b*x+a)^(1/2)+1/2*x/c*(c*x^2+b*x+a)^(1/2)*e^2-3/4*b/c^2*(c*x^
2+b*x+a)^(1/2)*e^2+3/8*b^2/c^(5/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/
2))*e^2-1/2*a/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*e^2-35/64
*f^2*b^3/c^4*(c*x^2+b*x+a)^(1/2)+35/128*f^2*b^4/c^(9/2)*ln((c*x+1/2*b)/c^(1
/2)+(c*x^2+b*x+a)^(1/2))+3/8*f^2*a^2/c^(5/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+
b*x+a)^(1/2))-5/6*e*f*b/c^2*x*(c*x^2+b*x+a)^(1/2)+1/4*f^2*x^3*(c*x^2+b*x+a)
^(1/2)/c+3/2*e*f*b/c^(5/2)*a*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+d^
2*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)-5/8*e*f*b^3/c^(7/2)*l
n((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-4/3*e*f*a/c^2*(c*x^2+b*x+a)^(1/2
)+x/c*(c*x^2+b*x+a)^(1/2)*d*f-3/2*b/c^2*(c*x^2+b*x+a)^(1/2)*d*f+3/4*b^2/c^(
5/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*d*f-a/c^(3/2)*ln((c*x+1/2*
b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*d*f-d*e*b/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c
*x^2+b*x+a)^(1/2))-3/8*f^2*a/c^2*x*(c*x^2+b*x+a)^(1/2)+55/48*f^2*b/c^3*a*(c
*x^2+b*x+a)^(1/2)-7/24*f^2*b/c^2*x^2*(c*x^2+b*x+a)^(1/2)+35/96*f^2*b^2/c^3*
x*(c*x^2+b*x+a)^(1/2)-15/16*f^2*b^2/c^(7/2)*a*ln((c*x+1/2*b)/c^(1/2)+(c*x^2
+b*x+a)^(1/2))+2/3*e*f*x^2/c*(c*x^2+b*x+a)^(1/2)+5/4*e*f*b^2/c^3*(c*x^2+b*x
+a)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(f\,x^2+e\,x+d\right)^2}{\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2)^2/(a + b*x + c*x^2)^(1/2),x)
```

```
[Out] int((d + e*x + f*x^2)^2/(a + b*x + c*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d+ex+fx^2\right)^2}{\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)**2/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x + f*x**2)**2/sqrt(a + b*x + c*x**2), x)
```

3.108 $\int \frac{d+ex+fx^2}{\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=116

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-4c(af+be)+3b^2f+8c^2d\right)}{8c^{5/2}}+\frac{\sqrt{a+bx+cx^2}\,(4ce-3bf)}{4c^2}+\frac{fx\sqrt{a+bx+cx^2}}{2c}$$

Rubi [A] time = 0.11, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1661, 640, 621, 206}

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-4c(af+be)+3b^2f+8c^2d\right)}{8c^{5/2}}+\frac{\sqrt{a+bx+cx^2}\,(4ce-3bf)}{4c^2}+\frac{fx\sqrt{a+bx+cx^2}}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] ((4*c*e - 3*b*f)*Sqrt[a + b*x + c*x^2])/(4*c^2) + (f*x*Sqrt[a + b*x + c*x^2
])/(2*c) + ((8*c^2*d + 3*b^2*f - 4*c*(b*e + a*f))*ArcTanh[(b + 2*c*x)/(2*Sq
rt[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{d+ex+fx^2}{\sqrt{a+bx+cx^2}}\,dx = \frac{fx\sqrt{a+bx+cx^2}}{2c} + \frac{\int \frac{2cd-af+\frac{1}{2}(4ce-3bf)x}{\sqrt{a+bx+cx^2}}\,dx}{2c}$$

$$= \frac{(4ce-3bf)\sqrt{a+bx+cx^2}}{4c^2} + \frac{fx\sqrt{a+bx+cx^2}}{2c} + \frac{\left(2c(2cd-af)-\frac{1}{2}b(4ce-3bf)\right)\int \frac{}{\sqrt{a+b}}}{4c^2}$$

$$= \frac{(4ce-3bf)\sqrt{a+bx+cx^2}}{4c^2} + \frac{fx\sqrt{a+bx+cx^2}}{2c} + \frac{\left(2c(2cd-af)-\frac{1}{2}b(4ce-3bf)\right)\text{Subst}}{2c^2}$$

$$= \frac{(4ce-3bf)\sqrt{a+bx+cx^2}}{4c^2} + \frac{fx\sqrt{a+bx+cx^2}}{2c} + \frac{\left(8c^2d+3b^2f-4c(be+af)\right)\tanh^{-1}\left(\frac{}{2}\right.}{8c^{5/2}}$$

Mathematica [A] time = 0.15, size = 96, normalized size = 0.83

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)\left(-4c(af+be)+3b^2f+8c^2d\right)}{8c^{5/2}} + \frac{\sqrt{a+x(b+cx)}\,(-3bf+4ce+2cfx)}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] ((4*c*e - 3*b*f + 2*c*f*x)*Sqrt[a + x*(b + c*x)])/(4*c^2) + ((8*c^2*d + 3*b
^2*f - 4*c*(b*e + a*f))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)
])])/(8*c^(5/2))
```

IntegrateAlgebraic [A] time = 0.48, size = 102, normalized size = 0.88

$$\frac{\log\left(-2c^{5/2}\sqrt{a+bx+cx^2}+bc^2+2c^3x\right)\left(4acf-3b^2f+4bce-8c^2d\right)}{8c^{5/2}} + \frac{\sqrt{a+bx+cx^2}\,(-3bf+4ce+2cfx)}{4c^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d + e*x + f*x^2)/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] ((4*c*e - 3*b*f + 2*c*f*x)*Sqrt[a + b*x + c*x^2])/(4*c^2) + ((-8*c^2*d + 4*
b*c*e - 3*b^2*f + 4*a*c*f)*Log[b*c^2 + 2*c^3*x - 2*c^(5/2)*Sqrt[a + b*x + c
*x^2]])/(8*c^(5/2))
```

fricas [A] time = 0.63, size = 227, normalized size = 1.96

$$\left[-\frac{\left(8\,c^{2}d-4\,bce+\left(3\,b^{2}-4\,ac\right)f\right)\sqrt{c}\,\log\left(-8\,c^{2}x^{2}-8\,bcx-b^{2}+4\,\sqrt{cx^{2}+bx+a}\,(2\,cx+b)\sqrt{c}\,-4\,ac\right)-4\left(2\,c^{2}fx+4\,c^{2}e-3\,bcf\right)\sqrt{cx^{2}+bx+a}}{16\,c^{3}},\,-\frac{\left(8\,c^{2}d-4\,bce+\left(3\,b^{2}-4\,ac\right)f\right)\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^{2}+bx+a}\,(2\,cx+b)\sqrt{-c}}{2\left(c^{2}x^{2}+bcx+ac\right)}\right)-2\left(2\,c^{2}fx+4\,c^{2}e-3\,bcf\right)\sqrt{cx^{2}+bx+a}}{8\,c^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*((8*c^2*d - 4*b*c*e + (3*b^2 - 4*a*c)*f)*sqrt(c)*log(-8*c^2*x^2 - 8*
b*c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(2*c
^2*f*x + 4*c^2*e - 3*b*c*f)*sqrt(c*x^2 + b*x + a))/c^3, -1/8*((8*c^2*d - 4*
b*c*e + (3*b^2 - 4*a*c)*f)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x
 + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(2*c^2*f*x + 4*c^2*e - 3*b*c*f)
*sqrt(c*x^2 + b*x + a))/c^3]
```

giac [A] time = 0.50, size = 98, normalized size = 0.84

$$\frac{1}{4}\,\sqrt{cx^{2}+bx+a}\left(\frac{2\,fx}{c}-\frac{3\,bf-4\,ce}{c^{2}}\right)-\frac{\left(8\,c^{2}d+3\,b^{2}f-4\,acf-4\,bce\right)\log\left(\left|-2\left(\sqrt{c}\,x-\sqrt{cx^{2}+bx+a}\right)\sqrt{c}\,-b\right|\right)}{8\,c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(c*x^2 + b*x + a)*(2*f*x/c - (3*b*f - 4*c*e)/c^2) - 1/8*(8*c^2*d +
3*b^2*f - 4*a*c*f - 4*b*c*e)*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
*sqrt(c) - b))/c^(5/2)
```

maple [A] time = 0.01, size = 185, normalized size = 1.59

$$-\frac{af\ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^{2}+bx+a}\right)}{2c^{\frac{3}{2}}}+\frac{3b^{2}f\ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^{2}+bx+a}\right)}{8c^{\frac{5}{2}}}-\frac{be\ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^{2}+bx+a}\right)}{2c^{\frac{3}{2}}}+\frac{d\ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^{2}+bx+a}\right)}{\sqrt{c}}+\frac{\sqrt{c\,x^{2}+bx+a}\;fx}{2c}-\frac{3\sqrt{c\,x^{2}+bx+a}\;bf}{4c^{2}}+\frac{\sqrt{c\,x^{2}+bx+a}\;e}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] 1/2*f*x*(c*x^2+b*x+a)^(1/2)/c-3/4*f*b/c^2*(c*x^2+b*x+a)^(1/2)+3/8*f*b^2/c^(
5/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-1/2*f*a/c^(3/2)*ln((c*x+1/
2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+e/c*(c*x^2+b*x+a)^(1/2)-1/2*e*b/c^(3/2)*l
```

```
n((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+d*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+
b*x+a)^(1/2))/c^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{f\,x^2+e\,x+d}{\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2)/(a + b*x + c*x^2)^(1/2),x)
```

```
[Out] int((d + e*x + f*x^2)/(a + b*x + c*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d+ex+fx^2}{\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x + f*x**2)/sqrt(a + b*x + c*x**2), x)
```

3.109 $\int \frac{1}{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}\,dx$

Optimal. Leaf size=374

$$\frac{\sqrt{2}\,f\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\,\sqrt{a+bx+cx^2}\,\sqrt{2af^2+\sqrt{e^2-4df}\,(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{e^2-4df}\,\sqrt{2af^2+\sqrt{e^2-4df}\,(ce-bf)-bef-2cdf+ce^2}}-\frac{\sqrt{2}\,f\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-\cdots}\right.}{2\sqrt{2}\,\sqrt{a+bx+cx^2}\,\sqrt{2af^2-\sqrt{e^2-4df}\,(ce-bf)-be\cdots}}\right.}{\sqrt{e^2-4df}\,\sqrt{2af^2-\sqrt{e^2-4df}\,(ce-bf)-bef-\cdots}}$$

Rubi [A] time = 0.58, antiderivative size = 374, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {983, 724, 206}

$$\frac{\sqrt{2}\,f\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\,\sqrt{a+bx+cx^2}\,\sqrt{2af^2+\sqrt{e^2-4df}\,(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{e^2-4df}\,\sqrt{2af^2+\sqrt{e^2-4df}\,(ce-bf)-bef-2cdf+ce^2}}-\frac{\sqrt{2}\,f\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\,\sqrt{a+bx+cx^2}\,\sqrt{2af^2-\sqrt{e^2-4df}\,(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{e^2-4df}\,\sqrt{2af^2-\sqrt{e^2-4df}\,(ce-bf)-bef-2cdf+ce^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] -((Sqrt[2]*f*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - S
qrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (
c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[e^2 - 4*d*f]*S
qrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]])) +
(Sqrt[2]*f*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqr
t[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*
e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[e^2 - 4*d*f]*Sqr
t[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 983

```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(
x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[1/((
b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[(2*c)/q, Int[1/((b + q
 + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[c*e - b*f, 0] && PosQ[b
^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}\,dx = \frac{(2f)\int \frac{1}{\left(e-\sqrt{e^2-4df}+2fx\right)\sqrt{a+bx+cx^2}}\,dx}{\sqrt{e^2-4df}} - \frac{(2f)\int \frac{1}{\left(e+\sqrt{e^2-4df}+2fx\right)\sqrt{a+bx+cx^2}}\,dx}{\sqrt{e^2-4df}}$$

$$= -\frac{(4f)\,\text{Subst}\left(\int \frac{1}{16af^2-8bf\left(e-\sqrt{e^2-4df}\right)+4c\left(e-\sqrt{e^2-4df}\right)^2-x^2}\,dx, x, \frac{4af-b\left(e-\sqrt{e^2-4df}\right.}{\sqrt{}}\right.}{\sqrt{e^2-4df}}$$

$$= -\frac{\sqrt{2}f\tanh^{-1}\left(\frac{4af-b\left(e-\sqrt{e^2-4df}\right)+2\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)x}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}\sqrt{a+bx+cx^2}}\right)}{\sqrt{e^2-4df}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}} + \frac{\sqrt{2}f\tan}{\sqrt{e^2-4d}}$$

Mathematica [A] time = 1.45, size = 376, normalized size = 1.01

$$\frac{\sqrt{2}f\left(\frac{\tanh^{-1}\left(\frac{4af-b\left(\sqrt{e^2-4df}+e-2fx\right)-2cx\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+x(b+cx)}\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{\tanh^{-1}\left(\frac{4af+b\left(\sqrt{e^2-4df}-e+2fx\right)+2cx\left(\sqrt{e^2-4df}-e\right)}{2\sqrt{2}\sqrt{a+x(b+cx)}\sqrt{f\left(2af+b\sqrt{e^2-4df}+b(-e)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{f\left(2af+b\sqrt{e^2-4df}+b(-e)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{e^2-4df}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] (Sqrt[2]*f*(ArcTanh[(4*a*f - 2*c*(e + Sqrt[e^2 - 4*d*f])*x - b*(e + Sqrt[e^
2 - 4*d*f] - 2*f*x))/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])
+ f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c*x)])]/Sqrt[c*(e^
2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))] -
 ArcTanh[(4*a*f + 2*c*(-e + Sqrt[e^2 - 4*d*f])*x + b*(-e + Sqrt[e^2 - 4*d*f
] + 2*f*x))/(2*Sqrt[2]*Sqrt[f*(-(b*e) + 2*a*f + b*Sqrt[e^2 - 4*d*f]) + c*(e
```

```
^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + x*(b + c*x)])]/Sqrt[f*(-(b*e) +
 2*a*f + b*Sqrt[e^2 - 4*d*f]) + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]))/Sq
rt[e^2 - 4*d*f]
```

IntegrateAlgebraic [C] time = 0.44, size = 211, normalized size = 0.56

$$-\text{RootSum}\left[\#1^4 f - 2\#1^3\sqrt{c}e - 2\#1^2 af + \#1^2 be + 4\#1^2 cd + 2\#1 a\sqrt{c}e - 4\#1 b\sqrt{c}d + a^2 f - abe + b^2 d\&, \frac{b\log\left(-\#1 + \sqrt{a+bx+cx^2} - \sqrt{c}x\right) - 2\#1\sqrt{c}\log\left(-\#1 + \sqrt{a+bx+cx^2} - \sqrt{c}x\right)}{-2\#1^3 f + 3\#1^2\sqrt{c}e + 2\#1 af - \#1 be - 4\#1 cd - a\sqrt{c}e + 2b\sqrt{c}d}\&\right]$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] -RootSum[b^2*d - a*b*e + a^2*f - 4*b*Sqrt[c]*d*#1 + 2*a*Sqrt[c]*e*#1 + 4*c*
d*#1^2 + b*e*#1^2 - 2*a*f*#1^2 - 2*Sqrt[c]*e*#1^3 + f*#1^4 & , (b*Log[-(Sqr
t[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*Sqrt[c]*Log[-(Sqrt[c]*x) + Sqrt[a
 + b*x + c*x^2] - #1]*#1)/(2*b*Sqrt[c]*d - a*Sqrt[c]*e - 4*c*d*#1 - b*e*#1
+ 2*a*f*#1 + 3*Sqrt[c]*e*#1^2 - 2*f*#1^3) & ]
```

fricas [B] time = 7.14, size = 11287, normalized size = 30.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*sqrt((c*e^2 + 2*a*f^2 - (2*c*d + b*e)*f + (c^2*d^2*e^2 - b*c*d*
e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f
^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)*sqrt((c^2
*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^
5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e
+ a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^
2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^
2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20
*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2
*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 +
6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*
e^4)*f)))/(c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a
^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b
^2 - 6*a*c)*d*e^2)*f))*log((2*(b^2*d - a*b*e)*f^2 + sqrt(2)*(c^2*d*e^3 - 4*
a*b*d*f^3 + (4*b*c*d^2 + 4*a*c*d*e + a*b*e^2)*f^2 - (4*c^2*d^2*e + b*c*d*e^
2 + a*c*e^3)*f - (c^3*d^3*e^3 - b*c^2*d^2*e^4 + a*c^2*d*e^5 + 4*(2*a^2*b*d^
2 - a^3*d*e)*f^4 + (2*a^2*b*d*e^2 + a^3*e^3 + 8*(b^3 - 2*a*b*c)*d^3 - 4*(3*
a*b^2 - a^2*c)*d^2*e)*f^3 + (8*b*c^2*d^4 - a^2*b*e^4 - 4*(3*b^2*c - a*c^2)*
d^3*e - 2*(b^3 - 10*a*b*c)*d^2*e^2 + (3*a*b^2 - 5*a^2*c)*d*e^3)*f^2 - (4*c^
3*d^4*e - 2*b*c^2*d^3*e^2 + 4*a*b*c*d*e^4 - a^2*c*e^5 - (3*b^2*c - 5*a*c^2)
*d^2*e^3)*f)*sqrt((c^2*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^
```

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3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d
^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b
*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a
^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*
c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c
)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2
*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a
*b^2*c - 2*a^2*c^2)*d*e^4)*f)))*sqrt(c*x^2 + b*x + a)*sqrt((c*e^2 + 2*a*f^2
 - (2*c*d + b*e)*f + (c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*
a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a
*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)*sqrt((c^2*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*
d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (
b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)
*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 -
 a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d
^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 +
2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 -
 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b
*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))/(c^2*d^2*e^2 - b*c*d*e^
3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2
 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)) - 2*(b*c*d
*e - a*c*e^2)*f + ((4*b*c*d - b^2*e)*f^2 - (4*c^2*d*e - b*c*e^2)*f)*x - (8*
a^3*d*f^4 - 2*(4*a^2*b*d*e + a^3*e^2 - 4*(a*b^2 - 2*a^2*c)*d^2)*f^3 + 2*(4*
a*c^2*d^3 - 4*a*b*c*d^2*e + a^2*b*e^3 - (a*b^2 - 6*a^2*c)*d*e^2)*f^2 - 2*(a
*c^2*d^2*e^2 - a*b*c*d*e^3 + a^2*c*e^4)*f + (4*a^2*b*d*f^4 - (4*a*b^2*d*e +
 a^2*b*e^2 - 4*(b^3 - 2*a*b*c)*d^2)*f^3 + (4*b*c^2*d^3 - 4*b^2*c*d^2*e + a*
b^2*e^3 - (b^3 - 6*a*b*c)*d*e^2)*f^2 - (b*c^2*d^2*e^2 - b^2*c*d*e^3 + a*b*c
*e^4)*f)*x)*sqrt((c^2*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3
*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^
2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*
e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^
2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c
^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)
*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*
b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*
b^2*c - 2*a^2*c^2)*d*e^4)*f)))/x) - 1/4*sqrt(2)*sqrt((c*e^2 + 2*a*f^2 - (2*
c*d + b*e)*f + (c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*
e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3
 - (b^2 - 6*a*c)*d*e^2)*f)*sqrt((c^2*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^
2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^
2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*
f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b
*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8
*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b
^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c
```

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^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*
d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))/(c^2*d^2*e^2 - b*c*d*e^3 + a*
c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*
c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f))*log((2*(b^2*d -
a*b*e)*f^2 - sqrt(2)*(c^2*d*e^3 - 4*a*b*d*f^3 + (4*b*c*d^2 + 4*a*c*d*e + a*
b*e^2)*f^2 - (4*c^2*d^2*e + b*c*d*e^2 + a*c*e^3)*f - (c^3*d^3*e^3 - b*c^2*d
^2*e^4 + a*c^2*d*e^5 + 4*(2*a^2*b*d^2 - a^3*d*e)*f^4 + (2*a^2*b*d*e^2 + a^3
*e^3 + 8*(b^3 - 2*a*b*c)*d^3 - 4*(3*a*b^2 - a^2*c)*d^2*e)*f^3 + (8*b*c^2*d^
4 - a^2*b*e^4 - 4*(3*b^2*c - a*c^2)*d^3*e - 2*(b^3 - 10*a*b*c)*d^2*e^2 + (3
*a*b^2 - 5*a^2*c)*d*e^3)*f^2 - (4*c^3*d^4*e - 2*b*c^2*d^3*e^2 + 4*a*b*c*d*e
^4 - a^2*c*e^5 - (3*b^2*c - 5*a*c^2)*d^2*e^3)*f)*sqrt((c^2*e^2 - 2*b*c*e*f
+ b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 -
 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^
2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*
d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*
c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2
*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2
- 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2
+ (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))*sqrt(c*
x^2 + b*x + a)*sqrt((c*e^2 + 2*a*f^2 - (2*c*d + b*e)*f + (c^2*d^2*e^2 - b*c
*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2
)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)*sqrt((
c^2*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d
*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d
*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a
*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d
*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 -
 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2
+ 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2
 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)
*d*e^4)*f)))/(c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e
+ a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 -
 (b^2 - 6*a*c)*d*e^2)*f)) - 2*(b*c*d*e - a*c*e^2)*f + ((4*b*c*d - b^2*e)*f^
2 - (4*c^2*d*e - b*c*e^2)*f)*x - (8*a^3*d*f^4 - 2*(4*a^2*b*d*e + a^3*e^2 -
4*(a*b^2 - 2*a^2*c)*d^2)*f^3 + 2*(4*a*c^2*d^3 - 4*a*b*c*d^2*e + a^2*b*e^3 -
 (a*b^2 - 6*a^2*c)*d*e^2)*f^2 - 2*(a*c^2*d^2*e^2 - a*b*c*d*e^3 + a^2*c*e^4)
*f + (4*a^2*b*d*f^4 - (4*a*b^2*d*e + a^2*b*e^2 - 4*(b^3 - 2*a*b*c)*d^2)*f^3
 + (4*b*c^2*d^3 - 4*b^2*c*d^2*e + a*b^2*e^3 - (b^3 - 6*a*b*c)*d*e^2)*f^2 -
(b*c^2*d^2*e^2 - b^2*c*d*e^3 + a*b*c*e^4)*f)*x)*sqrt((c^2*e^2 - 2*b*c*e*f +
 b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 -
4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2
*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d
^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c
^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*
c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 -
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 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 +
 (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))/x) + 1/4
*sqrt(2)*sqrt((c*e^2 + 2*a*f^2 - (2*c*d + b*e)*f - (c^2*d^2*e^2 - b*c*d*e^3
 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2
- (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)*sqrt((c^2*e^
2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 +
 a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a
^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c
 + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*
f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*
b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^
3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a
*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4
)*f)))/(c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*
e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2
- 6*a*c)*d*e^2)*f))*log((2*(b^2*d - a*b*e)*f^2 + sqrt(2)*(c^2*d*e^3 - 4*a*b
*d*f^3 + (4*b*c*d^2 + 4*a*c*d*e + a*b*e^2)*f^2 - (4*c^2*d^2*e + b*c*d*e^2 +
 a*c*e^3)*f + (c^3*d^3*e^3 - b*c^2*d^2*e^4 + a*c^2*d*e^5 + 4*(2*a^2*b*d^2 -
 a^3*d*e)*f^4 + (2*a^2*b*d*e^2 + a^3*e^3 + 8*(b^3 - 2*a*b*c)*d^3 - 4*(3*a*b
^2 - a^2*c)*d^2*e)*f^3 + (8*b*c^2*d^4 - a^2*b*e^4 - 4*(3*b^2*c - a*c^2)*d^3
*e - 2*(b^3 - 10*a*b*c)*d^2*e^2 + (3*a*b^2 - 5*a^2*c)*d*e^3)*f^2 - (4*c^3*d
^4*e - 2*b*c^2*d^3*e^2 + 4*a*b*c*d*e^4 - a^2*c*e^5 - (3*b^2*c - 5*a*c^2)*d^
2*e^3)*f)*sqrt((c^2*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e
^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*
e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^
3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*
b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2
)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d
*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*
c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^
2*c - 2*a^2*c^2)*d*e^4)*f)))*sqrt(c*x^2 + b*x + a)*sqrt((c*e^2 + 2*a*f^2 -
(2*c*d + b*e)*f - (c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b
*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*
e^3 - (b^2 - 6*a*c)*d*e^2)*f)*sqrt((c^2*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4
*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2
*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^
2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^
2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4
- 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(
a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*
b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^
2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))/(c^2*d^2*e^2 - b*c*d*e^3 +
 a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 -
(4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)) - 2*(b*c*d*e
- a*c*e^2)*f + ((4*b*c*d - b^2*e)*f^2 - (4*c^2*d*e - b*c*e^2)*f)*x + (8*a^3
```

```
*d*f^4 - 2*(4*a^2*b*d*e + a^3*e^2 - 4*(a*b^2 - 2*a^2*c)*d^2)*f^3 + 2*(4*a*c
^2*d^3 - 4*a*b*c*d^2*e + a^2*b*e^3 - (a*b^2 - 6*a^2*c)*d*e^2)*f^2 - 2*(a*c^
2*d^2*e^2 - a*b*c*d*e^3 + a^2*c*e^4)*f + (4*a^2*b*d*f^4 - (4*a*b^2*d*e + a^
2*b*e^2 - 4*(b^3 - 2*a*b*c)*d^2)*f^3 + (4*b*c^2*d^3 - 4*b^2*c*d^2*e + a*b^2
*e^3 - (b^3 - 6*a*b*c)*d*e^2)*f^2 - (b*c^2*d^2*e^2 - b^2*c*d*e^3 + a*b*c*e^
4)*f)*x)*sqrt((c^2*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^
3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e
^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3
 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b
^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)
*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*
e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c
*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2
*c - 2*a^2*c^2)*d*e^4)*f)))/x) - 1/4*sqrt(2)*sqrt((c*e^2 + 2*a*f^2 - (2*c*d
 + b*e)*f - (c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e +
 a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 -
(b^2 - 6*a*c)*d*e^2)*f)*sqrt((c^2*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^2 -
 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 +
 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4
 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)
*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b
^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3
- 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*
d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2
*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))/(c^2*d^2*e^2 - b*c*d*e^3 + a*c*e
^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2
*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f))*log((2*(b^2*d - a*b
*e)*f^2 - sqrt(2)*(c^2*d*e^3 - 4*a*b*d*f^3 + (4*b*c*d^2 + 4*a*c*d*e + a*b*e
^2)*f^2 - (4*c^2*d^2*e + b*c*d*e^2 + a*c*e^3)*f + (c^3*d^3*e^3 - b*c^2*d^2*
e^4 + a*c^2*d*e^5 + 4*(2*a^2*b*d^2 - a^3*d*e)*f^4 + (2*a^2*b*d*e^2 + a^3*e^
3 + 8*(b^3 - 2*a*b*c)*d^3 - 4*(3*a*b^2 - a^2*c)*d^2*e)*f^3 + (8*b*c^2*d^4 -
 a^2*b*e^4 - 4*(3*b^2*c - a*c^2)*d^3*e - 2*(b^3 - 10*a*b*c)*d^2*e^2 + (3*a*
b^2 - 5*a^2*c)*d*e^3)*f^2 - (4*c^3*d^4*e - 2*b*c^2*d^3*e^2 + 4*a*b*c*d*e^4
- a^2*c*e^5 - (3*b^2*c - 5*a*c^2)*d^2*e^3)*f)*sqrt((c^2*e^2 - 2*b*c*e*f + b
^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*
a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b
^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3
 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2
 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^
2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2
*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (
b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))*sqrt(c*x^2
 + b*x + a)*sqrt((c*e^2 + 2*a*f^2 - (2*c*d + b*e)*f - (c^2*d^2*e^2 - b*c*d*
e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f
^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)*sqrt((c^2
```

```
*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^
5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e
+ a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^
2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^
2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20
*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2
*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 +
6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*
e^4)*f)))/(c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a
^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b
^2 - 6*a*c)*d*e^2)*f)) - 2*(b*c*d*e - a*c*e^2)*f + ((4*b*c*d - b^2*e)*f^2 -
 (4*c^2*d*e - b*c*e^2)*f)*x + (8*a^3*d*f^4 - 2*(4*a^2*b*d*e + a^3*e^2 - 4*(
a*b^2 - 2*a^2*c)*d^2)*f^3 + 2*(4*a*c^2*d^3 - 4*a*b*c*d^2*e + a^2*b*e^3 - (a
*b^2 - 6*a^2*c)*d*e^2)*f^2 - 2*(a*c^2*d^2*e^2 - a*b*c*d*e^3 + a^2*c*e^4)*f
+ (4*a^2*b*d*f^4 - (4*a*b^2*d*e + a^2*b*e^2 - 4*(b^3 - 2*a*b*c)*d^2)*f^3 +
(4*b*c^2*d^3 - 4*b^2*c*d^2*e + a*b^2*e^3 - (b^3 - 6*a*b*c)*d*e^2)*f^2 - (b*
c^2*d^2*e^2 - b^2*c*d*e^3 + a*b*c*e^4)*f)*x)*sqrt((c^2*e^2 - 2*b*c*e*f + b^
2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a
^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^
2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3
- 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2
- 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2
)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*
(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b
^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))/x)
```

giac [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [**B**] time = 0.02, size = 761, normalized size = 2.03

$$\frac{\sqrt{2}\,\ln\left(\frac{\frac{\left(bf-ce-\sqrt{-4df+e^2}\,c\right)\left(x+\frac{e+\sqrt{-4df+e^2}}{2f}\right)}{f}+\frac{2af^2-bef-2cdf+ce^2-\sqrt{-4df+e^2}\,bf+\sqrt{-4df+e^2}\,ce}{f^2}+\frac{\sqrt{2}\,\sqrt{\frac{2af^2-bef-2cdf+ce^2-\sqrt{-4df+e^2}\,bf+\sqrt{-4df+e^2}\,ce}{f^2}}\,\sqrt{4\left(x+\frac{e+\sqrt{-4df+e^2}}{2f}\right)^2 c+\frac{4\left(bf-ce-\sqrt{-4df+e^2}\,c\right)\left(x+\frac{e+\sqrt{-4df+e^2}}{2f}\right)}{f}+\frac{4af^2-2bef-4cdf+2ce^2-2\sqrt{-4df+e^2}\,bf+2\sqrt{-4df+e^2}\,ce}{f^2}}}{2}}{x+\frac{e+\sqrt{-4df+e^2}}{2f}}\right)}{\sqrt{-4df+e^2}\,\sqrt{\frac{2af^2-bef-2cdf+ce^2-\sqrt{-4df+e^2}\,bf+\sqrt{-4df+e^2}\,ce}{f^2}}}-\frac{\sqrt{2}\,\ln\left(\frac{\frac{\left(bf-ce+\sqrt{-4df+e^2}\,c\right)\left(x-\frac{-e+\sqrt{-4df+e^2}}{2f}\right)}{f}+\frac{2af^2-bef-2cdf+ce^2+\sqrt{-4df+e^2}\,bf-\sqrt{-4df+e^2}\,ce}{f^2}+\frac{\sqrt{2}\,\sqrt{\frac{2af^2-bef-2cdf+ce^2+\sqrt{-4df+e^2}\,bf-\sqrt{-4df+e^2}\,ce}{f^2}}\,\sqrt{4\left(x-\frac{-e+\sqrt{-4df+e^2}}{2f}\right)^2 c+\frac{4\left(bf-ce+\sqrt{-4df+e^2}\,c\right)\left(x-\frac{-e+\sqrt{-4df+e^2}}{2f}\right)}{f}+\frac{4af^2-2bef-4cdf+2ce^2+2\sqrt{-4df+e^2}\,bf-2\sqrt{-4df+e^2}\,ce}{f^2}}}{2}}{x-\frac{-e+\sqrt{-4df+e^2}}{2f}}\right)}{\sqrt{-4df+e^2}\,\sqrt{\frac{2af^2-bef-2cdf+ce^2+\sqrt{-4df+e^2}\,bf-\sqrt{-4df+e^2}\,ce}{f^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x)
```

```
[Out] 1/(-4*d*f+e^2)^(1/2)*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c
*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*b*f+(-4
```

```
*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+1/f*(-c*(-4*d*f+e^2)^(
1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^
(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*(4
*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+4/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(
x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/
2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2
))/f))-1/(-4*d*f+e^2)^(1/2)*2^(1/2)/(((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(
1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*b*
f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+(c*(-4*d*f+e^2)^(
1/2)+b*f-c*e)/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2
)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*
(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c+4*(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f
*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(
1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(
1/2))/f))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re details)Is 4*d*f-e^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{c\,x^2+b\,x+a}\,\left(f\,x^2+e\,x+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)),x)
```

```
[Out] int(1/((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,\left(d+ex+fx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d),x)
[Out] Integral(1/(sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2)), x)
```

3.110 $\int \frac{1}{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)^2}\,dx$

Optimal. Leaf size=789

$$\frac{\left(f\left(e-\sqrt{e^2-4df}\right)(ce-bf)(2af-be+2cd)-2f\left(f\left(-4a^2f^2+3abef+b^2\left(e^2-6df\right)\right)-c\left(4af\left(e^2-3df\right)+b\right.\right.\right.}{2\sqrt{2}\left(e^2-4df\right)^{3/2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{2af^2-}}$$

Rubi [A] time = 8.21, antiderivative size = 787, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {974, 1032, 724, 206}

$$\frac{\left(f\left(e-\sqrt{e^2-4df}\right)(ce-bf)(2af-be+2cd)-2f\left(-4a^2f^3+3abef^2-4acf\left(e^2-3df\right)+b^2f\left(e^2-6df\right)-bc\left(e^3-5def\right)+2c^2d\left(e^2-4df\right)\right)\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{2\sqrt{2}\left(e^2-4df\right)^{3/2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}-\frac{\left(f\left(\sqrt{e^2-4df}+e\right)(ce-bf)(2af-be+2cd)-2f\left(-4a^2f^3+3abef^2-4acf\left(e^2-3df\right)+b^2f\left(e^2-6df\right)-bc\left(e^3-5def\right)+2c^2d\left(e^2-4df\right)\right)\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{2\sqrt{2}\left(e^2-4df\right)^{3/2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}+\frac{\sqrt{a+bx+cx^2}\left(fx\left(f(be-2af)-c\left(e^2-2df\right)\right)+f\left(-aef-2bdf+be^2\right)-c\left(e^3-3def\right)\right)}{\left(e^2-4df\right)\left(d+ex+fx^2\right)\left((cd-af)^2-(bd-ae)(ce-bf)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)^2),x]
```

```
[Out] ((f*(b*e^2 - 2*b*d*f - a*e*f) - c*(e^3 - 3*d*e*f) + f*(f*(b*e - 2*a*f) - c*
(e^2 - 2*d*f))*x)*Sqrt[a + b*x + c*x^2])/((e^2 - 4*d*f)*((c*d - a*f)^2 - (b
*d - a*e)*(c*e - b*f))*(d + e*x + f*x^2)) + ((f*(2*c*d - b*e + 2*a*f)*(c*e
- b*f)*(e - Sqrt[e^2 - 4*d*f]) - 2*f*(3*a*b*e*f^2 - 4*a^2*f^3 + b^2*f*(e^2
- 6*d*f) + 2*c^2*d*(e^2 - 4*d*f) - 4*a*c*f*(e^2 - 3*d*f) - b*c*(e^3 - 5*d*e
*f)))*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2
 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b
*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])]])/(2*Sqrt[2]*(e^2 - 4*d*f)^(3
/2)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqrt[c*e^2 - 2*c*d*f - b*e*f
+ 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]) - ((f*(2*c*d - b*e + 2*a*f)*(c*
e - b*f)*(e + Sqrt[e^2 - 4*d*f]) - 2*f*(3*a*b*e*f^2 - 4*a^2*f^3 + b^2*f*(e^
2 - 6*d*f) + 2*c^2*d*(e^2 - 4*d*f) - 4*a*c*f*(e^2 - 3*d*f) - b*c*(e^3 - 5*d
*e*f)))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e
^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e -
 b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])]])/(2*Sqrt[2]*(e^2 - 4*d*f)^
(3/2)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqrt[c*e^2 - 2*c*d*f - b*e*
f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 974

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*
a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*(
d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q,
 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)^2}\,dx = \frac{\left(f\left(be^2-2bdf-aef\right)-c\left(e^3-3def\right)+f\left(f(be-2af)-c\left(e^2-2df\right)\right)\right.}{\left(e^2-4df\right)\left((cd-af)^2-(bd-ae)(ce-bf)\right)\left(d+ex+fx\right.}$$

$$= \frac{\left(f\left(be^2-2bdf-aef\right)-c\left(e^3-3def\right)+f\left(f(be-2af)-c\left(e^2-2df\right)\right)\right.}{\left(e^2-4df\right)\left((cd-af)^2-(bd-ae)(ce-bf)\right)\left(d+ex+fx\right.}$$

$$= \frac{\left(f\left(be^2-2bdf-aef\right)-c\left(e^3-3def\right)+f\left(f(be-2af)-c\left(e^2-2df\right)\right)\right.}{\left(e^2-4df\right)\left((cd-af)^2-(bd-ae)(ce-bf)\right)\left(d+ex+fx\right.}$$

$$= \frac{\left(f\left(be^2-2bdf-aef\right)-c\left(e^3-3def\right)+f\left(f(be-2af)-c\left(e^2-2df\right)\right)\right.}{\left(e^2-4df\right)\left((cd-af)^2-(bd-ae)(ce-bf)\right)\left(d+ex+fx\right.}$$

Mathematica [A] time = 6.69, size = 1377, normalized size = 1.75

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)^2),x]
```

```
[Out] (-8*f^3*(a + b*x + c*x^2))/((e^2 - 4*d*f)*(4*a*f^2 - 2*b*f*(e - Sqrt[e^2 -
4*d*f]) + c*(e - Sqrt[e^2 - 4*d*f])^2)*(e - Sqrt[e^2 - 4*d*f] + 2*f*x)*Sqrt
[a + x*(b + c*x)]) - (8*f^3*(a + b*x + c*x^2))/((e^2 - 4*d*f)*(4*a*f^2 - 2*
b*f*(e + Sqrt[e^2 - 4*d*f]) + c*(e + Sqrt[e^2 - 4*d*f])^2)*(e + Sqrt[e^2 -
4*d*f] + 2*f*x)*Sqrt[a + x*(b + c*x)]) + (2*Sqrt[2]*f^2*Sqrt[a + b*x + c*x^
2]*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 -
4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*
f - b*(e - Sqrt[e^2 - 4*d*f]))]*Sqrt[a + b*x + c*x^2])])/((e^2 - 4*d*f)^(3/
2)*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e - Sqrt[e^2
- 4*d*f]))]*Sqrt[a + x*(b + c*x)]) - (8*Sqrt[2]*f^2*Sqrt[c*e^2 - 2*c*d*f -
b*e*f + 2*a*f^2 - c*e*Sqrt[e^2 - 4*d*f] + b*f*Sqrt[e^2 - 4*d*f]]*(2*b*f + 2
*c*(-e + Sqrt[e^2 - 4*d*f]))*Sqrt[a + b*x + c*x^2]*ArcTanh[(-4*a*f - b*(-e
+ Sqrt[e^2 - 4*d*f]) - (2*b*f + 2*c*(-e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]
*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - c*e*Sqrt[e^2 - 4*d*f] + b*f*Sqrt[
e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/((e^2 - 4*d*f)*(4*a*f^2 + 2*b*f*(-e
+ Sqrt[e^2 - 4*d*f]) + c*(-e + Sqrt[e^2 - 4*d*f])^2)*(16*a*f^2 + 8*b*f*(-e
```

```
+ Sqrt[e^2 - 4*d*f]) + 4*c*(-e + Sqrt[e^2 - 4*d*f])^2)*Sqrt[a + x*(b + c*x)
]) - (2*Sqrt[2]*f^2*Sqrt[a + b*x + c*x^2]*ArcTanh[(4*a*f - b*(e + Sqrt[e^2
- 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*(e^2 -
 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]*Sqrt
[a + b*x + c*x^2])])/((e^2 - 4*d*f)^(3/2)*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2
- 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c*x)]) -
 (8*Sqrt[2]*f^2*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + c*e*Sqrt[e^2 - 4*d
*f] - b*f*Sqrt[e^2 - 4*d*f]]*(-2*b*f + 2*c*(e + Sqrt[e^2 - 4*d*f]))*Sqrt[a
+ b*x + c*x^2]*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) - (-2*b*f + 2*c*(
e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^
2 + c*e*Sqrt[e^2 - 4*d*f] - b*f*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])
/((e^2 - 4*d*f)*(4*a*f^2 - 2*b*f*(e + Sqrt[e^2 - 4*d*f]) + c*(e + Sqrt[e^2
- 4*d*f])^2)*(16*a*f^2 - 8*b*f*(e + Sqrt[e^2 - 4*d*f]) + 4*c*(e + Sqrt[e^2
- 4*d*f])^2)*Sqrt[a + x*(b + c*x)])
```

IntegrateAlgebraic [C] time = 175.85, size = 55521, normalized size = 70.37

Result too large to show

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)^2),x]
[Out] Result too large to show
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d)^2,x, algorithm="fricas")
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d)^2,x, algorithm="giac")
[Out] Timed out
```

maple [B] time = 0.03, size = 3858, normalized size = 4.89

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d)^2,x)
```

```
[Out] 2*f/(4*d*f-e^2)/(-4*d*f+e^2)^(1/2)*2^(1/2)/((2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-
4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*ln(((b*f-c*e-(-4*d*
f+e^2)^(1/2)*c)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)/f+(2*a*f^2-b*e*f-2*c*d*f+c
*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2+1/2*2^(1/2)*((2*a*f
^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^
(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+4*(b*f-c*e-(-4*d*f+e^2)^(1/2)
*c)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)/f+2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d
*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2
)^(1/2))/f))+2/(4*d*f-e^2)/(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*
b*f-(-4*d*f+e^2)^(1/2)*c*e)*f^2/(x-1/2/f*(-4*d*f+e^2)^(1/2)+1/2*e/f)*((x-1/
2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c+(b*f-c*e+(-4*d*f+e^2)^(1/2)*c)*(x-1/2*(-e+
(-4*d*f+e^2)^(1/2))/f)/f+1/2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2
)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)-1/(4*d*f-e^2)*f/(2*a*f^2-b*e*f-2*c
*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)*2^(1/2)/((2*a*f^2
-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1
/2)*ln(((b*f-c*e+(-4*d*f+e^2)^(1/2)*c)*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)/f+
(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)
/f^2+1/2*2^(1/2)*((2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d
*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c+4*(b
*f-c*e+(-4*d*f+e^2)^(1/2)*c)*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)/f+2*(2*a*f^2
-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1
/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))*c*(-4*d*f+e^2)^(1/2)-1/(4*d*f-e^2)*
f^2/(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*
c*e)*2^(1/2)/((2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e
^2)^(1/2)*c*e)/f^2)^(1/2)*ln(((b*f-c*e+(-4*d*f+e^2)^(1/2)*c)*(x-1/2*(-e+(-4
*d*f+e^2)^(1/2))/f)/f+(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(
-4*d*f+e^2)^(1/2)*c*e)/f^2+1/2*2^(1/2)*((2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*
f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e
^2)^(1/2))/f)^2*c+4*(b*f-c*e+(-4*d*f+e^2)^(1/2)*c)*(x-1/2*(-e+(-4*d*f+e^2)^
(1/2))/f)/f+2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e
^2)^(1/2)*c*e)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))*b+1/(4*d*f-e^
2)*f/(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)
*c*e)*2^(1/2)/((2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+
e^2)^(1/2)*c*e)/f^2)^(1/2)*ln(((b*f-c*e+(-4*d*f+e^2)^(1/2)*c)*(x-1/2*(-e+(-
4*d*f+e^2)^(1/2))/f)/f+(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-
(-4*d*f+e^2)^(1/2)*c*e)/f^2+1/2*2^(1/2)*((2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d
*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+
e^2)^(1/2))/f)^2*c+4*(b*f-c*e+(-4*d*f+e^2)^(1/2)*c)*(x-1/2*(-e+(-4*d*f+e^2)
^(1/2))/f)/f+2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+
e^2)^(1/2)*c*e)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))*c*e+2/(4*d*f
-e^2)/(2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2
```

```
)*c*e)*f^2/(x+1/2/f*(-4*d*f+e^2)^(1/2)+1/2*e/f)*((x+1/2*(e+(-4*d*f+e^2)^(1/
2))/f)^2*c+(b*f-c*e-(-4*d*f+e^2)^(1/2)*c)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)/
f+1/2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2
)*c*e)/f^2)^(1/2)+1/(4*d*f-e^2)*f/(2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)
^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)*2^(1/2)/((2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-
4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*ln(((b*f-c*e-(-4*d*
f+e^2)^(1/2)*c)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)/f+(2*a*f^2-b*e*f-2*c*d*f+c
*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2+1/2*2^(1/2)*((2*a*f
^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^
(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+4*(b*f-c*e-(-4*d*f+e^2)^(1/2)
*c)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)/f+2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d
*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2
)^(1/2))/f))*c*(-4*d*f+e^2)^(1/2)-1/(4*d*f-e^2)*f^2/(2*a*f^2-b*e*f-2*c*d*f+
c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)*2^(1/2)/((2*a*f^2-b*e*
f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*l
n(((b*f-c*e-(-4*d*f+e^2)^(1/2)*c)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)/f+(2*a*f
^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2+1
/2*2^(1/2)*((2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2
)^(1/2)*c*e)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+4*(b*f-c*e-
(-4*d*f+e^2)^(1/2)*c)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)/f+2*(2*a*f^2-b*e*f-2
*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2))/(x+
1/2*(e+(-4*d*f+e^2)^(1/2))/f))*b+1/(4*d*f-e^2)*f/(2*a*f^2-b*e*f-2*c*d*f+c*e
^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)*2^(1/2)/((2*a*f^2-b*e*f-2
*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*ln((
(b*f-c*e-(-4*d*f+e^2)^(1/2)*c)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)/f+(2*a*f^2-
b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2+1/2*
2^(1/2)*((2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(
1/2)*c*e)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+4*(b*f-c*e-(-4
*d*f+e^2)^(1/2)*c)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)/f+2*(2*a*f^2-b*e*f-2*c*
d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2))/(x+1/2
*(e+(-4*d*f+e^2)^(1/2))/f))*c*e-2*f/(4*d*f-e^2)/(-4*d*f+e^2)^(1/2)*2^(1/2)/
((2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e
)/f^2)^(1/2)*ln(((b*f-c*e+(-4*d*f+e^2)^(1/2)*c)*(x-1/2*(-e+(-4*d*f+e^2)^(1/
2))/f)/f+(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(
1/2)*c*e)/f^2+1/2*2^(1/2)*((2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*
b*f-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)
^2*c+4*(b*f-c*e+(-4*d*f+e^2)^(1/2)*c)*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)/f+2
*(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e
)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d)^2,x, algorithm="maxima")
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*(f*x^2 + e*x + d)^2), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{c\,x^2+b\,x+a}\,\left(f\,x^2+e\,x+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)^2),x)
[Out] int(1/((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)^2), x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d)**2,x)
[Out] Timed out
```

3.111 $\int \frac{(d+ex+fx^2)^3}{(a+bx+cx^2)^{3/2}}\,dx$

Optimal. Leaf size=649

$$\frac{3\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(80c^2f\left(a^2f^2+6abef+3b^2\left(df+e^2\right)\right)-280b^2cf^2(af+be)-64c^3\left(3af\left(df+e^2\right)+b\left(\right.\right.\right.}{128c^{11/2}}$$

Rubi [A] time = 2.11, antiderivative size = 649, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1660, 1661, 640, 621, 206}

$$\frac{2\left(-x\left(-2acf+b^2f-bce+2c^2d\right)\left(a^2c^2f^2-4ab^2cf^2+7abc^2ef-2ac^3df-3ac^3e^2+b^2c^2df+b^2c^2e^2-2b^3cef+b^4f^2-bc^3de+c^4d^2\right)-bc^2\left(-9a^2cf\left(df+e^2\right)+5a^3f^3+3ac^2d\left(df+e^2\right)+c^3d^3\right)+2ac^3e\left(3a^2f^2-ac\left(6df+e^2\right)+3c^2d^2\right)-ab^2c^2e\left(12af^2-c\left(6df+e^2\right)\right)+ab^3cf\left(5af^2-3c\left(df+e^2\right)\right)+3ab^4cef^2-ab^5f^3\right)}{c^5\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}+\frac{3\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(80c^2f\left(a^2f^2+6abef+3b^2\left(df+e^2\right)\right)-280b^2cf^2(af+be)-64c^3\left(3af\left(df+e^2\right)+b\left(6def+e^3\right)\right)+105b^4f^3+128c^4d\left(df+e^2\right)\right)}{128c^{11/2}}+\frac{fx\sqrt{a+bx+cx^2}\left(-4cf(7af+22be)+41b^2f^2+48c^2\left(df+e^2\right)\right)}{32c^4}-\frac{\sqrt{a+bx+cx^2}\left(16c^2f\left(20aef+21b\left(df+e^2\right)\right)-4bcf^2(73af+114be)+187b^3f^3-64c^3\left(6def+e^3\right)\right)}{64c^5}+\frac{f^2x^2\sqrt{a+bx+cx^2}(8ce-5bf)}{8c^3}+\frac{f^3x^3\sqrt{a+bx+cx^2}}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)^3/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] (2*(3*a*b^4*c*e*f^2 - a*b^5*f^3 + a*b^3*c*f*(5*a*f^2 - 3*c*(e^2 + d*f)) - b
*c^2*(c^3*d^3 + 5*a^3*f^3 + 3*a*c^2*d*(e^2 + d*f) - 9*a^2*c*f*(e^2 + d*f))
- a*b^2*c^2*e*(12*a*f^2 - c*(e^2 + 6*d*f)) + 2*a*c^3*e*(3*c^2*d^2 + 3*a^2*f
^2 - a*c*(e^2 + 6*d*f)) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*(c^4*d^2 - b*
c^3*d*e + b^2*c^2*e^2 - 3*a*c^3*e^2 + b^2*c^2*d*f - 2*a*c^3*d*f - 2*b^3*c*e
*f + 7*a*b*c^2*e*f + b^4*f^2 - 4*a*b^2*c*f^2 + a^2*c^2*f^2)*x))/(c^5*(b^2 -
 4*a*c)*Sqrt[a + b*x + c*x^2]) - ((187*b^3*f^3 - 4*b*c*f^2*(114*b*e + 73*a*
f) - 64*c^3*(e^3 + 6*d*e*f) + 16*c^2*f*(20*a*e*f + 21*b*(e^2 + d*f)))*Sqrt[
a + b*x + c*x^2])/(64*c^5) + (f*(41*b^2*f^2 - 4*c*f*(22*b*e + 7*a*f) + 48*c
^2*(e^2 + d*f))*x*Sqrt[a + b*x + c*x^2])/(32*c^4) + (f^2*(8*c*e - 5*b*f)*x^
2*Sqrt[a + b*x + c*x^2])/(8*c^3) + (f^3*x^3*Sqrt[a + b*x + c*x^2])/(4*c^2)
+ (3*(105*b^4*f^3 - 280*b^2*c*f^2*(b*e + a*f) + 128*c^4*d*(e^2 + d*f) + 80*
c^2*f*(6*a*b*e*f + a^2*f^2 + 3*b^2*(e^2 + d*f)) - 64*c^3*(3*a*f*(e^2 + d*f)
 + b*(e^3 + 6*d*e*f)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2]
)])/(128*c^(11/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
```

```
b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(d+ex+fx^2\right)^3}{\left(a+bx+cx^2\right)^{3/2}}\,dx = \frac{2\left(3ab^4cef^2-ab^5f^3+ab^3cf\left(5af^2-3c\left(e^2+df\right)\right)-bc^2\left(c^3d^3+5a^3f^3+3ac^2d\left(e^2+d\right.\right.\right.}{}$$

$$= \frac{2\left(3ab^4cef^2-ab^5f^3+ab^3cf\left(5af^2-3c\left(e^2+df\right)\right)-bc^2\left(c^3d^3+5a^3f^3+3ac^2d\left(e^2+d\right.\right.\right.}{}$$

$$= \frac{2\left(3ab^4cef^2-ab^5f^3+ab^3cf\left(5af^2-3c\left(e^2+df\right)\right)-bc^2\left(c^3d^3+5a^3f^3+3ac^2d\left(e^2+d\right.\right.\right.}{}$$

$$= \frac{2\left(3ab^4cef^2-ab^5f^3+ab^3cf\left(5af^2-3c\left(e^2+df\right)\right)-bc^2\left(c^3d^3+5a^3f^3+3ac^2d\left(e^2+d\right.\right.\right.}{}$$

$$= \frac{2\left(3ab^4cef^2-ab^5f^3+ab^3cf\left(5af^2-3c\left(e^2+df\right)\right)-bc^2\left(c^3d^3+5a^3f^3+3ac^2d\left(e^2+d\right.\right.\right.}{}$$

$$= \frac{2\left(3ab^4cef^2-ab^5f^3+ab^3cf\left(5af^2-3c\left(e^2+df\right)\right)-bc^2\left(c^3d^3+5a^3f^3+3ac^2d\left(e^2+d\right.\right.\right.}{}$$

$$= \frac{2\left(3ab^4cef^2-ab^5f^3+ab^3cf\left(5af^2-3c\left(e^2+df\right)\right)-bc^2\left(c^3d^3+5a^3f^3+3ac^2d\left(e^2+d\right.\right.\right.}{}$$

Mathematica [A] time = 1.66, size = 745, normalized size = 1.15

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)^3/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] (315*b^6*f^3*x + 105*b^5*f^2*(3*a*f + c*x*(-8*e + f*x)) - 2*b^4*c*f*(105*a*
f*(4*e + 9*f*x) + c*x*(-360*e^2 - 360*d*f + 140*e*f*x + 21*f^2*x^2)) - 8*b^
3*c*(210*a^2*f^3 - c^2*x*(-24*e^3 + 30*e^2*f*x + 3*f^2*x*(10*d + f*x^2) + 2
*e*f*(-72*d + 7*f*x^2)) + a*c*f*(-90*e^2 - 530*e*f*x + f*(-90*d + 77*f*x^2)
)) - 16*b^2*c^2*(-(a^2*f^2*(230*e + 169*f*x)) + a*c*(12*e^3 + 186*e^2*f*x +
 2*e*f*(36*d - 43*f*x^2) + f^2*x*(186*d - 13*f*x^2)) + c^2*x*(-24*d^2*f + 6
```

```
*d*(-4*e^2 + 4*e*f*x + f^2*x^2) + x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*
x^3))) + 32*c^3*(8*c^3*d^3*x - a^3*f^2*(64*e + 15*f*x) + a^2*c*(16*e^3 + 36
*e^2*f*x + f^2*x*(36*d - 5*f*x^2) - 32*e*f*(-3*d + f*x^2)) + 2*a*c^2*(-12*d
^2*(e + f*x) + 6*d*x*(-2*e^2 + 4*e*f*x + f^2*x^2) + x^2*(4*e^3 + 6*e^2*f*x
+ 4*e*f^2*x^2 + f^3*x^3))) + 16*b*c^2*(113*a^3*f^3 + 8*c^3*d^2*(d - 3*e*x)
+ a^2*c*f*(-156*e^2 - 244*e*f*x + f*(-156*d + 49*f*x^2)) + 2*a*c^2*(12*d^2*
f + 6*d*(2*e^2 + 20*e*f*x - 5*f^2*x^2) - x*(-20*e^3 + 30*e^2*f*x + 14*e*f^2
*x^2 + 3*f^3*x^3)))))/(64*c^5*(-b^2 + 4*a*c)*Sqrt[a + x*(b + c*x)]) + (3*(10
5*b^4*f^3 - 280*b^2*c*f^2*(b*e + a*f) + 128*c^4*d*(e^2 + d*f) + 80*c^2*f*(6
*a*b*e*f + a^2*f^2 + 3*b^2*(e^2 + d*f)) - 64*c^3*(3*a*f*(e^2 + d*f) + b*(e^
3 + 6*d*e*f)))*Log[b + 2*c*x + 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])/(128*c^(11
/2))
```

IntegrateAlgebraic [A] time = 5.98, size = 1100, normalized size = 1.69

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d + e*x + f*x^2)^3/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] -1/64*(-128*b*c^5*d^3 + 768*a*c^5*d^2*e - 384*a*b*c^4*d*e^2 + 192*a*b^2*c^3
*e^3 - 512*a^2*c^4*e^3 - 384*a*b*c^4*d^2*f + 1152*a*b^2*c^3*d*e*f - 3072*a^
2*c^4*d*e*f - 720*a*b^3*c^2*e^2*f + 2496*a^2*b*c^3*e^2*f - 720*a*b^3*c^2*d*
f^2 + 2496*a^2*b*c^3*d*f^2 + 840*a*b^4*c*e*f^2 - 3680*a^2*b^2*c^2*e*f^2 + 2
048*a^3*c^3*e*f^2 - 315*a*b^5*f^3 + 1680*a^2*b^3*c*f^3 - 1808*a^3*b*c^2*f^3
 - 256*c^6*d^3*x + 384*b*c^5*d^2*e*x - 384*b^2*c^4*d*e^2*x + 768*a*c^5*d*e^
2*x + 192*b^3*c^3*e^3*x - 640*a*b*c^4*e^3*x - 384*b^2*c^4*d^2*f*x + 768*a*c
^5*d^2*f*x + 1152*b^3*c^3*d*e*f*x - 3840*a*b*c^4*d*e*f*x - 720*b^4*c^2*e^2*
f*x + 2976*a*b^2*c^3*e^2*f*x - 1152*a^2*c^4*e^2*f*x - 720*b^4*c^2*d*f^2*x +
 2976*a*b^2*c^3*d*f^2*x - 1152*a^2*c^4*d*f^2*x + 840*b^5*c*e*f^2*x - 4240*a
*b^3*c^2*e*f^2*x + 3904*a^2*b*c^3*e*f^2*x - 315*b^6*f^3*x + 1890*a*b^4*c*f^
3*x - 2704*a^2*b^2*c^2*f^3*x + 480*a^3*c^3*f^3*x + 64*b^2*c^4*e^3*x^2 - 256
*a*c^5*e^3*x^2 + 384*b^2*c^4*d*e*f*x^2 - 1536*a*c^5*d*e*f*x^2 - 240*b^3*c^3
*e^2*f*x^2 + 960*a*b*c^4*e^2*f*x^2 - 240*b^3*c^3*d*f^2*x^2 + 960*a*b*c^4*d*
f^2*x^2 + 280*b^4*c^2*e*f^2*x^2 - 1376*a*b^2*c^3*e*f^2*x^2 + 1024*a^2*c^4*e
*f^2*x^2 - 105*b^5*c*f^3*x^2 + 616*a*b^3*c^2*f^3*x^2 - 784*a^2*b*c^3*f^3*x^
2 + 96*b^2*c^4*e^2*f*x^3 - 384*a*c^5*e^2*f*x^3 + 96*b^2*c^4*d*f^2*x^3 - 384
*a*c^5*d*f^2*x^3 - 112*b^3*c^3*e*f^2*x^3 + 448*a*b*c^4*e*f^2*x^3 + 42*b^4*c
^2*f^3*x^3 - 208*a*b^2*c^3*f^3*x^3 + 160*a^2*c^4*f^3*x^3 + 64*b^2*c^4*e*f^2
*x^4 - 256*a*c^5*e*f^2*x^4 - 24*b^3*c^3*f^3*x^4 + 96*a*b*c^4*f^3*x^4 + 16*b
^2*c^4*f^3*x^5 - 64*a*c^5*f^3*x^5)/(c^5*(-b^2 + 4*a*c)*Sqrt[a + b*x + c*x^2
]) - (3*(128*c^4*d*e^2 - 64*b*c^3*e^3 + 128*c^4*d^2*f - 384*b*c^3*d*e*f + 2
40*b^2*c^2*e^2*f - 192*a*c^3*e^2*f + 240*b^2*c^2*d*f^2 - 192*a*c^3*d*f^2 -
280*b^3*c*e*f^2 + 480*a*b*c^2*e*f^2 + 105*b^4*f^3 - 280*a*b^2*c*f^3 + 80*a^
2*c^2*f^3)*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + b*x + c*x^2]])/(128*c^(11/2))
```

fricas [B] time = 1.95, size = 3143, normalized size = 4.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)^3/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/256*(3*(128*(a*b^2*c^4 - 4*a^2*c^5)*d*e^2 - 64*(a*b^3*c^3 - 4*a^2*b*c^4)
*e^3 + 5*(21*a*b^6 - 140*a^2*b^4*c + 240*a^3*b^2*c^2 - 64*a^4*c^3)*f^3 + 8*
(6*(5*a*b^4*c^2 - 24*a^2*b^2*c^3 + 16*a^3*c^4)*d - 5*(7*a*b^5*c - 40*a^2*b^
3*c^2 + 48*a^3*b*c^3)*e)*f^2 + (128*(b^2*c^5 - 4*a*c^6)*d*e^2 - 64*(b^3*c^4
 - 4*a*b*c^5)*e^3 + 5*(21*b^6*c - 140*a*b^4*c^2 + 240*a^2*b^2*c^3 - 64*a^3*
c^4)*f^3 + 8*(6*(5*b^4*c^3 - 24*a*b^2*c^4 + 16*a^2*c^5)*d - 5*(7*b^5*c^2 -
40*a*b^3*c^3 + 48*a^2*b*c^4)*e)*f^2 + 16*(8*(b^2*c^5 - 4*a*c^6)*d^2 - 24*(b
^3*c^4 - 4*a*b*c^5)*d*e + 3*(5*b^4*c^3 - 24*a*b^2*c^4 + 16*a^2*c^5)*e^2)*f)
*x^2 + 16*(8*(a*b^2*c^4 - 4*a^2*c^5)*d^2 - 24*(a*b^3*c^3 - 4*a^2*b*c^4)*d*e
 + 3*(5*a*b^4*c^2 - 24*a^2*b^2*c^3 + 16*a^3*c^4)*e^2)*f + (128*(b^3*c^4 - 4
*a*b*c^5)*d*e^2 - 64*(b^4*c^3 - 4*a*b^2*c^4)*e^3 + 5*(21*b^7 - 140*a*b^5*c
+ 240*a^2*b^3*c^2 - 64*a^3*b*c^3)*f^3 + 8*(6*(5*b^5*c^2 - 24*a*b^3*c^3 + 16
*a^2*b*c^4)*d - 5*(7*b^6*c - 40*a*b^4*c^2 + 48*a^2*b^2*c^3)*e)*f^2 + 16*(8*
(b^3*c^4 - 4*a*b*c^5)*d^2 - 24*(b^4*c^3 - 4*a*b^2*c^4)*d*e + 3*(5*b^5*c^2 -
 24*a*b^3*c^3 + 16*a^2*b*c^4)*e^2)*f)*x)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x -
 b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(128*b*c^6*
d^3 - 768*a*c^6*d^2*e + 384*a*b*c^5*d*e^2 - 16*(b^2*c^5 - 4*a*c^6)*f^3*x^5
- 8*(8*(b^2*c^5 - 4*a*c^6)*e*f^2 - 3*(b^3*c^4 - 4*a*b*c^5)*f^3)*x^4 - 64*(3
*a*b^2*c^4 - 8*a^2*c^5)*e^3 + (315*a*b^5*c - 1680*a^2*b^3*c^2 + 1808*a^3*b*
c^3)*f^3 - 2*(48*(b^2*c^5 - 4*a*c^6)*e^2*f + (21*b^4*c^3 - 104*a*b^2*c^4 +
80*a^2*c^5)*f^3 + 8*(6*(b^2*c^5 - 4*a*c^6)*d - 7*(b^3*c^4 - 4*a*b*c^5)*e)*f
^2)*x^3 + 8*(6*(15*a*b^3*c^3 - 52*a^2*b*c^4)*d - (105*a*b^4*c^2 - 460*a^2*b
^2*c^3 + 256*a^3*c^4)*e)*f^2 - (64*(b^2*c^5 - 4*a*c^6)*e^3 - 7*(15*b^5*c^2
- 88*a*b^3*c^3 + 112*a^2*b*c^4)*f^3 - 8*(30*(b^3*c^4 - 4*a*b*c^5)*d - (35*b
^4*c^3 - 172*a*b^2*c^4 + 128*a^2*c^5)*e)*f^2 + 48*(8*(b^2*c^5 - 4*a*c^6)*d*
e - 5*(b^3*c^4 - 4*a*b*c^5)*e^2)*f)*x^2 + 48*(8*a*b*c^5*d^2 - 8*(3*a*b^2*c^
4 - 8*a^2*c^5)*d*e + (15*a*b^3*c^3 - 52*a^2*b*c^4)*e^2)*f + (256*c^7*d^3 -
384*b*c^6*d^2*e + 384*(b^2*c^5 - 2*a*c^6)*d*e^2 - 64*(3*b^3*c^4 - 10*a*b*c^
5)*e^3 + (315*b^6*c - 1890*a*b^4*c^2 + 2704*a^2*b^2*c^3 - 480*a^3*c^4)*f^3
+ 8*(6*(15*b^4*c^3 - 62*a*b^2*c^4 + 24*a^2*c^5)*d - (105*b^5*c^2 - 530*a*b^
3*c^3 + 488*a^2*b*c^4)*e)*f^2 + 48*(8*(b^2*c^5 - 2*a*c^6)*d^2 - 8*(3*b^3*c^
4 - 10*a*b*c^5)*d*e + (15*b^4*c^3 - 62*a*b^2*c^4 + 24*a^2*c^5)*e^2)*f)*x)*s
qrt(c*x^2 + b*x + a))/(a*b^2*c^6 - 4*a^2*c^7 + (b^2*c^7 - 4*a*c^8)*x^2 + (b
^3*c^6 - 4*a*b*c^7)*x), -1/128*(3*(128*(a*b^2*c^4 - 4*a^2*c^5)*d*e^2 - 64*(
a*b^3*c^3 - 4*a^2*b*c^4)*e^3 + 5*(21*a*b^6 - 140*a^2*b^4*c + 240*a^3*b^2*c^
2 - 64*a^4*c^3)*f^3 + 8*(6*(5*a*b^4*c^2 - 24*a^2*b^2*c^3 + 16*a^3*c^4)*d -
5*(7*a*b^5*c - 40*a^2*b^3*c^2 + 48*a^3*b*c^3)*e)*f^2 + (128*(b^2*c^5 - 4*a*
c^6)*d*e^2 - 64*(b^3*c^4 - 4*a*b*c^5)*e^3 + 5*(21*b^6*c - 140*a*b^4*c^2 + 2
```

```
40*a^2*b^2*c^3 - 64*a^3*c^4)*f^3 + 8*(6*(5*b^4*c^3 - 24*a*b^2*c^4 + 16*a^2*
c^5)*d - 5*(7*b^5*c^2 - 40*a*b^3*c^3 + 48*a^2*b*c^4)*e)*f^2 + 16*(8*(b^2*c^
5 - 4*a*c^6)*d^2 - 24*(b^3*c^4 - 4*a*b*c^5)*d*e + 3*(5*b^4*c^3 - 24*a*b^2*c
^4 + 16*a^2*c^5)*e^2)*f)*x^2 + 16*(8*(a*b^2*c^4 - 4*a^2*c^5)*d^2 - 24*(a*b^
3*c^3 - 4*a^2*b*c^4)*d*e + 3*(5*a*b^4*c^2 - 24*a^2*b^2*c^3 + 16*a^3*c^4)*e^
2)*f + (128*(b^3*c^4 - 4*a*b*c^5)*d*e^2 - 64*(b^4*c^3 - 4*a*b^2*c^4)*e^3 +
5*(21*b^7 - 140*a*b^5*c + 240*a^2*b^3*c^2 - 64*a^3*b*c^3)*f^3 + 8*(6*(5*b^5
*c^2 - 24*a*b^3*c^3 + 16*a^2*b*c^4)*d - 5*(7*b^6*c - 40*a*b^4*c^2 + 48*a^2*
b^2*c^3)*e)*f^2 + 16*(8*(b^3*c^4 - 4*a*b*c^5)*d^2 - 24*(b^4*c^3 - 4*a*b^2*c
^4)*d*e + 3*(5*b^5*c^2 - 24*a*b^3*c^3 + 16*a^2*b*c^4)*e^2)*f)*x)*sqrt(-c)*a
rctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c
)) + 2*(128*b*c^6*d^3 - 768*a*c^6*d^2*e + 384*a*b*c^5*d*e^2 - 16*(b^2*c^5 -
 4*a*c^6)*f^3*x^5 - 8*(8*(b^2*c^5 - 4*a*c^6)*e*f^2 - 3*(b^3*c^4 - 4*a*b*c^5
)*f^3)*x^4 - 64*(3*a*b^2*c^4 - 8*a^2*c^5)*e^3 + (315*a*b^5*c - 1680*a^2*b^3
*c^2 + 1808*a^3*b*c^3)*f^3 - 2*(48*(b^2*c^5 - 4*a*c^6)*e^2*f + (21*b^4*c^3
- 104*a*b^2*c^4 + 80*a^2*c^5)*f^3 + 8*(6*(b^2*c^5 - 4*a*c^6)*d - 7*(b^3*c^4
 - 4*a*b*c^5)*e)*f^2)*x^3 + 8*(6*(15*a*b^3*c^3 - 52*a^2*b*c^4)*d - (105*a*b
^4*c^2 - 460*a^2*b^2*c^3 + 256*a^3*c^4)*e)*f^2 - (64*(b^2*c^5 - 4*a*c^6)*e^
3 - 7*(15*b^5*c^2 - 88*a*b^3*c^3 + 112*a^2*b*c^4)*f^3 - 8*(30*(b^3*c^4 - 4*
a*b*c^5)*d - (35*b^4*c^3 - 172*a*b^2*c^4 + 128*a^2*c^5)*e)*f^2 + 48*(8*(b^2
*c^5 - 4*a*c^6)*d*e - 5*(b^3*c^4 - 4*a*b*c^5)*e^2)*f)*x^2 + 48*(8*a*b*c^5*d
^2 - 8*(3*a*b^2*c^4 - 8*a^2*c^5)*d*e + (15*a*b^3*c^3 - 52*a^2*b*c^4)*e^2)*f
 + (256*c^7*d^3 - 384*b*c^6*d^2*e + 384*(b^2*c^5 - 2*a*c^6)*d*e^2 - 64*(3*b
^3*c^4 - 10*a*b*c^5)*e^3 + (315*b^6*c - 1890*a*b^4*c^2 + 2704*a^2*b^2*c^3 -
 480*a^3*c^4)*f^3 + 8*(6*(15*b^4*c^3 - 62*a*b^2*c^4 + 24*a^2*c^5)*d - (105*
b^5*c^2 - 530*a*b^3*c^3 + 488*a^2*b*c^4)*e)*f^2 + 48*(8*(b^2*c^5 - 2*a*c^6)
*d^2 - 8*(3*b^3*c^4 - 10*a*b*c^5)*d*e + (15*b^4*c^3 - 62*a*b^2*c^4 + 24*a^2
*c^5)*e^2)*f)*x)*sqrt(c*x^2 + b*x + a))/(a*b^2*c^6 - 4*a^2*c^7 + (b^2*c^7 -
 4*a*c^8)*x^2 + (b^3*c^6 - 4*a*b*c^7)*x)]
```

giac [A] time = 0.42, size = 1099, normalized size = 1.69

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)^3/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/64*(((2*(4*(2*(b^2*c^4*f^3 - 4*a*c^5*f^3)*x/(b^2*c^5 - 4*a*c^6) - (3*b^3*
c^3*f^3 - 12*a*b*c^4*f^3 - 8*b^2*c^4*f^2*e + 32*a*c^5*f^2*e)/(b^2*c^5 - 4*a
*c^6))*x + (48*b^2*c^4*d*f^2 - 192*a*c^5*d*f^2 + 21*b^4*c^2*f^3 - 104*a*b^2
*c^3*f^3 + 80*a^2*c^4*f^3 - 56*b^3*c^3*f^2*e + 224*a*b*c^4*f^2*e + 48*b^2*c
^4*f*e^2 - 192*a*c^5*f*e^2)/(b^2*c^5 - 4*a*c^6))*x - (240*b^3*c^3*d*f^2 - 9
60*a*b*c^4*d*f^2 + 105*b^5*c*f^3 - 616*a*b^3*c^2*f^3 + 784*a^2*b*c^3*f^3 -
384*b^2*c^4*d*f*e + 1536*a*c^5*d*f*e - 280*b^4*c^2*f^2*e + 1376*a*b^2*c^3*f
^2*e - 1024*a^2*c^4*f^2*e + 240*b^3*c^3*f*e^2 - 960*a*b*c^4*f*e^2 - 64*b^2*
```

```
c^4*e^3 + 256*a*c^5*e^3)/(b^2*c^5 - 4*a*c^6))*x - (256*c^6*d^3 + 384*b^2*c^
4*d^2*f - 768*a*c^5*d^2*f + 720*b^4*c^2*d*f^2 - 2976*a*b^2*c^3*d*f^2 + 1152
*a^2*c^4*d*f^2 + 315*b^6*f^3 - 1890*a*b^4*c*f^3 + 2704*a^2*b^2*c^2*f^3 - 48
0*a^3*c^3*f^3 - 384*b*c^5*d^2*e - 1152*b^3*c^3*d*f*e + 3840*a*b*c^4*d*f*e -
 840*b^5*c*f^2*e + 4240*a*b^3*c^2*f^2*e - 3904*a^2*b*c^3*f^2*e + 384*b^2*c^
4*d*e^2 - 768*a*c^5*d*e^2 + 720*b^4*c^2*f*e^2 - 2976*a*b^2*c^3*f*e^2 + 1152
*a^2*c^4*f*e^2 - 192*b^3*c^3*e^3 + 640*a*b*c^4*e^3)/(b^2*c^5 - 4*a*c^6))*x
- (128*b*c^5*d^3 + 384*a*b*c^4*d^2*f + 720*a*b^3*c^2*d*f^2 - 2496*a^2*b*c^3
*d*f^2 + 315*a*b^5*f^3 - 1680*a^2*b^3*c*f^3 + 1808*a^3*b*c^2*f^3 - 768*a*c^
5*d^2*e - 1152*a*b^2*c^3*d*f*e + 3072*a^2*c^4*d*f*e - 840*a*b^4*c*f^2*e + 3
680*a^2*b^2*c^2*f^2*e - 2048*a^3*c^3*f^2*e + 384*a*b*c^4*d*e^2 + 720*a*b^3*
c^2*f*e^2 - 2496*a^2*b*c^3*f*e^2 - 192*a*b^2*c^3*e^3 + 512*a^2*c^4*e^3)/(b^
2*c^5 - 4*a*c^6))/sqrt(c*x^2 + b*x + a) - 3/128*(128*c^4*d^2*f + 240*b^2*c^
2*d*f^2 - 192*a*c^3*d*f^2 + 105*b^4*f^3 - 280*a*b^2*c*f^3 + 80*a^2*c^2*f^3
- 384*b*c^3*d*f*e - 280*b^3*c*f^2*e + 480*a*b*c^2*f^2*e + 128*c^4*d*e^2 + 2
40*b^2*c^2*f*e^2 - 192*a*c^3*f*e^2 - 64*b*c^3*e^3)*log(abs(-2*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(11/2)
```

maple [B] time = 0.03, size = 2827, normalized size = 4.36

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)^3/(c*x^2+b*x+a)^(3/2),x)
```

```
[Out] 2*a/c^2/(c*x^2+b*x+a)^(1/2)*e^3+2*d^3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(
1/2)+315/128*f^3*b^4/c^(11/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+1
5/8*f^3*a^2/c^(7/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+315/256*f^3
*b^5/c^6/(c*x^2+b*x+a)^(1/2)+1/4*f^3*x^5/c/(c*x^2+b*x+a)^(1/2)+3/c^(3/2)*ln
((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*f*d^2+3/c^(3/2)*ln((c*x+1/2*b)/c^
(1/2)+(c*x^2+b*x+a)^(1/2))*e^2*d-3*d^2*e/c/(c*x^2+b*x+a)^(1/2)+x^2/c/(c*x^2
+b*x+a)^(1/2)*e^3-3/4*b^2/c^3/(c*x^2+b*x+a)^(1/2)*e^3-3/2*b/c^(5/2)*ln((c*x
+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*e^3+115/4*e*f^2*b^3/c^3*a/(4*a*c-b^2)/
(c*x^2+b*x+a)^(1/2)*x-9*b^3/c^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*d*e*f+12*
a/c^2*b^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*d*e*f-39/2*b^2/c^2*a/(4*a*c-b^2)/
(c*x^2+b*x+a)^(1/2)*x*d*f^2-39/2*b^2/c^2*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*
x*e^2*f-16*e*f^2*a^2/c^2*b/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x+21/32*f^3*b^2/
c^3*x^3/(c*x^2+b*x+a)^(1/2)-105/64*f^3*b^3/c^4*x^2/(c*x^2+b*x+a)^(1/2)-315/
128*f^3*b^4/c^5*x/(c*x^2+b*x+a)^(1/2)+315/256*f^3*b^7/c^6/(4*a*c-b^2)/(c*x^
2+b*x+a)^(1/2)-105/16*f^3*b^2/c^(9/2)*a*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a
)^(1/2))-5/8*f^3*a/c^2*x^3/(c*x^2+b*x+a)^(1/2)-15/8*f^3*a^2/c^3*x/(c*x^2+b*
x+a)^(1/2)-3*x/c/(c*x^2+b*x+a)^(1/2)*f*d^2-3*x/c/(c*x^2+b*x+a)^(1/2)*e^2*d+
3/2*x^3/c/(c*x^2+b*x+a)^(1/2)*d*f^2+3/2*x^3/c/(c*x^2+b*x+a)^(1/2)*e^2*f+45/
16*b^3/c^4/(c*x^2+b*x+a)^(1/2)*d*f^2+45/16*b^3/c^4/(c*x^2+b*x+a)^(1/2)*e^2*
f+45/8*b^2/c^(7/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*d*f^2+45/8*b
```

```
^2/c^(7/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*e^2*f-9/2*a/c^(5/2)*
ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*d*f^2-9/2*a/c^(5/2)*ln((c*x+1/2
*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*e^2*f+3/2*b/c^2*x/(c*x^2+b*x+a)^(1/2)*e^3-
3/4*b^4/c^3/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*e^3+e*f^2*x^4/c/(c*x^2+b*x+a)^(
1/2)-105/32*e*f^2*b^4/c^5/(c*x^2+b*x+a)^(1/2)-105/16*e*f^2*b^3/c^(9/2)*ln((
c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-8*e*f^2*a^2/c^3/(c*x^2+b*x+a)^(1/2)
+3/2*b/c^2/(c*x^2+b*x+a)^(1/2)*f*d^2+3/2*b/c^2/(c*x^2+b*x+a)^(1/2)*e^2*d-10
5/16*f^3*b^3/c^5*a/(c*x^2+b*x+a)^(1/2)+113/16*f^3*b/c^4*a^2/(c*x^2+b*x+a)^(
1/2)+24*a/c*b/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*d*e*f+9*b/c^2*x/(c*x^2+b*x+
a)^(1/2)*d*e*f-9/2*b^4/c^3/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*d*e*f+4*a/c*b/(4
*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*e^3+3*b^2/c/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)
*x*f*d^2+3*b^2/c/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*e^2*d-8*e*f^2*a^2/c^3*b^
2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+45/8*b^4/c^3/(4*a*c-b^2)/(c*x^2+b*x+a)^(1
/2)*x*d*f^2+45/8*b^4/c^3/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*e^2*f-39/4*b^3/c
^3*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*d*f^2-39/4*b^3/c^3*a/(4*a*c-b^2)/(c*x^
2+b*x+a)^(1/2)*e^2*f-105/16*e*f^2*b^5/c^4/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x
+115/8*e*f^2*b^4/c^4*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)-45/4*e*f^2*b/c^3*a*x
/(c*x^2+b*x+a)^(1/2)+113/8*f^3*b^2/c^3*a^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*
x-105/8*f^3*b^4/c^4*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x+105/16*e*f^2*b^3/c^
4*x/(c*x^2+b*x+a)^(1/2)-105/32*e*f^2*b^6/c^5/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2
)+115/8*e*f^2*b^2/c^4*a/(c*x^2+b*x+a)^(1/2)+45/4*e*f^2*b/c^(7/2)*a*ln((c*x+
1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-4*e*f^2*a/c^2*x^2/(c*x^2+b*x+a)^(1/2)+3
15/128*f^3*b^6/c^5/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x-105/16*f^3*b^5/c^5*a/(
4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+105/16*f^3*b^2/c^4*a*x/(c*x^2+b*x+a)^(1/2)+4
9/16*f^3*b/c^3*a*x^2/(c*x^2+b*x+a)^(1/2)+113/16*f^3*b^3/c^4*a^2/(4*a*c-b^2)
/(c*x^2+b*x+a)^(1/2)+6*x^2/c/(c*x^2+b*x+a)^(1/2)*d*e*f-9/2*b^2/c^3/(c*x^2+b
*x+a)^(1/2)*d*e*f-3/2*b^3/c^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*e^3-9*b/c^(
5/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*d*e*f+12*a/c^2/(c*x^2+b*x+
a)^(1/2)*d*e*f+2*a/c^2*b^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*e^3+3/2*b^3/c^2/
(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*f*d^2+3/2*b^3/c^2/(4*a*c-b^2)/(c*x^2+b*x+a)
^(1/2)*e^2*d-6*d^2*e*b/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x-3*d^2*e*b^2/c/(4*a
*c-b^2)/(c*x^2+b*x+a)^(1/2)-15/4*b/c^2*x^2/(c*x^2+b*x+a)^(1/2)*d*f^2-15/4*b
/c^2*x^2/(c*x^2+b*x+a)^(1/2)*e^2*f-45/8*b^2/c^3*x/(c*x^2+b*x+a)^(1/2)*d*f^2
-45/8*b^2/c^3*x/(c*x^2+b*x+a)^(1/2)*e^2*f-7/4*e*f^2*b/c^2*x^3/(c*x^2+b*x+a)
^(1/2)+35/8*e*f^2*b^2/c^3*x^2/(c*x^2+b*x+a)^(1/2)+45/16*b^5/c^4/(4*a*c-b^2)
/(c*x^2+b*x+a)^(1/2)*e^2*f-39/4*b/c^3*a/(c*x^2+b*x+a)^(1/2)*d*f^2-39/4*b/c^
3*a/(c*x^2+b*x+a)^(1/2)*e^2*f+9/2*a/c^2*x/(c*x^2+b*x+a)^(1/2)*d*f^2+9/2*a/c
^2*x/(c*x^2+b*x+a)^(1/2)*e^2*f-3/8*f^3*b/c^2*x^4/(c*x^2+b*x+a)^(1/2)+45/16*
b^5/c^4/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*d*f^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)^3/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(f\,x^2+e\,x+d\right)^3}{\left(c\,x^2+b\,x+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2)^3/(a + b*x + c*x^2)^(3/2),x)
```

```
[Out] int((d + e*x + f*x^2)^3/(a + b*x + c*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d+ex+fx^2\right)^3}{\left(a+bx+cx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)**3/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Integral((d + e*x + f*x**2)**3/(a + b*x + c*x**2)**(3/2), x)
```

3.112 $\int \frac{\left(d+ex+fx^2\right)^2}{\left(a+bx+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=309

$$\frac{2\left(-x\left(c^2\left(2a^2f^2+6abef+b^2\left(2df+e^2\right)\right)-2b^2cf(2af+be)-2c^3\left(a\left(2df+e^2\right)+bde\right)+b^4f^2+2c^4d^2\right)-bc\left(\cdots\right.\right.}{c^3\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

Rubi [A] time = 0.45, antiderivative size = 309, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1660, 1661, 640, 621, 206}

$$\frac{2\left(-x\left(c^2\left(2a^2f^2+6abef+b^2\left(2df+e^2\right)\right)-2b^2cf(2af+be)-2c^3\left(a\left(2df+e^2\right)+bde\right)+b^4f^2+2c^4d^2\right)-bc\left(-3a^2f^2+ac\left(2df+e^2\right)+c^2d^2\right)+2ab^2cef-ab^3f^2+4ac^2e(cd-af)\right)}{c^3\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}+\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-12cf(af+2be)+15b^2f^2+8c^2\left(2df+e^2\right)\right)}{8c^{7/2}}+\frac{f\sqrt{a+bx+cx^2}(8ce-7bf)}{4c^3}+\frac{f^2x\sqrt{a+bx+cx^2}}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)^2/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] (2*(2*a*b^2*c*e*f - a*b^3*f^2 + 4*a*c^2*e*(c*d - a*f) - b*c*(c^2*d^2 - 3*a^
2*f^2 + a*c*(e^2 + 2*d*f)) - (2*c^4*d^2 + b^4*f^2 - 2*b^2*c*f*(b*e + 2*a*f)
 - 2*c^3*(b*d*e + a*(e^2 + 2*d*f)) + c^2*(6*a*b*e*f + 2*a^2*f^2 + b^2*(e^2
+ 2*d*f)))*x))/(c^3*(b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]) + (f*(8*c*e - 7*b*
f)*Sqrt[a + b*x + c*x^2])/(4*c^3) + (f^2*x*Sqrt[a + b*x + c*x^2])/(2*c^2) +
 ((15*b^2*f^2 - 12*c*f*(2*b*e + a*f) + 8*c^2*(e^2 + 2*d*f))*ArcTanh[(b + 2*
c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
```

```
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(d+ex+fx^2\right)^2}{\left(a+bx+cx^2\right)^{3/2}}\,dx = \frac{2\left(2ab^2cef-ab^3f^2+4ac^2e(cd-af)-bc\left(c^2d^2-3a^2f^2+ac\left(e^2+2df\right)\right)-\left(2c^4d^2+b\right.\right.}{c^3\left(b^2-4a\right.}$$

$$= \frac{2\left(2ab^2cef-ab^3f^2+4ac^2e(cd-af)-bc\left(c^2d^2-3a^2f^2+ac\left(e^2+2df\right)\right)-\left(2c^4d^2+b\right.\right.}{c^3\left(b^2-4a\right.}$$

$$= \frac{2\left(2ab^2cef-ab^3f^2+4ac^2e(cd-af)-bc\left(c^2d^2-3a^2f^2+ac\left(e^2+2df\right)\right)-\left(2c^4d^2+b\right.\right.}{c^3\left(b^2-4a\right.}$$

$$= \frac{2\left(2ab^2cef-ab^3f^2+4ac^2e(cd-af)-bc\left(c^2d^2-3a^2f^2+ac\left(e^2+2df\right)\right)-\left(2c^4d^2+b\right.\right.}{c^3\left(b^2-4a\right.}$$

$$= \frac{2\left(2ab^2cef-ab^3f^2+4ac^2e(cd-af)-bc\left(c^2d^2-3a^2f^2+ac\left(e^2+2df\right)\right)-\left(2c^4d^2+b\right.\right.}{c^3\left(b^2-4a\right.}$$

Mathematica [A] time = 0.73, size = 288, normalized size = 0.93

$$\frac{4bc\left(-13a^2f^2+ac\left(4df+2e^2+20efx-5f^2x^2\right)+2c^2d(d-2ex)\right)+8c^2\left(a^2f(8e+3fx)+ac\left(x\left(-2e^2+4efx+f^2x^2\right)-4d(e+fx)\right)+2c^2d^2x\right)+b^3f(15af+cx(5fx-24e))-2b^2c\left(af(12e+31fx)+cx\left(-8df-4e^2+4efx+f^2x^2\right)\right)+15b^4f^2x}{4c^3\left(4ac-b^2\right)\sqrt{a+x(b+cx)}}+\frac{\log\left(2\sqrt{c}\sqrt{a+x(b+cx)}+b+2cx\right)\left(-12cf(af+2be)+15b^2f^2+8c^2\left(2df+e^2\right)\right)}{8c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)^2/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] (15*b^4*f^2*x + b^3*f*(15*a*f + c*x*(-24*e + 5*f*x)) + 4*b*c*(-13*a^2*f^2 +
 2*c^2*d*(d - 2*e*x) + a*c*(2*e^2 + 4*d*f + 20*e*f*x - 5*f^2*x^2)) - 2*b^2*
c*(a*f*(12*e + 31*f*x) + c*x*(-4*e^2 - 8*d*f + 4*e*f*x + f^2*x^2)) + 8*c^2*
(2*c^2*d^2*x + a^2*f*(8*e + 3*f*x) + a*c*(-4*d*(e + f*x) + x*(-2*e^2 + 4*e*
f*x + f^2*x^2))))/(4*c^3*(-b^2 + 4*a*c)*Sqrt[a + x*(b + c*x)]) + ((15*b^2*f
^2 - 12*c*f*(2*b*e + a*f) + 8*c^2*(e^2 + 2*d*f))*Log[b + 2*c*x + 2*Sqrt[c]*
Sqrt[a + x*(b + c*x)]])/(8*c^(7/2))
```

IntegrateAlgebraic [A] time = 2.16, size = 375, normalized size = 1.21

$$\frac{\log\left(-2c^{7/2}\sqrt{a+bx+cx^2}+bc^3+2c^4x\right)\left(12acf^2-15b^2f^2+24bcef-16c^2df-8c^2e^2\right)}{8c^{7/2}}-\frac{52a^2bcf^2-64a^2c^2ef-24a^2c^2f^2x-15ab^3f^2+24ab^2cef+62ab^2cf^2x-16abc^2df-8abc^2e^2-80abc^2efx+20abc^2f^2x^2+32ac^3de+32ac^3dfx+16ac^3e^2x-32ac^3efx^2-8ac^3f^2x^3-15b^4f^2x+24b^3cefx-5b^3cf^2x^2-16b^2c^2dfx-8b^2c^2e^2x+8b^2c^2efx^2+2b^2c^2f^2x^3-8bc^3d^2+16bc^3dex-16c^4d^2x}{4c^3\left(4ac-b^2\right)\sqrt{a+bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d + e*x + f*x^2)^2/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] -1/4*(-8*b*c^3*d^2 + 32*a*c^3*d*e - 8*a*b*c^2*e^2 - 16*a*b*c^2*d*f + 24*a*b
^2*c*e*f - 64*a^2*c^2*e*f - 15*a*b^3*f^2 + 52*a^2*b*c*f^2 - 16*c^4*d^2*x +
16*b*c^3*d*e*x - 8*b^2*c^2*e^2*x + 16*a*c^3*e^2*x - 16*b^2*c^2*d*f*x + 32*a
*c^3*d*f*x + 24*b^3*c*e*f*x - 80*a*b*c^2*e*f*x - 15*b^4*f^2*x + 62*a*b^2*c*
f^2*x - 24*a^2*c^2*f^2*x + 8*b^2*c^2*e*f*x^2 - 32*a*c^3*e*f*x^2 - 5*b^3*c*f
^2*x^2 + 20*a*b*c^2*f^2*x^2 + 2*b^2*c^2*f^2*x^3 - 8*a*c^3*f^2*x^3)/(c^3*(-b
^2 + 4*a*c)*Sqrt[a + b*x + c*x^2]) + ((-8*c^2*e^2 - 16*c^2*d*f + 24*b*c*e*f
 - 15*b^2*f^2 + 12*a*c*f^2)*Log[b*c^3 + 2*c^4*x - 2*c^(7/2)*Sqrt[a + b*x +
c*x^2]])/(8*c^(7/2))
```

fricas [B] time = 1.24, size = 1305, normalized size = 4.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)^2/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/16*((8*(a*b^2*c^2 - 4*a^2*c^3)*e^2 + 3*(5*a*b^4 - 24*a^2*b^2*c + 16*a^3
*c^2)*f^2 + (8*(b^2*c^3 - 4*a*c^4)*e^2 + 3*(5*b^4*c - 24*a*b^2*c^2 + 16*a^2
*c^3)*f^2 + 8*(2*(b^2*c^3 - 4*a*c^4)*d - 3*(b^3*c^2 - 4*a*b*c^3)*e)*f)*x^2
+ 8*(2*(a*b^2*c^2 - 4*a^2*c^3)*d - 3*(a*b^3*c - 4*a^2*b*c^2)*e)*f + (8*(b^3
*c^2 - 4*a*b*c^3)*e^2 + 3*(5*b^5 - 24*a*b^3*c + 16*a^2*b*c^2)*f^2 + 8*(2*(b
^3*c^2 - 4*a*b*c^3)*d - 3*(b^4*c - 4*a*b^2*c^2)*e)*f)*x)*sqrt(c)*log(-8*c^2
*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c)
 + 4*(8*b*c^4*d^2 - 32*a*c^4*d*e + 8*a*b*c^3*e^2 - 2*(b^2*c^3 - 4*a*c^4)*f^
2*x^3 + (15*a*b^3*c - 52*a^2*b*c^2)*f^2 - (8*(b^2*c^3 - 4*a*c^4)*e*f - 5*(b
^3*c^2 - 4*a*b*c^3)*f^2)*x^2 + 8*(2*a*b*c^3*d - (3*a*b^2*c^2 - 8*a^2*c^3)*e
)*f + (16*c^5*d^2 - 16*b*c^4*d*e + 8*(b^2*c^3 - 2*a*c^4)*e^2 + (15*b^4*c -
62*a*b^2*c^2 + 24*a^2*c^3)*f^2 + 8*(2*(b^2*c^3 - 2*a*c^4)*d - (3*b^3*c^2 -
10*a*b*c^3)*e)*f)*x)*sqrt(c*x^2 + b*x + a))/(a*b^2*c^4 - 4*a^2*c^5 + (b^2*c
^5 - 4*a*c^6)*x^2 + (b^3*c^4 - 4*a*b*c^5)*x), -1/8*((8*(a*b^2*c^2 - 4*a^2*c
^3)*e^2 + 3*(5*a*b^4 - 24*a^2*b^2*c + 16*a^3*c^2)*f^2 + (8*(b^2*c^3 - 4*a*c
^4)*e^2 + 3*(5*b^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*f^2 + 8*(2*(b^2*c^3 - 4*a
*c^4)*d - 3*(b^3*c^2 - 4*a*b*c^3)*e)*f)*x^2 + 8*(2*(a*b^2*c^2 - 4*a^2*c^3)*
d - 3*(a*b^3*c - 4*a^2*b*c^2)*e)*f + (8*(b^3*c^2 - 4*a*b*c^3)*e^2 + 3*(5*b^
5 - 24*a*b^3*c + 16*a^2*b*c^2)*f^2 + 8*(2*(b^3*c^2 - 4*a*b*c^3)*d - 3*(b^4*
c - 4*a*b^2*c^2)*e)*f)*x)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x
+ b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(8*b*c^4*d^2 - 32*a*c^4*d*e + 8*
a*b*c^3*e^2 - 2*(b^2*c^3 - 4*a*c^4)*f^2*x^3 + (15*a*b^3*c - 52*a^2*b*c^2)*f
^2 - (8*(b^2*c^3 - 4*a*c^4)*e*f - 5*(b^3*c^2 - 4*a*b*c^3)*f^2)*x^2 + 8*(2*a
*b*c^3*d - (3*a*b^2*c^2 - 8*a^2*c^3)*e)*f + (16*c^5*d^2 - 16*b*c^4*d*e + 8*
(b^2*c^3 - 2*a*c^4)*e^2 + (15*b^4*c - 62*a*b^2*c^2 + 24*a^2*c^3)*f^2 + 8*(2
*(b^2*c^3 - 2*a*c^4)*d - (3*b^3*c^2 - 10*a*b*c^3)*e)*f)*x)*sqrt(c*x^2 + b*x
 + a))/(a*b^2*c^4 - 4*a^2*c^5 + (b^2*c^5 - 4*a*c^6)*x^2 + (b^3*c^4 - 4*a*b*
c^5)*x)]
```

giac [A] time = 0.35, size = 407, normalized size = 1.32

$$\frac{\left(\left(\frac{2\left(b^2c^2f^2-4\,ac^3f^2\right)x}{b^2c^3-4\,ac^4}-\frac{5\,b^3cf^2-20\,abc^2f^2-8\,b^2c^2fe+32\,ac^3fe}{b^2c^3-4\,ac^4}\right)x-\frac{16\,c^4d^2+16\,b^2c^2df-32\,ac^3df+15\,b^4f^2-62\,ab^2cf^2+24\,a^2c^2f^2-16\,bc^3de-24\,b^3cfe+80\,abc^2fe+8\,b^2c^2e^2-16\,ac^3e^2}{b^2c^3-4\,ac^4}\right)x-\frac{8\,bc^3d^2+16\,abc^2df+15\,ab^3f^2-52\,a^2bcf^2-32\,ac^3de-24\,ab^2cfe+64\,a^2c^2fe+8\,abc^2e^2}{b^2c^3-4\,ac^4}}{4\,\sqrt{cx^2+bx+a}}-\frac{\left(16\,c^2df+15\,b^2f^2-12\,acf^2-24\,bcfe+8\,c^2e^2\right)\log\left(\left|-2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}-b\right|\right)}{8\,c^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)^2/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*(((2*(b^2*c^2*f^2 - 4*a*c^3*f^2)*x/(b^2*c^3 - 4*a*c^4) - (5*b^3*c*f^2 -
 20*a*b*c^2*f^2 - 8*b^2*c^2*f*e + 32*a*c^3*f*e)/(b^2*c^3 - 4*a*c^4))*x - (1
6*c^4*d^2 + 16*b^2*c^2*d*f - 32*a*c^3*d*f + 15*b^4*f^2 - 62*a*b^2*c*f^2 + 2
4*a^2*c^2*f^2 - 16*b*c^3*d*e - 24*b^3*c*f*e + 80*a*b*c^2*f*e + 8*b^2*c^2*e^
2 - 16*a*c^3*e^2)/(b^2*c^3 - 4*a*c^4))*x - (8*b*c^3*d^2 + 16*a*b*c^2*d*f +
15*a*b^3*f^2 - 52*a^2*b*c*f^2 - 32*a*c^3*d*e - 24*a*b^2*c*f*e + 64*a^2*c^2*
f*e + 8*a*b*c^2*e^2)/(b^2*c^3 - 4*a*c^4))/sqrt(c*x^2 + b*x + a) - 1/8*(16*c
^2*d*f + 15*b^2*f^2 - 12*a*c*f^2 - 24*b*c*f*e + 8*c^2*e^2)*log(abs(-2*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(7/2)
```

maple [B] time = 0.02, size = 1011, normalized size = 3.27

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)^2/(c*x^2+b*x+a)^(3/2),x)
```

```
[Out] -2*d*e/c/(c*x^2+b*x+a)^(1/2)-x/c/(c*x^2+b*x+a)^(1/2)*e^2+1/2*b/c^2/(c*x^2+b
*x+a)^(1/2)*e^2+2/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*d*f+1
/2*f^2*x^3/c/(c*x^2+b*x+a)^(1/2)+15/16*f^2*b^3/c^4/(c*x^2+b*x+a)^(1/2)+15/8
*f^2*b^2/c^(7/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-3/2*f^2*a/c^(5
/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+2*d^2*(2*c*x+b)/(4*a*c-b^2)
/(c*x^2+b*x+a)^(1/2)+4*e*f*a/c^2*b^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+2*b^2/
c/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*d*f-13/2*f^2*b^2/c^2*a/(4*a*c-b^2)/(c*x
^2+b*x+a)^(1/2)*x-3*e*f*b^3/c^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x+8*e*f*a/c
*b/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x-13/4*f^2*b^3/c^3*a/(4*a*c-b^2)/(c*x^2+
b*x+a)^(1/2)-4*d*e*b/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x+15/8*f^2*b^4/c^3/(4*
a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x-2*d*e*b^2/c/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+
3*e*f*b/c^2*x/(c*x^2+b*x+a)^(1/2)-3/2*e*f*b^4/c^3/(4*a*c-b^2)/(c*x^2+b*x+a)
^(1/2)+b^2/c/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*e^2+b^3/c^2/(4*a*c-b^2)/(c*x
^2+b*x+a)^(1/2)*d*f+1/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*e
^2+1/2*b^3/c^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*e^2+2*e*f*x^2/c/(c*x^2+b*x+a
)^(1/2)-3/2*e*f*b^2/c^3/(c*x^2+b*x+a)^(1/2)-3*e*f*b/c^(5/2)*ln((c*x+1/2*b)/
c^(1/2)+(c*x^2+b*x+a)^(1/2))+4*e*f*a/c^2/(c*x^2+b*x+a)^(1/2)-5/4*f^2*b/c^2*
x^2/(c*x^2+b*x+a)^(1/2)-15/8*f^2*b^2/c^3*x/(c*x^2+b*x+a)^(1/2)+15/16*f^2*b^
5/c^4/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)-13/4*f^2*b/c^3*a/(c*x^2+b*x+a)^(1/2)+
```

```
3/2*f^2*a/c^2*x/(c*x^2+b*x+a)^(1/2)-2*x/c/(c*x^2+b*x+a)^(1/2)*d*f+b/c^2/(c*
x^2+b*x+a)^(1/2)*d*f
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)^2/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(f\,x^2+e\,x+d\right)^2}{\left(c\,x^2+b\,x+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2)^2/(a + b*x + c*x^2)^(3/2),x)
```

```
[Out] int((d + e*x + f*x^2)^2/(a + b*x + c*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d+ex+fx^2\right)^2}{\left(a+bx+cx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)**2/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Integral((d + e*x + f*x**2)**2/(a + b*x + c*x**2)**(3/2), x)
```

3.113 $\int \frac{d+ex+fx^2}{\left(a+bx+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=111

$$\frac{2\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x\left(-2acf+b^2f-bce+2c^2d\right)\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}+\frac{f\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{3/2}}$$

Rubi [A] time = 0.08, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1660, 12, 621, 206}

$$\frac{2\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x\left(-2acf+b^2f-bce+2c^2d\right)\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}+\frac{f\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] (2*(c*(2*a*e - b*(d + (a*f)/c)) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*x))/(
c*(b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]) + (f*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*
Sqrt[a + b*x + c*x^2])])/c^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{d+ex+fx^2}{\left(a+bx+cx^2\right)^{3/2}}\,dx = \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}-\frac{2\int -\frac{\left(b^2-4ac\right)f}{2c\sqrt{a+bx+cx^2}}\,dx}{b^2-4ac}$$

$$= \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}+\frac{f\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{c}$$

$$= \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}+\frac{(2f)\,\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx,x,\frac{b}{\sqrt{a+}}\right.}{c}$$

$$= \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}+\frac{f\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{3/2}}$$

Mathematica [A] time = 0.31, size = 113, normalized size = 1.02

$$\frac{\frac{2\sqrt{c}\left(abf-2ac(e+fx)+b^2fx+bc(d-ex)+2c^2dx\right)}{\sqrt{a+x(b+cx)}}-f\left(b^2-4ac\right)\log\left(2\sqrt{c}\sqrt{a+x(b+cx)}+b+2cx\right)}{c^{3/2}\left(4ac-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] ((2*Sqrt[c]*(a*b*f + 2*c^2*d*x + b^2*f*x + b*c*(d - e*x) - 2*a*c*(e + f*x))
)/Sqrt[a + x*(b + c*x)] - (b^2 - 4*a*c)*f*Log[b + 2*c*x + 2*Sqrt[c]*Sqrt[a
+ x*(b + c*x)]])/(c^(3/2)*(-b^2 + 4*a*c))
```

IntegrateAlgebraic [A] time = 0.55, size = 111, normalized size = 1.00

$$\frac{2\left(abf-2ace-2acfx+b^2fx+bcd-bcex+2c^2dx\right)}{c\left(4ac-b^2\right)\sqrt{a+bx+cx^2}}-\frac{f\log\left(-2c^{3/2}\sqrt{a+bx+cx^2}+bc+2c^2x\right)}{c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d + e*x + f*x^2)/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] (2*(b*c*d - 2*a*c*e + a*b*f + 2*c^2*d*x - b*c*e*x + b^2*f*x - 2*a*c*f*x))/(
c*(-b^2 + 4*a*c)*Sqrt[a + b*x + c*x^2]) - (f*Log[b*c + 2*c^2*x - 2*c^(3/2)*
Sqrt[a + b*x + c*x^2]])/c^(3/2)
```

fricas [B] time = 0.78, size = 429, normalized size = 3.86

$$\left[\frac{\left(\left(b^2c-4ac^2\right)fx^2+\left(b^3-4abc\right)fx+\left(ab^2-4a^2c\right)f\right)\sqrt{c}\log\left(-8c^2x^2-8bcx-b^2-4\sqrt{cx^2+bx+a}(2cx+b)\sqrt{c}-4ac\right)-4\left(bc^2d-2ac^2e+abcf+\left(2c^3d-bc^2e+\left(b^2c-2ac^2\right)f\right)x\right)\sqrt{cx^2+bx+a}}{2\left(ab^2c^2-4a^2c^3+\left(b^2c^3-4ac^4\right)x^2+\left(b^3c^2-4abc^3\right)x\right)},-\frac{\left(\left(b^2c-4ac^2\right)fx^2+\left(b^3-4abc\right)fx+\left(ab^2-4a^2c\right)f\right)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^2+bx+a}(2cx+b)\sqrt{-c}}{2\left(c^2x^2+bcx+ac\right)}\right)+2\left(bc^2d-2ac^2e+abcf+\left(2c^3d-bc^2e+\left(b^2c-2ac^2\right)f\right)x\right)\sqrt{cx^2+bx+a}}{ab^2c^2-4a^2c^3+\left(b^2c^3-4ac^4\right)x^2+\left(b^3c^2-4abc^3\right)x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(((b^2*c - 4*a*c^2)*f*x^2 + (b^3 - 4*a*b*c)*f*x + (a*b^2 - 4*a^2*c)*f)
*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x +
b)*sqrt(c) - 4*a*c) - 4*(b*c^2*d - 2*a*c^2*e + a*b*c*f + (2*c^3*d - b*c^2*e
 + (b^2*c - 2*a*c^2)*f)*x)*sqrt(c*x^2 + b*x + a))/(a*b^2*c^2 - 4*a^2*c^3 +
(b^2*c^3 - 4*a*c^4)*x^2 + (b^3*c^2 - 4*a*b*c^3)*x), -(((b^2*c - 4*a*c^2)*f*
x^2 + (b^3 - 4*a*b*c)*f*x + (a*b^2 - 4*a^2*c)*f)*sqrt(-c)*arctan(1/2*sqrt(c
*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(b*c^2*d
- 2*a*c^2*e + a*b*c*f + (2*c^3*d - b*c^2*e + (b^2*c - 2*a*c^2)*f)*x)*sqrt(c
*x^2 + b*x + a))/(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^2 + (b^3*c^
2 - 4*a*b*c^3)*x)]
```

giac [A] time = 0.31, size = 122, normalized size = 1.10

$$-\frac{2\left(\frac{\left(2c^2d+b^2f-2acf-bce\right)x}{b^2c-4ac^2}+\frac{bcd+abf-2ace}{b^2c-4ac^2}\right)}{\sqrt{cx^2+bx+a}}-\frac{f\log\left(\left|-2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}-b\right|\right)}{c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] -2*((2*c^2*d + b^2*f - 2*a*c*f - b*c*e)*x/(b^2*c - 4*a*c^2) + (b*c*d + a*b*
f - 2*a*c*e)/(b^2*c - 4*a*c^2))/sqrt(c*x^2 + b*x + a) - f*log(abs(-2*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(3/2)
```

maple [B] time = 0.01, size = 249, normalized size = 2.24

$$\frac{b^2 f x}{\left(4ac-b^2\right)\sqrt{c\,x^2+bx+a}\;c}-\frac{2bex}{\left(4ac-b^2\right)\sqrt{c\,x^2+bx+a}}+\frac{b^3 f}{2\left(4ac-b^2\right)\sqrt{c\,x^2+bx+a}\;c^2}-\frac{b^2 e}{\left(4ac-b^2\right)\sqrt{c\,x^2+bx+a}\;c}-\frac{fx}{\sqrt{c\,x^2+bx+a}\;c}+\frac{2\left(2cx+b\right)d}{\left(4ac-b^2\right)\sqrt{c\,x^2+bx+a}}+\frac{f\ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{c^{\frac{3}{2}}}+\frac{bf}{2\sqrt{c\,x^2+bx+a}\;c^2}-\frac{e}{\sqrt{c\,x^2+bx+a}\;c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x)
```

```
[Out] -f*x/c/(c*x^2+b*x+a)^(1/2)+1/2*f*b/c^2/(c*x^2+b*x+a)^(1/2)+f*b^2/c/(4*a*c-b
^2)/(c*x^2+b*x+a)^(1/2)*x+1/2*f*b^3/c^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+f/c
^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-e/c/(c*x^2+b*x+a)^(1/2)-
2*e*b/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x-e*b^2/c/(4*a*c-b^2)/(c*x^2+b*x+a)^(
1/2)+2*d*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

mupad [B] time = 3.73, size = 143, normalized size = 1.29

$$\frac{f\,\ln\left(\frac{\frac{b}{2}+c\,x}{\sqrt{c}}+\sqrt{c\,x^2+b\,x+a}\right)}{c^{3/2}}-\frac{e\,\left(4\,a+2\,b\,x\right)}{\left(4\,a\,c-b^2\right)\,\sqrt{c\,x^2+b\,x+a}}+\frac{d\,\left(\frac{b}{2}+c\,x\right)}{\left(a\,c-\frac{b^2}{4}\right)\,\sqrt{c\,x^2+b\,x+a}}+\frac{f\,\left(\frac{a\,b}{2}-x\,\left(a\,c-\frac{b^2}{2}\right)\right)}{c\,\left(a\,c-\frac{b^2}{4}\right)\,\sqrt{c\,x^2+b\,x+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2)/(a + b*x + c*x^2)^(3/2),x)
```

```
[Out] (f*log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2)))/c^(3/2) - (e*(4*a +
2*b*x))/((4*a*c - b^2)*(a + b*x + c*x^2)^(1/2)) + (d*(b/2 + c*x))/((a*c - b
^2/4)*(a + b*x + c*x^2)^(1/2)) + (f*((a*b)/2 - x*(a*c - b^2/2)))/(c*(a*c -
b^2/4)*(a + b*x + c*x^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + ex + fx^2}{\left(a + bx + cx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Integral((d + e*x + f*x**2)/(a + b*x + c*x**2)**(3/2), x)
```

3.114 $\int \frac{1}{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\,dx$

Optimal. Leaf size=666

$$\frac{2\left(-cx\left(-2acf+b^2f-bce+2c^2d\right)-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}-\frac{f\left(f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\ \cdots}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2-(bd\ \cdots\right.}$$

Rubi [A] time = 1.83, antiderivative size = 666, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {974, 1032, 724, 206}

$$\frac{2\left(-cx\left(-2acf+b^2f-bce+2c^2d\right)-bc(cd-3af)-2ac^2e+b^2ce+b^3(-f)\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}-\frac{f\left(f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{f\left(2af-b\left(e-\sqrt{e^2-4df}\right)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{f\left(f\left(2af-b\left(e-\sqrt{e^2-4df}\right)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] (2*(b^2*c*e - 2*a*c^2*e - b^3*f - b*c*(c*d - 3*a*f) - c*(2*c^2*d - b*c*e +
b^2*f - 2*a*c*f)*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f
))*Sqrt[a + b*x + c*x^2]) - (f*(c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(
2*a*f - b*(e + Sqrt[e^2 - 4*d*f])))*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*
f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*
f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])
])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqr
t[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e - Sqrt[e^2 - 4*d*
f]))]) + (f*(c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e - Sqrt
[e^2 - 4*d*f])))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e
 + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2
 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*Sqrt[e^
2 - 4*d*f]*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqrt[c*(e^2 - 2*d*f +
e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
```

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 974

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*
a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*(
d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q,
 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\,dx = \frac{2\left(b^2ce-2ac^2e-b^3f-bc(cd-3af)-c\left(2c^2d-bce+b^2f-2acf\right)x\right)}{\left(b^2-4ac\right)\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{a+bx+cx^2}} +$$

$$= \frac{2\left(b^2ce-2ac^2e-b^3f-bc(cd-3af)-c\left(2c^2d-bce+b^2f-2acf\right)x\right)}{\left(b^2-4ac\right)\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{a+bx+cx^2}} +$$

$$= \frac{2\left(b^2ce-2ac^2e-b^3f-bc(cd-3af)-c\left(2c^2d-bce+b^2f-2acf\right)x\right)}{\left(b^2-4ac\right)\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{a+bx+cx^2}} -$$

$$= \frac{2\left(b^2ce-2ac^2e-b^3f-bc(cd-3af)-c\left(2c^2d-bce+b^2f-2acf\right)x\right)}{\left(b^2-4ac\right)\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{a+bx+cx^2}} -$$

Mathematica [A] time = 5.34, size = 700, normalized size = 1.05

$$\frac{2f\left(-\frac{2\left(-2c\left(2af+cx\left(\sqrt{e^2-4df}+e\right)\right)+2b^2f-bc\left(\sqrt{e^2-4df}+e-2fx\right)\right)}{\left(b^2-4ac\right)\sqrt{a+x(b+cx)}\left(4af^2-2bf\left(\sqrt{e^2-4df}+e\right)+c\left(\sqrt{e^2-4df}+e\right)^2\right)}+\frac{2c\left(cx\left(\sqrt{e^2-4df}-e\right)-2af\right)+2b^2f+bc\left(\sqrt{e^2-4df}-e+2fx\right)}{\left(b^2-4ac\right)\sqrt{a+x(b+cx)}\left(f\left(2af+b\left(\sqrt{e^2-4df}-e\right)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)\right)}+\frac{\sqrt{2}f^2\tanh^{-1}\left(\frac{4af-b\left(\sqrt{e^2-4df}+e-2fx\right)-2cx\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+x(b+cx)}\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\left(f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)\right)^{3/2}}-\frac{\sqrt{2}f^2\sqrt{f\left(2af+b\left(\sqrt{e^2-4df}-e\right)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}\tanh^{-1}\left(\frac{4af+b\left(\sqrt{e^2-4df}-e+2fx\right)+2cx\left(\sqrt{e^2-4df}-e\right)}{2\sqrt{2}\sqrt{a+x(b+cx)}\sqrt{f\left(2af+b\left(\sqrt{e^2-4df}-e\right)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\left(f\left(b\left(e-\sqrt{e^2-4df}\right)-2af\right)+c\left(e\sqrt{e^2-4df}+2df-e^2\right)\right)^2}\right)}{\sqrt{e^2-4df}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] (2*f*((2*b^2*f + b*c*(-e + Sqrt[e^2 - 4*d*f] + 2*f*x) + 2*c*(-2*a*f + c*(-e
 + Sqrt[e^2 - 4*d*f])*x))/((b^2 - 4*a*c)*(c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d
*f]) + f*(2*a*f + b*(-e + Sqrt[e^2 - 4*d*f])))*Sqrt[a + x*(b + c*x)]) - (2*
(2*b^2*f - b*c*(e + Sqrt[e^2 - 4*d*f] - 2*f*x) - 2*c*(2*a*f + c*(e + Sqrt[e
^2 - 4*d*f])*x)))/((b^2 - 4*a*c)*(4*a*f^2 - 2*b*f*(e + Sqrt[e^2 - 4*d*f]) +
 c*(e + Sqrt[e^2 - 4*d*f])^2)*Sqrt[a + x*(b + c*x)]) + (Sqrt[2]*f^2*ArcTanh
[(4*a*f - 2*c*(e + Sqrt[e^2 - 4*d*f])*x - b*(e + Sqrt[e^2 - 4*d*f] - 2*f*x)
)/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e +
 Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c*x)])])/(c*(e^2 - 2*d*f + e*Sqrt[e^2
 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f])))^(3/2) - (Sqrt[2]*f^2*Sq
rt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f + b*(-e + Sqrt[e^2 - 4*
d*f]))]*ArcTanh[(4*a*f + 2*c*(-e + Sqrt[e^2 - 4*d*f])*x + b*(-e + Sqrt[e^2
- 4*d*f] + 2*f*x))/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) +
f*(2*a*f + b*(-e + Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c*x)])])/(c*(-e^2 +
 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(-2*a*f + b*(e - Sqrt[e^2 - 4*d*f])))^2))
/Sqrt[e^2 - 4*d*f]
```

IntegrateAlgebraic [C] time = 1.61, size = 730, normalized size = 1.10

$$\frac{\text{RootSum}\left[\#1^4 f-2\#1^3\sqrt{c}e-2\#1^2af+\#1^2be+4\#1^2cd+2\#1a\sqrt{c}e-4\#1b\sqrt{c}d+a^2f-abe+b^2d\&,\frac{-\#1^2cef\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)+\#1^2bf^2\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)+b^2ef\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)-2\#1c^{3/2}df\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)+2\#1c^{3/2}e^2\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)+bcdf\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)-bce^2\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)-2\#1b\sqrt{c}ef\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)+acef\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)-2abf^2\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)+2\#1a\sqrt{c}f^2\log\left(-\#1+\sqrt{a+bx+cx^2}-\sqrt{c}x\right)}{-2\#1^3f+3\#1^2\sqrt{c}e+2\#1af-\#1be-4\#1cd-a\sqrt{c}e+2b\sqrt{c}d}\&\right]}{a^2f^2-abef-2acdf+ace^2+b^2df-bcde+c^2d^2}-\frac{2\left(-3abcf+2ac^2e-2ac^2fx+b^3f-b^2ce+b^2cfx+bc^2d-bc^2ex+2c^3dx\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(a^2f^2-abef-2acdf+ace^2+b^2df-bcde+c^2d^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] (-2*(b*c^2*d - b^2*c*e + 2*a*c^2*e + b^3*f - 3*a*b*c*f + 2*c^3*d*x - b*c^2*
e*x + b^2*c*f*x - 2*a*c^2*f*x))/((b^2 - 4*a*c)*(c^2*d^2 - b*c*d*e + a*c*e^2
 + b^2*d*f - 2*a*c*d*f - a*b*e*f + a^2*f^2)*Sqrt[a + b*x + c*x^2]) + RootSu
m[b^2*d - a*b*e + a^2*f - 4*b*Sqrt[c]*d*#1 + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2
+ b*e*#1^2 - 2*a*f*#1^2 - 2*Sqrt[c]*e*#1^3 + f*#1^4 & , (-(b*c*e^2*Log[-(Sq
rt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]) + b*c*d*f*Log[-(Sqrt[c]*x) + Sqrt[a
 + b*x + c*x^2] - #1] + b^2*e*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] -
#1] + a*c*e*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*a*b*f^2*Lo
g[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + 2*c^(3/2)*e^2*Log[-(Sqrt[c]*
x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - 2*c^(3/2)*d*f*Log[-(Sqrt[c]*x) + Sqrt
[a + b*x + c*x^2] - #1]*#1 - 2*b*Sqrt[c]*e*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*
x + c*x^2] - #1]*#1 + 2*a*Sqrt[c]*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x
^2] - #1]*#1 - c*e*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2 +
b*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2)/(2*b*Sqrt[c]*d -
 a*Sqrt[c]*e - 4*c*d*#1 - b*e*#1 + 2*a*f*#1 + 3*Sqrt[c]*e*#1^2 - 2*f*#1^3)
& ]/(c^2*d^2 - b*c*d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*e*f + a^2*f^2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$sage_2$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] sage2
```

maple [B] time = 0.03, size = 4099, normalized size = 6.15

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x)

```
[Out] -2/(-4*d*f+e^2)^(1/2)/(2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(
-4*d*f+e^2)^(1/2)*c*e)*f^2/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+(b*f-c*e-(
-4*d*f+e^2)^(1/2)*c)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)/f+1/2*(2*a*f^2-b*e*f-
2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)-4*f
/(2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e
)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2-1/f^2*(-4*d*f+e^2)*c^2-b^2)/((x+1/2*(e+(-4*d
*f+e^2)^(1/2))/f)^2*c+(b*f-c*e-(-4*d*f+e^2)^(1/2)*c)*(x+1/2*(e+(-4*d*f+e^2)
^(1/2))/f)/f+1/2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*
f+e^2)^(1/2)*c*e)/f^2)^(1/2)*x*c^2+4/(-4*d*f+e^2)^(1/2)*f^2/(2*a*f^2-b*e*f-
2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/(4*a*c-4*c^2/f
*d+c^2/f^2*e^2-1/f^2*(-4*d*f+e^2)*c^2-b^2)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f
)^2*c+(b*f-c*e-(-4*d*f+e^2)^(1/2)*c)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)/f+1/2
*(2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e
)/f^2)^(1/2)*x*b*c-4/(-4*d*f+e^2)^(1/2)*f/(2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*
d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2-1/f
^2*(-4*d*f+e^2)*c^2-b^2)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+(b*f-c*e-(-4
*d*f+e^2)^(1/2)*c)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)/f+1/2*(2*a*f^2-b*e*f-2*
c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*x*c^2
*e-2*f/(2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/
2)*c*e)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2-1/f^2*(-4*d*f+e^2)*c^2-b^2)/((x+1/2*(e
+(-4*d*f+e^2)^(1/2))/f)^2*c+(b*f-c*e-(-4*d*f+e^2)^(1/2)*c)*(x+1/2*(e+(-4*d*
f+e^2)^(1/2))/f)/f+1/2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+
(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*b*c+2/(-4*d*f+e^2)^(1/2)*f^2/(2*a*f^2-b*
e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/(4*a*c-4*c
^2/f*d+c^2/f^2*e^2-1/f^2*(-4*d*f+e^2)*c^2-b^2)/((x+1/2*(e+(-4*d*f+e^2)^(1/2
))/f)^2*c+(b*f-c*e-(-4*d*f+e^2)^(1/2)*c)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)/f
+1/2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)
*c*e)/f^2)^(1/2)*b^2-2/(-4*d*f+e^2)^(1/2)*f/(2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-
4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2-1
/f^2*(-4*d*f+e^2)*c^2-b^2)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+(b*f-c*e-(
-4*d*f+e^2)^(1/2)*c)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)/f+1/2*(2*a*f^2-b*e*f-
2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*b*c
*e+2/(-4*d*f+e^2)^(1/2)/(2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f
+(-4*d*f+e^2)^(1/2)*c*e)*f^2*2^(1/2)/((2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+
e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*ln(((b*f-c*e-(-4*d*f+e^2)
^(1/2)*c)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)/f+(2*a*f^2-b*e*f-2*c*d*f+c*e^2-(
-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2+1/2*2^(1/2)*((2*a*f^2-b*e
*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*
(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+4*(b*f-c*e-(-4*d*f+e^2)^(1/2)*c)*(x
+1/2*(e+(-4*d*f+e^2)^(1/2))/f)/f+2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2
)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2
```

```
))/f))+2/(-4*d*f+e^2)^(1/2)/(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)
*b*f-(-4*d*f+e^2)^(1/2)*c*e)*f^2/((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c+(b*
f-c*e+(-4*d*f+e^2)^(1/2)*c)*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)/f+1/2*(2*a*f^
2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(
1/2)-4*f/(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(
1/2)*c*e)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2-1/f^2*(-4*d*f+e^2)*c^2-b^2)/((x-1/2*
(-e+(-4*d*f+e^2)^(1/2))/f)^2*c+(b*f-c*e+(-4*d*f+e^2)^(1/2)*c)*(x-1/2*(-e+(-
4*d*f+e^2)^(1/2))/f)/f+1/2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*
b*f-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*x*c^2-4/(-4*d*f+e^2)^(1/2)*f^2/(2*a*
f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/(4*a
*c-4*c^2/f*d+c^2/f^2*e^2-1/f^2*(-4*d*f+e^2)*c^2-b^2)/((x-1/2*(-e+(-4*d*f+e^
2)^(1/2))/f)^2*c+(b*f-c*e+(-4*d*f+e^2)^(1/2)*c)*(x-1/2*(-e+(-4*d*f+e^2)^(1/
2))/f)/f+1/2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^
2)^(1/2)*c*e)/f^2)^(1/2)*x*b*c+4/(-4*d*f+e^2)^(1/2)*f/(2*a*f^2-b*e*f-2*c*d*
f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/(4*a*c-4*c^2/f*d+c^2
/f^2*e^2-1/f^2*(-4*d*f+e^2)*c^2-b^2)/((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c
+(b*f-c*e+(-4*d*f+e^2)^(1/2)*c)*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)/f+1/2*(2*
a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/f^
2)^(1/2)*x*c^2*e-2*f/(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-
4*d*f+e^2)^(1/2)*c*e)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2-1/f^2*(-4*d*f+e^2)*c^2-b
^2)/((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c+(b*f-c*e+(-4*d*f+e^2)^(1/2)*c)*(
x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)/f+1/2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f
+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*b*c-2/(-4*d*f+e^2)^(1/2)
*f^2/(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)
*c*e)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2-1/f^2*(-4*d*f+e^2)*c^2-b^2)/((x-1/2*(-e+
(-4*d*f+e^2)^(1/2))/f)^2*c+(b*f-c*e+(-4*d*f+e^2)^(1/2)*c)*(x-1/2*(-e+(-4*d*
f+e^2)^(1/2))/f)/f+1/2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-
(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*b^2+2/(-4*d*f+e^2)^(1/2)*f/(2*a*f^2-b*e*
f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/(4*a*c-4*c^2
/f*d+c^2/f^2*e^2-1/f^2*(-4*d*f+e^2)*c^2-b^2)/((x-1/2*(-e+(-4*d*f+e^2)^(1/2)
)/f)^2*c+(b*f-c*e+(-4*d*f+e^2)^(1/2)*c)*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)/f
+1/2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)
*c*e)/f^2)^(1/2)*b*c*e-2/(-4*d*f+e^2)^(1/2)/(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-
4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)*f^2*2^(1/2)/((2*a*f^2-b*e*f-2*
c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*ln(((
b*f-c*e+(-4*d*f+e^2)^(1/2)*c)*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)/f+(2*a*f^2-
b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/f^2+1/2*
2^(1/2)*((2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(
1/2)*c*e)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c+4*(b*f-c*e+(-
4*d*f+e^2)^(1/2)*c)*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)/f+2*(2*a*f^2-b*e*f-2*
c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2))/(x-1
/2*(-e+(-4*d*f+e^2)^(1/2))/f))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re details)Is 4*d*f-e^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(c\,x^2+b\,x+a\right)^{3/2}\,\left(f\,x^2+e\,x+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x)
```

```
[Out] int(1/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x+a)**(3/2)/(f*x**2+e*x+d),x)
```

```
[Out] Timed out
```

3.115 $\int \frac{\left(d+ex+fx^2\right)^3}{\left(a+bx+cx^2\right)^{5/2}} \, dx$

Optimal. Leaf size=891

$$\frac{x\sqrt{cx^2+bx+a}\,f^3}{2c^3}+\frac{(12ce-11bf)\sqrt{cx^2+bx+a}\,f^2}{4c^4}+\frac{\left(24\left(e^2+df\right)c^2-20f(3be+af)c+35b^2f^2\right)\tanh^{-1}\left(\frac{\cdots}{2\sqrt{c}\cdots}\right.}{8c^{9/2}}$$

Rubi [A] time = 1.77, antiderivative size = 891, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1660, 1661, 640, 621, 206}

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)^3/(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] (2*(3*a*b^4*c*e*f^2 - a*b^5*f^3 + a*b^3*c*f*(5*a*f^2 - 3*c*(e^2 + d*f)) - b
*c^2*(c^3*d^3 + 5*a^3*f^3 + 3*a*c^2*d*(e^2 + d*f) - 9*a^2*c*f*(e^2 + d*f))
- a*b^2*c^2*e*(12*a*f^2 - c*(e^2 + 6*d*f)) + 2*a*c^3*e*(3*c^2*d^2 + 3*a^2*f
^2 - a*c*(e^2 + 6*d*f)) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*(c^4*d^2 - b*
c^3*d*e + b^2*c^2*e^2 - 3*a*c^3*e^2 + b^2*c^2*d*f - 2*a*c^3*d*f - 2*b^3*c*e
*f + 7*a*b*c^2*e*f + b^4*f^2 - 4*a*b^2*c*f^2 + a^2*c^2*f^2)*x))/(3*c^5*(b^2
 - 4*a*c)*(a + b*x + c*x^2)^(3/2)) - (2*(3*b^6*c*e*f^2 - b^7*f^3 + 3*b^5*c*
f*(6*a*f^2 - c*(e^2 + d*f)) - 3*b^3*c^2*(29*a^2*f^3 + c^2*d*(e^2 + d*f) - 1
0*a*c*f*(e^2 + d*f)) - 4*b*c^3*(2*c^3*d^3 - 29*a^3*f^3 + 3*a*c^2*d*(e^2 + d
*f) + 24*a^2*c*f*(e^2 + d*f)) - 24*a^2*c^4*e*(6*a*f^2 - c*(e^2 + 6*d*f)) -
b^4*c^2*e*(42*a*f^2 - c*(e^2 + 6*d*f)) + 6*b^2*c^3*e*(2*c^2*d^2 + 28*a^2*f^
2 - a*c*(e^2 + 6*d*f)) - c*(16*c^6*d^3 - 10*b^6*f^3 + 3*b^4*c*f^2*(7*b*e +
26*a*f) - 24*c^5*d*(b*d*e - a*(e^2 + d*f)) - 6*b^2*c^2*f*(25*a*b*e*f + 27*a
^2*f^2 + 2*b^2*(e^2 + d*f)) + 6*c^4*(b^2*d*(e^2 + d*f) - 16*a^2*f*(e^2 + d*
f) - 2*a*b*e*(e^2 + 6*d*f)) + c^3*(240*a^2*b*e*f^2 + 56*a^3*f^3 + 84*a*b^2*
f*(e^2 + d*f) + b^3*(e^3 + 6*d*e*f)))*x))/(3*c^5*(b^2 - 4*a*c)^2*Sqrt[a + b
*x + c*x^2]) + (f^2*(12*c*e - 11*b*f)*Sqrt[a + b*x + c*x^2])/(4*c^4) + (f^3
*x*Sqrt[a + b*x + c*x^2])/(2*c^3) + (f*(35*b^2*f^2 - 20*c*f*(3*b*e + a*f) +
 24*c^2*(e^2 + d*f))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])]
)/(8*c^(9/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(d+ex+fx^2\right)^3}{\left(a+bx+cx^2\right)^{5/2}}\,dx = \frac{2\left(3ab^4cef^2 - ab^5f^3 + ab^3cf\left(5af^2 - 3c\left(e^2+df\right)\right) - bc^2\left(c^3d^3 + 5a^3f^3 + 3ac^2d\left(e^2 + \right.\right.\right.}{}$$

$$= \frac{2\left(3ab^4cef^2 - ab^5f^3 + ab^3cf\left(5af^2 - 3c\left(e^2+df\right)\right) - bc^2\left(c^3d^3 + 5a^3f^3 + 3ac^2d\left(e^2 + \right.\right.\right.}{}$$

$$= \frac{2\left(3ab^4cef^2 - ab^5f^3 + ab^3cf\left(5af^2 - 3c\left(e^2+df\right)\right) - bc^2\left(c^3d^3 + 5a^3f^3 + 3ac^2d\left(e^2 + \right.\right.\right.}{}$$

$$= \frac{2\left(3ab^4cef^2 - ab^5f^3 + ab^3cf\left(5af^2 - 3c\left(e^2+df\right)\right) - bc^2\left(c^3d^3 + 5a^3f^3 + 3ac^2d\left(e^2 + \right.\right.\right.}{}$$

$$= \frac{2\left(3ab^4cef^2 - ab^5f^3 + ab^3cf\left(5af^2 - 3c\left(e^2+df\right)\right) - bc^2\left(c^3d^3 + 5a^3f^3 + 3ac^2d\left(e^2 + \right.\right.\right.}{}$$

$$= \frac{2\left(3ab^4cef^2 - ab^5f^3 + ab^3cf\left(5af^2 - 3c\left(e^2+df\right)\right) - bc^2\left(c^3d^3 + 5a^3f^3 + 3ac^2d\left(e^2 + \right.\right.\right.}{}$$

Mathematica [A] time = 2.19, size = 872, normalized size = 0.98

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)^3/(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] (-105*b^7*f^3*x^2 - 10*b^6*f^2*x*(21*a*f + 2*c*x*(-9*e + 7*f*x)) + 6*b^4*c*
f*(5*a^2*f*(6*e + 53*f*x) - 6*a*c*x*(4*e^2 + 4*d*f + 30*e*f*x - 31*f^2*x^2)
 + c^2*x^3*(-16*e^2 - 16*d*f + 6*e*f*x + f^2*x^2)) - 3*b^5*f*(35*a^2*f^2 -
10*a*c*f*x*(12*e + 23*f*x) + c^2*x^2*(24*e^2 + 24*d*f - 80*e*f*x + 7*f^2*x^
2)) - 48*b*c^2*(27*a^4*f^3 - 4*c^4*d^2*x^2*(d - e*x) + a^2*c^2*(-4*d^2*f +
4*e^3*x - 64*e*f^2*x^3 + 7*f^3*x^4 - 4*d*e*(e - 6*f*x)) - 2*a*c^3*(d^3 - e^
3*x^3 + 3*d*e*x^2*(e - 2*f*x) + 3*d^2*x*(-e + f*x)) - 2*a^3*c*f*(5*e^2 + 39
*e*f*x + f*(5*d - 14*f*x^2))) - 8*b^3*c*(-95*a^3*f^3 + c^3*(d^3 - e^3*x^3 +
```

```
 9*d^2*x*(e - f*x) - 3*d*e*x^2*(3*e + 2*f*x)) - 3*a*c^2*f*x^2*(18*e^2 - 74*
e*f*x + f*(18*d + 7*f*x^2)) + 3*a^2*c*f*(3*e^2 + 105*e*f*x + f*(3*d + 29*f*
x^2))) + 32*c^3*(4*c^4*d^3*x^3 + 3*a^4*f^2*(16*e + 5*f*x) + 6*a*c^3*d*x*(d^
2 + e^2*x^2 + d*f*x^2) - 2*a^3*c*(2*e^3 + 9*e^2*f*x + f^2*x*(9*d - 10*f*x^2
) + 12*e*f*(d - 3*f*x^2)) - 3*a^2*c^2*(2*d^2*e + 4*d*f*x^2*(3*e + 2*f*x) +
x^2*(2*e^3 + 8*e^2*f*x - 6*e*f^2*x^2 - f^3*x^3))) - 48*b^2*c^2*(a^3*f^2*(25
*e + 63*f*x) - c^3*d*x*(d^2 + e^2*x^2 + d*x*(-6*e + f*x)) + a^2*c*f*x*(-21*
e^2 - 12*e*f*x + 7*f*(-3*d + 7*f*x^2)) + a*c^2*(d^2*(e - 6*f*x) - 2*d*x*(3*
e^2 - 3*e*f*x + 7*f^2*x^2) + x^2*(e^3 - 14*e^2*f*x + 6*e*f^2*x^2 + f^3*x^3)
)))/(12*c^4*(b^2 - 4*a*c)^2*(a + x*(b + c*x))^(3/2)) + (f*(35*b^2*f^2 - 20*
c*f*(3*b*e + a*f) + 24*c^2*(e^2 + d*f))*Log[b + 2*c*x + 2*Sqrt[c]*Sqrt[a +
x*(b + c*x)]])/(8*c^(9/2))
```

IntegrateAlgebraic [A] time = 37.60, size = 1411, normalized size = 1.58

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d + e*x + f*x^2)^3/(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] (-8*b^3*c^4*d^3 + 96*a*b*c^5*d^3 - 48*a*b^2*c^4*d^2*e - 192*a^2*c^5*d^2*e +
 192*a^2*b*c^4*d*e^2 - 128*a^3*c^4*e^3 + 192*a^2*b*c^4*d^2*f - 768*a^3*c^4*
d*e*f - 72*a^2*b^3*c^2*e^2*f + 480*a^3*b*c^3*e^2*f - 72*a^2*b^3*c^2*d*f^2 +
 480*a^3*b*c^3*d*f^2 + 180*a^2*b^4*c*e*f^2 - 1200*a^3*b^2*c^2*e*f^2 + 1536*
a^4*c^3*e*f^2 - 105*a^2*b^5*f^3 + 760*a^3*b^3*c*f^3 - 1296*a^4*b*c^2*f^3 +
48*b^2*c^5*d^3*x + 192*a*c^6*d^3*x - 72*b^3*c^4*d^2*e*x - 288*a*b*c^5*d^2*e
*x + 288*a*b^2*c^4*d*e^2*x - 192*a^2*b*c^4*e^3*x + 288*a*b^2*c^4*d^2*f*x -
1152*a^2*b*c^4*d*e*f*x - 144*a*b^4*c^2*e^2*f*x + 1008*a^2*b^2*c^3*e^2*f*x -
 576*a^3*c^4*e^2*f*x - 144*a*b^4*c^2*d*f^2*x + 1008*a^2*b^2*c^3*d*f^2*x - 5
76*a^3*c^4*d*f^2*x + 360*a*b^5*c*e*f^2*x - 2520*a^2*b^3*c^2*e*f^2*x + 3744*
a^3*b*c^3*e*f^2*x - 210*a*b^6*f^3*x + 1590*a^2*b^4*c*f^3*x - 3024*a^3*b^2*c
^2*f^3*x + 480*a^4*c^3*f^3*x + 192*b*c^6*d^3*x^2 - 288*b^2*c^5*d^2*e*x^2 +
72*b^3*c^4*d*e^2*x^2 + 288*a*b*c^5*d*e^2*x^2 - 48*a*b^2*c^4*e^3*x^2 - 192*a
^2*c^5*e^3*x^2 + 72*b^3*c^4*d^2*f*x^2 + 288*a*b*c^5*d^2*f*x^2 - 288*a*b^2*c
^4*d*e*f*x^2 - 1152*a^2*c^5*d*e*f*x^2 - 72*b^5*c^2*e^2*f*x^2 + 432*a*b^3*c^
3*e^2*f*x^2 - 72*b^5*c^2*d*f^2*x^2 + 432*a*b^3*c^3*d*f^2*x^2 + 180*b^6*c*e*
f^2*x^2 - 1080*a*b^4*c^2*e*f^2*x^2 + 576*a^2*b^2*c^3*e*f^2*x^2 + 2304*a^3*c
^4*e*f^2*x^2 - 105*b^7*f^3*x^2 + 690*a*b^5*c*f^3*x^2 - 696*a^2*b^3*c^2*f^3*
x^2 - 1344*a^3*b*c^3*f^3*x^2 + 128*c^7*d^3*x^3 - 192*b*c^6*d^2*e*x^3 + 48*b
^2*c^5*d*e^2*x^3 + 192*a*c^6*d*e^2*x^3 + 8*b^3*c^4*e^3*x^3 - 96*a*b*c^5*e^3
*x^3 + 48*b^2*c^5*d^2*f*x^3 + 192*a*c^6*d^2*f*x^3 + 48*b^3*c^4*d*e*f*x^3 -
576*a*b*c^5*d*e*f*x^3 - 96*b^4*c^3*e^2*f*x^3 + 672*a*b^2*c^4*e^2*f*x^3 - 76
8*a^2*c^5*e^2*f*x^3 - 96*b^4*c^3*d*f^2*x^3 + 672*a*b^2*c^4*d*f^2*x^3 - 768*
a^2*c^5*d*f^2*x^3 + 240*b^5*c^2*e*f^2*x^3 - 1776*a*b^3*c^3*e*f^2*x^3 + 3072
*a^2*b*c^4*e*f^2*x^3 - 140*b^6*c*f^3*x^3 + 1116*a*b^4*c^2*f^3*x^3 - 2352*a^
```

```
2*b^2*c^3*f^3*x^3 + 640*a^3*c^4*f^3*x^3 + 36*b^4*c^3*e*f^2*x^4 - 288*a*b^2*
c^4*e*f^2*x^4 + 576*a^2*c^5*e*f^2*x^4 - 21*b^5*c^2*f^3*x^4 + 168*a*b^3*c^3*
f^3*x^4 - 336*a^2*b*c^4*f^3*x^4 + 6*b^4*c^3*f^3*x^5 - 48*a*b^2*c^4*f^3*x^5
+ 96*a^2*c^5*f^3*x^5)/(12*c^4*(-b^2 + 4*a*c)^2*(a + b*x + c*x^2)^(3/2)) + (
(-24*c^2*e^2*f - 24*c^2*d*f^2 + 60*b*c*e*f^2 - 35*b^2*f^3 + 20*a*c*f^3)*Log
[b*c^4 + 2*c^5*x - 2*c^(9/2)*Sqrt[a + b*x + c*x^2]])/(8*c^(9/2))
```

fricas [B] time = 5.38, size = 3995, normalized size = 4.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)^3/(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/48*(3*((24*(b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*e^2*f + 5*(7*b^6*c^2 -
60*a*b^4*c^3 + 144*a^2*b^2*c^4 - 64*a^3*c^5)*f^3 + 12*(2*(b^4*c^4 - 8*a*b^2
*c^5 + 16*a^2*c^6)*d - 5*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*e)*f^2)*x^4
 + 24*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*e^2*f + 5*(7*a^2*b^6 - 60*
a^3*b^4*c + 144*a^4*b^2*c^2 - 64*a^5*c^3)*f^3 + 2*(24*(b^5*c^3 - 8*a*b^3*c^
4 + 16*a^2*b*c^5)*e^2*f + 5*(7*b^7*c - 60*a*b^5*c^2 + 144*a^2*b^3*c^3 - 64*
a^3*b*c^4)*f^3 + 12*(2*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*d - 5*(b^6*c^
2 - 8*a*b^4*c^3 + 16*a^2*b^2*c^4)*e)*f^2)*x^3 + 12*(2*(a^2*b^4*c^2 - 8*a^3*
b^2*c^3 + 16*a^4*c^4)*d - 5*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*e)*f
^2 + (24*(b^6*c^2 - 6*a*b^4*c^3 + 32*a^3*c^5)*e^2*f + 5*(7*b^8 - 46*a*b^6*c
 + 24*a^2*b^4*c^2 + 224*a^3*b^2*c^3 - 128*a^4*c^4)*f^3 + 12*(2*(b^6*c^2 - 6
*a*b^4*c^3 + 32*a^3*c^5)*d - 5*(b^7*c - 6*a*b^5*c^2 + 32*a^3*b*c^4)*e)*f^2)
*x^2 + 2*(24*(a*b^5*c^2 - 8*a^2*b^3*c^3 + 16*a^3*b*c^4)*e^2*f + 5*(7*a*b^7
- 60*a^2*b^5*c + 144*a^3*b^3*c^2 - 64*a^4*b*c^3)*f^3 + 12*(2*(a*b^5*c^2 - 8
*a^2*b^3*c^3 + 16*a^3*b*c^4)*d - 5*(a*b^6*c - 8*a^2*b^4*c^2 + 16*a^3*b^2*c^
3)*e)*f^2)*x)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x +
 a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(192*a^2*b*c^5*d*e^2 - 128*a^3*c^5*e^3
 + 6*(b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*f^3*x^5 + 3*(12*(b^4*c^4 - 8*a*b^
2*c^5 + 16*a^2*c^6)*e*f^2 - 7*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*f^3)*x
^4 - 8*(b^3*c^5 - 12*a*b*c^6)*d^3 - 48*(a*b^2*c^5 + 4*a^2*c^6)*d^2*e - (105
*a^2*b^5*c - 760*a^3*b^3*c^2 + 1296*a^4*b*c^3)*f^3 + 4*(32*c^8*d^3 - 48*b*c
^7*d^2*e + 12*(b^2*c^6 + 4*a*c^7)*d*e^2 + 2*(b^3*c^5 - 12*a*b*c^6)*e^3 - (3
5*b^6*c^2 - 279*a*b^4*c^3 + 588*a^2*b^2*c^4 - 160*a^3*c^5)*f^3 - 12*(2*(b^4
*c^4 - 7*a*b^2*c^5 + 8*a^2*c^6)*d - (5*b^5*c^3 - 37*a*b^3*c^4 + 64*a^2*b*c^
5)*e)*f^2 + 12*((b^2*c^6 + 4*a*c^7)*d^2 + (b^3*c^5 - 12*a*b*c^6)*d*e - 2*(b
^4*c^4 - 7*a*b^2*c^5 + 8*a^2*c^6)*e^2)*f)*x^3 - 12*(2*(3*a^2*b^3*c^3 - 20*a
^3*b*c^4)*d - (15*a^2*b^4*c^2 - 100*a^3*b^2*c^3 + 128*a^4*c^4)*e)*f^2 + 3*(
64*b*c^7*d^3 - 96*b^2*c^6*d^2*e + 24*(b^3*c^5 + 4*a*b*c^6)*d*e^2 - 16*(a*b^
2*c^5 + 4*a^2*c^6)*e^3 - (35*b^7*c - 230*a*b^5*c^2 + 232*a^2*b^3*c^3 + 448*
a^3*b*c^4)*f^3 - 12*(2*(b^5*c^3 - 6*a*b^3*c^4)*d - (5*b^6*c^2 - 30*a*b^4*c^
3 + 16*a^2*b^2*c^4 + 64*a^3*c^5)*e)*f^2 + 24*((b^3*c^5 + 4*a*b*c^6)*d^2 - 4
*(a*b^2*c^5 + 4*a^2*c^6)*d*e - (b^5*c^3 - 6*a*b^3*c^4)*e^2)*f)*x^2 + 24*(8*
```

a^2*b*c^5*d^2 - 32*a^3*c^5*d*e - (3*a^2*b^3*c^3 - 20*a^3*b*c^4)*e^2)*f + 6*
(48*a*b^2*c^5*d*e^2 - 32*a^2*b*c^5*e^3 + 8*(b^2*c^6 + 4*a*c^7)*d^3 - 12*(b^
3*c^5 + 4*a*b*c^6)*d^2*e - (35*a*b^6*c - 265*a^2*b^4*c^2 + 504*a^3*b^2*c^3
- 80*a^4*c^4)*f^3 - 12*(2*(a*b^4*c^3 - 7*a^2*b^2*c^4 + 4*a^3*c^5)*d - (5*a*
b^5*c^2 - 35*a^2*b^3*c^3 + 52*a^3*b*c^4)*e)*f^2 + 24*(2*a*b^2*c^5*d^2 - 8*a
^2*b*c^5*d*e - (a*b^4*c^3 - 7*a^2*b^2*c^4 + 4*a^3*c^5)*e^2)*f)*x)*sqrt(c*x^
2 + b*x + a))/(a^2*b^4*c^5 - 8*a^3*b^2*c^6 + 16*a^4*c^7 + (b^4*c^7 - 8*a*b^
2*c^8 + 16*a^2*c^9)*x^4 + 2*(b^5*c^6 - 8*a*b^3*c^7 + 16*a^2*b*c^8)*x^3 + (b
^6*c^5 - 6*a*b^4*c^6 + 32*a^3*c^8)*x^2 + 2*(a*b^5*c^5 - 8*a^2*b^3*c^6 + 16*
a^3*b*c^7)*x), -1/24*(3*((24*(b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*e^2*f + 5
*(7*b^6*c^2 - 60*a*b^4*c^3 + 144*a^2*b^2*c^4 - 64*a^3*c^5)*f^3 + 12*(2*(b^4
*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*d - 5*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^
5)*e)*f^2)*x^4 + 24*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*e^2*f + 5*(7
*a^2*b^6 - 60*a^3*b^4*c + 144*a^4*b^2*c^2 - 64*a^5*c^3)*f^3 + 2*(24*(b^5*c^
3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*e^2*f + 5*(7*b^7*c - 60*a*b^5*c^2 + 144*a^2
*b^3*c^3 - 64*a^3*b*c^4)*f^3 + 12*(2*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)
d - 5(b^6*c^2 - 8*a*b^4*c^3 + 16*a^2*b^2*c^4)*e)*f^2)*x^3 + 12*(2*(a^2*b^
4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d - 5*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a
^4*b*c^3)*e)*f^2 + (24*(b^6*c^2 - 6*a*b^4*c^3 + 32*a^3*c^5)*e^2*f + 5*(7*b^
8 - 46*a*b^6*c + 24*a^2*b^4*c^2 + 224*a^3*b^2*c^3 - 128*a^4*c^4)*f^3 + 12*(
2*(b^6*c^2 - 6*a*b^4*c^3 + 32*a^3*c^5)*d - 5*(b^7*c - 6*a*b^5*c^2 + 32*a^3*
b*c^4)*e)*f^2)*x^2 + 2*(24*(a*b^5*c^2 - 8*a^2*b^3*c^3 + 16*a^3*b*c^4)*e^2*f
 + 5*(7*a*b^7 - 60*a^2*b^5*c + 144*a^3*b^3*c^2 - 64*a^4*b*c^3)*f^3 + 12*(2*
(a*b^5*c^2 - 8*a^2*b^3*c^3 + 16*a^3*b*c^4)*d - 5*(a*b^6*c - 8*a^2*b^4*c^2 +
 16*a^3*b^2*c^3)*e)*f^2)*x)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*
x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(192*a^2*b*c^5*d*e^2 - 128*a^3
*c^5*e^3 + 6*(b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*f^3*x^5 + 3*(12*(b^4*c^4
- 8*a*b^2*c^5 + 16*a^2*c^6)*e*f^2 - 7*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5
)*f^3)*x^4 - 8*(b^3*c^5 - 12*a*b*c^6)*d^3 - 48*(a*b^2*c^5 + 4*a^2*c^6)*d^2*
e - (105*a^2*b^5*c - 760*a^3*b^3*c^2 + 1296*a^4*b*c^3)*f^3 + 4*(32*c^8*d^3
- 48*b*c^7*d^2*e + 12*(b^2*c^6 + 4*a*c^7)*d*e^2 + 2*(b^3*c^5 - 12*a*b*c^6)*
e^3 - (35*b^6*c^2 - 279*a*b^4*c^3 + 588*a^2*b^2*c^4 - 160*a^3*c^5)*f^3 - 12
(2(b^4*c^4 - 7*a*b^2*c^5 + 8*a^2*c^6)*d - (5*b^5*c^3 - 37*a*b^3*c^4 + 64*
a^2*b*c^5)*e)*f^2 + 12*((b^2*c^6 + 4*a*c^7)*d^2 + (b^3*c^5 - 12*a*b*c^6)*d*
e - 2*(b^4*c^4 - 7*a*b^2*c^5 + 8*a^2*c^6)*e^2)*f)*x^3 - 12*(2*(3*a^2*b^3*c^
3 - 20*a^3*b*c^4)*d - (15*a^2*b^4*c^2 - 100*a^3*b^2*c^3 + 128*a^4*c^4)*e)*f
^2 + 3*(64*b*c^7*d^3 - 96*b^2*c^6*d^2*e + 24*(b^3*c^5 + 4*a*b*c^6)*d*e^2 -
16*(a*b^2*c^5 + 4*a^2*c^6)*e^3 - (35*b^7*c - 230*a*b^5*c^2 + 232*a^2*b^3*c^
3 + 448*a^3*b*c^4)*f^3 - 12*(2*(b^5*c^3 - 6*a*b^3*c^4)*d - (5*b^6*c^2 - 30*
a*b^4*c^3 + 16*a^2*b^2*c^4 + 64*a^3*c^5)*e)*f^2 + 24*((b^3*c^5 + 4*a*b*c^6)
d^2 - 4(a*b^2*c^5 + 4*a^2*c^6)*d*e - (b^5*c^3 - 6*a*b^3*c^4)*e^2)*f)*x^2
+ 24*(8*a^2*b*c^5*d^2 - 32*a^3*c^5*d*e - (3*a^2*b^3*c^3 - 20*a^3*b*c^4)*e^2
)*f + 6*(48*a*b^2*c^5*d*e^2 - 32*a^2*b*c^5*e^3 + 8*(b^2*c^6 + 4*a*c^7)*d^3
- 12*(b^3*c^5 + 4*a*b*c^6)*d^2*e - (35*a*b^6*c - 265*a^2*b^4*c^2 + 504*a^3*
b^2*c^3 - 80*a^4*c^4)*f^3 - 12*(2*(a*b^4*c^3 - 7*a^2*b^2*c^4 + 4*a^3*c^5)*d

```
 - (5*a*b^5*c^2 - 35*a^2*b^3*c^3 + 52*a^3*b*c^4)*e)*f^2 + 24*(2*a*b^2*c^5*d
^2 - 8*a^2*b*c^5*d*e - (a*b^4*c^3 - 7*a^2*b^2*c^4 + 4*a^3*c^5)*e^2)*f)*x)*s
qrt(c*x^2 + b*x + a))/(a^2*b^4*c^5 - 8*a^3*b^2*c^6 + 16*a^4*c^7 + (b^4*c^7
- 8*a*b^2*c^8 + 16*a^2*c^9)*x^4 + 2*(b^5*c^6 - 8*a*b^3*c^7 + 16*a^2*b*c^8)*
x^3 + (b^6*c^5 - 6*a*b^4*c^6 + 32*a^3*c^8)*x^2 + 2*(a*b^5*c^5 - 8*a^2*b^3*c
^6 + 16*a^3*b*c^7)*x)]
```

giac [A] time = 0.45, size = 1401, normalized size = 1.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)^3/(c*x^2+b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/12*((((3*(2*(b^4*c^3*f^3 - 8*a*b^2*c^4*f^3 + 16*a^2*c^5*f^3)*x/(b^4*c^4 -
 8*a*b^2*c^5 + 16*a^2*c^6) - (7*b^5*c^2*f^3 - 56*a*b^3*c^3*f^3 + 112*a^2*b*
c^4*f^3 - 12*b^4*c^3*f^2*e + 96*a*b^2*c^4*f^2*e - 192*a^2*c^5*f^2*e)/(b^4*c
^4 - 8*a*b^2*c^5 + 16*a^2*c^6))*x + 4*(32*c^7*d^3 + 12*b^2*c^5*d^2*f + 48*a
*c^6*d^2*f - 24*b^4*c^3*d*f^2 + 168*a*b^2*c^4*d*f^2 - 192*a^2*c^5*d*f^2 - 3
5*b^6*c*f^3 + 279*a*b^4*c^2*f^3 - 588*a^2*b^2*c^3*f^3 + 160*a^3*c^4*f^3 - 4
8*b*c^6*d^2*e + 12*b^3*c^4*d*f*e - 144*a*b*c^5*d*f*e + 60*b^5*c^2*f^2*e - 4
44*a*b^3*c^3*f^2*e + 768*a^2*b*c^4*f^2*e + 12*b^2*c^5*d*e^2 + 48*a*c^6*d*e^
2 - 24*b^4*c^3*f*e^2 + 168*a*b^2*c^4*f*e^2 - 192*a^2*c^5*f*e^2 + 2*b^3*c^4*
e^3 - 24*a*b*c^5*e^3)/(b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6))*x + 3*(64*b*c^6
*d^3 + 24*b^3*c^4*d^2*f + 96*a*b*c^5*d^2*f - 24*b^5*c^2*d*f^2 + 144*a*b^3*c
^3*d*f^2 - 35*b^7*f^3 + 230*a*b^5*c*f^3 - 232*a^2*b^3*c^2*f^3 - 448*a^3*b*c
^3*f^3 - 96*b^2*c^5*d^2*e - 96*a*b^2*c^4*d*f*e - 384*a^2*c^5*d*f*e + 60*b^6
*c*f^2*e - 360*a*b^4*c^2*f^2*e + 192*a^2*b^2*c^3*f^2*e + 768*a^3*c^4*f^2*e
+ 24*b^3*c^4*d*e^2 + 96*a*b*c^5*d*e^2 - 24*b^5*c^2*f*e^2 + 144*a*b^3*c^3*f*
e^2 - 16*a*b^2*c^4*e^3 - 64*a^2*c^5*e^3)/(b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^
6))*x + 6*(8*b^2*c^5*d^3 + 32*a*c^6*d^3 + 48*a*b^2*c^4*d^2*f - 24*a*b^4*c^2
*d*f^2 + 168*a^2*b^2*c^3*d*f^2 - 96*a^3*c^4*d*f^2 - 35*a*b^6*f^3 + 265*a^2*
b^4*c*f^3 - 504*a^3*b^2*c^2*f^3 + 80*a^4*c^3*f^3 - 12*b^3*c^4*d^2*e - 48*a*
b*c^5*d^2*e - 192*a^2*b*c^4*d*f*e + 60*a*b^5*c*f^2*e - 420*a^2*b^3*c^2*f^2*
e + 624*a^3*b*c^3*f^2*e + 48*a*b^2*c^4*d*e^2 - 24*a*b^4*c^2*f*e^2 + 168*a^2
*b^2*c^3*f*e^2 - 96*a^3*c^4*f*e^2 - 32*a^2*b*c^4*e^3)/(b^4*c^4 - 8*a*b^2*c^
5 + 16*a^2*c^6))*x - (8*b^3*c^4*d^3 - 96*a*b*c^5*d^3 - 192*a^2*b*c^4*d^2*f
+ 72*a^2*b^3*c^2*d*f^2 - 480*a^3*b*c^3*d*f^2 + 105*a^2*b^5*f^3 - 760*a^3*b^
3*c*f^3 + 1296*a^4*b*c^2*f^3 + 48*a*b^2*c^4*d^2*e + 192*a^2*c^5*d^2*e + 768
*a^3*c^4*d*f*e - 180*a^2*b^4*c*f^2*e + 1200*a^3*b^2*c^2*f^2*e - 1536*a^4*c^
3*f^2*e - 192*a^2*b*c^4*d*e^2 + 72*a^2*b^3*c^2*f*e^2 - 480*a^3*b*c^3*f*e^2
+ 128*a^3*c^4*e^3)/(b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6))/(c*x^2 + b*x + a)^
(3/2) - 1/8*(24*c^2*d*f^2 + 35*b^2*f^3 - 20*a*c*f^3 - 60*b*c*f^2*e + 24*c^2
*f*e^2)*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(9/2
)
```

maple [B] time = 0.03, size = 4635, normalized size = 5.20

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)^3/(c*x^2+b*x+a)^(5/2),x)
```

```
[Out] 5/6*f^3*a/c^2*x^3/(c*x^2+b*x+a)^(3/2)+5/2*f^3*a/c^3*x/(c*x^2+b*x+a)^(1/2)-6
*b/c*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*x*d*e*f-5/2*f^3*a/c^3*b^2/(4*a*c-b^2
)/(c*x^2+b*x+a)^(1/2)*x-33/4*f^3*b^2/c^3*a^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2
)*x-66*f^3*b^2/c^2*a^2/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x-1/8*b^4/c^3/(4*a
*c-b^2)/(c*x^2+b*x+a)^(3/2)*x*e^2*f+4*b^3/c/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/
2)*x*d*e*f-3*b^2/c^2*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*d*e*f-48*b*a/(4*a*c-
b^2)^2/(c*x^2+b*x+a)^(1/2)*x*d*e*f-24*b^2/c*a/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(
1/2)*d*e*f+12*b^2/c*a/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x*d*f^2+12*b^2/c*a/
(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x*e^2*f+3/2*b^2/c^2*a/(4*a*c-b^2)/(c*x^2+
b*x+a)^(3/2)*x*d*f^2+3/2*b^2/c^2*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*x*e^2*f+
12*e*f^2*a^2/c^2*b/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*x+96*e*f^2*a^2/c*b/(4*a*
c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x-19/4*e*f^2*b^3/c^3*a/(4*a*c-b^2)/(c*x^2+b*x+
a)^(3/2)*x-38*e*f^2*b^3/c^2*a/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x+1/2*b^3/c
^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*x*d*e*f+2/3*d^3/(4*a*c-b^2)/(c*x^2+b*x+a
)^(3/2)*b-x^2/c/(c*x^2+b*x+a)^(3/2)*e^3+1/24*b^2/c^3/(c*x^2+b*x+a)^(3/2)*e^
3-2/3*a/c^2/(c*x^2+b*x+a)^(3/2)*e^3+3/c^(5/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2
+b*x+a)^(1/2))*d*f^2+3/c^(5/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*
e^2*f+35/16*f^3*b^3/c^5/(c*x^2+b*x+a)^(1/2)-5/2*f^3*a/c^(7/2)*ln((c*x+1/2*b
)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+35/8*f^3*b^2/c^(9/2)*ln((c*x+1/2*b)/c^(1/2)+
(c*x^2+b*x+a)^(1/2))+1/2*f^3*x^5/c/(c*x^2+b*x+a)^(3/2)-35/384*f^3*b^5/c^6/(
c*x^2+b*x+a)^(3/2)-d^2*e/c/(c*x^2+b*x+a)^(3/2)-33/16*f^3*b^2/c^4*a*x/(c*x^2
+b*x+a)^(3/2)-33/8*f^3*b^3/c^4*a^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+23/16*f^
3*b^5/c^5*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+23/2*f^3*b^5/c^4*a/(4*a*c-b^2)^
2/(c*x^2+b*x+a)^(1/2)-5/4*f^3*a/c^4*b^3/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)-35/
192*f^3*b^6/c^5/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*x+35/8*f^3*b^4/c^4/(4*a*c-b
^2)/(c*x^2+b*x+a)^(1/2)*x+1/4*b^3/c^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*f*d^2
+1/4*b^3/c^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*e^2*d+4*b^2/(4*a*c-b^2)^2/(c*x
^2+b*x+a)^(1/2)*x*f*d^2+4*b^2/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x*e^2*d+2*b
^3/c/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*f*d^2+2*b^3/c/(4*a*c-b^2)^2/(c*x^2+b
*x+a)^(1/2)*e^2*d+2*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*x*f*d^2-15/4*e*f^2*b^
4/c^4/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+3/2*b/c^2*x^2/(c*x^2+b*x+a)^(3/2)*d*f
^2+3/2*b/c^2*x^2/(c*x^2+b*x+a)^(3/2)*e^2*f+3/8*b^2/c^3*x/(c*x^2+b*x+a)^(3/2
)*d*f^2+3/8*b^2/c^3*x/(c*x^2+b*x+a)^(3/2)*e^2*f-1/16*b^5/c^4/(4*a*c-b^2)/(c
*x^2+b*x+a)^(3/2)*d*f^2-1/16*b^5/c^4/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*e^2*f-
1/2*b^5/c^3/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*d*f^2-1/2*b^5/c^3/(4*a*c-b^2)
^2/(c*x^2+b*x+a)^(1/2)*e^2*f+b/c^3*a/(c*x^2+b*x+a)^(3/2)*d*f^2+b/c^3*a/(c*x
^2+b*x+a)^(3/2)*e^2*f+3/2/c^3*b^3/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*d*f^2+3/2
```

/c^3*b^3/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*e^2*f+12*e*f^2*a/c^2*x^2/(c*x^2+b*x+a)^(3/2)+5/2*e*f^2*b/c^2*x^3/(c*x^2+b*x+a)^(3/2)-15/4*e*f^2*b^2/c^3*x^2/(c*x^2+b*x+a)^(3/2)-15/16*e*f^2*b^3/c^4*x/(c*x^2+b*x+a)^(3/2)+5/32*e*f^2*b^6/c^5/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+5/4*e*f^2*b^6/c^4/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)-3*e*f^2*b^2/c^4*a/(c*x^2+b*x+a)^(3/2)+15/2*e*f^2*b/c^3*x/(c*x^2+b*x+a)^(1/2)-35/24*f^3*b^6/c^4/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x-33/4*f^3*b/c^3*a*x^2/(c*x^2+b*x+a)^(3/2)-33*f^3*b^3/c^3*a^2/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)+2*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*x*e^2*d+8*a/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*b*f*d^2+8*a/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*b*e^2*d-2*d^2*e*b/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*x-d^2*e*b^2/c/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)-6*x^2/c/(c*x^2+b*x+a)^(3/2)*d*e*f+1/4*b^2/c^3/(c*x^2+b*x+a)^(3/2)*d*e*f+1/12*b^3/c^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*x*e^3+2/3*b^3/c/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x*e^3-1/2*b^2/c^2*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*e^3-8*b*a/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x*e^3-4*b^2/c*a/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*e^3-4*a/c^2/(c*x^2+b*x+a)^(3/2)*d*e*f-16*d^2*e*b*c/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x-b^4/c^2/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x*e^2*f+3/4*b^3/c^3*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*d*f^2+3/4*b^3/c^3*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*e^2*f+6*b^3/c^2*a/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*d*f^2+6*b^3/c^2*a/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*e^2*f+3/c^2*b^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*d*f^2-b^4/c^2/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x*d*f^2-3/2*b/c^2*x/(c*x^2+b*x+a)^(3/2)*d*e*f+1/4*b^4/c^3/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*d*e*f+2*b^4/c^2/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*d*e*f-b/c*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*x*e^3+5/2*e*f^2*b^5/c^3/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x+48*e*f^2*a^2/c^2*b^2/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)+3*e*f^2*a/c^3*b*x/(c*x^2+b*x+a)^(3/2)+6*e*f^2*a^2/c^3*b^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)-19/8*e*f^2*b^4/c^4*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)-19*e*f^2*b^4/c^3*a/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)-15/2*e*f^2*b^3/c^3/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x+5/16*e*f^2*b^5/c^4/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*x+23/8*f^3*b^4/c^4*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*x+23*f^3*b^4/c^3*a/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x+1/2*b^2/c/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*x*e^2*d+a/c/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*b*f*d^2+a/c/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*b*e^2*d+16*a*c/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x*f*d^2+16*a*c/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x*e^2*d+3/c^2*b^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*e^2*f-1/8*b^4/c^3/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*x*d*f^2+1/2*b^2/c/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*x*f*d^2-5/4*f^3*a/c^4*b/(c*x^2+b*x+a)^(1/2)+173/96*f^3*b^3/c^5*a/(c*x^2+b*x+a)^(3/2)-35/8*f^3*b^2/c^4*x/(c*x^2+b*x+a)^(1/2)+35/16*f^3*b^5/c^5/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)-11/2*f^3*b/c^4*a^2/(c*x^2+b*x+a)^(3/2)-7/4*f^3*b/c^2*x^4/(c*x^2+b*x+a)^(3/2)-35/24*f^3*b^2/c^3*x^3/(c*x^2+b*x+a)^(3/2)+35/16*f^3*b^3/c^4*x^2/(c*x^2+b*x+a)^(3/2)+35/64*f^3*b^4/c^5*x/(c*x^2+b*x+a)^(3/2)-35/384*f^3*b^7/c^6/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)-35/48*f^3*b^7/c^5/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)+8*e*f^2*a^2/c^3/(c*x^2+b*x+a)^(3/2)-15/2*e*f^2*b/c^(7/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+3*e*f^2*x^4/c/(c*x^2+b*x+a)^(3/2)+5/32*e*f^2*b^4/c^5/(c*x^2+b*x+a)^(3/2)-15/4*e*f^2*b^2/c^4/(c*x^2+b*x+a)^(1/2)-x^3/c/(c*x^2+b*x+a)^(3/2)*d*f^2-x^3/c/(c*x^2+b*x+a)^(3/2)*e^2*f-1/16*b^3/c^4/(c*x^2+

```
b*x+a)^(3/2)*d*f^2-1/16*b^3/c^4/(c*x^2+b*x+a)^(3/2)*e^2*f-3/c^2*x/(c*x^2+b*
x+a)^(1/2)*d*f^2-3/c^2*x/(c*x^2+b*x+a)^(1/2)*e^2*f+3/2/c^3*b/(c*x^2+b*x+a)^
(1/2)*d*f^2+3/2/c^3*b/(c*x^2+b*x+a)^(1/2)*e^2*f-1/4*b/c^2*x/(c*x^2+b*x+a)^(
3/2)*e^3+1/24*b^4/c^3/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*e^3+1/3*b^4/c^2/(4*a*
c-b^2)^2/(c*x^2+b*x+a)^(1/2)*e^3-3/2*x/c/(c*x^2+b*x+a)^(3/2)*f*d^2-3/2*x/c/
(c*x^2+b*x+a)^(3/2)*e^2*d+1/4*b/c^2/(c*x^2+b*x+a)^(3/2)*f*d^2+1/4*b/c^2/(c*
x^2+b*x+a)^(3/2)*e^2*d-8*d^2*e*b^2/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)+4/3*d^
3/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*x*c+32/3*d^3*c^2/(4*a*c-b^2)^2/(c*x^2+b*x
+a)^(1/2)*x+16/3*d^3*c/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*b
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)^3/(c*x^2+b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(f\,x^2+e\,x+d\right)^3}{\left(c\,x^2+b\,x+a\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2)^3/(a + b*x + c*x^2)^(5/2),x)
```

```
[Out] int((d + e*x + f*x^2)^3/(a + b*x + c*x^2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)**3/(c*x**2+b*x+a)**(5/2),x)
```

```
[Out] Timed out
```

3.116 $\int \frac{(d+ex+fx^2)^2}{(a+bx+cx^2)^{5/2}}\,dx$

Optimal. Leaf size=444

$$\frac{2\left(-x\left(c^2\left(2a^2f^2+6abef+b^2\left(2df+e^2\right)\right)-2b^2cf(2af+be)-2c^3\left(a\left(2df+e^2\right)+bde\right)+b^4f^2+2c^4d^2\right)-bc\left(- \cdots\right.\right.}{3c^3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}}$$

Rubi [A] time = 0.45, antiderivative size = 444, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1660, 12, 621, 206}

$$-\frac{2\left(-2cx\left(-c^2\left(16a^2f^2+12abef+b^2\left(-\left(2df+e^2\right)\right)\right)+b^2cf(14af+be)-c^3\left(8bde-4a\left(2df+e^2\right)\right)-2b^4f^2+8c^4d^2\right)-4bc^2\left(8a^2f^2+ac\left(2df+e^2\right)+2c^2d^2\right)+48a^2c^3ef+4b^2c^2e(2cd-3af)+b^3c\left(10af^2-c\left(2df+e^2\right)\right)+2b^4cef+b^5\left(-f^2\right)\right)}{3c^3\left(b^2-4ac\right)^2\sqrt{a+bx+cx^2}}+\frac{2\left(-x\left(c^2\left(2a^2f^2+6abef+b^2\left(2df+e^2\right)\right)-2b^2cf(2af+be)-2c^3\left(a\left(2df+e^2\right)+bde\right)+b^4f^2+2c^4d^2\right)-bc\left(-3a^2f^2+ac\left(2df+e^2\right)+c^2d^2\right)+2ab^2cef-ab^3f^2+4ac^2e(cd-af)\right)}{3c^3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}}+\frac{f^2\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)^2/(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] (2*(2*a*b^2*c*e*f - a*b^3*f^2 + 4*a*c^2*e*(c*d - a*f) - b*c*(c^2*d^2 - 3*a^
2*f^2 + a*c*(e^2 + 2*d*f)) - (2*c^4*d^2 + b^4*f^2 - 2*b^2*c*f*(b*e + 2*a*f)
 - 2*c^3*(b*d*e + a*(e^2 + 2*d*f)) + c^2*(6*a*b*e*f + 2*a^2*f^2 + b^2*(e^2
+ 2*d*f)))*x))/(3*c^3*(b^2 - 4*a*c)*(a + b*x + c*x^2)^(3/2)) - (2*(2*b^4*c*
e*f + 48*a^2*c^3*e*f - b^5*f^2 + 4*b^2*c^2*e*(2*c*d - 3*a*f) + b^3*c*(10*a*
f^2 - c*(e^2 + 2*d*f)) - 4*b*c^2*(2*c^2*d^2 + 8*a^2*f^2 + a*c*(e^2 + 2*d*f)
) - 2*c*(8*c^4*d^2 - 2*b^4*f^2 + b^2*c*f*(b*e + 14*a*f) - c^3*(8*b*d*e - 4*
a*(e^2 + 2*d*f)) - c^2*(12*a*b*e*f + 16*a^2*f^2 - b^2*(e^2 + 2*d*f)))*x))/(
3*c^3*(b^2 - 4*a*c)^2*Sqrt[a + b*x + c*x^2]) + (f^2*ArcTanh[(b + 2*c*x)/(2*
Sqrt[c]*Sqrt[a + b*x + c*x^2])])/c^(5/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
```

```
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{\left(d+ex+fx^2\right)^2}{\left(a+bx+cx^2\right)^{5/2}}\,dx = \frac{2\left(2ab^2cef-ab^3f^2+4ac^2e(cd-af)-bc\left(c^2d^2-3a^2f^2+ac\left(e^2+2df\right)\right)-\left(2c^4d^2+b^4\right.\right.}{3c^3\left(b^2-4ac\right)}$$

$$= \frac{2\left(2ab^2cef-ab^3f^2+4ac^2e(cd-af)-bc\left(c^2d^2-3a^2f^2+ac\left(e^2+2df\right)\right)-\left(2c^4d^2+b^4\right.\right.}{3c^3\left(b^2-4ac\right)}$$

$$= \frac{2\left(2ab^2cef-ab^3f^2+4ac^2e(cd-af)-bc\left(c^2d^2-3a^2f^2+ac\left(e^2+2df\right)\right)-\left(2c^4d^2+b^4\right.\right.}{3c^3\left(b^2-4ac\right)}$$

$$= \frac{2\left(2ab^2cef-ab^3f^2+4ac^2e(cd-af)-bc\left(c^2d^2-3a^2f^2+ac\left(e^2+2df\right)\right)-\left(2c^4d^2+b^4\right.\right.}{3c^3\left(b^2-4ac\right)}$$

$$= \frac{2\left(2ab^2cef-ab^3f^2+4ac^2e(cd-af)-bc\left(c^2d^2-3a^2f^2+ac\left(e^2+2df\right)\right)-\left(2c^4d^2+b^4\right.\right.}{3c^3\left(b^2-4ac\right)}$$

Mathematica [A] time = 1.26, size = 387, normalized size = 0.87

$$\frac{2\left(b^3\left(-3a^2f^2+18acf^2x^2+c^2\left(-d^2+6dx(fx-e)+ex^2(3e+2fx)\right)\right)+2b^2c\left(21a^2f^2x-2ac\left(d(e-6fx)+x\left(-3e^2+3efx-7f^2x^2\right)\right)+c^2x\left(3d^2+2dx(fx-6e)+e^2x^2\right)\right)+4bc\left(5a^3f^2+2a^2c\left(2df+e^2-6efx\right)+3ac^2(d-ex)(d+x(2fx-e))+2c^3dx^2(3d-2ex)\right)+8c^2\left(a^3(-f)(4e+3fx)-2a^2c\left(de+fx^2(3e+2fx)\right)+ac^2x\left(3d^2+2dfx^2+e^2x^2\right)+2c^3d^2x^3\right)-2b^4f^2x\left(3a+2cx^2\right)-3b^5f^2x^2\right)}{3c^2\left(b^2-4ac\right)^2(a+x(b+cx))^{3/2}}+\frac{f^2\log\left(2\sqrt{c}\sqrt{a+x(b+cx)}+b+2cx\right)}{c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)^2/(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] (2*(-3*b^5*f^2*x^2 - 2*b^4*f^2*x*(3*a + 2*c*x^2) + 4*b*c*(5*a^3*f^2 + 2*c^3
*d*x^2*(3*d - 2*e*x) + 2*a^2*c*(e^2 + 2*d*f - 6*e*f*x) + 3*a*c^2*(d - e*x)*
(d + x*(-e + 2*f*x))) + b^3*(-3*a^2*f^2 + 18*a*c*f^2*x^2 + c^2*(-d^2 + 6*d*
x*(-e + f*x) + e*x^2*(3*e + 2*f*x))) + 8*c^2*(2*c^3*d^2*x^3 - a^3*f*(4*e +
3*f*x) + a*c^2*x*(3*d^2 + e^2*x^2 + 2*d*f*x^2) - 2*a^2*c*(d*e + f*x^2*(3*e
+ 2*f*x))) + 2*b^2*c*(21*a^2*f^2*x + c^2*x*(3*d^2 + e^2*x^2 + 2*d*x*(-6*e +
 f*x)) - 2*a*c*(d*(e - 6*f*x) + x*(-3*e^2 + 3*e*f*x - 7*f^2*x^2)))))/(3*c^2
*(b^2 - 4*a*c)^2*(a + x*(b + c*x))^(3/2)) + (f^2*Log[b + 2*c*x + 2*Sqrt[c]*
Sqrt[a + x*(b + c*x)]])/c^(5/2)
```

IntegrateAlgebraic [A] time = 3.16, size = 555, normalized size = 1.25

$$-\frac{2\left(-20a^3bcf^2+32a^3c^2ef+24a^3c^2f^2x+3a^2b^3f^2-42a^2b^2cf^2x-16a^2bc^2df-8a^2bc^2e^2+48a^2bc^2efx+16a^2c^3de+48a^2c^3efx^2+32a^2c^3f^2x^3+6ab^4f^2x-18ab^3cf^2x^2+4ab^2c^2de-24ab^2c^2dfx-12ab^2c^2e^2x+12ab^2c^2efx^2-28ab^2c^2f^2x^3-12abc^3d^2+24abc^3dex-24abc^3dfx^2-12abc^3e^2x^2+24abc^3efx^3-24ac^4d^2x-16ac^4dfx^3-8ac^4e^2x^3+3b^5f^2x^2+4b^4cf^2x^3+b^3c^2d^2+6b^3c^2dex-6b^3c^2dfx^2-3b^3c^2e^2x^2-2b^3c^2efx^3-6b^2c^3d^2x+24b^2c^3dex^2-4b^2c^3dfx^3-2b^2c^3e^2x^3-24bc^4d^2x^2+16bc^4dex^3-16c^5d^2x^3\right)}{3c^2\left(4ac-b^2\right)^2\left(a+bx+cx^2\right)^{3/2}}-\frac{f^2\log\left(-2c^{5/2}\sqrt{a+bx+cx^2}+bc^2+2c^3x\right)}{c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d + e*x + f*x^2)^2/(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] (-2*(b^3*c^2*d^2 - 12*a*b*c^3*d^2 + 4*a*b^2*c^2*d*e + 16*a^2*c^3*d*e - 8*a^
2*b*c^2*e^2 - 16*a^2*b*c^2*d*f + 32*a^3*c^2*e*f + 3*a^2*b^3*f^2 - 20*a^3*b*
c*f^2 - 6*b^2*c^3*d^2*x - 24*a*c^4*d^2*x + 6*b^3*c^2*d*e*x + 24*a*b*c^3*d*e
*x - 12*a*b^2*c^2*e^2*x - 24*a*b^2*c^2*d*f*x + 48*a^2*b*c^2*e*f*x + 6*a*b^4
*f^2*x - 42*a^2*b^2*c*f^2*x + 24*a^3*c^2*f^2*x - 24*b*c^4*d^2*x^2 + 24*b^2*
c^3*d*e*x^2 - 3*b^3*c^2*e^2*x^2 - 12*a*b*c^3*e^2*x^2 - 6*b^3*c^2*d*f*x^2 -
24*a*b*c^3*d*f*x^2 + 12*a*b^2*c^2*e*f*x^2 + 48*a^2*c^3*e*f*x^2 + 3*b^5*f^2*
x^2 - 18*a*b^3*c*f^2*x^2 - 16*c^5*d^2*x^3 + 16*b*c^4*d*e*x^3 - 2*b^2*c^3*e^
2*x^3 - 8*a*c^4*e^2*x^3 - 4*b^2*c^3*d*f*x^3 - 16*a*c^4*d*f*x^3 - 2*b^3*c^2*
e*f*x^3 + 24*a*b*c^3*e*f*x^3 + 4*b^4*c*f^2*x^3 - 28*a*b^2*c^2*f^2*x^3 + 32*
a^2*c^3*f^2*x^3))/(3*c^2*(-b^2 + 4*a*c)^2*(a + b*x + c*x^2)^(3/2)) - (f^2*L
og[b*c^2 + 2*c^3*x - 2*c^(5/2)*Sqrt[a + b*x + c*x^2]])/c^(5/2)
```

fricas [A] time = 3.50, size = 1581, normalized size = 3.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)^2/(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*f^2*x^4 + 2*(b^5*c - 8*a*b^3*
c^2 + 16*a^2*b*c^3)*f^2*x^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*f^2*x^2 + 2*(a
*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*f^2*x + (a^2*b^4 - 8*a^3*b^2*c + 16*a^4*
c^2)*f^2)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*
(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(8*a^2*b*c^3*e^2 + 2*(8*c^6*d^2 - 8*b*c^5*
d*e + (b^2*c^4 + 4*a*c^5)*e^2 - 2*(b^4*c^2 - 7*a*b^2*c^3 + 8*a^2*c^4)*f^2 +
 (2*(b^2*c^4 + 4*a*c^5)*d + (b^3*c^3 - 12*a*b*c^4)*e)*f)*x^3 - (b^3*c^3 - 1
```

```
2*a*b*c^4)*d^2 - 4*(a*b^2*c^3 + 4*a^2*c^4)*d*e - (3*a^2*b^3*c - 20*a^3*b*c^
2)*f^2 + 3*(8*b*c^5*d^2 - 8*b^2*c^4*d*e + (b^3*c^3 + 4*a*b*c^4)*e^2 - (b^5*
c - 6*a*b^3*c^2)*f^2 + 2*((b^3*c^3 + 4*a*b*c^4)*d - 2*(a*b^2*c^3 + 4*a^2*c^
4)*e)*f)*x^2 + 16*(a^2*b*c^3*d - 2*a^3*c^3*e)*f + 6*(2*a*b^2*c^3*e^2 + (b^2
*c^4 + 4*a*c^5)*d^2 - (b^3*c^3 + 4*a*b*c^4)*d*e - (a*b^4*c - 7*a^2*b^2*c^2
+ 4*a^3*c^3)*f^2 + 4*(a*b^2*c^3*d - 2*a^2*b*c^3*e)*f)*x)*sqrt(c*x^2 + b*x +
 a))/(a^2*b^4*c^3 - 8*a^3*b^2*c^4 + 16*a^4*c^5 + (b^4*c^5 - 8*a*b^2*c^6 + 1
6*a^2*c^7)*x^4 + 2*(b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6)*x^3 + (b^6*c^3 -
6*a*b^4*c^4 + 32*a^3*c^6)*x^2 + 2*(a*b^5*c^3 - 8*a^2*b^3*c^4 + 16*a^3*b*c^5
)*x), -1/3*(3*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*f^2*x^4 + 2*(b^5*c - 8*
a*b^3*c^2 + 16*a^2*b*c^3)*f^2*x^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*f^2*x^2
+ 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*f^2*x + (a^2*b^4 - 8*a^3*b^2*c + 1
6*a^4*c^2)*f^2)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(
-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(8*a^2*b*c^3*e^2 + 2*(8*c^6*d^2 - 8*b*c^5*
d*e + (b^2*c^4 + 4*a*c^5)*e^2 - 2*(b^4*c^2 - 7*a*b^2*c^3 + 8*a^2*c^4)*f^2 +
 (2*(b^2*c^4 + 4*a*c^5)*d + (b^3*c^3 - 12*a*b*c^4)*e)*f)*x^3 - (b^3*c^3 - 1
2*a*b*c^4)*d^2 - 4*(a*b^2*c^3 + 4*a^2*c^4)*d*e - (3*a^2*b^3*c - 20*a^3*b*c^
2)*f^2 + 3*(8*b*c^5*d^2 - 8*b^2*c^4*d*e + (b^3*c^3 + 4*a*b*c^4)*e^2 - (b^5*
c - 6*a*b^3*c^2)*f^2 + 2*((b^3*c^3 + 4*a*b*c^4)*d - 2*(a*b^2*c^3 + 4*a^2*c^
4)*e)*f)*x^2 + 16*(a^2*b*c^3*d - 2*a^3*c^3*e)*f + 6*(2*a*b^2*c^3*e^2 + (b^2
*c^4 + 4*a*c^5)*d^2 - (b^3*c^3 + 4*a*b*c^4)*d*e - (a*b^4*c - 7*a^2*b^2*c^2
+ 4*a^3*c^3)*f^2 + 4*(a*b^2*c^3*d - 2*a^2*b*c^3*e)*f)*x)*sqrt(c*x^2 + b*x +
 a))/(a^2*b^4*c^3 - 8*a^3*b^2*c^4 + 16*a^4*c^5 + (b^4*c^5 - 8*a*b^2*c^6 + 1
6*a^2*c^7)*x^4 + 2*(b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6)*x^3 + (b^6*c^3 -
6*a*b^4*c^4 + 32*a^3*c^6)*x^2 + 2*(a*b^5*c^3 - 8*a^2*b^3*c^4 + 16*a^3*b*c^5
)*x)]
```

giac [**A**] time = 0.34, size = 587, normalized size = 1.32

$$-\frac{f^2\log\left(\left|-2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}-b\right|\right)}{c^{\frac{5}{2}}}+\frac{2\left(\left(\left(\frac{2\left(8c^5d^2+2b^2c^3df+8ac^4df-2b^4cf^2+14ab^2c^2f^2-16a^2c^3f^2-8bc^4de+b^3c^2fe-12abc^3fe+b^2c^3e^2+4ac^4e^2\right)x}{b^4c^2-8ab^2c^3+16a^2c^4}+\frac{3\left(8bc^4d^2+2b^3c^2df+8abc^3df-b^5f^2+6ab^3cf^2-8b^2c^3de-4ab^2c^2fe-16a^2c^3fe+b^3c^2e^2+4abc^3e^2\right)}{b^4c^2-8ab^2c^3+16a^2c^4}\right)x+\frac{6\left(b^2c^3d^2+4ac^4d^2+4ab^2c^2df-ab^4f^2+7a^2b^2cf^2-4a^3c^2f^2-b^3c^2de-4abc^3de-8a^2bc^2fe+2ab^2c^2e^2\right)}{b^4c^2-8ab^2c^3+16a^2c^4}\right)x-\frac{b^3c^2d^2-12abc^3d^2-16a^2bc^2df+3a^2b^3f^2-20a^3bcf^2+4ab^2c^2de+16a^2c^3de+32a^3c^2fe-8a^2bc^2e^2}{b^4c^2-8ab^2c^3+16a^2c^4}\right)}{3\left(cx^2+bx+a\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)^2/(c*x^2+b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] -f^2*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(5/2) +
 2/3*(((2*(8*c^5*d^2 + 2*b^2*c^3*d*f + 8*a*c^4*d*f - 2*b^4*c*f^2 + 14*a*b^2
*c^2*f^2 - 16*a^2*c^3*f^2 - 8*b*c^4*d*e + b^3*c^2*f*e - 12*a*b*c^3*f*e + b^
2*c^3*e^2 + 4*a*c^4*e^2)*x/(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4) + 3*(8*b*c^
4*d^2 + 2*b^3*c^2*d*f + 8*a*b*c^3*d*f - b^5*f^2 + 6*a*b^3*c*f^2 - 8*b^2*c^3
*d*e - 4*a*b^2*c^2*f*e - 16*a^2*c^3*f*e + b^3*c^2*e^2 + 4*a*b*c^3*e^2)/(b^4
*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4))*x + 6*(b^2*c^3*d^2 + 4*a*c^4*d^2 + 4*a*b^
2*c^2*d*f - a*b^4*f^2 + 7*a^2*b^2*c*f^2 - 4*a^3*c^2*f^2 - b^3*c^2*d*e - 4*a
*b*c^3*d*e - 8*a^2*b*c^2*f*e + 2*a*b^2*c^2*e^2)/(b^4*c^2 - 8*a*b^2*c^3 + 16
*a^2*c^4))*x - (b^3*c^2*d^2 - 12*a*b*c^3*d^2 - 16*a^2*b*c^2*d*f + 3*a^2*b^3
*f^2 - 20*a^3*b*c*f^2 + 4*a*b^2*c^2*d*e + 16*a^2*c^3*d*e + 32*a^3*c^2*f*e -
```

```
 8*a^2*b*c^2*e^2)/(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4))/(c*x^2 + b*x + a)^(
3/2)
```

maple [B] time = 0.01, size = 1786, normalized size = 4.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)^2/(c*x^2+b*x+a)^(5/2),x)
```

```
[Out] -2/3*d*e/c/(c*x^2+b*x+a)^(3/2)+2/3*d^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*b-1/
2*x/c/(c*x^2+b*x+a)^(3/2)*e^2+1/12*b/c^2/(c*x^2+b*x+a)^(3/2)*e^2-1/3*f^2*x^
3/c/(c*x^2+b*x+a)^(3/2)-1/48*f^2*b^3/c^4/(c*x^2+b*x+a)^(3/2)-f^2/c^2*x/(c*x
^2+b*x+a)^(1/2)+1/2*f^2/c^3*b/(c*x^2+b*x+a)^(1/2)-2*e*f*b/c*a/(4*a*c-b^2)/(
c*x^2+b*x+a)^(3/2)*x+16/3*a/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*b*d*f-1/24*f^
2*b^4/c^3/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*x-1/3*f^2*b^4/c^2/(4*a*c-b^2)^2/(
c*x^2+b*x+a)^(1/2)*x+1/4*f^2*b^3/c^3*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+2*f^
2*b^3/c^2*a/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)+f^2/c^2*b^2/(4*a*c-b^2)/(c*x^
2+b*x+a)^(1/2)*x-4/3*d*e*b/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*x-2/3*d*e*b^2/c/
(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)-1/2*e*f*b/c^2*x/(c*x^2+b*x+a)^(3/2)+1/12*e*
f*b^4/c^3/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+2/3*e*f*b^4/c^2/(4*a*c-b^2)^2/(c*
x^2+b*x+a)^(1/2)+1/6*b^2/c/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*x*e^2+1/6*b^3/c^
2/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*d*f+8/3*b^2/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(
1/2)*x*d*f+4/3*b^3/c/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*d*f+4/3*a/(4*a*c-b^2
)/(c*x^2+b*x+a)^(3/2)*x*d*f+1/3*a/c/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*b*e^2+1
6/3*a*c/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x*e^2+f^2/c^(5/2)*ln((c*x+1/2*b)/
c^(1/2)+(c*x^2+b*x+a)^(1/2))-2*e*f*x^2/c/(c*x^2+b*x+a)^(3/2)+1/12*e*f*b^2/c
^3/(c*x^2+b*x+a)^(3/2)-4/3*e*f*a/c^2/(c*x^2+b*x+a)^(3/2)+1/2*f^2*b/c^2*x^2/
(c*x^2+b*x+a)^(3/2)+1/8*f^2*b^2/c^3*x/(c*x^2+b*x+a)^(3/2)-1/48*f^2*b^5/c^4/
(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)-1/6*f^2*b^5/c^3/(4*a*c-b^2)^2/(c*x^2+b*x+a)
^(1/2)+1/3*f^2*b/c^3*a/(c*x^2+b*x+a)^(3/2)+1/2*f^2/c^3*b^3/(4*a*c-b^2)/(c*x
^2+b*x+a)^(1/2)+2/3*b^3/c/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*e^2+2/3*a/(4*a*
c-b^2)/(c*x^2+b*x+a)^(3/2)*x*e^2+8/3*a/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*b*
e^2-16/3*d*e*b^2/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)+4/3*d^2/(4*a*c-b^2)/(c*x
^2+b*x+a)^(3/2)*x*c+32/3*d^2*c^2/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x+16/3*d
^2*c/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*b-x/c/(c*x^2+b*x+a)^(3/2)*d*f+1/6*b/
c^2/(c*x^2+b*x+a)^(3/2)*d*f+32/3*a*c/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x*d*
f-32/3*d*e*b*c/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x-16*e*f*b*a/(4*a*c-b^2)^2
/(c*x^2+b*x+a)^(1/2)*x-8*e*f*b^2/c*a/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)+1/6*
e*f*b^3/c^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*x+1/2*f^2*b^2/c^2*a/(4*a*c-b^2)
/(c*x^2+b*x+a)^(3/2)*x+4*f^2*b^2/c*a/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x+1/
3*b^2/c/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*x*d*f+2/3*a/c/(4*a*c-b^2)/(c*x^2+b*
x+a)^(3/2)*b*d*f+1/12*b^3/c^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*e^2+4/3*b^2/(
4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x*e^2+4/3*e*f*b^3/c/(4*a*c-b^2)^2/(c*x^2+b
*x+a)^(1/2)*x-e*f*b^2/c^2*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)^2/(c*x^2+b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(f\,x^2+e\,x+d\right)^2}{\left(c\,x^2+b\,x+a\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2)^2/(a + b*x + c*x^2)^(5/2),x)
```

```
[Out] int((d + e*x + f*x^2)^2/(a + b*x + c*x^2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)**2/(c*x**2+b*x+a)**(5/2),x)
```

```
[Out] Timed out
```

3.117 $\int \frac{d+ex+fx^2}{\left(a+bx+cx^2\right)^{5/2}} dx$

Optimal. Leaf size=131

$$\frac{2\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x\left(-2acf+b^2f-bce+2c^2d\right)\right)}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}}+\frac{2(b+2cx)\left(4af+\frac{b^2f}{c}-4be+8cd\right)}{3\left(b^2-4ac\right)^2\sqrt{a+bx+cx^2}}$$

Rubi [A] time = 0.09, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {1660, 12, 613}

$$\frac{2\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x\left(-2acf+b^2f-bce+2c^2d\right)\right)}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}}+\frac{2(b+2cx)\left(4af+\frac{b^2f}{c}-4be+8cd\right)}{3\left(b^2-4ac\right)^2\sqrt{a+bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)/(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] (2*(c*(2*a*e - b*(d + (a*f)/c)) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*x))/(
3*c*(b^2 - 4*a*c)*(a + b*x + c*x^2)^(3/2)) + (2*(8*c*d - 4*b*e + 4*a*f + (b
^2*f)/c)*(b + 2*c*x))/(3*(b^2 - 4*a*c)^2*Sqrt[a + b*x + c*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
```

```
- 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{d+ex+fx^2}{\left(a+bx+cx^2\right)^{5/2}}\,dx = \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}} - \frac{2\int \frac{8cd-4be+4af+\frac{b^2f}{c}}{2\left(a+bx+cx^2\right)^{3/2}}\,dx}{3\left(b^2-4ac\right)}$$

$$= \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}} - \frac{\left(8cd-4be+4af+\frac{b^2f}{c}\right)\int \frac{}{(a+}}{3\left(b^2-4ac\right)}$$

$$= \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}} + \frac{2\left(8cd-4be+4af+\frac{b^2f}{c}\right)(b}{3\left(b^2-4ac\right)^2\sqrt{a+bx+c}}$$

Mathematica [A] time = 0.38, size = 147, normalized size = 1.12

$$\frac{8b\left(2a^2f+3ac\left(d-ex+fx^2\right)-2c^2x^2(ex-3d)\right)+16c\left(-a^2e+acx\left(3d+fx^2\right)+2c^2dx^3\right)-4b^2\left(a(e-6fx)-cx\left(3d-6ex+fx^2\right)\right)-2b^3(d+3x(e-fx))}{3\left(b^2-4ac\right)^2(a+x(b+cx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)/(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] (-2*b^3*(d + 3*x*(e - f*x)) + 16*c*(-(a^2*e) + 2*c^2*d*x^3 + a*c*x*(3*d + f
*x^2)) - 4*b^2*(a*(e - 6*f*x) - c*x*(3*d - 6*e*x + f*x^2)) + 8*b*(2*a^2*f -
 2*c^2*x^2*(-3*d + e*x) + 3*a*c*(d - e*x + f*x^2)))/(3*(b^2 - 4*a*c)^2*(a +
 x*(b + c*x))^(3/2))
```

IntegrateAlgebraic [A] time = 1.17, size = 176, normalized size = 1.34

$$-\frac{2\left(-8a^2bf+8a^2ce+2ab^2e-12ab^2fx-12abcd+12abcex-12abcfx^2-24ac^2dx-8ac^2fx^3+b^3d+3b^3ex-3b^3fx^2-6b^2cdx+12b^2cex^2-2b^2cfx^3-24bc^2dx^2+8bc^2ex^3-16c^3dx^3\right)}{3\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d + e*x + f*x^2)/(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] (-2*(b^3*d - 12*a*b*c*d + 2*a*b^2*e + 8*a^2*c*e - 8*a^2*b*f - 6*b^2*c*d*x -
 24*a*c^2*d*x + 3*b^3*e*x + 12*a*b*c*e*x - 12*a*b^2*f*x - 24*b*c^2*d*x^2 +
12*b^2*c*e*x^2 - 3*b^3*f*x^2 - 12*a*b*c*f*x^2 - 16*c^3*d*x^3 + 8*b*c^2*e*x^
```

```
3 - 2*b^2*c*f*x^3 - 8*a*c^2*f*x^3))/(3*(b^2 - 4*a*c)^2*(a + b*x + c*x^2)^(3
/2))
```

fricas [B] time = 2.60, size = 286, normalized size = 2.18

$$\frac{2\left(8a^2bf+2\left(8c^3d-4bc^2e+\left(b^2c+4ac^2\right)f\right)x^3+3\left(8bc^2d-4b^2ce+\left(b^3+4abc\right)f\right)x^2-\left(b^3-12abc\right)d-2\left(ab^2+4a^2c\right)e+3\left(4ab^2f+2\left(b^2c+4ac^2\right)d-\left(b^3+4abc\right)e\right)x\right)\sqrt{cx^2+bx+a}}{3\left(a^2b^4-8a^3b^2c+16a^4c^2+\left(b^4c^2-8ab^2c^3+16a^2c^4\right)x^4+2\left(b^5c-8ab^3c^2+16a^2bc^3\right)x^3+\left(b^6-6ab^4c+32a^3c^3\right)x^2+2\left(ab^5-8a^2b^3c+16a^3bc^2\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(8*a^2*b*f + 2*(8*c^3*d - 4*b*c^2*e + (b^2*c + 4*a*c^2)*f)*x^3 + 3*(8*b
*c^2*d - 4*b^2*c*e + (b^3 + 4*a*b*c)*f)*x^2 - (b^3 - 12*a*b*c)*d - 2*(a*b^2
 + 4*a^2*c)*e + 3*(4*a*b^2*f + 2*(b^2*c + 4*a*c^2)*d - (b^3 + 4*a*b*c)*e)*x
)*sqrt(c*x^2 + b*x + a)/(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^4*c^2 - 8*
a*b^2*c^3 + 16*a^2*c^4)*x^4 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^3 +
(b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^2 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)
*x)
```

giac [A] time = 0.32, size = 240, normalized size = 1.83

$$\frac{2\left(\left(\left(\frac{2\left(8c^3d+b^2cf+4ac^2f-4bc^2e\right)x}{b^4-8ab^2c+16a^2c^2}+\frac{3\left(8bc^2d+b^3f+4abcf-4b^2ce\right)}{b^4-8ab^2c+16a^2c^2}\right)x+\frac{3\left(2b^2cd+8ac^2d+4ab^2f-b^3e-4abce\right)}{b^4-8ab^2c+16a^2c^2}\right)x-\frac{b^3d-12abcd-8a^2bf+2ab^2e+8a^2ce}{b^4-8ab^2c+16a^2c^2}\right)}{3\left(cx^2+bx+a\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] 2/3*(((2*(8*c^3*d + b^2*c*f + 4*a*c^2*f - 4*b*c^2*e)*x/(b^4 - 8*a*b^2*c + 1
6*a^2*c^2) + 3*(8*b*c^2*d + b^3*f + 4*a*b*c*f - 4*b^2*c*e)/(b^4 - 8*a*b^2*c
 + 16*a^2*c^2))*x + 3*(2*b^2*c*d + 8*a*c^2*d + 4*a*b^2*f - b^3*e - 4*a*b*c*
e)/(b^4 - 8*a*b^2*c + 16*a^2*c^2))*x - (b^3*d - 12*a*b*c*d - 8*a^2*b*f + 2*
a*b^2*e + 8*a^2*c*e)/(b^4 - 8*a*b^2*c + 16*a^2*c^2))/(c*x^2 + b*x + a)^(3/2
)
```

maple [A] time = 0.01, size = 185, normalized size = 1.41

$$\frac{\frac{16}{3}ac^2fx^3+\frac{4}{3}b^2cfx^3-\frac{16}{3}bc^2ex^3+\frac{32}{3}c^3dx^3+8abcfx^2+2b^3fx^2-8b^2cex^2+16bc^2dx^2+8ab^2fx-8abcex+16ac^2dx-2b^3ex+4b^2cdx+\frac{16}{3}a^2bf-\frac{16}{3}a^2ce-\frac{4}{3}ab^2e+8abcd-\frac{2}{3}b^3d}{\left(cx^2+bx+a\right)^{\frac{3}{2}}\left(16a^2c^2-8ab^2c+b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)/(c*x^2+b*x+a)^(5/2),x)
```

```
[Out] 2/3/(c*x^2+b*x+a)^(3/2)*(8*a*c^2*f*x^3+2*b^2*c*f*x^3-8*b*c^2*e*x^3+16*c^3*d
*x^3+12*a*b*c*f*x^2+3*b^3*f*x^2-12*b^2*c*e*x^2+24*b*c^2*d*x^2+12*a*b^2*f*x-
```

```
12*a*b*c*e*x+24*a*c^2*d*x-3*b^3*e*x+6*b^2*c*d*x+8*a^2*b*f-8*a^2*c*e-2*a*b^2
*e+12*a*b*c*d-b^3*d)/(16*a^2*c^2-8*a*b^2*c+b^4)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

mupad [B] time = 3.71, size = 175, normalized size = 1.34

$$\frac{2\left(8fa^2b-8ea^2c+12fab^2x-2eab^2+12fabcx^2-12eabcx+12dabc+8fac^2x^3+24dac^2x+3fb^3x^2-3eb^3x-db^3+2fb^2cx^3-12eb^2cx^2+6db^2cx-8ebc^2x^3+24dbc^2x^2+16dc^3x^3\right)}{3\left(4ac-b^2\right)^2\left(cx^2+bx+a\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2)/(a + b*x + c*x^2)^(5/2),x)
```

```
[Out] (2*(16*c^3*d*x^3 - b^3*d + 3*b^3*f*x^2 - 2*a*b^2*e + 8*a^2*b*f - 8*a^2*c*e
- 3*b^3*e*x + 24*a*c^2*d*x + 12*a*b^2*f*x + 6*b^2*c*d*x + 24*b*c^2*d*x^2 -
12*b^2*c*e*x^2 + 8*a*c^2*f*x^3 - 8*b*c^2*e*x^3 + 2*b^2*c*f*x^3 + 12*a*b*c*d
 - 12*a*b*c*e*x + 12*a*b*c*f*x^2))/(3*(4*a*c - b^2)^2*(a + b*x + c*x^2)^(3/
2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(c*x**2+b*x+a)**(5/2),x)
```

```
[Out] Timed out
```

3.118 $\int \frac{1}{\sqrt{-7+2x+5x^2}\,(8+12x+5x^2)}\,dx$

Optimal. Leaf size=51

$$\frac{1}{10}\tan^{-1}\left(\frac{5(x+2)}{2\sqrt{5x^2+2x-7}}\right)+\frac{1}{5}\tanh^{-1}\left(\frac{5(x+1)}{\sqrt{5x^2+2x-7}}\right)$$

Rubi [A] time = 0.07, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {986, 1029, 203, 207}

$$\frac{1}{10}\tan^{-1}\left(\frac{5(x+2)}{2\sqrt{5x^2+2x-7}}\right)+\frac{1}{5}\tanh^{-1}\left(\frac{5(x+1)}{\sqrt{5x^2+2x-7}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-7 + 2*x + 5*x^2]*(8 + 12*x + 5*x^2)),x]
```

```
[Out] ArcTan[(5*(2 + x))/(2*Sqrt[-7 + 2*x + 5*x^2])]/10 + ArcTanh[(5*(1 + x))/Sqr
t[-7 + 2*x + 5*x^2]]/5
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 986

```
Int[1/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*
(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f)
, 2]}, Dist[1/(2*q), Int[(c*d - a*f + q + (c*e - b*f)*x)/((a + b*x + c*x^2)
*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[(c*d - a*f - q + (c*e -
 b*f)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b,
 c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[c*e -
 b*f, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1029

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
 c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-7+2x+5x^2}\,(8+12x+5x^2)}\,dx &= -\left(\frac{1}{50}\int \frac{-100-50x}{\sqrt{-7+2x+5x^2}\,(8+12x+5x^2)}\,dx\right)+\frac{1}{50}\int \frac{-50\ldots}{\sqrt{-7+2x+5x\ldots}} \\ &= 400\,\text{Subst}\left(\int \frac{1}{160000+100x^2}\,dx, x, \frac{200+100x}{\sqrt{-7+2x+5x^2}}\right)+1600\,\text{Subst}\ldots \\ &= \frac{1}{10}\tan^{-1}\left(\frac{5(2+x)}{2\sqrt{-7+2x+5x^2}}\right)+\frac{1}{5}\tanh^{-1}\left(\frac{5(1+x)}{\sqrt{-7+2x+5x^2}}\right)\end{aligned}$$

Mathematica [C] time = 0.04, size = 81, normalized size = 1.59

$$\left(\frac{1}{10}-\frac{i}{20}\right)\tanh^{-1}\left(\frac{\left(\frac{1}{100}+\frac{i}{50}\right)((100-40i)x+(164-8i))}{\sqrt{5x^2+2x-7}}\right)-\left(\frac{1}{20}-\frac{i}{10}\right)\tan^{-1}\left(\frac{\left(\frac{1}{50}+\frac{i}{100}\right)((-100-40i)x-(164+8i))}{\sqrt{5x^2+2x-7}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-7 + 2*x + 5*x^2]*(8 + 12*x + 5*x^2)),x]
```

```
[Out] (-1/20 + I/10)*ArcTan[((1/50 + I/100)*((-164 - 8*I) - (100 + 40*I)*x))/Sqrt
[-7 + 2*x + 5*x^2]] + (1/10 - I/20)*ArcTanh[((1/100 + I/50)*((164 - 8*I) +
(100 - 40*I)*x))/Sqrt[-7 + 2*x + 5*x^2]]
```

IntegrateAlgebraic [A] time = 0.34, size = 53, normalized size = 1.04

$$\frac{1}{10}\tan^{-1}\left(\frac{\frac{5x}{2}+5}{\sqrt{5x^2+2x-7}}\right)+\frac{1}{5}\tanh^{-1}\left(\frac{5x+5}{\sqrt{5x^2+2x-7}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[-7 + 2*x + 5*x^2]*(8 + 12*x + 5*x^2)),x]
```

```
[Out] ArcTan[(5 + (5*x)/2)/Sqrt[-7 + 2*x + 5*x^2]]/10 + ArcTanh[(5 + 5*x)/Sqrt[-7
 + 2*x + 5*x^2]]/5
```

fricas [B] time = 0.45, size = 154, normalized size = 3.02

$$\frac{1}{20}\arctan\left(\frac{27x^2+20\sqrt{5x^2+2x-7}(x+2)+36x}{31x^2+16x-56}\right)+\frac{1}{20}\arctan\left(-\frac{27x^2-20\sqrt{5x^2+2x-7}(x+2)+36x}{31x^2+16x-56}\right)+\frac{1}{20}\log\left(\frac{15x^2+5\sqrt{5x^2+2x-7}(x+1)+26x+9}{x^2}\right)-\frac{1}{20}\log\left(\frac{15x^2-5\sqrt{5x^2+2x-7}(x+1)+26x+9}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+12*x+8)/(5*x^2+2*x-7)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/20*arctan((27*x^2 + 20*sqrt(5*x^2 + 2*x - 7)*(x + 2) + 36*x)/(31*x^2 + 16
*x - 56)) + 1/20*arctan(-(27*x^2 - 20*sqrt(5*x^2 + 2*x - 7)*(x + 2) + 36*x)
/(31*x^2 + 16*x - 56)) + 1/20*log((15*x^2 + 5*sqrt(5*x^2 + 2*x - 7)*(x + 1)
 + 26*x + 9)/x^2) - 1/20*log((15*x^2 - 5*sqrt(5*x^2 + 2*x - 7)*(x + 1) + 26
*x + 9)/x^2)
```

giac [B] time = 0.28, size = 205, normalized size = 4.02

$$-\frac{1}{10}\arctan\left(-\frac{5\sqrt{5}x+6\sqrt{5}-5\sqrt{5x^2+2x-7}+5}{2(\sqrt{5}+5)}\right)-\frac{1}{10}\arctan\left(\frac{5\sqrt{5}x+6\sqrt{5}-5\sqrt{5x^2+2x-7}-5}{2(\sqrt{5}-5)}\right)+\frac{1}{10}\log\left(5\left(\sqrt{5}x-\sqrt{5x^2+2x-7}\right)^2+2\left(\sqrt{5}x-\sqrt{5x^2+2x-7}\right)(6\sqrt{5}+5)+20\sqrt{5}+65\right)-\frac{1}{10}\log\left(5\left(\sqrt{5}x-\sqrt{5x^2+2x-7}\right)^2+2\left(\sqrt{5}x-\sqrt{5x^2+2x-7}\right)(6\sqrt{5}-5)-20\sqrt{5}+65\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+12*x+8)/(5*x^2+2*x-7)^(1/2),x, algorithm="giac")
```

```
[Out] -1/10*arctan(-1/2*(5*sqrt(5)*x + 6*sqrt(5) - 5*sqrt(5*x^2 + 2*x - 7) + 5)/(
sqrt(5) + 5)) - 1/10*arctan(1/2*(5*sqrt(5)*x + 6*sqrt(5) - 5*sqrt(5*x^2 + 2
*x - 7) - 5)/(sqrt(5) - 5)) + 1/10*log(5*(sqrt(5)*x - sqrt(5*x^2 + 2*x - 7)
)^2 + 2*(sqrt(5)*x - sqrt(5*x^2 + 2*x - 7))*(6*sqrt(5) + 5) + 20*sqrt(5) +
65) - 1/10*log(5*(sqrt(5)*x - sqrt(5*x^2 + 2*x - 7))^2 + 2*(sqrt(5)*x - sqr
t(5*x^2 + 2*x - 7))*(6*sqrt(5) - 5) - 20*sqrt(5) + 65)
```

maple [B] time = 0.02, size = 144, normalized size = 2.82

$$-\frac{\sqrt{-\frac{4(x+2)^2}{(-x-1)^2}+9}\left(2\,\mathrm{arctanh}\left(\frac{\sqrt{-\frac{4(x+2)^2}{(-x-1)^2}+9}}{5}\right)+\arctan\left(\frac{5\sqrt{-\frac{4(x+2)^2}{(-x-1)^2}+9}\,(x+2)}{2\left(\frac{4(x+2)^2}{(-x-1)^2}-9\right)(-x-1)}\right)\right)}{10\sqrt{-\frac{\frac{4(x+2)^2}{(-x-1)^2}-9}{\left(1+\frac{x+2}{-x-1}\right)^2}}\,\left(1+\frac{x+2}{-x-1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2+12*x+8)/(5*x^2+2*x-7)^(1/2),x)
```

```
[Out] -1/10*(-4*(x+2)^2/(-1-x)^2+9)^(1/2)*(2*arctanh(1/5*(-4*(x+2)^2/(-1-x)^2+9)^
(1/2))+arctan(5/2*(-4*(x+2)^2/(-1-x)^2+9)^(1/2)/(4*(x+2)^2/(-1-x)^2-9)*(x+2
)/(-1-x)))/(-(4*(x+2)^2/(-1-x)^2-9)/(1+(x+2)/(-1-x))^2)^(1/2)/(1+(x+2)/(-1-
x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(5\,x^2+12\,x+8\right)\sqrt{5\,x^2+2\,x-7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+12*x+8)/(5*x^2+2*x-7)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((5*x^2 + 12*x + 8)*sqrt(5*x^2 + 2*x - 7)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{5\,x^2+2\,x-7}\,\left(5\,x^2+12\,x+8\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*x + 5*x^2 - 7)^(1/2)*(12*x + 5*x^2 + 8)),x)
[Out] int(1/((2*x + 5*x^2 - 7)^(1/2)*(12*x + 5*x^2 + 8)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{(x-1)\,(5x+7)}\,\left(5x^2+12x+8\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x**2+12*x+8)/(5*x**2+2*x-7)**(1/2),x)
[Out] Integral(1/(sqrt((x - 1)*(5*x + 7))*(5*x**2 + 12*x + 8)), x)
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

This is a subset of Rubi test suite thanks to Albert Rich, which includes only the algebraic integrals with elementray optimal antiderivatives. It also includes a subset of a test file provided thanks to Sam Blake.

Mathematica format Mathematica_syntax_CAS_integration_elementary_version.zip

Maple and Mupad format Maple_syntax_CAS_integration_elementary_version.zip

Sympy format SYMPY_syntax_CAS_integration_elementary_version.zip

Sage math format SAGE_syntax_CAS_integration_elementary_version.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
```

```
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not
    as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
```

```
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
```

```
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:
```

```
#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```
def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)
   ))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type
   (expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
```

```
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
   expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,
   Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
```

```
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
    ']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
    sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
    sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
```

```
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
```

```
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.
    func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
    expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
```

```
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
        else: #result contains complex but optimal is not
            return "C"
    else: # result do not contain complex, this assumes optimal do not as
well
        if leaf_count_result <= 2*leaf_count_optimal:
            return "A"
        else:
            return "B"
else:
    return "C"
```